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# Robust Estimates of Okun's Coefficient for South Africa

J. Geldenhuys and M. Marinkov[*]

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[*] Department of Economics, University of the Free State

# Robust Estimates of Okun's Coefficient for South Africa

J Geldenhuys*and M Marinkov†

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**Abstract**

Persistently high unemployment in South Africa, especially in the face of improved economic conditions since 1994, begs the question: Does unemployment in South Africa respond to changes in output? When considering the linkages between output and unemployment, it is useful to decompose unemployment into its three components: structural, frictional and cyclical unemployment. Deficient aggregate demand gives rise to cyclical unemployment. Okun's law (1962) refers to the inverse relationship that exists between cyclical output and cyclical unemployment This paper estimates the relationship between economic activity (cyclical GDP) and changes in the unemployment rate (cyclical unemployment) for South Africa. A variety of detrending methods is used to decompose output and unemployment series into their trend and cyclical components. The paper also addresses the question of asymmetries in Okun's coefficient. The results indicate the presence of an Okun's law relationship in South Africa over the period 1970-2005 with more evidence in favour of asymmetries during recessions.

## 1 Introduction

Persistently high unemployment in South Africa, especially in the face of improved economic conditions since 1994, begs the question: Does unemployment in South Africa respond to changes in output? In many circles, it was believed that South Africa experienced jobless growth during the 1990s. Bhorat (2004:946) and Casale *et al* (2004:989) contend that the South African economy did not experience jobless growth, but that the increase in output and employment in post-1994 South Africa was insufficient to lead to lower unemployment, due to increased labour force participation. When considering the linkages between output and unemployment, it is useful to decompose unemployment into its three components: structural, frictional and cyclical unemployment. Deficient aggregate demand gives rise to cyclical unemployment, while microeconomic labour market imperfections give rise to frictional and structural unemployment (Grant, 2002: 98). Okun's law (1962) refers to the inverse relationship that exists between cyclical output and cyclical unemployment, relating activity in the goods market to activity in the labour market over the course of the business cycle (Silvapulle, *et al* 2004: 354).

A large body of literature investigates the presence of Okun's law for mainly OECD countries (especially the US). However, Okun's law has not yet (to the authors' knowledge) been investigated for South Africa. Examining whether or not Okun's law is valid for the South African

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*University of the Free State

†University of the Free State

economy has important policy implications. Okun's law, along with the Phillips curve, is used to construct an economy's aggregate supply curve (Prachowny, 1993:331; Moosa, 1997:335; Silvapulle, *et al*, 2004:354), thereby creating linkages between the inflation rate, unemployment rate and economic growth rate. These linkages are important to consider in South Africa, given that the SARB targets inflation, while the government is aiming to reduce the unemployment rate by half, through increased economic growth.

The objectives of the paper are, first, to estimate Okun's coefficient for the South African economy, using annual data for the period 1970-2005. Okun's law is specified as the relation between cyclical output (output gap) and cyclical unemployment (unemployment gap). Output and unemployment series were detrended using a variety of detrending techniques. These methods range from purely statistical techniques, such as the Hodrick-Prescott filter, the Baxter-King band-pass filter, the Beveridge-Nelson decomposition and linear detrending, to more a theoretically sound technique, the production function method. Furthermore, the paper accounts for the possibility that there might be asymmetries present in the South African Okun's relation, which would imply that the response of cyclical unemployment to cyclical output depends on the state of the economy.

The rest of the paper is organised as follows: Section 2 presents the literature review, while Sections 3 and 4 present the literature review and results, respectively. Section 5 addresses the issue of asymmetries while Section 6 concludes.

## 2 Literature Review

A review of the literature indicates that Okun's coefficient is an empirical regularity, irrespective of the countries or the sample periods used in the studies (Moosa, 1997; Lee, 2000; Cauresma, 2003; Silvapulle *et al*, 2004). Table 1 below contains a review of the recent literature on estimates of Okun's coefficient. Most authors estimate Okun's coefficient for the US economy, while other authors include OECD countries in their analysis. In terms of specification, all the papers, with the exception of Attfield and Silverstone (1998), specify Okun's law as the relation between cyclical output and cyclical unemployment. The methods used to obtain the cyclical components are mostly statistical in nature, including the Hodrick-Prescott filter, the Beveridge-Nelson decomposition, Harvey's structural time series model and linear detrending. Estimation methods include OLS, VAR and maximum likelihood. In addition, Lee (2000), Cauresma (2003), Silvapulle et al (2004), and Holmes and Silverstone (2005) investigate the presence of asymmetries and whether a linear specification is appropriate or not. If asymmetries are present, the asymmetric specification is the preferred specification.

TABLE 1 HERE

All of the studies find a statistically significant relationship between cyclical output and cyclical unemployment. The magnitude of the coefficient estimates differs in terms of the specification, variables used in the model, dynamic structure, as well as the econometric method used to estimate the model (although Weber (1995) and Moosa (1997:336) argue that this is not as important as model specification).

## 3 Research Method

According to Grant (2002:97-8) and Attfield and Silverstone (1998:625), the relationship between unemployment and output as put forward by Okun (1962) is a gap equation. Thus, the method used to estimate Okun's relation is based on the notion of the gap between observed and potential output as well as the gap between observed and potential (natural) unemployment – hence, the "gap" model (Lee, 2000:334). The "gap" model takes the following bivariate specification:

$$y_t^c \equiv y_t - y_t^p \tag{1}$$

$$u_t^c \equiv u_t - u_t^p \tag{2}$$

$$u_t^c = \gamma y_t^c + \xi_t \tag{3}$$

where $y^c$ denotes the logarithm of cyclical output (i.e. the output gap); $y$ denotes the logarithm of observed output; $y^p$ denotes the logarithm of potential output; $u^c$ denotes the cyclical unemployment rate; $u$ denotes the observed unemployment rate; $u^p$ denotes the potential unemployment rate; $\gamma$ denotes Okun's coefficient $(\gamma < 0)$; and $\xi$ is a stochastic error term (Weber, 1995:438; Moosa, 1997:337; 1999:296). Several authors also employ the gap specification to estimate Okun's coefficient (cf. Lee, 2000:334; Harris and Silverstone, 2001:2). However, these authors specify the Okun's law equation the other way around with the output gap as the dependent variable. Nonetheless, conclusions reached from this specification are qualitatively the same as those of the specification in equation (3) (Lee, 2000:333, footnote 2). Following Moosa (1997:337, 1999:296) and Weber (1995:438), some dynamics are added to equation (3) since equation (3) assumes a contemporaneous (static) relationship which may not be theoretically plausible. This yields equation (4):

$$u_t^c = \sum_{i=1}^{m} \beta_i u_{t\text{-}i}^c + \sum_{i=0}^{m} \gamma_i y_{t\text{-}\mathrm{i}}^c + \xi_t \tag{4}$$

where $\gamma_0$denotes the contemporaneous effect of output on unemployment. The specification in equation (4) can also be used to calculate the "medium" run effect of cyclical output on cyclical unemployment (Moosa, 1997:337; 1999:296). This "medium" run effect is obtained by calculating a function of the coefficients obtained from equation (4), i.e. $\omega$:

$$\omega = \frac{\sum_{i=0}^{m} \gamma_i}{1 - \sum_{i=1}^{m} \beta_i} \tag{5}$$

Given that empirical results for equation (4) might be sensitive to the choice of detrending technique (Grant, 2002:98; Moosa, 1997:336; 1999:293-4), this paper uses eight methods to obtain estimates of potential output (i.e. $y^p$) and five methods to obtain the estimates for potential unemployment (i.e. $u^p$). This results in the estimation of eight versions of equation (4) summarised in Table 2 below.

TABLE 2 HERE

The first difference method does not involve an estimation of the permanent components of the unemployment and output series. Instead, the cyclical components in equation 4 (i.e. $u^c$ and $y^c$) are merely taken to be the first differences of the $y$ and $u$ series (cf. Lee, 2000:333). The linear trend representation, on the other hand, assumes that an economic series contains a deterministic trend that grows at a constant rate.[1] Given a time series $x_t$ and a linear trend variable $t$, the detrended (cycle) series ($e_t$) can be obtained by estimating the following regression (cf. Grant, 2002:98):

$$x_t = a_0 + a_1 t + e_t \tag{6}$$

Other statistical techniques used to distinguish between permanent and transitory components of a time series include the Hodrick-Prescott (HP) Filter, the Baxter-King Band-Pass (BP) Filter and the Beveridge-Nelson (BN) decomposition.[2] The Hodrick-Prescott (HP) filter is a generalisation of a linear trend method that allows the slope of the trend to change gradually over time (Hodrick and Prescott, 1997). Suppose that the observed series is denoted by xt which can be decomposed into a trend ($\mu_t$) and a stationary component ($x_t - \mu_t$). The HP filter minimizes the sum of the squared deviations between the trend and the actual series, with a penalty for the curvature that keeps the trend smooth ($\lambda$). Thus, the HP function is given by:

$$Minimise: \quad 1/T\sum_{t=1}^{T}(x_t - \mu_t)^2 + \lambda/T\sum_{t=2}^{T}[(\mu_{t+1} - \mu_t) - (\mu_t - \mu_{t-1})]^2 \tag{7}$$

where $T$ denotes the number of observations. According to convention, $\lambda$ takes on a value of 100 for annual data, 1600 for quarterly data and 44000 for monthly data. If $\lambda = 0$, the HP filter would yield the original series and if $\lambda \to \infty$, the HP filter would result in a linear time trend (Enders, 2004:224). Given the use of annual data in this study, $\lambda$ is set to 100. Some criticisms of the HP Filter include that it requires users to choose a smoothing parameter which determines entirely how much variation the final estimate of the detrended series will display, that it can generate cycles that do not exist in the original series, as well as the end sample bias where the estimates of the trend components tend to rely excessively on the latest observations in the actual series. However, despite of its shortcomings, HP filter is still a very popular method of detrending and is used widely in economic literature. In addition, the paper also employs a frequency filter, the band-pass filter, proposed by Baxter and King (1995). This filter is used to isolate the cyclical component of a time series by specifying a range for its duration. Roughly speaking, the band-pass filter is a linear filter that takes a two-sided weighted moving average of the data where cycles in a "band" (given by a specified lower and upper bound) are "passed" through, or extracted, and the remaining cycles are "filtered" out. When applied to annual data, the band-pass filter proposed by Baxter and King (1995) takes the form of a 3-year moving average:

$$x_t^f = \sum_{b=-1}^{b=1} a_b x_{t-b} = a(L)x_t \tag{8}$$

where $L$ is the lag operator. The weights $a_b$ can be derived from the inverse Fourier transformation of the frequency response function. Baxter and King (1995) adjust the band-pass (BP) filter with

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[1] This is one of the major drawbacks of this approach as there is an increased interest in the stochastic nature of the long run trends in economic time series (Grant, 2002:99).

[2] See Hodrick and Prescott (1997), Baxter and King (1995) and Beveridge and Nelson (1981).

a constraint that the gain is zero on the zero frequency. This constraint implies that the sum of the moving average coefficients must be zero. In addition, when using the BP filter, one year is sacrificed at the beginning and the end of the time series. The advantage of the band-pass filter is that because the decompositions are based on moving averages they are easy to apply. However, much like the HP filter, the band-pass filter also smoothes the long-term component.

Beveridge and Nelson (1981) propose a different method of extracting a cycle form a series. A time series $x_t$ can be represented as the k-period ahead forecast of output at time $t$ by adding all forecastable future changes to the current observation (Grant 2002:100-2), i.e.:

$$E_t(x_{t+k}) = x_t + E_t\left(\sum_{i=1}^{k} \Delta x_{t+i}\right) \tag{9}$$

If $x_t$ series is I(1), $\Delta x_t$ is I(0) and hence has an estimable moving average representation through Wold decomposition:

$$\Delta x_t = \mu + \varepsilon_t + \lambda_1 \varepsilon_{t-1} + \lambda_2 \varepsilon_{t-2} + \ldots + \lambda_i \varepsilon_{t-i} \tag{10}$$

where $\mu$ is the mean forecastable change in $x_t$ and $\varepsilon_t \tilde{} NIID\left(0, \sigma^2\right)$. Combining (9) and (10) and extending the forecast function over a long period of time, the expected value of $x$ ($k$ periods into the future) is given by:

$$E_t(x_{t+k}) = x_t + k\mu + \sum_{i=1}^{\infty} \lambda_i \varepsilon_t + \sum_{i=2}^{\infty} \lambda_i \varepsilon_{t-1} + \ldots \tag{11}$$

The expected innovations (i.e. the $\varepsilon$ series) are obtained by fitting an ARIMA model to the first difference of the $x_t$ series and using the estimated parameters to forecast future changes in the $x_t$ series over a very long horizon. Assuming that future expected innovations have a mean of zero, equation (11) can be written as:

$$E_t(x_{t+k}) = x_t^p + k\mu \tag{12}$$

Thus, by rearranging equation (12) the current permanent component can be obtained as a long run forecast from equation (11):

$$x_t^p = E_t(x_{t+k}) - k\mu \tag{13}$$

The cyclical component of $x_t$ is then obtained as follows:

$$x_t^c = x_t - x_t^p \tag{14}$$

The BN decomposition method is thus a straightforward procedure to decompose any non-stationary series into a temporary and a permanent component – however, this method is not unique since it forces perfectly correlation between the innovation in the trend and stationary component (see Enders, 2004).

Given that the abovementioned detrending techniques are purely statistical in nature, the production function approach is also used to extract the cyclical component of the output series. The production function method entails the estimation of a production function to obtain the $y^p$series, hence a more economic approach (c.f. Smit and Burrows, 2002; Arora and Bhundia, 2003). Following Arora and Bhundia (2003:5-6), the study uses a constant returns-to-scale Cobb-Douglas production function:

$$Y = AL^a K^{1-a} \tag{15}$$

where the weights of labour and capital (i.e. $a$ and $1 - a$) are taken to be the average shares of labour and capital in national income for the period under consideration. Next, the total factor productivity is calculated as follows (Smit and Burrows, 2002:5; Arora and Bhundia, 2003:6):

$$A = Y / L^a K^{1-a} \tag{16}$$

The Hodrick-Prescott (HP), Beveridge-Nelson (BN) and Band-pass (BP) filters are then applied to both labour and total factor productivity (it is assumed that capital is always utilised at full capacity) (Burger and Marinkov, 2006:180). The smoothed values of labour and total factor productivity are then substituted into equation (16) to calculate the potential output:

$$Y_{HP}^{p} = A_{HP} L_{HP}^{a} K^{1-a} \tag{17}$$

$$Y_{BP}^{p} = A_{BP} L_{BP}^{a} K^{1-a} \tag{18}$$

$$Y_{BN}^{p} = A_{BN} L_{BN}^{a} K^{1-a} \tag{19}$$

The output gaps are then calculated as the difference between the natural logs of actual and potential outputs as calculated by the HP, BN and BP methods (Burger and Marinkov, 2006:180).

Annual data for the period 1970 to 2005 is used for estimation purposes. Data on real GDP was obtained from the SARB (2006). The unemployment series was obtained from Quantec (2006) and is constructed by taking the difference between the total labour force and the total number of employed persons (the latter includes both formal and informal sector employees). Employment data (total employment) is based on surveys adopted by Statistics South Africa, the Department of Manpower and the Central Statistical Service (Quantec, 2006).

# 4 Results

Figure 1 contains the estimates of the unemployment and output gaps obtained by using the methods listed in Table 2. A negative relationship between the unemployment gap and the output gap is apparent from all the figures. It is also interesting to note the chronology as well as the amplitude of the different estimates of the gaps. The Band-Pass Filter and the Beveridge-Nelson gaps have a much lower amplitude and a higher frequency than the other gaps. Furthermore, at the end of the sample, cyclical output exceeds cyclical unemployment for the BP and the BN estimations (whereas other estimations indicate the opposite). Grant (2002:104) has similar findings where different methods of detrending yield gaps (cycles) that differ both qualitatively and quantitatively. Specifically, he finds wide disparities between the Hodrick-Prescott estimate of the output gap (cycles with duration of about 4-6 years), linear trend measure of the output gap (long cycles with a high degree of variability) and the Beveridge-Nelson estimate of the output gap (cycles of high frequency and low amplitude) (Grant, 2002:104).

FIGURE 1

Next, the results from the estimation of equation (4) using the different measures of the gaps are summarised in Table 3 below. Equations (4.2), (4.3) and (4.6) suffered from serial correlation problems and will thus be excluded from further discussion. For the remaining equations (i.e. (4.1), (4.4), (4.5), (4.7) and (4.8)) estimates of the contemporaneous Okun's coefficient $\left(\gamma^0\right)$ are all statistically significant at the 5% significance level. Furthermore, they are all negative; though there are large differences in the magnitudes of these coefficients (this can be attributed to the different techniques used to estimate the gaps (cf. Lee, 2000:341)). The estimates of the "medium" run Okun's coefficient $(\omega)$ are on average double the contemporaneous Okun's coefficient $(\gamma_0)$, indicating that the Okun's relationship for South Africa is stronger in the medium run. Furthermore, the lags of the output gaps are only significant in equations (4.4) and (4.7) providing limited evidence of the persistence of output gap effect. It also seems that cyclical unemployment displays inertia, as indicated by the $\beta_1$ coefficient in equations (4.1) and (4.5). The $\beta_1$coefficient is positive and statistically significant in both cases indicating that the one period lag of cyclical unemployment is associated with an increase in the contemporaneous cyclical unemployment rate (as expected on *a priori* grounds).

The magnitude of the contemporaneous Okun's coefficient for equation (4.1) (i.e. first difference estimation) implies that a 1% increase in real GDP is associated with a 0.164 percentage point reduction of the change in the unemployment rate. On the other hand, the magnitudes of the contemporaneous Okun's coefficients for equations (4.4), (4.5), (4.7) and (4.8) imply that, holding other factors constant, a 1 percent increase in the output gap is associated with a decrease in South African *cyclical unemployment* of between 0.164 and 0.732 percentage points.

Figure IA (Appendix I) presents the recursive coefficient estimates of the $\gamma_0$coefficients estimated in equations (4.1), (4.4), (4.5), (4.7) and (4.8). It should be noted that all of the estimated recursive coefficients remained within the two standard error confidence bands for the entire period under consideration. Furthermore, all of the plots (except perhaps equations (4.5) and (4.8)) indicate stability, with the estimated coefficients ranging between -0.17 and -0.78.

# 5 Asymmetries in Okun's Law

According to various authors (cf. Lee, 2000; Harris and Silverstone, 2001; Cuaresma, 2003; Silvapulle *et al.*, 2004; Holmes and Silverstone, 2005) the symmetric specification presented above is a misspecification of Okun's law if cyclical unemployment responds differently to changes in cyclical output, depending on whether the economy is experiencing an upswing or a downswing. In essence, asymmetry would imply that unemployment is either more responsive to changes in output during upswings or more responsive to changes in output during downswings. This section investigates whether or not there are any asymmetries present in the South African Okun's law relationship.

An asymmetric specification of Okun's law is motivated on the following grounds (Silvapulle *et al.*, 2004:356; Harris and Silverstone, 2001:1): first, it helps to discriminate between competing theories of joint behaviour in labour and goods markets. Second, it strengthens the case for an asymmetric (convex) Phillips curve, where unemployment decreasing below the NAIRU ultimately leads to explosive inflation whilst unemployment increasing above the NAIRU has a waning effect. Third, the extent of asymmetries is useful for policymakers formulating structural and stabilisation policies. Fourth, forecasting errors would arise if an asymmetric relationship is specified and estimated as a symmetric relationship – hence, model misspecification.

Asymmetries in Okun's law are attributed to factor substitution over the course of the business cycle, fluctuations in multi-factor productivity, and changes in the distribution of sectoral growth (Silvapulle et al., 2004:356). The relationship will be stronger during a downswing if responses by heterogeneous plants in terms of job creation and job destruction were asymmetric; if there were substantial geographic and sectoral mismatches between the unemployed and available job opportunities; and if employers are more likely to lay off workers during downswings than hire new workers during upswings. Conversely, the relationship will be stronger during an upswing in the presence of labour market rigidities (if firing costs/restrictions exceed hiring costs/restrictions), and if employers invest substantially in the training of their workers.

Thus, to account for asymmetries, equations (4) and (5) become

$$u_t^c = \beta_1 u_{t-1}^c + \gamma_0^- y_t^{c-} + \gamma_1^- y_{t-1}^{c-} + \gamma_0^+ y_t^{c+} + \gamma_1^+ y_{t-1}^{c+} + \zeta_t \tag{20}$$

$$\omega^- = \frac{\gamma_0^- + \gamma_1^-}{1 - \beta_1} \tag{21}$$

$$\omega^+ = \frac{\gamma_0^+ + \gamma_1^+}{1 - \beta_1} \tag{22}$$

where $y_t^{c-}$ and $y_{t+1}^{c-}$denote the cyclical output values below the threshold value (with the long run coefficient given by $\omega^-$ ), and $y_t^{c+}$ and $y_{t-1}^{c+}$denote the cyclical output values above the threshold value (with the long run coefficient by $\omega^+$ ). The threshold value was first assumed to be zero (cf. Lee, 2000; Silvapulle *et al.*, 2004; Holmes and Silverstone, 2005), and was subsequently estimated with a grid search[3] to find the optimal threshold value for each case (cf. Harris and Silverstone, 2001; Cuaresma, 2003). Eight versions of equation (20) were estimated for the two types of thresholds (the equations are numbered corresponding to the methods summarised in Table 2 above, where a denotes an assumed threshold of zero and b denotes the thresholds estimated with a grid search). The results are contained in Tables 4 and 5.

TABLE 4

Table 4 above presents the estimation results for equation (20) when a threshold value of zero is assumed. Due to serial correlation problems, results from equations (20.2a), (20.3a) and (20.6a) will not be discussed below. For equations (20.1a), (20.4a) and (20.7a), only the negative contemporaneous output gap is found to be significant with the estimated values ranging from -0.18 to -0.535. This would seemingly indicate that Okun's relationship is present only during recessions in South Africa where a 1 percent increase in the "negative" output gap is associated with a decrease in South African *cyclical unemployment* of between 0.18 and 0.535 percentage points. However, equations (20.5a) and (20.8a) imply that Okun's relationship is equally strong during recessions

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[3] For more on the grid search see Enders and Silkos (2001).

*and* upswings as both the positive and negative contemporaneous output gaps are found to be statistically significant. The coefficient magnitudes for both the positive as well as the negative contemporaneous coefficients do not differ significantly in equations (20.5a) and (20.8a) – for the negative output gap, between -0.728 and -0.841; for the positive output gap between -0.727 and -0.748. This implies that irrespective of whether the South African economy is in an upswing or a recession, a 1 percent increase in the output gap is associated with a decrease in South African *cyclical unemployment* of just over 0.7 percentage points. Similar to the medium run coefficients from equation 4 (Table 3), the medium run $\omega^-$ coefficient for equations (20.1a), (20.4a) and (20.7a) is on average double the contemporaneous $\gamma_0^-$ coefficient. This is also true for the medium run estimates $\omega^-$ and $\omega^+$ for equation (20.5a), whereas the medium run coefficient estimates for equation (20.8a) are less than double the contemporaneous coefficient estimates and $\gamma_0^-$ and $\gamma_0^+$. Equations (20.1a) and (20.5a) indicate the presence of some inertia effects – the $\beta_1$ coefficient is positive and statistically significant in both cases.

Figure IB (Appendix I) presents the recursive estimates for $\gamma_0^-$ and $\gamma_0^+$ coefficients in equations (20.1a), (20.4a), (20.5a), (20.7a) and (20.8a) that were found to be statistically significant. Once again, all of the estimated recursive coefficients remained within the two standard error confidence bands for the entire period under consideration. Most of the plots indicate some instability towards the end of the sample (except perhaps equations (20.5a) and (20.8a)), with the estimated coefficients ranging between -0.55 and -0.85.

TABLE 5 HERE

Next, the results from Table 5 (above) are discussed. Due to the presence of serial correlation in most of the regressions, only the results for equations (20.4b), (20.7b) and (20.8b) will be discussed. The estimated threshold value for the three equations varies between -0.23 and -0.96. However, splitting of the output gaps according to the different thresholds produces mixed results. The contemporaneous output gaps *both* above and below the estimated threshold value are statistically significant for all three equations – however, the magnitudes of these coefficients differ. For the output gaps below the threshold, the Okun's coefficient varies from -0.149 to -0.904, whereas the Okun's coefficient estimates for output gaps above the threshold vary from -0.210 and -0.711. This implies that Okun's relationship is present during both upswings and recessions in South Africa. This implies that irrespective of whether the South African economy is in an upswing or a recession, a 1 percent increase in the output gap is associated with a decrease in South African *cyclical unemployment* of up to 0.7-0.9 percentage points. The medium run coefficients give mixed results – double for "positive" output gap estimates in equations (20.4b) and (20.7b), and considerably less than double otherwise (even positive, but close to 0, for the "negative" output gap estimates in equations (20.4b) and (20.7b)). No inertia effects were present in equations (20.4b), (20.7b) and (20.8b) – the $\beta_1$ coefficient is statistically insignificant in all cases. Furthermore, the lagged split gaps are significant in equations (20.4b) and (20.7b) – however, the signs of the lagged output gaps below the threshold are incorrect on *a priori* grounds.

Figure IC (Appendix I) presents the recursive estimates for the $\gamma_0^-$ and $\gamma_0^+$ coefficients in equations (20.4b), (20.7b) and (20.8b) that were found to be statistically significant. All of the estimated recursive coefficients remained within the two standard error confidence bands for the entire period under consideration. As with the results from Table 4, most of the plots indicate some instability towards the end of the sample, with the estimated coefficients ranging between -0.7 and -0.95.

## 6 Conclusion

This paper estimated the relationship between cyclical unemployment and cyclical output by using a variety of detrending methods to decompose output and unemployment series into their trend and cyclical components. The detrending methods used yielded unemployment and output cycles that differed substantially in terms of the chronology of the phases of the cycles as well as the amplitudes and frequencies of the cycles. However, irrespective of the detrending method used to estimate the dynamic relationship between cyclical output and cyclical unemployment, the contemporaneous relationship between these two variables was always found to be statistically significant. Estimates of the contemporaneous Okun's coefficient ranged between -0.17 and -0.78, while estimates of the "long-run" Okun's coefficient ranged between -0.24 and -1.09. For all specifications, the long run coefficient was found to be larger (often substantially) than the short run coefficient. These results seemingly indicate the presence of an Okun's law relationship in South Africa over the period 1970-2005. Recursive estimates of the contemporaneous Okun's coefficient further revealed that this relationship remained relatively stable over the sample period. In addition, the paper finds evidence of asymmetries in Okun's law, with a more pronounced relationship during recessions. For specifications in which both the above and below the threshold coefficients were significant, the size of Okun's coefficient did not vary considerably whether the economy was in an upswing or a recession.

The limitations of this study, which also provide scope for future research, are that a single equation model could present a misspecification of the Okun's relation and that a simultaneous equation model might be more appropriate. In addition, the asymmetric response of cyclical unemployment to cyclical output warrants further investigation.

The results though indicate that a statistically significant relationship exists between cyclical output and cyclical unemployment. This finding suggests that one method with which to combat unemployment in South Africa is through increased output, and therefore more expansionary fiscal and monetary policy. However, the extent to which total unemployment (and not just cyclical unemployment) responds to output should be investigated, as well as the factors associated with other types of unemployment (structural and frictional), before any definite policy recommendations can be made.

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*Table 1. Review of the Recent Literature on Okun's Law*

| **Author(s)** | **Country / Sample/ Data** | **Specification** | | **Method of extracting cyclical components** | **Estimation method** | **Asymmetries / thresholds** | **Results** |
|---|---|---|---|---|---|---|---|
| | | **Dependent variable(s)** | **Independent variable(s)** | | | | |
| **Prachowny (1993)** | US; 75:I-88:IV and 67:II-86:II; quarterly | Output gap | Capacity utilisation gap; labour-supply gap; unemployment gap; hours gap | Output and unemployment gaps as per Adams and Coe (1989), Gordon (1987); other gaps generated as difference between observed and second-degree polynomial time trend | OLS (first-difference production function) | No | Unemployment gap statistically significant; estimates varied between -0.62 and -0.67 |
| **Weber (1995)** | US;1948:I-1988:IV; quarterly | Unemployment gap; output gap | Output gap; unemployment gap | Perron's (1989) trend stationary with structural break (1973) decomposition (for output); residual of regression of unemployment rate on constant and a dummy | OLS, ARDL model proposed by Gordon (1984), bivariate VAR adapted from Blanchard (1989), Engle and Granger (1987) cointegration method | No | Unemployment and output gaps statistically significant; static estimates support Okun's original estimate (-0.32); dynamic specifications suggest a smaller value (between -0.22 and -0.260) |
| **Moosa (1997)** | US, Japan, Germany, France, UK, Italy, Canada; 1960-1995; annual | Unemployment gap | Lagged unemployment gap, contemporaneous output gap | Harvey's (1985, 1989) structural time series model | OLS; rolling OLS; SUR | No | Output gap statistically significant; Estimates varied between -0.49 and -0.09 |
| **Attfield and Silverstone (1998)** | UK; 1959:I-1994:I; quarterly | Output | Unemployment rate | Beveridge-Nelson decomposition (if output and unemployment are cointegrated) | VECM | No | Statistically significant coefficient on unemployment -1.45 |
| **Moosa (1998)** | US, 1947:I-1992:II; quarterly | Unemployment gap | Lagged unemployment gap, contemporaneous and lagged output gap | Harvey's (1985, 1989) structural time series model | ARDL/OLS | No | Output gap statistically significant; Estimates equal to -0.16 for 2-5 lags |
| **Apel and Jansson (1999)** | Canada, UK, US; 1970:I-1998:II; quarterly | Change in inflation rate; output gap; NAIRU; potential output; unemployment gap | Lagged change in inflation rate, contemporaneous and lagged unemployment gap; contemporaneous and lagged unemployment gap; lagged NAIRU; lagged potential output; lagged unemployment gap | Unobserved-components model/Kalman filter | Kalman filter and maximum likelihood | No | Contemporaneous unemployment gap statistically significant for all three countries; estimates range between -7.1 and -3.5 |
| **Dixon and Thomson (2000)** | Australia; 1964-1999; annual | Change in unemployment rate | Change in output | No detrending, first-differences used | OLS | No | Estimated slope coefficient of -0.22 (no indication of significance) |
| **Lee (2000)** | 16 OECD countries; 1955-1996 (1960-2006 for Germany); annual | Change in output (output gap) | Change in unemployment (unemployment gap) | First differences, HP filter, Beveridge-Nelson decomposition, unobserved-components model/Kalman filter | OLS, Kalman filter and maximum likelihood, VECM | Yes; threshold = 0 | Results generally support the validity of Okun's law as parameter estimates are statistically significant; however, the results are not as robust as those originally reported by Okun (differ across countries and across |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | | | | alternative detrending methods) |
| **Harris and Silverstone (2001)** | Australia, Canada, Germany, Japan, New Zealand, UK, US; 1978:I-1998:III (or 1998:IV or 1999:I, depending on the country's data availability; quarterly | Unemployment rate | Output | No detrending, first-differences used | Single equation ECM | Yes; thresholds estimated by means of TAR | Contemporaneous Okun's coefficient ranges between -0.09 and -0.5 (statistically significant); evidence found in favour of asymmetries for all countries except Canada. |

*Table 2. Variations of Equation (4)*

| Equation | $u^p$ | $y^p$ |
|---|---|---|
| **(4.1)** | First difference method | First difference method |
| **(4.2)** | Linear trend representation | Linear trend representation |
| **(4.3)** | Hodrick-Prescott (HP) Filter | Hodrick-Prescott (HP) Filter |
| **(4.4)** | Band-Pass (BP) Filter | Band-Pass (BP) Filter |
| **(4.5)** | Beveridge-Nelson (BN) decomposition | Beveridge-Nelson (BN) decomposition |
| **(4.6)** | Hodrick-Prescott (HP) Filter | Production Function Approach with HP |
| **(4.7)** | Band-Pass (BP) Filter | Production Function Approach with BP |
| **(4.8)** | Beveridge-Nelson (BN) decomposition | Production Function Approach with BN |

*Table 3. Estimation Results for Equation (4)*

| Equation | (4.1) | (4.2) | (4.3) | (4.4) | (4.5) | (4.6) | (4.7) | (4.8) |
|---|---|---|---|---|---|---|---|---|
| $\beta_1$ | 0.817*** | 0.839*** | 0.635*** | -0.131 | 0.329* | 0.654*** | -0.107 | 0.195 |
| | (6.305) | (13.330) | (3.960) | (-0.731) | (1.930) | (4.061) | (-0.597) | (1.128) |
| $\gamma_0$ | -0.164** | -0.276*** | -0.306*** | -0.297*** | -0.732*** | -0.295*** | -0.175*** | -0.772*** |
| | (-2.620) | (-4.233) | (-4.377) | (-2.987) | (-3.689) | (-3.904) | (-2.870) | (-9.278) |
| $\gamma_1$ | 0.105* | 0.120 | 0.053 | -0.157 | -0.311 | 0.023 | -0.092 | -0.085 |
| | (1.175) | (1.559) | (0.604) | (-1.445) | (-1.352) | (0.257) | (-1.381) | (-0.543) |
| $\omega$ | -0.322 | -0.969 | -0.693 | -0.401 | -1.554 | -0.786 | -0.241 | -1.065 |
| **Adj. $R^2$** | 0.439 | 0.919 | 0.588 | 0.182 | 0.427 | 0.560 | 0.166 | 0.778 |
| **Serial correlation** | 0.034 | 7.135** | 10.519*** | 7.016 | 1.982 | 11.916** | 7.209 | 1.499 |
| **LM test** | (0.983) | (0.028) | (0.001) | (0.135) | (0.159) | (0.018) | (0.125) | (0.827) |
| **Heteroscedasticity** | 9.759 | 6.595 | 3.972 | 12.564 | 5.705 | 3.088 | 12.823 | 9.292 |
| **test (White)** | (0.135) | (0.360) | (0.680) | (0.183) | (0.457) | (0.798) | (0.171) | (0.158) |
| **Normality test** | 0.646 | 0.025 | 2.075 | 1.913 | 0.515 | 5.638* | 1.605 | 1.427 |
| **(JB)** | (0.724) | (0.987) | (0.354) | (0.384) | (0.773) | (0.060) | (0.448) | (0.490) |
| **Ramsey's RESET** | 1.493 | 0.768 | 2.093 | 1.524 | 1.809 | 4.117 | 1.317 | 1.312 |
| **test** | (0.474) | (0.681) | (0.351) | (0.467) | (0.405) | (0.128) | (0.518) | (0.252) |

Note:
1. For the estimated coefficients t-statistics are included in parentheses
2. For serial correlation, heteroscedasticity test, normality as well as the Ramsey's RESET tests p-values are included in parentheses.
3. ***, ** and * denote significance at 1%, 5% and 10% levels, respectively.

## *Table 4. Estimation Results (threshold value = 0)*

| **Equation** | **(20.1a)** | **(20.2a)** | **(20.3a)** | **(20.4a)** | **(20.5a)** | **(20.6a)** | **(20.7a)** | **(20.8a)** |
|---|---|---|---|---|---|---|---|---|
| $\beta_1$ | 0.707*** | 0.831*** | 0.624*** | -0.134 | 0.332* | 0.648*** | -0.105 | 0.185 |
| | (4.623) | (12.750) | (3.820) | (-0.711) | (1.852) | (3.934) | (-0.556) | (1.040) |
| $\gamma_0^-$ | -0.535* | -0.249** | -0.210 | -0.297* | -0.728* | -0.215 | -0.180* | -0.841*** |
| | (-1.857) | (-2.052) | (-1.683) | (-1.781) | (-1.954) | (-1.612) | (-1.779) | (-5.239) |
| $\gamma_1^-$ | -0.020 | 0.129 | -0.008 | -0.184 | -0.376 | -0.015 | -0.090 | -0.110 |
| | (-0.064) | (1.062) | (-0.061) | (-0.923) | (-0.876) | (-0.107) | (-0.750) | (-0.496) |
| $\gamma_0^+$ | -0.125 | -0.279*** | -0.378*** | -0.302 | -0.748** | -0.361*** | -0.169 | -0.727*** |
| | (-1.650) | (-3.565) | (-3.587) | (-1.669) | (-2.142) | (-3.044) | (-1.485) | (-5.466) |
| $\gamma_1^+$ | 0.076 | 0.081 | 0.084 | -0.136 | -0.262 | 0.048 | -0.092 | -0.088 |
| | (1.033) | (0.863) | (0.702) | (-0.859) | (-0.783) | (0.368) | (-0.0912) | (-0.458) |
| $\omega^-$ | -1.601 | -0.704 | -0.580 | -0.424 | -1.653 | -0.653 | -0.244 | -1.167 |
| $\omega^+$ | -0.167 | -1.172 | -0.782 | -0.386 | -1.512 | -1.162 | -0.236 | -1.000 |
| **Adj. $R^2$** | 0.444 | 0.918 | 0.574 | 0.125 | 0.389 | 0.541 | 0.107 | 0.767 |
| **Serial correlation** | 0.236 | 7.857** | 11.649*** | 7.561 | 4.270 | 11.169*** | 8.061 | 1.153 |
| **LM test** | (0.889) | (0.020) | (0.009) | (0.109) | (0.118) | (0.004) | (0.234) | (0.764) |
| **Heteroscedasticity** | 8.737 | 12.391 | 7.131 | 12.342 | 11.340 | 5.300 | 11.998 | 14.336 |
| **test (White)** | (0.557) | (0.260) | (0.713) | (0.263) | (0.332) | (0.870) | (0.285) | (0.158) |
| **Normality test** | 1.022 | 0.031 | 1.741 | 1.720 | 0.545 | 5.762 | 1.639 | 1.544 |
| **(JB)** | (0.599) | (0.984) | (0.419) | (0.423) | (0.762) | (0.056) | (0.441) | (0.462) |
| **Ramsey's RESET** | 4.532 | 3.705 | 2.452 | 2.406 | 2.756 | 5.015 | 2.467 | 1.055 |
| **test** | (0.104) | (0.157) | (0.293) | (0.300) | (0.252) | (0.171) | (0.291) | (0.304) |

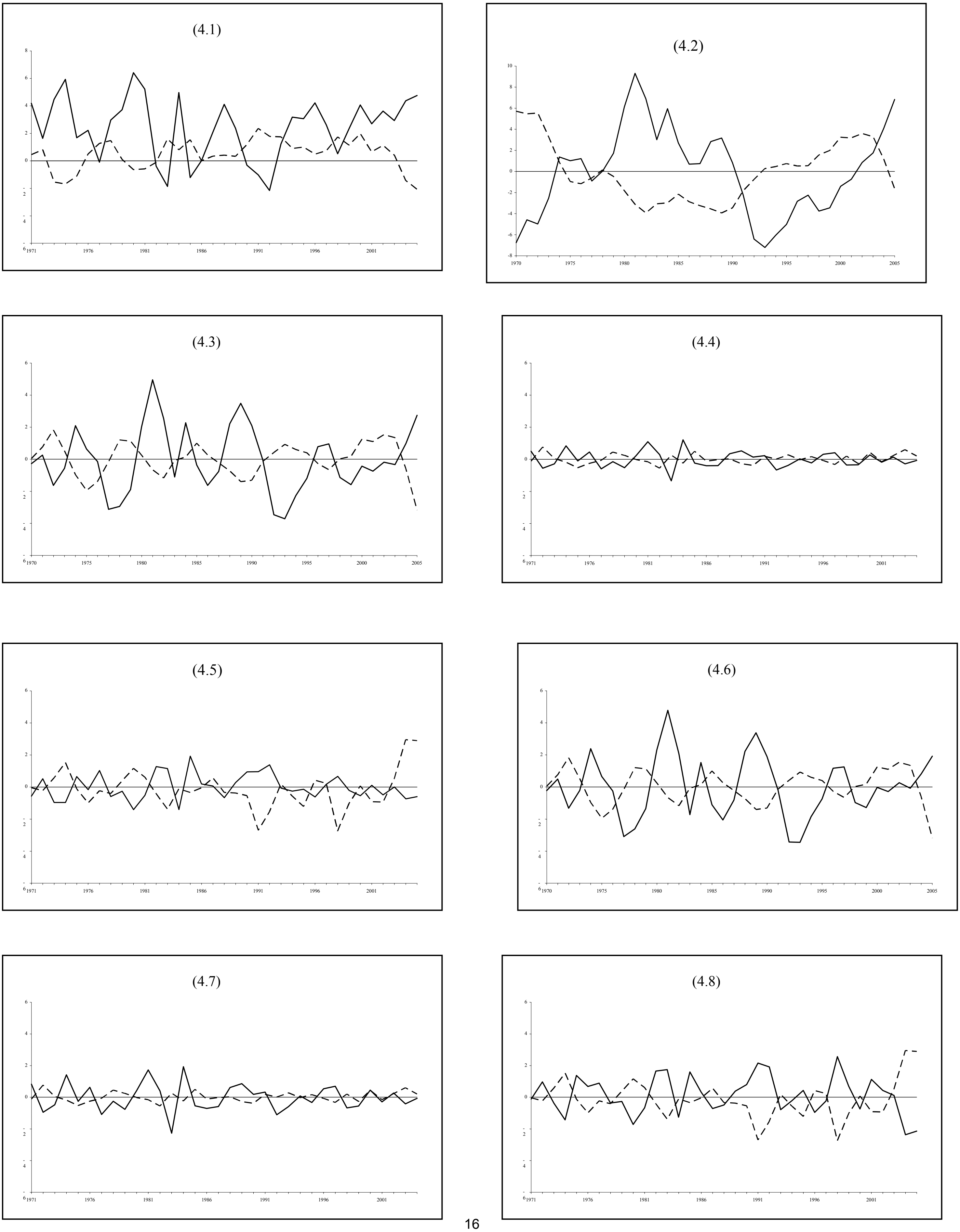
Note: 1. For the estimated coefficients t-statistics are included in parentheses
2. For serial correlation, heteroscedasticity test, normality as well as the Ramsey's RESET tests p-values are included in parentheses.
3. ***, ** and * denote significance at 1%, 5% and 10% levels, respectively.

## *Table 5. Estimation Results (threshold values obtained with a grid search)*

| **Equation** | **(20.1b)** | **(20.2b)** | **(20.3b)** | **(20.4b)** | **(20.5b)** | **(20.6b)** | **(20.7b)** | **(20.8b)** |
|---|---|---|---|---|---|---|---|---|
| $\beta_1$ | 0.705*** | 0.831*** | 0.628*** | -0.030 | 0.404** | 0.671*** | 0.007 | 0.082 |
| | (4.922) | (13.146) | (3.891) | (-0.192) | (2.307) | (4.249) | (0.045) | (0.461) |
| $\gamma_0^-$ | -0.060 | -0.295** | -0.275** | -0.245* | -1.006*** | -0.306*** | -0.149** | -0.904*** |
| | (-0.687) | (-2.400) | (-2.459) | (-2.007) | (-3.869) | (-2.832) | (-2.075) | (-6.192) |
| $\gamma_1^-$ | 0.100 | 0.189 | 0.112 | 0.261* | -0.224 | 0.149 | 0.171* | -0.399 |
| | (1.516) | (1.515) | (0.896) | (1.758) | (-0.887) | (1.283) | (1.893) | (-1.515) |
| $\gamma_0^+$ | -0.187* | -0.270*** | -0.340*** | -0.347*** | -0.361 | -0.286*** | -0.210*** | -0.711*** |
| | (-1.802) | (-3.796) | (-3.825) | (-2.942) | (-1.178) | (-2.924) | (-2.819) | (-7.414) |
| $\gamma_1^+$ | 0.026 | 0.058 | -0.005 | -0.371*** | -0.313 | -0.102 | -0.225*** | -0.115 |
| | (0.183) | (0.679) | (-0.052) | (-3.346) | (-0.725) | (-0.903) | (-3.338) | (-0.742) |
| $\omega^-$ | 0.136 | -0.627 | -0.438 | 0.016 | -2.064 | -0.477 | 0.022 | -1.419 |
| $\omega^+$ | -0.532 | -1.254 | -0.927 | -0.697 | -1.131 | -1.179 | -0.438 | -0.900 |
| **Optimal Threshold** | 4.37 | -1.42 | -0.43 | -0.23 | 1.02 | -0.03 | -0.30 | -0.96 |
| **Adj. $R^2$** | 0.465 | 0.922 | 0.586 | 0.404 | 0.438 | 0.581 | 0.402 | 0.789 |
| **Serial correlation** | 12.818*** | 10.685*** | 18.006*** | 8.619 | 7.863** | 18.021*** | 9.476 | 1.382 |
| **LM test** | (0.002) | (0.005) | (0.001) | (0.125) | (0.020) | (0.001) | (0.149) | (0.710) |
| **Heteroscedasticity** | 18.170 | 8.494 | 28.871*** | 9.185 | 11.521 | 31.116*** | 8.727 | 14.277 |
| **test (White)** | (0.199) | (0.581) | (0.001) | (0.515) | (0.318) | (0.005) | (0.558) | (0.161) |
| **Normality test** | 2.131 | 0.331 | 1.796 | 3.171 | 1.473 | 3.161 | 2.191 | 1.131 |
| **(JB)** | (0.345) | (0.847) | (0.407) | (0.205) | (0.479) | (0.206) | (0.334) | (0.568) |
| **Ramsey's RESET** | 4.702 | 0.091 | 11.652*** | 6.319 | 0.234 | 15.745*** | 2.621 | 4.467 |
| **test** | (0.319) | (0.956) | (0.003) | (0.788) | (0.889) | (0.000) | (0.998) | (0.107) |

Note: 1. For the estimated coefficients t-statistics are included in parentheses
2. For serial correlation, heteroscedasticity test, normality as well as the Ramsey's RESET tests p-values are included in parentheses.
3. ***, ** and * denote significance at 1%, 5% and 10% levels, respectively.

*Figure 1 Cyclical GDP and Cyclical Unemployment Estimates*



APPENDIX I: Recursive Coefficient Estimates

*Figure IA. Recursive Coefficient Estimates for Equation (4).*

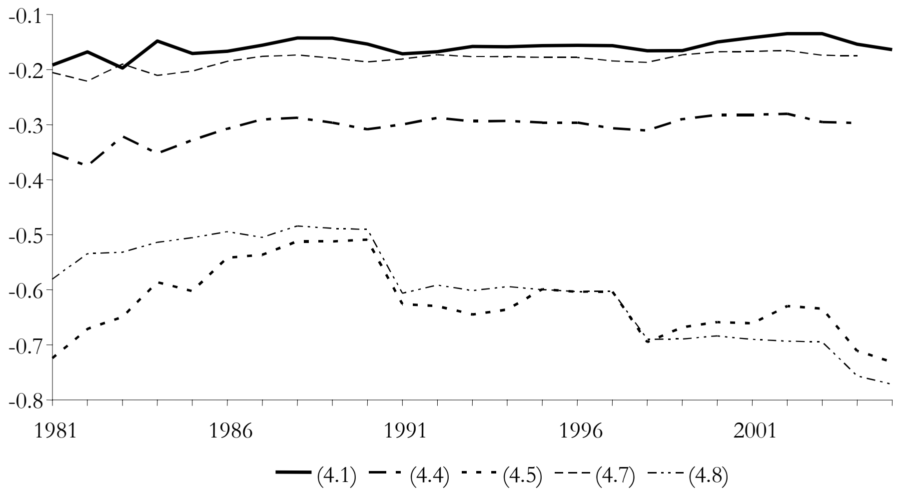


*Figure IB. Recursive Coefficient Estimates for Equation (20a) – Threshold Value = 0.*

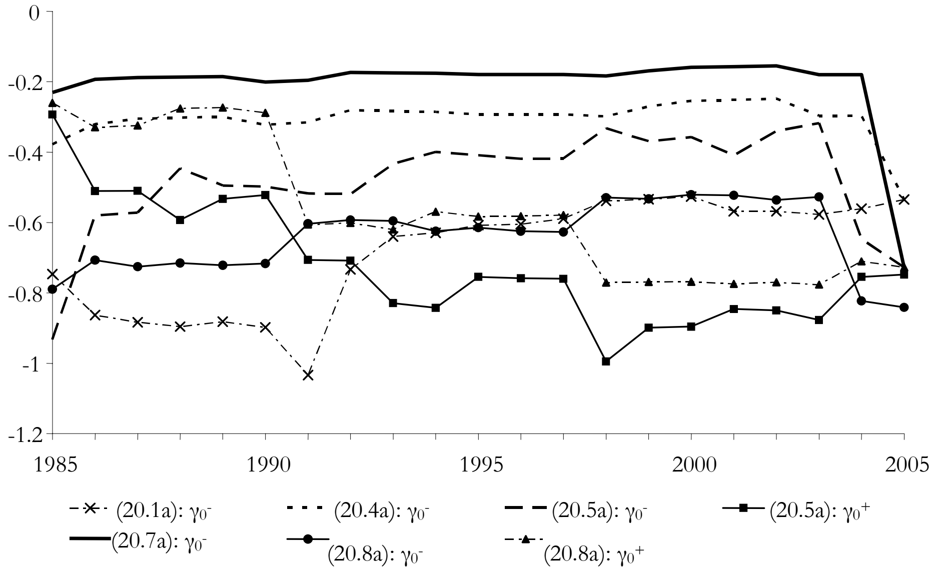


*Figure IC. Recursive Coefficient Estimates for Equation (20b) – Threshold Value Obtained Through Grid Search.*