

Informative Advertising: Competition or Cooperation?

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Abstract

I compare the outcome when firms semicollude on advertising to the outcome in the Grossman and Shapiro (1984) model of informative advertising. I show that advertising is lower but prices and profits are higher under semicollusion on advertising. I also show that semicollusion on advertising is detrimental to welfare. Although firms earn higher profits when colluding on advertising, fewer consumers are informed, and as a result, welfare is lower. Compared to semicollusion on price, semicollusion on advertising is not always less profitable. Hence I lend theoretical support to empirical studies that find evidence of collusion on advertising rather than price.

Keywords: Informative advertising, semicollusion, competition, product differentiation

JEL Classification: D43, L13, L15, M37

1 Introduction

The importance of advertising as a competitive weapon in sellers' interactions has long been recognized. Typically, a firm that advertises more can expect higher demand and hence higher revenues, other things being equal. In multifirm industries, this possibility to steal customers from competitors often results in costly "advertising wars" as firms try to regain lost market share. If, in addition, advertising conveys price information, such advertising wars inadvertently lead to lower prices – a double blow!

Indeed, Grossman and Shapiro (1984), Christou and Vettas (2003), among others, show that increased price advertising raises demand elasticity and thus lowers prices. Hence, excessive advertising may actually hurt firms. Therefore, if firms are sophisticated, they ought to realize the folly of unbridled price advertising. Yet, the analysis of price advertising has been framed exclusively in terms of fully noncooperative interaction. While in many countries price collusion is per se illegal (which may explain nonprice collusion), collusion

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on advertising is not.¹ If anything, the existence of advertising agencies – that often handle advertising from several competing firms – provides scope for collusion on advertising (Bernheim and Whinston, 1985).²

What is more, empirical evidence (on price and advertising strategies in different industries) seem to support the hypothesis of collusion on advertising. Gasmi et al (1992) investigate possible market configurations in the Cola market (Nash behaviour, Stackelberg leadership and several possible configurations of collusion). They use data for the period 1968-1986 to test their hypotheses and thus to select a model of strategic behavior that best fits the data. Noncooperative behaviour in both advertising and prices is rejected by the data. They find support for collusion on advertising (but not price). In a similar study, but for the US butter and margarine industry, Wang et al (2004) reach a similar conclusion. A related study is that of the US cigarette market by Roberts and Samuelson (1988). They find that, particularly for low tar cigarettes, the data does not seem to support the hypothesis of combative advertising. Moreover, they cannot reject the hypothesis of joint profit maximizing choice of advertising.

In this paper, I examine firms' incentives to collude on advertising when advertising is purely informative. More precisely, I compare the equilibrium under collusion on advertising to the fully noncooperative equilibrium as well as to the equilibrium under price collusion. I also investigate the welfare implications of collusion on advertising.

I adopt the framework of Grossman and Shapiro (1984) and postulate a linear city in which firms sell a differentiated product. Consumers are uniformly distributed along the unit interval and do not search. Firms advertise to inform consumers. I analyze three cases: no collusion, collusion on advertising only and collusion on price only.³

I find that, compared to the noncooperative equilibrium outcome, collusion on advertising leads to reduced advertising but higher prices and profits. By lowering the advertising intensity, collusion on advertising raises informational product differentiation and this relaxes price competition. This allows the firms to charge higher prices. Also, lower advertising has a positive direct effect on profit – lower advertising outlay. The lower advertising outlay, coupled with the induced higher prices, enable firms to earn higher profits.

Although firms earn higher profits, semicollusion on advertising is bad for welfare. Consumers not only pay higher prices, rather, in addition to higher prices, fewer consumers get informed when firms collude on advertising – and this exacerbates the loss of consumer surplus. In comparing price collusion to collusion on advertising, I find that the former dominates the latter in terms of revenues. Firms advertise more and charge higher prices

¹In the US, the pertinent case is California Dental Association (CDA) vs Federal Trade Commission (FTC). While the FTC and the Ninth Circuit Court of Appeals condemned the CDA's advertising restrictions as per se illegal, the US Supreme Court ruled that it was not "intuitively obvious" that the restrictions were anticompetitive. Instead, the Court instructed that the restrictions be examined (by the Ninth Circuit) under the rule of reason – where the potential benefits are contrasted to the costs (Lande and Marvel, 2000). When a particular conduct is deemed per se illegal, the FTC /Court will move directly to the punishment phase.

²In the US for example, promotion of milk products is cooperatively managed (Blisard, *Undated*; Lande and Marvel, 2000).

³I deliberately omit the case of full collusion (collusion on both advertising and prices). It is well understood that the monopoly profit is at least as large as the sum of the duopoly profits. Therefore, there is nothing much to be gleaned from studying this case.

when colluding on price. However, price collusion is not, in general, more profitable.

This paper adds to the growing literature on semicollusion. Semicollusion obtains whenever economic agents choose to cooperate along some dimension(s) while at the same time competing along another dimension. The only previous work on semicollusion on advertising that I am aware of is Aluf and Shy (2001).⁴ They study comparison advertising in a duopoly market where products are, in the absence of advertising, homogeneous. In their model, advertising serves to differentiate products in the eyes of the consumers (spurious product differentiation). They show that semicollusion leads to higher advertising, prices and profits relative to the noncooperative outcome.

In an interesting contribution, Fershtman and Gandal (1994) challenge the widely accepted view that price collusion is always beneficial to firms. They argue that semicollusion can be disadvantageous. In particular, they show that when firms noncooperatively choose capacity in the first stage of the game and then collude on price in the second stage, they earn lower profits compared to the fully noncooperative outcome. Steen and Sørsgard (1999) adapt the Fershtman and Gandal (1994) model to suit the Norwegian cement market. In their model, firms can also export excess output at the prevailing world price. They show that if each firm's domestic market share is determined by the firm's share of total industry capacity and firms collude on price, a higher domestic demand may induce overinvestment in capacity and this in turn will lead to an increase in exports. They label this effect the "semicollusion effect". They empirically test for and find support for this effect in the Norwegian cement cartel.

The paper closest to mine in scope is Aluf and Shy (2001). However, in our framework, unlike Aluf and Shy (2001), advertising does not change consumers' tastes. That is, advertising is purely informative. I also differ with them in that I allow for semicollusion on price. This enables me to make comparisons between semicollusion on advertising and semicollusion on price.

The paper is organized as follows: Section 2 sets out the model. I derive the noncooperative and the semicollusive equilibria in section 3 and section 4 studies semicollusion on price. I contrast the equilibrium when firms semicollude on price to the equilibrium when they semicollude on advertising in section 5. Section 6 discusses the pros and cons of semicollusion and section 7 concludes the paper.

2 Model and Preliminaries

I adopt Tirole (1988)'s model – a simplification of the Grossman and Shapiro (1984) model of informative advertising with differentiated products. Two firms, firm 1 and firm 2, sell a horizontally differentiated good. The firms are located at the end points of a linear city of unit length with firm 1 located at point 0 and firm 2 at point 1. Firms randomly send out advertisements (ads) to inform consumers of the prices they charge. That is, every

⁴The literature has focused on situations where firms cooperate on price while at the same time competing on some other variable – for example capacity (Steen and Sørsgard, 1999), (Fershtman and Gandal, 1994); R&D (Fershtman and Gandal, 1994); location (Friedman and Thisse, 1993). See also Steen and Sørsgard, (1999; footnote 1).

consumer has an equal chance of receiving any ad that is sent by any firm. Let ϕ_i denote the advertising intensity of firm i ; $i = 1, 2$ (fraction of the consumer population that is exposed, at least once, to the advertising message of firm i). The cost of reaching fraction ϕ_i of consumers is denoted $A(\phi_i)$, where $A(\phi) = a\phi^2/2$; $a > t/2$.⁵ Each good is produced at a constant marginal cost which I normalize to zero. There is no entry or exit.

Consumers are uniformly distributed according to taste on $[0, 1]$, have unit demands and attach a dollar value of v to the consumption of a unit of the good. Consumers are uninformed about prices and firm locations unless they are reached by advertising. Thus, uninformed consumers do not participate in the market. Informed consumers incur a shopping cost of t per unit of distance travelled.⁶

Given the firms' advertising intensities, ϕ_1 and ϕ_2 , and the consumers' (passive) behavior, the market is delineated as follows; fraction $\phi_1\phi_2$ of consumers receive advertising messages from both firms (fully informed); fraction $\phi_i(1 - \phi_j)$; $i, j = 1, 2$; $j \neq i$ receive ads from firm i but not firm j (partially informed); and fraction $(1 - \phi_1)(1 - \phi_2)$ receive no ads from either firm (uninformed). I assume that $\phi_1\phi_2$ is large enough so that firms find it worthwhile to compete for the fully informed consumers.⁷

Fully informed consumers purchase from whichever firm guarantees them the greatest surplus. A consumer located at $x \in (0, 1)$ gets surplus $v - p_1 - tx$ buying from firm 1 and surplus $v - p_2 - t(1 - x)$ buying from firm 2. Let $\hat{x}$ denote the location of the consumer who is indifferent between buying from firm 1 and buying from firm 2; then, $\hat{x} = (p_2 - p_1 + t)/2t$. Consumers with locations $x \in [0, \hat{x})$ buy from firm 1 while those with locations $x \in (\hat{x}, 1]$ buy from firm 2. Thus, firm i faces the demand $D_{full}^i = (p_j - p_i + t)/2t$ from the fully informed consumers.

For partially informed consumers, demand is determined by individual rationality. Let x_i denote the location of the consumer who receives advertising only from firm i . Buying from firm i yields surplus $v - p_i - tx_i$ while the consumer gets surplus zero when not purchasing. Hence the demand from partially informed consumers is given by $x_i = \frac{v - p_i}{t}$. All partially informed consumers with locations less than x_i find it worthwhile to purchase while those with locations greater than x_i will not purchase. However, if $v - p_i \geq t$, all consumers who receive at least one ad from firm i will make a purchase, that is, $x_i = 1$.

Thus, each firm's demand is a sum of the demands by the partially informed and the fully informed consumers. That is;

$$D_i(\phi_1, \phi_2; p_1, p_2) = \phi_i \left((1 - \phi_j) \frac{v - p_i}{t} + \phi_j \frac{p_j - p_i + t}{2t} \right); i \neq j.$$

In the sequel, I assume that the market is fully covered. For the market to be fully covered, it is necessary and sufficient that the partially informed consumer who travels the

⁵I assume $a > t/2$ to allow for some consumers to be uninformed in equilibrium, so that it is possible to study the effects of varying the advertising level. For $a \leq t/2$, the advertising cost is too low and, as a result, we have full information in equilibrium. That is, $\phi_1 = \phi_2 = 1$. See also Tirole (1988; p. 292).

⁶Since consumers are distributed according to taste, t can also be interpreted as the disutility from consuming a good that is different from the ideal.

⁷A necessary condition for firms to compete for the fully informed consumers is that advertising costs are low. In an appendix available from the author, I derive the exact conditions on the advertising cost, a .

entire unit distance gets nonnegative surplus. That is, $p + t \leq v$.⁸

With this assumption, the demand facing firm i reduces to:

$$(1) \quad D_i(\phi_1, \phi_2; p_1, p_2) = \phi_i \left(1 - \phi_j + \phi_j \frac{p_j - p_i + t}{2t} \right).$$

3 Competition or Collusion?

In subsections 3.1 and 3.2 I derive, respectively, the noncooperative and the semicollusive equilibria and contrast them in subsection 3.3.

3.1 Noncooperative Equilibrium

Firms simultaneously and noncooperatively choose both advertising levels and prices (Nash equilibrium). Firm i has the following maximization problem:

$$(2) \quad \pi_i = \max_{p_i, \phi_i} \left\{ p_i \phi_i \left(1 - \phi_j + \phi_j \frac{p_j - p_i + t}{2t} \right) - \frac{a\phi_i^2}{2} \right\}.$$

The first order necessary conditions are

$$(3) \quad \frac{\partial \pi_i}{\partial p_i} = 1 - \phi_j + \phi_j \frac{p_j - p_i + t}{2t} - \frac{p_i \phi_j}{2t} = 0,$$

$$(4) \quad \frac{\partial \pi_i}{\partial \phi_i} = p_i \left(1 - \phi_j + \phi_j \frac{p_j - p_i + t}{2t} \right) - a\phi_i = 0.$$

Equation (4) equates the marginal revenue and the marginal cost of raising the advertising reach marginally.

Solving (3) for p at the symmetric equilibrium gives

$$(5) \quad p = \frac{2t}{\phi} - t.$$

It is immediate from (5) that higher advertising is associated with lower prices. This is explained by the fact that when the market is covered, fully informed consumers are price sensitive while partially informed consumers are not. A higher advertising intensity implies a higher proportion of fully informed consumers in the market and this puts pressure on prices.

Substituting (5) back into the objective function yields,

$$(6) \quad \pi = 2t - 2t\phi + \frac{1}{2}t\phi^2 - \frac{a\phi^2}{2}$$

as the firm's profit for any given level of advertising. One can easily show that;

⁸Full market coverage does not require **all** consumers to make a purchase but rather, it only requires that all **informed** consumers make a purchase.

Lemma 1 *Profits are strictly decreasing in the advertising intensity.*

Proof. $\partial\pi/\partial\phi = (\phi - 2)t - a\phi < 0$. ■

To understand why profits decrease with advertising at all levels, we write the profit function as: $\pi = R(\phi) - C(\phi)$, where the revenue, $R(\phi) = 2t - 2t\phi + t\phi^2/2$ and the cost, $C(\phi) = a\phi^2/2$. Differentiating the revenue and cost functions with respect to ϕ gives; $R'(\phi) = (\phi - 2)t < 0$ and respectively, $C'(\phi) = a\phi > 0$. That is, a small increase in advertising lowers the firm's revenues but raises the firm's costs. Although demand increases with advertising, the negative effect on price of an increase in ϕ dominates the total effect on revenues. Since revenues fall while costs rise with advertising, it follows that profit decreases with increases in advertising.

Solving (3) and (4) simultaneously gives;

$$(7) \quad \phi^{nc} = 2 / \left(1 + \sqrt{2a/t} \right); \quad p^{nc} = \sqrt{2at}$$

and substituting (7) back into the objective function gives

$$(8) \quad \pi^{nc} = 2a / \left(1 + \sqrt{2a/t} \right)^2$$

where nc is a mnemonic for noncooperative.

3.2 Semicollusion

In this section, I study a two period game where firms collude on advertising but compete on prices. The timing of the game is as follows: In the first stage, firms noncooperatively set their prices, and in the second stage, knowing the equilibrium prices chosen in the first stage, they collusively decide on advertising.

Our timing needs some dressing. Although the standard approach in the literature is to let firms set the less flexible variable in the first stage and then set the more flexible choice variable (typically prices) in the second stage (see for example, Aluf and Shy, 2001; Salvanes et al, 2003), our timing is not without merit. In the case of print advertising, our timing is natural. When firms advertise their prices, they need to know the prices before they can print them and send out the fliers. That is, firms choose prices first.

To bring more realism to this game, we can recast the game as follows: firms noncooperatively choose their prices while delegating the decision on advertising to a third party – the advertising agency.⁹ In the first stage, firms set prices and in the second stage, knowing the prices chosen by the firms in the first period, the advertising agency chooses the advertising level to maximize firms' joint profits.

⁹Bernheim and Whinston (1985) show that indeed the use of common marketing agents facilitates collusion. The role of advertising agencies include market analysis, media buying services, consultation on promotion strategies and techniques (design and packaging) among others (Printadvertising.com; Utah Firms Staff, 2003). Many firms nowadays employ advertising agencies to do the advertising on their behalf. For example, EURO RSCG Worldwide has, among its clients, Volvo, Citroen and Peugeot – firms competing in the same market! Catalpha Advertising and Design has among its clients; Black & Decker, DeWalt, Craftsman – firms selling similar products.

Introducing an agency into the game potentially creates an agency problem. A question that arises is whether the advertising agency will have incentives to act in the interest of the firms. However, it is not difficult to see that, in the present setting, compensation (incentive) schemes can be easily designed to induce the agency to act in the joint interest of the firms. For example, to align the agency and the firms' incentives, one can imagine Nash bargaining over the total industry profits between the agency and the firms, or, the agency can be paid a commission (as in Bernheim and Whinston, 1985) that is a fixed proportion of each firm's profit. Because higher advertising results in lower profit, in either case the agency will have incentives to reduce advertising. However, to simplify the analysis, I will assume that the advertising agency gets no share of the profits.

As is typical in two stage games, I solve the problem backwards, starting with the collusive phase.

3.2.1 Collusion phase

In the collusive phase, the advertising agency sets $\phi_1 = \phi_2 = \phi$, knowing the equilibrium prices, p_1 and p_2 , chosen by the firms in the prior (noncooperative) phase.¹⁰ The agency maximizes the following objective function:

$$(9) \quad \Pi \equiv \pi_1 + \pi_2 = \max_{\phi} \left\{ (p_1 + p_2) \phi (1 - \phi) + \frac{\phi^2}{2t} \left(t(p_1 + p_2) - (p_1 - p_2)^2 \right) - a\phi^2 \right\}.$$

The first order condition yields

$$(10) \quad \phi = \frac{t(p_1 + p_2)}{2at + t(p_1 + p_2) + (p_1 - p_2)^2}.$$

3.2.2 Competition phase

In the competition phase, firms noncooperatively set their prices, knowing that they will collude on advertising afterwards. Given the collusive advertising level in (10), firm i 's problem is described by:

$$(11) \quad \pi_i = \max_{p_i} \left\{ p_i \phi \left(1 - \phi + \phi \frac{p_j - p_i + t}{2t} \right) - \frac{a\phi^2}{2} \right\} \text{ subject to (10).}$$

Differentiating with respect to p_i , and solving for a symmetric equilibrium gives:

$$(12) \quad p^{ac} = t/2 + \sqrt{2at + t^2/4}.$$

Substituting (12) into (10) gives the semicollusive advertising level as:

$$(13) \quad \phi^{ac} = \frac{t/2 + \sqrt{2at + t^2/4}}{a + t/2 + \sqrt{2at + t^2/4}}.$$

¹⁰We assume that side payments are not feasible. Hence, $\phi_1 = \phi_2$ in the collusive equilibrium.

Finally, substituting (12) and (13) into (9) gives the semicollusive profit as:

$$(14) \quad \pi^{ac} = \frac{\left(t/2 + \sqrt{2at + t^2/4}\right)^2}{2\left(a + t/2 + \sqrt{2at + t^2/4}\right)}$$

where ac is a mnemonic for collusion on advertising.

The full coverage assumption implies that $p^{ac} = t/2 + \sqrt{2at + t^2/4} \leq v - t$. For given t and v , simplifying gives $a \leq \bar{a} \equiv \frac{(v-2t)(v-t)}{2t}$, where $\bar{a}$ is the highest advertising cost compatible with full market coverage.

3.3 Comparison

The question I seek to address here is the following: Does collusion on advertising and competition on price entail higher or lower prices; higher or lower advertising intensities; higher or lower profits compared to the noncooperative outcome? Comparing equations (7) and (12), it is immediate that $p^{ac} > p^{nc}$. That is, equilibrium prices are higher under collusion on advertising. Also, from (7) and (13), I get (after a bit of algebraic manipulation) that $\phi^{ac} < \phi^{nc}$. Since higher advertising has a negative direct effect on profit, collusion on advertising unambiguously raises profits relative to the noncooperative outcome.¹¹

The discussion following Lemma 1 gives a concise statement of why firms may want to constrain informative advertising. The mechanism works as follows; Consumers who receive advertising from both firms (fully informed) can make across firm price comparisons and, as a result, they buy from the firm quoting the lowest "delivered" price. Competition to sell to these consumers drives the price down. In contrast, consumers who receive advertising from a single firm only (partially informed) are totally price insensitive (for all prices $p \leq v - t$). Hence, the optimal price applicable to this group is higher compared to that applicable to the fully informed group. Intuitively, because an increase in advertising raises the proportion of fully informed consumers in the market, it elevates the importance of the fully informed consumers and this puts pressure on prices and by Lemma 1, lowers profits. The idea of collusion on advertising is precisely to try to minimize such competition by constraining the proportion of fully informed consumers. To summarize;

Proposition 1 *Collusion on advertising (and competition on price) gives lower equilibrium advertising but higher equilibrium prices and profits relative to the fully noncooperative equilibrium. That is, $\phi^{ac} < \phi^{nc}$, $p^{ac} > p^{nc}$ and $\pi^{ac} > \pi^{nc}$.*

Proof. (See Appendix A). ■

Given that price fixing is per se illegal, the fact that it is possible to sustain higher prices and profits without resorting to price fixing should be comforting for firms. Collusion on advertising is difficult to detect and /or prosecute (unlike price collusion).¹² As a matter of

¹¹ More precisely, since $\phi^{ac} < \phi^{nc}$ and $\frac{\partial \pi}{\partial \phi} < 0$ (Lemma 1), it follows that $\pi^{ac} > \pi^{nc}$.

¹² When the Ninth Circuit Court of Appeals concurred that the CDA code of conduct was a "naked" restraint on price competition, the Commission thought they had nailed the CDA. However, on appeal, the Supreme Court instructed that the rules be examined under the rule of reason. Upon reconsideration, the

fact, advertising agencies openly handle business on behalf of competing firms (see footnote 9, see also Bernheim and Whinston, p. 269).

From a welfare perspective, an important question is whether semicollusion on advertising improves welfare. Grossman and Shapiro (1984), Hamilton (2004) and Simbanegavi (2005) show that the market may overprovide informative advertising relative to the socially optimal level. Hence collusion on advertising, by restricting advertising, is potentially welfare improving, especially for low advertising costs. On the one hand, when firms collude on advertising, fewer consumers get informed and this lowers aggregate consumer surplus since uninformed consumers do not purchase. On the other hand, because prices are higher and because firms advertise less when colluding on advertising, they earn higher profits. So, which direction will the welfare effect go? The following Proposition answers this question:

Proposition 2 *Semicollusion on advertising is detrimental to welfare.*

Intuitively, when firms collude on advertising, they restrict advertising "too much". In fact, it can be shown that for all advertising costs in the relevant range, the collusive level is lower than the socially optimal level. That welfare in the semicollusive equilibrium is lower than in the Nash equilibrium is an important result, particularly for competition policy. Although firms may overprovide informative advertising in the noncooperative equilibrium (particularly for low advertising costs), uncontrolled collusion is not a remedy. It is even more inefficient. Under collusion on advertising, the collusive advertising level is "too low" and as a result, too few consumers are informed. This exacerbates the loss of consumer surplus. Since firms have incentives to collude on advertising, there is clearly need for monitoring.¹³

Although in the present model, just as in Aluf and Shy (2001), firms charge higher prices and earn higher profits when colluding on advertising, there are significant differences between the two models. First, in our framework, firms advertise less when colluding. Second, the mechanism through which advertising affects prices and profits is different. In my model, advertising does not change consumers' tastes. Instead, it affects informational product differentiation and hence the toughness of price competition. That is, it alters the proportion of fully informed consumers in the market and hence the price elasticity of demand.

In what follows, I contrast price collusion to collusion on advertising. This is motivated by the fact that the analysis of semicollusion to date has largely been framed as collusion on price and competition on a nonprice variable. The question I address is the following: Does price collusion lead to higher profits compared to nonprice collusion – in particular, to collusion on advertising?

decision of the Ninth Circuit was least expected. The Court concluded that there was not enough evidence to show that the said restrictions were indeed anticompetitive (Lande and Marvel, 2000; pages 956-957).

¹³This conclusion may not be very robust to variations in the models. For example, in a model in which TV channels sell advertising time to firms, Kind et al (2005) find that when TV channels collude on advertising, equilibrium advertising levels are higher and the TV channels earn higher profits than when they compete on advertising.

4 Price Collusion

For firms with multiple strategic variables, semicollusion on price may trigger more competitive behaviour in other choice variables (Fershtman and Gandal, 1994). I consider here a setting where firms cooperatively set the price at which their merchandise will be sold. However, each firm independently decides on the "measure" of fliers to send out to consumers. I derive the price collusion equilibrium and compare it to the advertising collusion equilibrium derived earlier.

As before, I model the firms' behaviour as a two stage game. In the first stage, firms collude on price and in the second stage, firms compete on advertising.

When firms collude on price, firm i 's demand is given by:

$$(15) \quad D_i(\phi_1, \phi_2; p) = \phi_i(1 - \phi_j) + \frac{\phi_i \phi_j}{2}; j \neq i.$$

A peculiar feature of our model is that when firms collude on price, demand is independent of price. This independence is a direct consequence of the full market coverage assumption. Because the market is fully covered, the demand by partially informed consumers is independent of price. Prices only matter for the partitioning of the fully informed segment of the market (see equation 1). Therefore, when firms collude on price, they divide the fully informed consumer population equally between them – independent of the price.

I solve the problem backwards, starting with the second stage. Given the collusive price, p , chosen in the first stage, firm i 's second stage maximization program is given by:

$$(16) \quad \pi_i^{pc} = \max_{\phi_i} \left\{ p \left(\phi_i(1 - \phi_j) + \frac{\phi_i \phi_j}{2} \right) - \frac{a\phi_i^2}{2} \right\}.$$

Differentiating (16) with respect to ϕ_i and evaluating the first order condition gives

$$(17) \quad \phi_i = \frac{2p}{2a + p}.$$

In the first period, anticipating competition on advertising in the second period, firms collude on price. Given full market coverage, there is a unique focal price. Let p^{pc} be the collusive price, where pc is a mnemonic for price collusion. Then:

Lemma 2 $p^{pc} = v - t$.

Proof. I prove by contradiction. Let p^{pc} be the profit maximizing collusive price and suppose $p^{pc} \neq v - t$. Then, either $p^{pc} < v - t$ or $p^{pc} > v - t$. First, suppose $p^{pc} < v - t$. Then (by continuity of price), $\exists \varepsilon > 0 : p^{pc} + \varepsilon < v - t$ and $\pi(p^{pc} + \varepsilon, \phi) > \pi(p^{pc}, \phi)$. Hence, any collusive price, $p^{pc} : p^{pc} < v - t$ cannot be profit maximizing – a contradiction.¹⁴ Therefore, we must have $p^{pc} > v - t$. However, observe that $p^{pc} > v - t$ violates the full market coverage assumption – a contradiction. Hence, it must be the case that $p^{pc} = v - t$. ■

¹⁴Raising the price to $p = p^{pc} + \varepsilon < v - t$, does not violate any consumer's individual rationality constraint – hence demand is unchanged.

Given Lemma 2, evaluating (16) and (17) gives:

$$(18) \quad \phi^{pc} = \begin{cases} 1 & \text{if } a \leq \frac{v-t}{2} \\ \frac{2(v-t)}{2a+v-t} & \text{if } a > \frac{v-t}{2} \end{cases} \quad \text{and} \quad \pi^{pc} = \begin{cases} \frac{v-t-a}{2} & \text{if } a \leq \frac{v-t}{2} \\ \frac{2a(v-t)^2}{(2a+v-t)^2} & \text{if } a > \frac{v-t}{2} \end{cases}.$$

We see from (18) that price collusion gives rise to a full information equilibrium for lower levels of the advertising cost while it gives rise to a partial information equilibrium for higher levels of the advertising cost. Because the price is given (price is unaffected by advertising), each firm wants to inform as many consumers as possible (demand effect). When a is small relative to price, it pays to inform all consumers. However, when a increases beyond $\frac{v-t}{2}$, the advertising outlay becomes large relative to the revenues and the firm responds by reducing the advertising intensity.

5 Collusion on Price or Advertising?

Below I relate the price collusion equilibrium to the advertising collusion equilibrium. The first result in this section comes from comparing the equilibrium prices and advertising intensities under the two collusive regimes.

Proposition 3 *Let $a \in \left(\frac{t}{2}, \frac{(v-2t)(v-t)}{2t}\right)$. Compared to semicollusion on advertising, the equilibrium price and advertising intensity are higher under semicollusion on price. That is, $p^{ac} < p^{pc}$ and $\phi^{ac} < \phi^{pc}$.*

Proof. By full market coverage, $p+t \leq v$. Therefore from (12), we must have that $p^{ac}+t = t/2 + \sqrt{2at+t^2/4} + t \leq v$. For given t and v , I can solve for a to get; $a \leq \frac{(v-2t)(v-t)}{2t} = \bar{a}$. Furthermore, by assumption, this model is valid for $a > t/2$. Hence, $a \in \left(\frac{t}{2}, \frac{(v-2t)(v-t)}{2t}\right]$. It follows then that for $a \in \left(\frac{t}{2}, \frac{(v-2t)(v-t)}{2t}\right)$, $p^{ac} = t/2 + \sqrt{2at+t^2/4} < v-t = p^{pc}$ as required. The proof of the second claim (that $\phi^{ac} < \phi^{pc}$) is given in Appendix A. ■

First, note that $p^{pc} = v-t$ is the highest possible price consistent with full market coverage. Secondly, as I argued in Proposition 1, collusion on advertising is a "proxy" for collusion on price. Being an indirect way of colluding on price, it is sensible that $p^{ac} < p^{pc}$.

That $\phi^{ac} < \phi^{pc}$ is intuitive. First, advertising is important in this model in that it raises demand. Hence, other things being equal, firms always want to increase advertising. Second, when firms collude on price, the negative relationship between price and advertising is broken. Clearly therefore, when firms collude on price, they have greater incentives to advertise than when they collude on advertising. It follows therefore that price collusion induces more advertising.

Since both the price and the advertising level are higher under price collusion, it follows immediately from Proposition 3 that:

Corollary 1 *Revenues and advertising outlays are higher when firms collude on price.*

Proof. Let R denote revenues and D denote the demand. At equilibrium, $\frac{\partial D}{\partial \phi} = 1 - \phi > 0$. That is, demand is increasing in the advertising intensity. Since $p^{pc} > p^{ac}$ and $\phi^{pc} > \phi^{ac}$,

it follows that $R^{pc} \equiv p^{pc} D^{pc} > p^{pc} D^{ac} > p^{ac} D^{ac} \equiv R^{ac}$ – where the first inequality follows from the fact that $\phi^{pc} > \phi^{ac}$ and $\frac{\partial D}{\partial \phi} > 0$ and the second inequality follows from the fact that $p^{pc} > p^{ac}$. That the advertising outlay is higher under price collusion follows from convexity of the advertising cost function (and the fact that $\phi^{pc} > \phi^{ac}$). ■

A closer look at Propositions 1 and 3 brings to the fore an important difference between price and nonprice collusion. Price collusion exacerbates competition on the variable that is chosen noncooperatively (see also Fershtman and Gandal, 1994 and Steen and Sørsgard, 1999). In contrast, nonprice collusion (collusion on advertising or capacity) does not intensify price competition. If anything, it relaxes price competition. In other words, the "semicollusion effect" (the competition intensifying effect of semicollusion) only kicks in under price collusion. To help explain this observation, I invoke Fudenberg and Tirole (1984)'s "taxonomy of business strategies".

As I have shown, when firms collude on advertising, they advertise less. By voluntarily restricting its advertising, each firm signals that it will not be aggressive in the price competition game. This is so because, with low advertising, fewer consumers are informed and with fewer informed consumers, demand is low. Hence profits can only be enhanced by charging a higher (and not a lower) price. Because prices are strategic compliments, collusion on advertising softens the rival firm's pricing behaviour¹⁵. In this sense, collusion on advertising is a "puppy dog" strategy.

In contrast, collusion on price induces more aggressive behaviour in the advertising competition game. Because prices are fixed, the larger the demand that a firm can generate, the higher the revenues it expects to get. However, since the price is fixed, demand can only be increased by informing more consumers – since uninformed consumers do not purchase. In this sense, price collusion makes each firm tough in the advertising game¹⁶. In the animal jargon of Fudenberg and Tirole (1984), price collusion is a "top dog" strategy.

The use of the animal terminology here needs to be qualified. Fudenberg and Tirole (1984) use the animal jargon in a setting in which firms move sequentially, with the first mover committing to a particular action, an action which is observed by the follower firm prior to making its own move. In our setting however, firms move simultaneously (rather than sequentially) at each stage, but still the commitment issue comes into play since firms' second period choices will only be made after both firms observe the first period choices.¹⁷

From the present analysis, together with the analyses of Fershtman and Gandal (1994) and Steen and Sørsgard (1999), it appears that the semicollusion effect can be explained by whether the collusion and competition instruments are strategic complements or substitutes. When firms collude on a strategic substitute (advertising or capacity/quantity) and compete on a strategic compliment (price), competition is relaxed. However, when firms collude

¹⁵From the first order conditions (see equation (3)), $p_i = (2t - t\phi_j + \phi_j p_j) / 2\phi_j$ and $\partial p_i / \partial p_j > 0$.

¹⁶Though not apparent, the firms' advertising intensities are strategic substitutes. From (4), $\phi_i = p_i (2t + (p_j - p_i - t) \phi_j) / 2at$. In any equilibrium in which firms compete for the fully informed consumers, it must be the case that for $i, j = 1, 2; j \neq i$, $p_j < p_i + t$. For if $p_j > p_i + t$, then all fully informed consumers will buy from firm i – violating the assumption that firms compete for the fully informed consumers. Hence, $\partial \phi_i / \partial \phi_j = p_i (p_j - p_i - t) / 2at < 0$.

¹⁷The commitment argument is made "as if" firms set advertising in the first stage. The premise for this argument is that the firms' expectations are fulfilled along the equilibrium path and therefore, the expectation to collude in the second stage already conditions behaviour in the first stage.

on a strategic complement (price) and compete on a strategic substitute (advertising or capacity/quantity), competition is exacerbated. I therefore conjecture that a necessary condition for the semicollusion effect to kick in is that the competition variable is a strategic substitute.

The observation that price collusion intensifies competition on the nonprice variable but not the other way round has important implications for firm conduct. If firms are "sophisticated" and have multiple choice variables, they ought to realize that price collusion is more likely to hurt them compared to nonprice collusion. Moreover, price collusion is per se illegal and is heavily punished for when discovered. This suggests then that firms ought to shift focus from price to nonprice collusion. They seem to. There is an increasing number of nonprice collusion cases that the US Federal Trade Commission has had to deal with in recent years. Examples include; the California Dental Association case in which the association instituted rules and regulations that restrict price and quality advertising (FTC Docket No. 9259); the Arizona Automobile Dealers Association case in which the association agreed with some of its members to "restrain truthful and nondeceptive advertising" (FTC File No. 931 0056); collusion on advertising by PolyGram (predecessor to Vivendi Universal) and Warner in order to reduce intrabrand competition – competition between the Three Tenors' third album and video and the first and second albums and video (FTC File No. 001 0231; Goldberg, 2005).

To recapitulate, the main question I address in this section is the following: If they had a choice, which strategic variable (price or advertising) would firms use as the collusion instrument? To answer this question, I compare π^{ac} and π^{pc} . As a prelude, it is instructive to analyze the relationship between profits and the advertising cost, a , under semicollusion on price and respectively, advertising. Differentiating equations (14) and (18) with respect to a , I find that:

Lemma 3 *Semicollusive profits are decreasing (increasing) in the advertising cost under price (advertising) collusion.*

Proof. See Appendix A. ■

Under both collusion on price and collusion on advertising, the effect of an increase in the advertising cost on profit can be decomposed into a direct effect and an indirect effect. Notice that the advertising cost, a , enters directly into the advertising cost function but only enters into the revenue function indirectly – via price and /or advertising level (see equations (9) and (16)). The direct effect of an increase in a is to raise the advertising outlay, other things being equal. However, other things will not remain equal. An increase in a induces firms to reduce advertising and this increases informational product differentiation – a strategic effect.

Under collusion on advertising, this strategic effect allows firms to raise prices and consequently revenues. The effect on revenues outweighs the direct effect on the advertising outlay and hence profits increase with the advertising cost.

Under semicollusion on price, there are two cases to consider. First, when $a \leq (v - t)/2$, we have full information (that is, $\phi^{pc} = 1$). Moreover, since $p^{pc} = v - t$, it follows that the

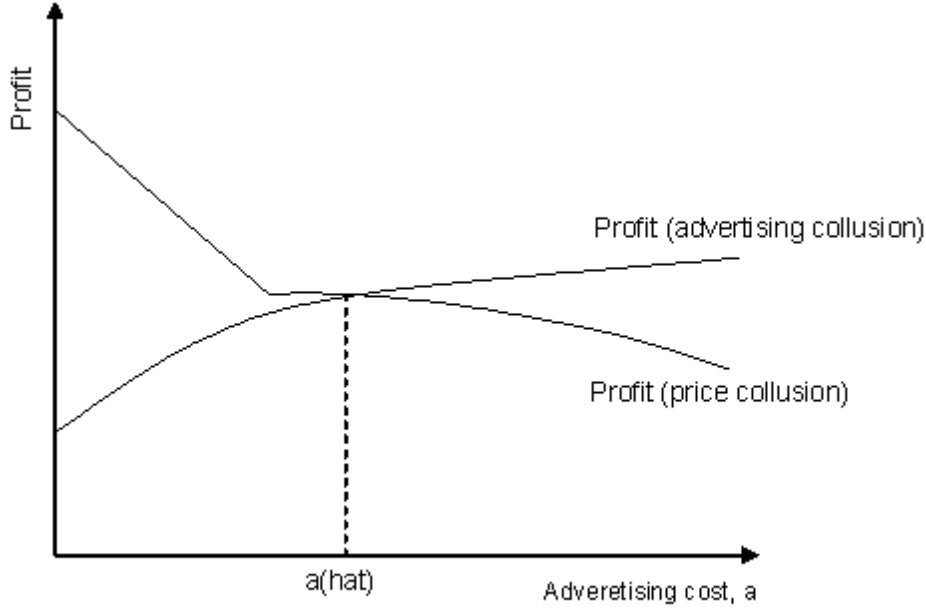


Figure1. Profits from price and advertising collusion

revenue function is independent of a . Therefore, when a increases, the only component of the profit function that changes is the advertising outlay (which increases with a). Hence, for $a \leq (v - t)/2$, profit necessarily decreases with a . For $a > (v - t)/2$, $\phi^{pc} = \frac{2(v-t)}{2a+v-t} < 1$ and, when the advertising cost increases, firms respond by advertising less. Although informational product differentiation increases, prices cannot be increased and hence revenues must of necessity decrease. Since the direct effect of an increase in a is to raise the advertising outlay, profits fall when the advertising cost, a , increases.

Following Lemma 3, one may conjecture that there exists an a , (call it $\hat{a}$) for which the two profit functions intersect. If indeed such an a exists, then, for $a < \hat{a}$, semicollusion on price should yield higher profits while for $a > \hat{a}$, semicollusion on advertising should yield higher profits.

Let α denote the ratio of transportation costs to the gross surplus, that is, $\alpha \equiv t/v$. Below I plot π^{ac} and π^{pc} as functions of a , for $\alpha = 0.25$. From Figure 1, we see that for "low" values of a , $\pi^{ac}(a) < \pi^{pc}(a)$ while the opposite is true for "high" values of a . In fact, it can be shown that, for a wide range of the parameter α , π^{ac} and π^{pc} intersect. Below I state the main result of this section;

Proposition 4 *Semicollusion on price does not always lead to higher profits compared to semicollusion on advertising. More precisely, let $a \in \left(\frac{\alpha}{2}, \frac{(1-2\alpha)(1-\alpha)}{2\alpha}\right)v$ and let $\hat{a} \equiv a(\alpha)$ such that $\pi^{pc}(a(\alpha)) = \pi^{ac}(a(\alpha))$. Then, $\forall \alpha \in (\underline{\alpha}, \frac{1}{3})$, $\underline{\alpha} > 0$; $\pi^{pc}(a) > \pi^{ac}(a)$ for $a < \hat{a}$ and $\pi^{pc}(a) < \pi^{ac}(a)$ for $a > \hat{a}$.*

Proof. See Appendix A. ■

Although price collusion dominates collusion on advertising in terms of revenues (Corollary 1), it is, in general, not superior to the latter. As was shown in Proposition 3, firms

advertise rather "excessively" when they collude on price (which increases demand and hence revenues). However, because the advertising cost function is convex, the firms incur higher advertising costs under price collusion (bad for profits). When the advertising cost is *low*, the revenue effect dominates in the profit function and this makes price collusion more profitable. However, for *higher* advertising costs, the revenue effect is weakened by the ballooning advertising outlays. Because firms advertise less when they collude on advertising, they incur lower advertising outlays. As a result, collusion on advertising yields higher profits compared to price collusion when the advertising cost is higher. In summary, collusion on price is not always more profitable compared to collusion on advertising. Depending on parameter values, sometimes price collusion dominates and sometimes it is dominated.

A principal assumption of this paper is that *equilibrium prices are such that the market is fully covered*. One might wonder what the effect of assuming full coverage is on profits, particularly under semicollusion on price. As I have presented it, Proposition 4 is predicated on the assumption that the market is covered. However, it is quite reasonable to conjecture that when firms collude on price, the optimal price may be such that some consumers find it profitable not to purchase. If this is the case, then, by assuming full coverage, I restrict the collusive profits under semicollusion on price, π^{pc} . Thus, it is imperative that I undertake a robustness check to see to what extent Proposition 4 depends on the full coverage assumption. I solve this exercise in Appendix B.

I show that although indeed there exist some profitable collusive prices for which the market will not be covered, qualitatively, Proposition 4 is unaltered. The unrestricted collusive price (and hence the associated unrestricted profit) exceeds the restricted collusive price (and hence the associated restricted profit) only for a "narrow" range of the advertising cost, a . For the most part, the restricted profits are higher! More importantly, in terms of comparisons with profits under semicollusion on advertising, π^{ac} , allowing for some prices that lead to less than full coverage is of no consequence. For reasonable parameter values, π^{pc} and π^{ac} intersect, with π^{ac} intersecting π^{pc} from below – which establishes the result.

The main findings thus far are that (i) collusion on advertising and competition on price is more profitable than competition on both price and advertising and (ii) collusion on price does not always lead to higher profits compared to collusion on advertising. Empirical evidence seem to support both our findings. As stated in the introduction, studies of price and advertising strategies find support for collusion on advertising but not price, which is supportive of my findings¹⁸.

6 Is Semicollusion Disadvantageous?

Fershtman and Gandal (1994) argue that semicollusion typically induces intense competition on the choice variable(s) chosen noncooperatively. If the competitive pressure is sufficiently intense, semicollusion results in lower profits compared to the fully noncooperative outcome. Does this thesis hold in our framework?

¹⁸The empirical evidence on this issue, however, is mixed. For example, Kadiyali (1996) find evidence supportive of collusion on both prices and advertising while Slade (1995) find evidence of competition on advertising and collusion on prices.

To answer this question, I compare the noncooperative profits to the collusive profits. Specifically, I compare π^{nc} and π^{ac} on the one hand and π^{nc} and π^{pc} on the other. I find that;

Proposition 5 *Semicollusion (price/advertising) yields higher equilibrium profits than when firms compete in both price and advertising.*

Proof. See Appendix A. ■

Proposition 5 supports the *conventional wisdom* that, overall, firms are better off colluding rather than competing. A question that arises is: Why is semicollusion disadvantageous in the models of Fershtman and Gandal (1994), but not in the present model? In Fershtman and Gandal (1994), when firms collude on price, they overinvest in capacity hoping to use the excess capacity as a bargaining chip in the division of the collusive profits. However, in equilibrium, excess capacity is *totally* redundant.^{19,20} Since capacity is costly to install, price collusion may hurt firms compared to fully noncooperative interaction.

Unlike capacity, advertising has a positive direct effect for the advertising firm – it raises demand. In fact, the reason for "excessive" advertising (under price collusion) is to increase demand. Thus, even though advertising (just like capacity) is costly, it is not totally redundant. This demand expansion effect mitigates the negative effect of higher advertising intensities. Hence, notwithstanding the fact that firms advertise excessively under price collusion, they still earn higher profits compared to competition on both price and advertising.

7 Conclusion

I analyze firms' incentives to collude on advertising when advertising is purely informative. I find that semicollusion on advertising is more profitable than competition on both price and advertising. From a welfare perspective, collusion on advertising is bad. When firms collude on advertising, "too few" consumers are informed and, as a result, welfare is lower than when firms compete on both prices and advertising. This result is important for policy. Although advertising is only informative, there is need for monitoring – more so with the advent of advertising agencies. Left unchecked, firms will be tempted to connive against consumers.

I also compare price collusion to collusion on advertising. In general, price collusion does not dominate collusion on advertising. In this sense, there is no justification for the theoretical literature's exclusive focus on price collusion. Hence I lend theoretical support to the empirical literature that largely find evidence of collusion on advertising rather than on price.

In this paper, I use a static model to study firms' incentives to collude on advertising. But, will the firms actually collude on advertising? To answer this question, we need a

¹⁹In a symmetric equilibrium, both firms overinvest to the same extent – hence the excess capacity will not in any way enhance a particular firm's bargaining position.

²⁰The same mechanism also operates in Steen and Sörgard (1999) to induce overinvestment in capacity. However, the export market provides a leeway that reduces the redundancy of excess capacity.

dynamic/ repeated setting which permits firms to respond to the actions of competitors. Collusion on advertising is sustainable only if the incentives to deviate are outweighed by the benefits from conforming. This, however, is left for future research.

Appendix

Appendix A: Proofs of Lemmas and Propositions

Proof of Proposition 1

Proof. Proposition 1 states that $\phi^{nc} > \phi^{ac}$; $p^{nc} < p^{ac}$ and $\pi^{nc} < \pi^{ac}$. From (7) and (12), $p^{nc} = \sqrt{2at} < \frac{t}{2} + \sqrt{2at + t^2/4} = p^{ac}$. By Lemma 1, $\partial\pi/\partial\phi < 0$. Therefore, if $\phi^{ac} < \phi^{nc}$, it follows that $\pi^{ac} > \pi^{nc}$. Hence, I only need to show that $\phi^{nc} > \phi^{ac}$, for all $a \in \left(t/2, \frac{(v-2t)(v-t)}{2t}\right)$; where $\phi^{nc} = \frac{2t}{t+\sqrt{2at}} = \frac{1}{1+\frac{\sqrt{2at}}{2t}-\frac{1}{2}}$ and $\phi^{ac} = \frac{\frac{t}{2}+\sqrt{2at+t^2/4}}{a+\frac{t}{2}+\sqrt{2at+t^2/4}} = \frac{1}{\frac{2a}{t+2\sqrt{2at+t^2/4}}+1}$. To show that $\phi^{ac} < \phi^{nc}$, it suffices to show that $\frac{\sqrt{2at}}{2t} - \frac{1}{2} < \frac{2a}{t+2\sqrt{2at+t^2/4}}$. Let $\Psi(a) \equiv \frac{\sqrt{2at}}{2t} - \frac{1}{2} - \frac{2a}{t+2\sqrt{2at+t^2/4}} = \frac{(\sqrt{2at}-t)(t+2\sqrt{2at+t^2/4})-4at}{2t(t+2\sqrt{2at+t^2/4})}$. $\phi^{ac} < \phi^{nc} \iff \Psi(a) < 0 \iff (\sqrt{2at}-t)(t+2\sqrt{2at+t^2/4})-4at < 0$. Expanding, I get that; $(\sqrt{2at}-t)(t+2\sqrt{2at+t^2/4})-4at = t\sqrt{2at}-t^2-2t\sqrt{2at+t^2/4}+2\sqrt{2at}\sqrt{2at+t^2/4}-4at$. Since $\sqrt{2at}\sqrt{2at+t^2/4} < 2at+t^2/4$, it follows that; $\Psi(a) = \frac{t\sqrt{2at}-t^2-2t\sqrt{2at+t^2/4}+2\sqrt{2at}\sqrt{2at+t^2/4}-4at}{2t(t+2\sqrt{2at+t^2/4})} < \frac{-\frac{1}{2}t-(2\sqrt{2at+t^2/4}-\sqrt{2at})}{2(t+2\sqrt{2at+t^2/4})} < 0 \forall a$. I conclude that $\phi^{nc} > \phi^{ac}$, for all $a \in \left(t/2, \frac{(v-2t)(v-t)}{2t}\right)$. ■

Proof of Proposition 2

Proof. Let W^{ac} (CS^{ac}) be the welfare (consumer surplus) when firms collude on advertising but compete on prices and W^{nc} (CS^{nc}) be the welfare (consumer surplus) when firms compete on both prices and advertising. Because the market is covered, $CS = v - p$. Defining welfare as profits plus consumer surplus ($W = \pi + CS$), I get that

$$\begin{aligned} W^{ac} &= \pi^{ac} + v - p^{ac} \\ &= \frac{\left(\frac{t}{2} + \sqrt{2at + \frac{t^2}{4}}\right)^2 + \left(v - \left(\frac{t}{2} + \sqrt{2at + \frac{t^2}{4}}\right)\right) 2 \left(a + \frac{t}{2} + \sqrt{2at + \frac{t^2}{4}}\right)}{2 \left(a + \frac{t}{2} + \sqrt{2at + \frac{t^2}{4}}\right)}, \end{aligned}$$

$$\begin{aligned} W^{nc} &= \pi^{nc} + v - p^{nc} \\ &= \frac{2at^2 + (v - \sqrt{2at})(t + \sqrt{2at})^2}{(t + \sqrt{2at})^2}. \end{aligned}$$

Subtracting the latter from the former gives

$$W^{ac} - W^{nc} = \frac{-2at^3 - 2a^2t^2 - \frac{1}{2}t^4 - 2at^2\sqrt{2at} - t^3\sqrt{2at + \frac{1}{4}t^2} - 4a^2t(\sqrt{2at + \frac{1}{4}t^2} - \sqrt{2at})}{2\left(a + \frac{t}{2} + \sqrt{2at + \frac{t^2}{4}}\right)(t + \sqrt{2at})^2} < 0.$$

I conclude therefore that welfare is lower when firms collude on advertising rather than compete. ■

Proof of Proposition 3

Proof. There are two cases to prove. First, I show that $\phi^{ac} < \phi^{pc}$ whenever $a \leq \frac{v-t}{2}$. Second, I show that $\phi^{ac} < \phi^{pc}$ for $a > \frac{v-t}{2}$. The first case is trivial. From Lemma 3, $\phi^{pc} = 1$ for $a \leq \frac{v-t}{2}$ and from (13), $\phi^{ac} = \left(\frac{t}{2} + \sqrt{2at + t^2/4}\right) / \left(a + \frac{t}{2} + \sqrt{2at + t^2/4}\right) < 1 \forall a$ – which proves the first claim. To prove the second claim, write the advertising intensities as; $\phi^{ac} = \frac{1}{\frac{2a}{t+2\sqrt{2at+t^2/4}} + 1}$ and (from (18)) $\phi^{pc} = \frac{2(v-t)}{2a+v-t} = \frac{1}{1 + \frac{2a}{2(v-t)} - \frac{1}{2}}$. First note that as a converges to $\left(\frac{v-t}{2}\right)^+$, $\phi^{pc} = 1 > \phi^{ac}$. Notice also that $\phi^{pc}(\bar{a}) > \phi^{ac}(\bar{a})$ if and only if $\frac{2\bar{a}}{2(v-t)} - \frac{1}{2} < \frac{2\bar{a}}{t+2\sqrt{2\bar{a}t+t^2/4}}$. Substituting $\bar{a} = \frac{(v-2t)(v-t)}{2t}$, I get; $\frac{2\bar{a}}{2(v-t)} - \frac{1}{2} = \frac{v-2t}{2t} - \frac{1}{2} < \frac{v-2t}{2t} = \frac{2\bar{a}}{t+2\sqrt{2\bar{a}t+t^2/4}}$. It follows therefore that $\phi^{pc}(\bar{a}) > \phi^{ac}(\bar{a})$. Second, because both ϕ^{pc} and ϕ^{ac} are everywhere continuous, if ϕ^{pc} and ϕ^{ac} never cross (intersect) for $a \in (t/2, \bar{a})$, then it must be the case that ϕ^{pc} lies everywhere above ϕ^{ac} . From the above expressions for ϕ^{pc} and ϕ^{ac} , it is obvious that at the point where they intersect, $\frac{2a}{2(v-t)} - \frac{1}{2} = \frac{2a}{t+2\sqrt{2at+t^2/4}}$. The only solution to the equation $\frac{2a}{2(v-t)} - \frac{1}{2} - \frac{2a}{t+2\sqrt{2at+t^2/4}} = 0$ is $a^* = \frac{v(v-t)}{2t}$. However, $a^* > \bar{a}$. Hence, in the interval $(t/2, \bar{a})$, ϕ^{pc} and ϕ^{ac} do not intersect. It follows therefore that, in the interval $(t/2, \bar{a})$, $\phi^{pc} > \phi^{ac}$. ■

Proof of Lemma 3

Proof. Lemma 3 claims that $\frac{\partial \pi^{ac}}{\partial a} > 0$ and $\frac{\partial \pi^{pc}}{\partial a} < 0 \forall a$. Under collusion on advertising, profits are given by; $\pi^{ac} = \frac{1}{2} \frac{\left(\frac{t}{2} + \sqrt{2at + t^2/4}\right)^2}{a + \frac{t}{2} + \sqrt{2at + t^2/4}}$ and $\frac{\partial \pi^{ac}}{\partial a} = \frac{t(t+2\sqrt{2at+t^2/4})}{\sqrt{2at+t^2/4}\left(a + \frac{1}{2}t + \sqrt{2at+t^2/4}\right)} - \frac{t + \sqrt{2at+t^2/4}}{\sqrt{2at+t^2/4}} \frac{\left(\frac{1}{2}t + \sqrt{2at+t^2/4}\right)^2}{\left(a + \frac{1}{2}t + \sqrt{2at+t^2/4}\right)^2} = \frac{\frac{1}{4}t^3 + at^2 + \frac{1}{2}t^2\sqrt{2at+t^2/4}}{\sqrt{2at+t^2/4}\left(a + \frac{1}{2}t + \sqrt{2at+t^2/4}\right)^2} > 0$ as required. As for price collusion, $\pi^{pc} = \frac{v-t-a}{2}$ for $a \leq \frac{v-t}{2}$ and $\frac{\partial \pi^{pc}}{\partial a} = -\frac{1}{2} < 0$ as required. Secondly, for $a > \frac{v-t}{2}$, $\pi^{pc} = \frac{2a(v-t)^2}{(2a+v-t)^2}$ and $\frac{\partial \pi^{pc}}{\partial a} = \frac{2(v-t)^2}{(2a+v-t)^2} + \frac{2a(v-t)^2(-2)}{(2a+v-t)^3} = \frac{2(v-t)^2}{(2a+v-t)^3} (v-t-2a) < 0$ since

$a > \frac{v-t}{2}$. Hence the lemma. ■

Proof of Proposition 4

Proof. Whenever $\hat{a}$ exists, the result follows directly from Lemma 3. Hence I only need to show that $\hat{a}$ indeed exists $\forall \alpha \in (\underline{\alpha}, \frac{1}{3})$, for some $\underline{\alpha} > 0$.²¹ Notice that for any given α , a is constrained to the interval $(\underline{a}(\alpha), \bar{a}(\alpha))$, where $\underline{a}(\alpha) = \frac{\alpha}{2}v$ and $\bar{a}(\alpha) = \frac{(1-2\alpha)(1-\alpha)}{2\alpha}v$. Since (by Lemma 3) $\pi^{pc}(a)$ is continuous and decreasing in a and $\pi^{ac}(a)$ is continuous and increasing in a , to show that $\hat{a}$ exists, it suffices to show that $\lim_{a \rightarrow \underline{a}(\alpha)^+} \pi^{pc}(a) \geq \lim_{a \rightarrow \underline{a}(\alpha)^+} \pi^{ac}(a)$ and $\lim_{a \rightarrow \bar{a}(\alpha)} \pi^{pc}(a) \leq \lim_{a \rightarrow \bar{a}(\alpha)} \pi^{ac}(a) \forall \alpha \in (\underline{\alpha}, \frac{1}{3})$. Substituting αv for t , the profit functions (14) and (18) reduce to $\pi^{ac} = \frac{(\alpha v/2 + \sqrt{2a\alpha v + (\alpha v)^2/4})^2}{2(a + \alpha v/2 + \sqrt{2a\alpha v + (\alpha v)^2/4})}$ and $\pi^{pc} = \frac{(1-\alpha)v-a}{2}$ for $a \leq \frac{(1-\alpha)v}{2}$ and $\pi^{pc} = \frac{2a((1-\alpha)v)^2}{(2a+(1-\alpha)v)^2}$ for $a > \frac{(1-\alpha)v}{2}$. To begin with, notice that $\lim_{a \rightarrow (\frac{\alpha v}{2})^+} \left(\frac{(1-\alpha)v-a}{2} \right) = \left(\frac{1}{2} - \frac{3}{4}\alpha \right)v > \alpha(\alpha-1)^2v = \lim_{a \rightarrow (\frac{\alpha v}{2})^+} \left(\frac{2a((1-\alpha)v)^2}{(2a+(1-\alpha)v)^2} \right) \forall \alpha \in (0, \frac{1}{2})$. Hence, the relevant part of π^{pc} as $a \rightarrow \underline{a}(\alpha)$ is $\pi^{pc} = \frac{(1-\alpha)v-a}{2}$. Notice also that $\lim_{a \rightarrow \frac{(1-2\alpha)(1-\alpha)v}{2\alpha}} \left(\frac{2a((1-\alpha)v)^2}{(2a+(1-\alpha)v)^2} \right) = \frac{2\alpha^2-\alpha}{\alpha-1}v > \frac{(1-\alpha)(4\alpha-1)v}{4\alpha} = \lim_{a \rightarrow \frac{(1-2\alpha)(1-\alpha)v}{2\alpha}} \left(\frac{(1-\alpha)v-a}{2} \right) \forall \alpha \in (0, \frac{1}{3})$. Hence, the relevant part of π^{pc} as $a \rightarrow \bar{a}(\alpha)$ is $\pi^{pc} = \frac{2a((1-\alpha)v)^2}{(2a+(1-\alpha)v)^2}$. First, $\lim_{a \rightarrow (\frac{\alpha v}{2})^+} \pi^{pc}(a) - \lim_{a \rightarrow (\frac{\alpha v}{2})^+} \pi^{ac}(a) = \frac{2-(\sqrt{20}+1)\alpha}{4}v > 0$ for $\alpha \in (0, 0.365)$. Since for our purposes $\alpha < 1/3$, I conclude that $\lim_{a \rightarrow \underline{a}(\alpha)^+} \pi^{pc}(a) > \lim_{a \rightarrow \underline{a}(\alpha)^+} \pi^{ac}(a) \forall \alpha \in (\underline{\alpha}, \frac{1}{3})$ as required. Second, $\lim_{a \rightarrow \bar{a}(\alpha)} \pi^{ac}(a) - \lim_{a \rightarrow \bar{a}(\alpha)} \pi^{pc}(a) = \left(\frac{\alpha(\alpha + \sqrt{(3\alpha-2)^2})^2}{4(3\alpha^2+1-3\alpha)+\alpha\sqrt{2(3\alpha-2)^2}} - \frac{2\alpha^2-\alpha}{\alpha-1} \right)v > 0 \forall \alpha \in (0, 1)$. I conclude that $\lim_{a \rightarrow \bar{a}(\alpha)} \pi^{pc}(a) < \lim_{a \rightarrow \bar{a}(\alpha)} \pi^{ac}(a) \forall \alpha \in (\underline{\alpha}, \frac{1}{3})$. In summary, for $\alpha \in (\underline{\alpha}, \frac{1}{3})$, $\lim_{a \rightarrow \underline{a}(\alpha)^+} \pi^{pc}(a) > \lim_{a \rightarrow \underline{a}(\alpha)^+} \pi^{ac}(a)$ and $\lim_{a \rightarrow \bar{a}(\alpha)} \pi^{pc}(a) < \lim_{a \rightarrow \bar{a}(\alpha)} \pi^{ac}(a)$. Hence, $\pi^{pc}(a)$ and $\pi^{ac}(a)$ intersect. ■

Proof of Proposition 5

Proof. Proposition 5 claims that $\pi^{ac} > \pi^{nc}$ and $\pi^{pc} > \pi^{nc}$, where $\pi^{pc} = \frac{v-t-a}{2}$ for $a \leq \frac{v-t}{2}$ and $\pi^{pc} = \frac{2a(v-t)^2}{(2a+v-t)^2}$ for $a > \frac{v-t}{2}$ while $\pi^{nc} = \frac{2at^2}{(t+\sqrt{2at})^2} \forall a \in (t/2, \bar{a})$. The first claim is

covered under Proposition 1. Hence, here I only prove the second claim. The idea of the proof

²¹The restriction that $\alpha < 1/3$ is a direct consequence of the full market coverage assumption. Under low differentiation, $t < v/2$, which implies $\alpha < 1/2$. However, internal consistencies (within the model) impose further restrictions on α . Because $\bar{a} = \frac{(v-2t)(v-t)}{2t}$ and a also takes values greater than $\frac{v-t}{2}$ (see equation (18)), consistency requires that $\frac{v-t}{2} < \bar{a}$. Simplifying this inequality gives the upper bound to the ratio t/v . More precisely, $\alpha \equiv \frac{t}{v} < 1/3$.

is to show that π^{pc} lies everywhere above π^{nc} in the interval $(t/2, \bar{a})$. To proceed, observe that

$$\frac{\partial \pi^{nc}}{\partial a} = \frac{2t^3}{(t + \sqrt{2at})^3} > 0 \text{ and that } \pi^{pc}(a) \text{ is continuous and decreasing (Lemma 3). Therefore,}$$

to show that $\pi^{pc} > \pi^{nc}$ for all $a \in (t/2, \bar{a})$, it suffices to show that $\pi^{pc}(\bar{a}) > \pi^{nc}(\bar{a})$. Noting

$$\text{that } \pi^{pc} = \frac{2a(v-t)^2}{(2a+v-t)^2} \text{ for } a > \frac{v-t}{2}, \text{ evaluating at } \bar{a} = \frac{(v-2t)(v-t)}{2t} \text{ gives } \pi^{pc}(\bar{a}) = \frac{t(v-2t)}{v-t}$$

$$\text{and } \pi^{nc}(\bar{a}) = \frac{t(v-2t)(v-t)}{(t + \sqrt{(v-2t)(v-t)})^2}. \text{ Subtracting the latter from the former gives, } \pi^{pc}(\bar{a}) -$$

$$\pi^{nc}(\bar{a}) = \frac{t^2(v-2t)(2t-v+2\sqrt{(v-2t)(v-t)})}{(v-t)(t + \sqrt{(v-2t)(v-t)})^2} > 0 \text{ if and only if } 2t - v + 2\sqrt{(v-2t)(v-t)} > 0.$$

Since $\sqrt{(v-2t)(v-t)} > v - 2t$, it follows that $2t^2 - tv + 2t\sqrt{(v-2t)(v-t)} > 0$. Hence,

$\pi^{pc}(\bar{a}) > \pi^{nc}(\bar{a})$. I conclude therefore that $\pi^{pc} > \pi^{nc}$ for all $a \in (t/2, \bar{a})$. ■

Appendix B: Unconstrained Collusive Price and Profits

As I mentioned before, it is possible (and plausible) that the firms may collude on a "high" price – a price which may induce some consumers not to purchase. The objective of this appendix is to show that assuming full coverage is not very restrictive and in particular, that our results are robust.

Suppose then that firms collude on a price that induces some consumers not to purchase. Then, for such a price, firm i 's demand is given by²²:

$$(B1) \quad D_i(\phi_1, \phi_2; p) = \phi_i \left(1 - \phi_j\right) \frac{v-p}{t} + \frac{\phi_i \phi_j}{2}; i, j = 1, 2; j \neq i,$$

where $(v-p)/t < 1$ denotes the purchase probability by a consumer who receives the advertising message from only one of the firms (partially informed). Because firms charge the same price, the purchase decision of the fully informed consumers is not governed by prices (as long as the price does not exceed the reservation price). That is, independent of the price, firms equally split (between them) the population of fully informed consumers.

As before, firms set prices in the first stage and advertising in the second stage. I start by solving for the optimal advertising level in the second stage. Given the collusive price, p , chosen in the first stage, firm i 's second stage maximization program is given by:

$$(B2) \quad \pi_i^{pc} = \max_{\phi_i} \left\{ p \left(\phi_i (1 - \phi_j) \frac{v-p}{t} + \frac{\phi_i \phi_j}{2} \right) - \frac{a \phi_i^2}{2} \right\}.$$

Differentiating with respect to ϕ_i and simplifying gives,

$$(B3) \quad \phi_i = \frac{2pv - 2p^2}{2at - pt + 2pv - 2p^2}$$

In the first stage, firms choose the collusive price to maximize joint profits. Their problem

²²The demand function in (B1) is valid **only** when the collusive price exceeds $v - t$. If the collusive price is less than or equal to $v - t$, then $(v-p)/t = 1$ and the demand is independent of prices.

is described by,

$$\Pi^{pc} = \pi_1^{pc} + \pi_2^{pc} = \max_p \left\{ 2p \left(\phi(1-\phi) \frac{v-p}{t} + \frac{\phi^2}{2} \right) - a\phi^2 \right\} \text{ s.t. (B3).}$$

Substituting for ϕ from (B3), this simplifies to

$$(B4) \quad \Pi^{pc} = \frac{4p^2 a (v-p)^2}{(2at - pt + 2pv - 2p^2)^2}$$

Differentiating with respect to p and solving the first order condition yields,

$$p \in \left\{ v, 2a + \sqrt{2a(2a-v)}, 2a - \sqrt{2a(2a-v)} \right\}.$$

Since consumers' reservation price is v , prices higher than v cannot be optimal. Hence, $p \leq v$. But, can firms collude on the price $p = v$? The answer is no! At the price $p = v$, each firm has demand zero and profits can be increased by lowering the price²³. Therefore I rule out $p = v$. Hence

$$p \in \left\{ 2a + \sqrt{2a(2a-v)}, 2a - \sqrt{2a(2a-v)} \right\}$$

Clearly, for p to be an equilibrium, we must have $a \geq v/2$. Suppose first that $a = v/2$. Then, $p = 2a = v$. But, $p = v$ is impossible. Hence, $a > v/2$. But, if $a > v/2$, then, $2a + \sqrt{2a(2a-v)} > v$, and hence cannot be an equilibrium collusive price. This leaves

$$(B5) \quad p_{ur}^{pc} = 2a - \sqrt{2a(2a-v)}$$

as the collusive price, where the subscript ur stands for unrestricted. Observe that $p_{ur}^{pc}(a)$ is decreasing in a .²⁴

Substituting (B5) back into (B3) gives

$$(B6) \quad \phi_{ur}^{pc} = \frac{8av - 16a^2 + (8a - 2v) \sqrt{a(4a - 2v)}}{8av - 16a^2 + (8a + t - 2v) \sqrt{a(4a - 2v)}} < 1.$$

Note that $\lim_{a \rightarrow \infty} p_{ur}^{pc} = v/2$. In fact, p_{ur}^{pc} quickly converges to $v/2$. For example, for $a = 8v$, $p_{ur}^{pc} = 0.50807v$. Since $\lim_{a \rightarrow v/2} p_{ur}^{pc}(a) = v$ and since p_{ur}^{pc} is decreasing in a and intersects p_r^{pc} (subscript r stands for restricted (I am abusing notation here) – this is the case studied in the main text), there exists a^* such that $p_{ur}^{pc}(a^*)$ ensures that the market is just fully covered²⁵. That is, the equation $2a - \sqrt{2a(2a-v)} - (v-t) = 0$ has a solution

²³Note that because consumers are distributed according to a continuous density, the probability that an ad sent by firm 1 (firm 2) will reach a consumer located at point 0 (1) is zero. Because it is costly to visit a store, consumers other than the ones located at 0 and 1 will not purchase at price $p = v$ even if they are informed.

²⁴ $\partial p_{ur}^{pc} / \partial a = \left(2\sqrt{2a(2a-v)} - (4a-v) \right) / \sqrt{a(2a-v)} < 0$ if and only if $2\sqrt{2a(2a-v)} < (4a-v)$.

Squaring both sides and simplifying we get; $16a^2 - 8av = \left(2\sqrt{a(4a-2v)} \right)^2 < (4a-v)^2 = 16a^2 - 8av + v^2$, as required. Hence, $\partial p / \partial a < 0$.

²⁵That p_{ur}^{pc} and p_r^{pc} intersect follows from the fact that p_{ur}^{pc} converges to $v/2$, together with the condition for low differentiation ($t < v/2$). The condition $t < v/2$ implies that $p_r^{pc} = v - t > v/2 = \lim_{a \rightarrow \infty} p_{ur}^{pc}$. Hence, p_{ur}^{pc} and p_r^{pc} intersect.

and this solution is given by,

$$(B7) \quad a^* = \frac{1}{2v - 4t} (t^2 - 2tv + v^2)$$

Remark 6 $p_{ur}^{pc} > p_r^{pc}$ for $a \in (v/2, a^*)$ and $p_{ur}^{pc} < p_r^{pc}$ for $a > a^*$. Because $\lim_{t \rightarrow 0} a^* = v/2$ it follows that $\lim_{t \rightarrow 0} (p_{ur}^{pc} - p_r^{pc}) = 0$. That is, when t is small, the restricted collusive price is close to the unrestricted price. This shows that the full coverage assumption does not constrain the collusive price "too much".

I next derive the firms' optimal profits. The objective is to show that the unrestricted profits do not differ much from the "restricted" profits. As we saw above, for $a \leq a^*$, the collusive price is given by $p_{ur}^{pc} = 2a - \sqrt{2a(2a - v)}$ and the collusive profits are given by

$$(B8) \quad \pi_{ur}^{pc} = p_{ur}^{pc} \left(\phi(1 - \phi) \frac{v - p_{ur}^{pc}}{t} + \frac{\phi^2}{2} \right) - \frac{a\phi^2}{2}$$

where ϕ is given by (B6). Observe that for $a \in (v/2, a^*)$, the market is not covered. That is, $\frac{v - p_{ur}^{pc}}{t} < 1$.

I claim that; For $a > a^*$, $\pi^{pc} = p_r^{pc} D(\phi_r^{pc}) - a \frac{(\phi_r^{pc})^2}{2}$, where $D(\phi_r^{pc}) = \phi_r^{pc} - (\phi_r^{pc})^2/2$. This is the case studied in section 4.

Observe that for $a > a^*$, $p_{ur}^{pc} < p_r^{pc}$ and the market is covered. If the firm charges the collusive price p_{ur}^{pc} , profits are given by $\pi_{ur}^{fullcov} = p_{ur}^{pc} D(\phi_{ur}^{pc}) - a \frac{(\phi_{ur}^{pc})^2}{2}$, where $D(\phi_{ur}^{pc}) = \phi_{ur}^{pc} - (\phi_{ur}^{pc})^2/2$. Since the market is covered and $p_{ur}^{pc} < p_r^{pc}$, it follows that $\pi_{ur}^{fullcov}$ is strictly dominated by $\pi^{naive} \equiv p_r^{pc} D(\phi_{ur}^{pc}) - a \frac{(\phi_{ur}^{pc})^2}{2}$. Hence, for $a > a^*$, firms cannot collude on price p_{ur}^{pc} . I next show that π^{naive} can be improved upon. Notice that at price p_r^{pc} , ϕ_{ur}^{pc} is not optimal. At this price, the optimal advertising level is given by ϕ_r^{pc} . It follows therefore that π^{naive} is dominated by $\pi_r^{pc} \equiv p_r^{pc} D(\phi_r^{pc}) - a \frac{(\phi_r^{pc})^2}{2}$. Hence the claim.

To summarize, let (with some abuse of notation), π^{pc} denote the optimal collusive profits when there are no a priori restrictions on the collusive price. Then, $\pi^{pc} = \pi_{ur}^{pc}$ for $a \in (v/2, a^*)$ and $\pi^{pc} = \pi_r^{pc}$ for $a > a^*$.

Remark 7 $a^* - v/2 = \frac{t^2}{2(v-2t)}$. We see that when t is small, the range of a over which π_{ur}^{pc} is relevant is very "narrow" and moreover, this range diminishes as t converges to zero. Hence, when t is small, π_{ur}^{pc} is largely irrelevant and $\pi^{pc} \approx \pi_r^{pc}$. That is, when t is small, assuming full coverage is not very restrictive.

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Author's Appendix

8 When will firms compete for the fully informed consumers?

In the main text, I assumed that firms compete for the fully informed consumers. Here, I provide a necessary condition for firms to compete for the fully informed consumers [the existence of a symmetric equilibrium in pure strategies].

Each firm has two choices – to compete for the fully informed consumers or to sell only to its captive consumers. When firms compete for the fully informed consumers, in equilibrium, each firm gets profit $\pi^c = 2at^2 / (t + \sqrt{2at})^2$, where superscript c stands for competition.

What if firm 1, for instance, deviates from the symmetric equilibrium and opts to serve only its partially informed consumers? In that case, since differentiation is low, firm 1 will advertise the price $p^m = v - t$ so that it extracts all the consumer surplus of the (partially informed) consumer who travels the farthest distance. In equilibrium, firm 1 faces the demand $D^m = \phi(1 - \phi)$ and its profit is given by $\pi^m = (v - t)\phi(1 - \phi) - a\phi^2/2$, where superscript m stands for monopoly. Substituting for the equilibrium advertising level (7), I get:

$$(19) \quad \pi^m = \frac{2t}{(t + \sqrt{2at})^2} \left((v - t)(\sqrt{2at} - t) - at \right)$$

For given v and t , firm 1 will compete for the fully informed consumers, rather than serve only its captive consumers if and only if $\pi^c \geq \pi^m$. However, $\pi^c \geq \pi^m$ if and only if $2at - (v - t)(\sqrt{2at} - t) \geq 0$. Solving for a gives;

$$(20) \quad a_1^* = (v - t) \left(v - 3t - \sqrt{(v - 5t)(v - t)} \right) / 4t$$

$$(21) \quad a_2^* = (v - t) \left(v - 3t + \sqrt{(v - 5t)(v - t)} \right) / 4t$$

For $a < a_1^*$, firms earn higher profits when they both serve all informed consumers. Intuitively, when the advertising cost is "low" ($a \in (t/2, a_1^*)$), the advertising intensity will be high. A higher advertising intensity implies that the majority of consumers are fully informed about the advertised prices, i.e., they have seen advertising from both firms. Because the majority of consumers have seen advertising from both firms, it follows that the market composed only of the partially informed consumers is "too thin". Consequently, firms find it profitable to compete for the fully informed consumers. For $a \in (a_1^*, a_2^*)$, the advertising levels are not high, which implies that not many consumers receive advertising messages from both firms. Because fewer consumers are fully informed, the offsetting benefit from competing for the fully informed consumers (market size) is small.²⁶ On the other hand, selling only to the captive consumers allows the firm to charge the monopoly price. Since the share of fully informed consumers is relatively small, the demand loss from ignoring the fully informed consumer segment is small. Hence, for $a \in (a_1^*, a_2^*)$, at least one firm finds it profitable to "defect" and only serve its partially informed consumers.²⁷

²⁶Remember that competition for the fully informed consumers leads to lower prices.

²⁷In the asymmetric equilibrium, when firm i , for instance, defects and only serve its captive consumers,

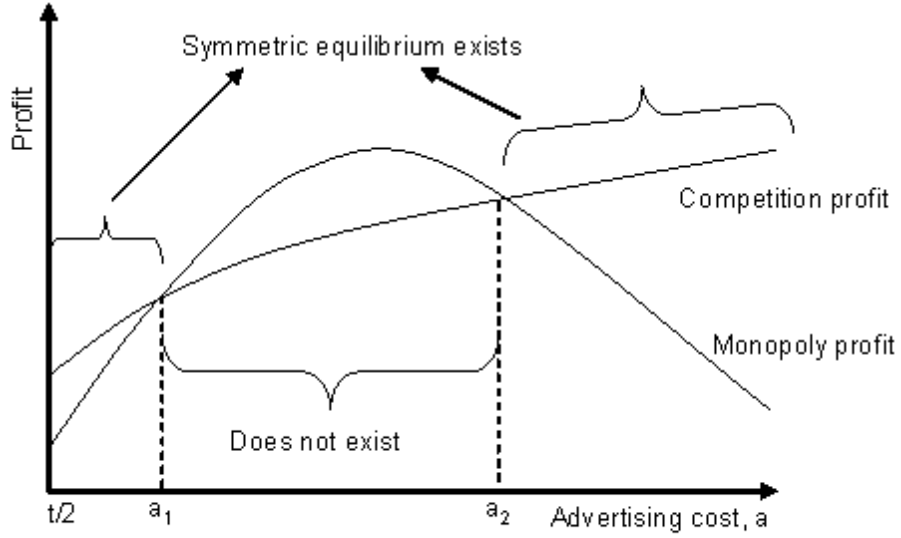


Figure 2. Existence of a symmetric equilibrium

For $a > a_2^*$, the advertising levels are low, but the equilibrium price when firms compete for the fully informed consumers is high ($\frac{\partial p}{\partial a} > 0$). Thus, although fewer consumers receive advertising from both firms, the price applicable to this group is not very different from the monopoly price. As a result, it pays to compete for the fully informed consumers. Hence, for $a > a_2^*$, firms find it profitable to compete for the fully informed consumers.

To summarize (see Figure 2. above), a symmetric equilibrium in pure strategies exists for $a \in (t/2, a_1^*)$ and for $a > a_2^*$. For $a \in (a_1^*, a_2^*)$, a symmetric equilibrium in pure strategies does not exist. Firms have incentives to defect from the symmetric equilibrium when $a \in (a_1^*, a_2^*)$.

it charges price $p = v - t$ for the advertised good while the other firm, firm j , charges price $v - 2t$. This price, $v - 2t$, ensures that the fully informed consumer who travels the farthest distance (unit interval) is just indifferent between buying from firm i at price $v - t$ and buying from firm j .