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EFFICIENCY OF OPTIMAL TAXATION IN A DYNAMIC STOCHASTIC ENVIRONMENT: Case of South Africa

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Abstract

This study investigates the optimality hypothesis of taxation and the volatility thereof in South Africa when using appropriate tax rates within a dynamic stochastic environment. Using a Marshallian macroeconomic model disaggregated by sectors (MMM-DA) several features of the South African economy are analysed that may contribute to the efficiency of the optimal taxation hypothesis. The results show that within a tax regime where revenue from labour and capital income constitutes the most significant source of government income, both such taxes distort the economy but that the distortion from a tax on capital exceeds that of a tax on income. This study has twofold implications. It highlights the impact of efficient optimal taxation on both overall economic growth and fiscal policy in the country.

Keywords: Optimality hypothesis; Dynamic stochastic environment; Marshallian macroeconomic model.

1 INTRODUCTION

In the most recent literature on optimal taxation policies, the authors have noticed an increasing interest in the use of dynamic stochastic optimization models to test for the effect of different forms of taxes on economic performance. This is an interesting phenomenon given the traditional view that optimally chosen tax rates are much more persistent than regular tax rates (Barro 1979). Different models are being used such as general equilibrium models (Lucas and Stokey, 1983) with a focus on state-contingent government-issued securities. Also, several tests of the optimality hypothesis have been used although no synchronized results have as yet been provided. Many studies support the random walk hypothesis (Barro et al. 1984, Skinner 1989) which shows strong persistence in

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tax rates. Judd (1992) introduced non-contemporaneous effects mainly through capital taxation with other interesting features previously omitted in optimal taxation.

Most authors have defined optimal taxation under a dynamic environment as a way to spread tax distortions considering a social welfare function. In this paper, we analyse optimal taxation under a dynamic stochastic environment and assess its functioning and its time-consistency during major economic shocks using data on the South African economy. We have designed a tax structure with labour and capital income in order to minimise any form of distortions and marginal deadweight losses using a quadratic loss function. All of that operate under a dynamic stochastic environment.

The optimal tax theory has fast evolved in substance over the past few years opening room for more quantitative approaches. The aim has always been to maximise a social welfare function condition on several constraints. However, difficulties spread across various features of the problem. First, identification and mostly estimation of a social welfare function has had its demurrers especially with major data constraints often faced. Habitually, the social planner is treated as utilitarian, i.e. the welfare function is derived from people's utilities. Many of the recent studies have identified and estimated social welfare functions as non-linear models. In this paper, we are not 'utilitarian'. We rather approach the question the other way round: we minimise the loss function. The loss function is quadratic and accounts for all the distortions that labour and capital income tax rates bring into the economy. Also, in this study, we clearly dissociate labour and capital income and their related effects from taxation. Despite evidence provided from deterministic models; see Atkinson et al (1980) and Judd (1987), with a few exceptions only (Judd, 1983), most studies on optimal taxation did not provide a clear distinction between income from labour and income from capital. In the present study, effects generated from the two types of income are clearly distinguished.

Our main motivation to conduct a study of this kind lies on a few epic points. First, there is a rising interest in characterizing optimal tax policies in a dynamic stochastic environment. From a policy point of view, optimally chosen tax rates display strong persistence. Also, the deterministic approach is very restrictive when it comes to lumping capital and labour income effects which raise the need to incorporate various distortions of taxation under a stochastic environment.

While primarily focusing on growth enhancing effects of optimal taxation in South Africa and how sustainable it is during major reforms and economic outbreaks, we also look at its effects on labour and capital markets, allocation of resources, and consumption. Thus, an attempt is made to present a comprehensive analysis of market interactions, ie. inputs and outputs markets, also including other effects such as wealth, labour and capital income as derived from the MMM (see Ngoie and Zellner (2011)). We make use of a Marshallian Macroeconomic model disaggregated (MMM-DA) by sectors. Each sector is composed by two markets, (1) the products market and (2) the factors (labour and capital) market. We distinguish between different features of taxation on labour and capital. Capital as well as labour formation is included in our analysis. Both capital and labour income constitutes a significant share of the country's national income and a significant source of national revenue.

In our modelling approach, we include the market value of current, past and expected tax liabilities and their impact on optimal taxation. The higher is the level of uncertainty within the economic environment, the more complex is the impact of optimal tax policy on labour and capital income. The MMM captures flows between labour and capital markets but also flows from and to the product markets. We allow for state-contingent securities. Besides, we define and model optimal taxation using a quadratic loss function. Effects on labour and capital income are clearly dissociated. The MMM includes a labour market where labour income is determined by contemporaneous rate of taxation. Also, we allow capital income to be sensitive to tax rates. Moreover, the way factors and products markets are interconnected in the Marshallian literature, we allow economic actors to shift distortions from taxation from labour to capital and vice-versa and from factors to products markets and vice-versa.

Our results suggest that the optimal tax theory defined using a quadratic loss function (Judd, 1989) breaks during major economic downturns such as the 2008 recession. Likewise, this study finds that expected state-contingent capital market distortions are higher than the labour market distortions.

The rest of the paper is structured as follows. Section II describes the methodology used in this study. This section includes a detailed elaboration of the optimization approach as well as the Marshallian model used to derive labor and capital income effects from optimal taxation. Subsequently, section III presents the empirical results and their explanation. And finally, concluding remarks are presented in the last section.

2 METHODOLOGY

First, we define a probability space under which stochastic processes will take place and determine the environment where government expenditures happen.

2.1 Optimization of tax policy

The government's dynamic optimization problem can be described using the following objective function and its related constraint (Judd, 1989).

$$Min_{\bar{K}\ \bar{L}} E\left\{\sum_{t=0}^{\infty} \lambda^t \Gamma(\bar{K}_t, \bar{L}_t)\right\} \tag{1}$$

$$s.t \sum_{t=0}^{\infty} (1+r)^{-t} (\bar{K}_t + \bar{L}_t - \bar{g}_t) = 0, a.s. \tag{2}$$

Using a dynamic programming approach, the permanent financial state of the economy is described by the following equation

$$z = (1\Delta g)'$$

with Δ representing the stock of debt obeying to the law of motion as

$$\Delta_{t+1} = (1+r)\Delta_t + g_t - K_t - L_t \tag{3}$$

As in the related literature, we assume two states of the economy with shocks Ω in each period which determine values of all the parameters. In period $t-1$, the government commits to state-contingent tax rates to be effective the next period.

$\theta_{t-1} = (K_{1,t}, K_{2,t}, L_{1,t}, L_{2,t})'$ are tax rates when shocks $\omega \epsilon \{1,2\}$ occur.

The problem can be solved conventionally using the following Bellman's equation for the value function $V(\Delta, g)$.

2.2 Bellman equation

$$V(\Delta, g) = \max_{\bar{K},\bar{L}} \lambda E\{\Gamma(\bar{K}, \bar{L}) + v(\bar{\Delta}, \bar{g})\tag{4}$$

$$s.t. \bar{\Delta} = \Delta(1+r) - \bar{K} - \bar{L} + \bar{g} \tag{5}$$

From the Bellman equation, skipping details about the derived Ricatti equation (Judd, 1989), we can obtain the loss function as follows.

The loss function associated to our specification can be written as follows.

$$\Xi_t = E_{t-1}\left\{\frac{1}{2}\eta L_t^2\right\} + E_{t-1}\left\{\frac{1}{2}\mu K_t^2\right\} + \frac{1}{2}\phi\{E_{t-1}\{\pi K_t\}\}^2 \tag{6}$$

In this equation, we have:

- π: the state-contingent marginal rate of substitution between consumption at time t and $t-1$;

- ηL^2: the total value of the distortions generated by labour income taxation;

- μK^2: the total value of the distortions generated by capital income taxation;

- $\phi\{E\{\pi K\}\}^2$: the total cost to the planner generated by the reduction of today's investment below its efficient level.

Considering some assumptions at the extreme such as negligeable *expost* state-contingent costs of capital income taxation, this yields a loss function as follows:

$$\Gamma_t = E_{t-1}\left\{\frac{1}{2}\eta_t \bar{L}_t^2\right\} + \frac{1}{2}\phi_t\left\{E_{t-1}\{\bar{K}_t\}\right\}^2 \tag{7}$$

with the social planner's discount rate $\lambda = (1+r)^{-1}$.

Establishing the first-order condition for trading between increasing labour tax revenue in state i and period t and increasing capital $L_{i,t}$ and increasing it in all subsequent states at $t+1$ is as follows.

$$\eta_t \bar{L}_t = E_t\{\eta_t \bar{L}_{t+1}\} \tag{8}$$

Most empirical tests of the optimality hypothesis are based on (8).

In period $t-1$, the first order condition between labour and capital taxation implies that

$$\eta_t L_t = \phi_t E_{t-1}\{\bar{K}_t\} \quad (9)$$

We can then combine all the trading conditions and obtain

$$\frac{\bar{K}}{\bar{L}} = \frac{\eta}{\phi} \quad (10)$$

In our estimations, we use (10) as an optimal condition to set optimal values in our loss function.

As the literature does not provide much of a formal theory of government expenditure, in this study we assume that government expenditures are determined using an AR (1) process (Judd, 1989).

$$G_{t+1} = \rho G_t + \mu_{t+1} \qquad |\rho| < 1 \quad (11)$$

Where G_t is the amount of government expenditures in period t and μ_t is an iid innovation process in period t.

Considering the amount on uncertainty surrounding government expenditures, we analyse the process under a stochastic environment as follows:

$$\tilde{G}_{t+1} = \delta \tilde{G}_t + \tilde{\mu}_{t+1} \quad (12)$$

Where the $\sim$ represents variables under the stochastic environment, i.e. we include randomness in the equation using the probability integral transform as it permits transformation of a given random variable into its uniform counterpart (Robert et al, 2010). Tax revenue originating from labour and capital income constitutes the most significant source of government revenue in our assumptions and it is suspected *ex-ante* that such taxes have a distorting effect on the economy. Other sources of government revenue are purposely ignored in this analysis; something that we propose to investigate in future studies. Also, in our model, we assume that government issues risk-free debt. Labour and capital income presents specific behaviour while distortionary effects are interlinked.

2.3 The Marshallian model (Ref. Ngoie and Zellner 2011)

As we mentioned earlier, in order to present a comprehensive analysis of market interactions (i.e. inputs and outputs markets) on for example wealth, labour and capital income are derived from the MMM; see Ngoie and Zellner (2011). We make use of a Marshallian Macroeconomic model disaggregated (MMM-DA) by sectors. Each sector is composed by two markets, (1) the products market and (2) the factors (labour and capital) market. We distinguish different features of taxation on labour and capital. Capital as well as labour formation is included in our analysis. Both capital and labour income constitutes a significant share of the country's national income and a significant source of national revenue.

The Marshallian model includes a product market for innovative outcomes, while a market for each production factor is endogenized in the modelwith an entry/exit equation. The entry/exit equation will be used to capture movement of firms in the market. Along with Alfred Marshall and many others, the MMM have introduced a product market involving demand and supply equations derived from assumed optimizing behavior of production units and consumers. On aggregating over production units, we obtain the industry supply equations that depend on the number of units in operation. The production units demand production factors which have their respective markets with supply and demand functions. For more on theoretical foundations and empirical evidence of the MMM; see Zellner (2000), Zellner and Chen (2001), Zellner and Palm (2004),Zellner and Israilevich (2005), Zellner and Ngoie (2011), and Banerjee et al. (2011).

2.4 Production function

Let's assume the ith firm in the industry with a Cobb-Douglas production function. We multiply individual firms' physical output functions by the total number of firms (N) and by the product market price index and we obtain the industry's sales supply equation at time t written as follows[1].

$$\begin{aligned} S_{st} &= N_t A_t P_t^{\frac{1}{\delta}} W_t^{\frac{-\alpha}{\delta}} R_t^{\frac{-\beta}{\delta}} \\ \text{with} &: A_t = A^{*\frac{1}{\delta}} \\ \text{and} &: \delta = 1-\alpha-\beta-\varphi, 0<\delta<1 \end{aligned} \tag{13}$$

where:
- N: Number of firms;
- A: Technological factor productivity;
- P: Output price;
- W: Wage rate;
- R: Interest rate.

Logging both sides of (13) we obtain the following sales equation in growth terms.

$$\frac{\dot{S}_s}{S_s} = \frac{\dot{N}}{N} + \frac{\dot{A}}{A} + \frac{1}{\delta}\cdot\frac{\dot{P}}{P} - \frac{\alpha}{\delta}\cdot\frac{\dot{W}}{W} - \frac{\beta}{\delta}\cdot\frac{\dot{R}}{R} \tag{14}$$

Similarly, we have a product demand function

$$S_{Dt} = P_t Q_t = D_t P_t^{1-\theta} Y_t^{\theta_s} H_t^{\theta_H} \prod_{j=1}^{n} X_{jt}^{\theta_j} \tag{15}$$

Logging both sides and differentiating with respect to time we obtain the sales demand equation expressed in growth terms.

$$\frac{\dot{S}_{Dt}}{S_{Dt}} = (1-\theta)\frac{\dot{P}_t}{P_t} + \theta_s\frac{\dot{S}_{St}}{S_{St}} + \theta_H\frac{\dot{H}_t}{H_t} + \sum_{j=1}^{n}\theta_j\frac{\dot{X}_{jt}}{X_{jt}} \tag{16}$$

[1] We assume firms with identical size within the industry.

- P: personal disposable income;
- H: number of households;
- X: demand shifters (with j, the number of demand shifters);
- Y: disposable income (income after tax);
- D: constant.

2.5 Entry/Exit Equation

As mentioned earlier, entry/exit which constitutes the third equation in our model is a basis for long run equilibrium modeling. In the long run firms enter and exit the market based on their profit mark-ups. As explained in most price theory textbooks, firms movement leads to long run shifts in the aggregate supply curve.

$$\frac{\dot{N}_t}{N_t} = C_E(\pi_t^a - \bar{\pi}_t) \tag{17}$$

The market equilibrium profit within a given sector at time t is represented by $\overline{\pi}_t$. Assuming that a firm's actual profit π^a constitutes a proportion ℓ of its sales supply S_S and $\pi_t^a = \ell S_{St}$. Further transformation leads to the following.

$$\frac{\dot{N}_t}{N_t} = C_E(S_{St} - \pi_t^e) \tag{18}$$

To this regard, we assume that $\pi^e = \frac{\bar{\pi}}{\ell}$ and $C_E = C_E'\ell$. The corporate profit tax appears at this level of our model. Indeed $\pi^e = (1 - ctax)\Pi$ with Π being the firm's profit before tax. Further in our analysis, we suggest a set of policy shocks that includes a tax cut on corporate profit tax and personal income tax. Personal income tax affects disposable income $y = (1 - pitax)Y$, where Y is the personal income before tax.

2.6 Factor markets

2.6.1 Labour

Labour Supply Equation

$$L_t = D_{Lt}\left(\frac{W_t}{P_t}\right)^{\gamma}\left(\frac{Y_t}{P_t}\right)^{\gamma_s} H_t^{\gamma_H}\sum_{j=1}^{d} z_{jt}^{\gamma_j} \tag{19}$$

where $D_L$is the total number of labour providers (mainly households) within the sector and the v variables are labour supply shifters.

$$\frac{\dot{L}_t}{L_t} = \frac{\dot{D}_{Lt}}{D_{Lt}} + \gamma\left(\frac{\dot{W}_t}{P_t} - \frac{\dot{P}_t}{P_t}\right) + \gamma_s\left(\frac{\dot{Y}_t}{Y_t} - \frac{\dot{P}_t}{P_t}\right) + \gamma_H\frac{\dot{H}_t}{H_t} + \sum_{j=1}^{d}\gamma_j z_{jt} \tag{20}$$

Labour Demand Equation The demand for efficient labour, derived from profit maximization on the part of firms is given by

$$\begin{aligned} L_t &= \frac{\alpha N_t P_t Q_t}{W_t} \\ L_t &= \alpha \frac{S_t}{W_t} \end{aligned} \tag{21}$$

Assuming fixed parameters and logging both sides and derive w.r.t. time we obtain the following.

$$\frac{\dot{L}_t}{L_t} = \frac{\dot{S}_t}{S_t} - \frac{\dot{W}_t}{W_t} \tag{22}$$

Capital As in the case of labour, capital equations are obtained from firms' profit maximization.

Capital Supply Equation

$$K_t = E_t \left(\frac{R_t}{P_t}\right)^{\phi} \left(\frac{Y_t}{P_t}\right)^{\phi_s} H_t^{\phi_H} \sum_{j=1}^{n} v_{jt}^{\phi_j} \tag{23}$$

$$\frac{\dot{K}_t}{K_t} = \frac{\dot{E}_t}{E_t} + \phi \left(\frac{\dot{R}_t}{R_t} - \frac{\dot{P}_t}{P_t}\right) + \phi_s \left(\frac{\dot{Y}_t}{Y_t} - \frac{\dot{P}_t}{P_t}\right) + \phi_H \frac{\dot{H}_t}{H_t} + \sum_{j=1}^{n} \phi_j v_{jt} \tag{24}$$

where:
- E represents the total number of capital providers that includes (1) Government, (2) Domestic providers, and (3) Foreign providers;
- v represents the capital supply shifters; and
- R represents the real interest rate.

Capital Demand Equation

$$\begin{aligned} K_t &= \frac{\beta N_t P_t Q_t}{R_t} \\ K &= \beta \frac{S_t}{R_t} \end{aligned} \tag{25}$$

that can be transformed as follows:

$$\frac{\dot{K}_t}{K_t} = \frac{\dot{S}_t}{S_t} - \frac{\dot{R}_t}{R_t} \tag{26}$$

Solving analytically for the reduced form equations in the factor markets:
For labour:

$$\frac{\dot{S}_t}{S_t} - \frac{\dot{W}_t}{W_t} = \frac{\dot{D}_{Lt}}{D_{Lt}} + \gamma \frac{\dot{W}_t}{W_t} - \gamma \frac{\dot{P}_t}{P_t} + \gamma_s \frac{\dot{Y}_t}{Y_t} - \gamma_s \frac{\dot{P}_t}{P_t} + \gamma_H \frac{\dot{H}_t}{H_t} + \sum_{j=1}^{d} \gamma_j z_{jt} \tag{27}$$

$$(1+\gamma)\frac{\dot{W}_t}{W_t} = \frac{\dot{S}_t}{S_t} - \frac{\dot{D}_{Lt}}{D_{Lt}} + \gamma\frac{\dot{P}_t}{P_t} - \gamma_s\frac{\dot{Y}_t}{Y_t} + \gamma_s\frac{\dot{P}_t}{P_t} - \gamma_H\frac{\dot{H}_t}{H_t} - \sum_{j=1}^{d}\gamma_j z_{jt} \tag{28}$$

$$\frac{\dot{W}_t}{W_t} = \left(\frac{1}{1+\gamma}\right)\frac{\dot{S}_t}{S_t} + \left(\frac{\gamma+\gamma_s}{1+\gamma}\right)\frac{\dot{P}_t}{P_t} - \left(\frac{\gamma_s}{1+\gamma}\right)\frac{\dot{Y}_t}{Y_t} - \left(\frac{\gamma_H}{1+\gamma}\right)\frac{\dot{H}_t}{H_t} - \sum_{j=1}^{d}\left(\frac{\gamma_j}{1+\gamma}\right)z_{jt} \tag{29}$$

For capital:

$$\frac{\dot{S}_t}{S_t} - \frac{\dot{R}_t}{R_t} = \frac{\dot{E}_t}{E_t} + \phi\frac{\dot{R}_t}{R_t} - \phi\frac{\dot{P}_t}{P_t} + \phi_S\frac{\dot{Y}_t}{Y_t} - \phi_S\frac{\dot{P}_t}{P_t} + \phi_H\frac{\dot{H}_t}{H_t} + \sum_{j=1}^{n}\phi_j v_{jt} \tag{30}$$

$$(1+\phi)\frac{\dot{R}_t}{R_t} = \frac{\dot{S}_t}{S_t} - \frac{\dot{E}_t}{E_t} + (\phi+\phi_S)\frac{\dot{P}_t}{P_t} - \phi_S\frac{\dot{Y}_t}{Y_t} - \phi_S\frac{\dot{P}_t}{P_t} + \phi_H\frac{\dot{H}_t}{H_t} + \sum_{j=1}^{n}\phi_j v_{jt} \tag{31}$$

$$\frac{\dot{R}_t}{R_t} = \left(\frac{1}{(1+\phi)}\right)\left[\frac{\dot{S}_t}{S_t} - \frac{\dot{E}_t}{E_t} + (\phi+\phi_S)\frac{\dot{P}_t}{P_t} - \phi_S\frac{\dot{Y}_t}{Y_t} - \phi_H\frac{\dot{H}_t}{H_t} + \sum_{j=1}^{n}\phi_j v_{jt}\right] \tag{32}$$

Solving analytically for the reduced form equations in the product market:

$$(1-\theta)\frac{\dot{P}_t}{P_t} = (1-\theta_S)\frac{\dot{S}_t}{S_t} - \theta_H\frac{\dot{H}_t}{H_t} - \sum_{j=1}^{n}\theta_j\frac{\dot{X}_{jt}}{X_{jt}} \tag{33}$$

$$\frac{\dot{P}_t}{P_t} = \left(\frac{1}{(1-\theta)}\right)\left[(1-\theta_S)\frac{\dot{S}_t}{S_t} - \theta_H\frac{\dot{H}_t}{H_t} - \sum_{j=1}^{n}\theta_j\frac{\dot{X}_{jt}}{X_{jt}}\right] \tag{34}$$

The Reduced Form Equation The final reduced form sales supply equation is formulated as follows.

$$\frac{\dot{S}_t}{S_t} = b\frac{\dot{S}_t}{S_t} + C_E(S_t - \pi_t^e) + c \tag{35}$$

$$\text{Or} \qquad \frac{\dot{S}_t}{S_t} = b\frac{\dot{S}_t}{S_t} + C_E(S_t - (1-ctax_t)\Pi_t + c \tag{36}$$

with:

$$\begin{aligned} c \;=\; & \frac{\dot{A}_t}{A_t} + \{\alpha[\gamma_h(1-\theta)+\gamma+\gamma_S]/(1+\gamma) + \beta[\phi_h(1-\theta)+\phi+\phi_S]/(1+\phi) - \theta_h/\delta(1-\theta) \\ & ..... + \alpha\sum_{j=1}^{d}\gamma_j\frac{\dot{z}_j}{z_j}/\delta(1+\gamma) + \beta\sum_{j=1}^{n}\delta_j\frac{\dot{v}_j}{v_j}/\delta(1+\phi) + \sum_{j=1}^{m}[\theta_j/\delta(1-\theta)\left(\frac{\dot{X}_j}{X_j}\right)[\alpha(\gamma+\gamma_s)/(1+\gamma) \\ & ..... + \beta(\phi+\phi_s)/(1+\phi) - 1] \end{aligned} \tag{37}$$

And,

$$b = \left\{ \begin{array}{c} 1-\theta_s-\alpha[(1-\theta)(1-\gamma_s)+(1-\theta_s)(\gamma+\gamma_s)]/(1+\gamma) \\ -\beta[(1-\theta)(1-\phi_s)+(1-\theta_s)(\phi+\phi_s)]/(1+\phi) \end{array} \right\} /\delta(1-\theta) \quad (38)$$

Note: when there is no money illusion $\eta = \eta_s, f = 1$, and the final reduced form sales supply equation can be written as follows.

$$S_t = \pi_t^e - \frac{C}{C_E} \text{ or } S_t = (1 - ctax_t)\Pi_t - \frac{C}{C_E}$$

Also, and most importantly, stock prices (SP) and money supply (M2) will be included in our estimations as leading indicators.

Transfer functions

$$\begin{bmatrix} 1 & -\lambda(L) & -1 \\ 1 & -\gamma(L) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} s_t \\ p_t \\ n_t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \delta_0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \delta_1 \end{bmatrix} S_{t-1} + \begin{bmatrix} \kappa_1 \\ 0 \\ 0 \end{bmatrix} w_t + \begin{bmatrix} \kappa_2 \\ 0 \\ 0 \end{bmatrix} r_t$$

$$+ \begin{bmatrix} \kappa_3 \\ 0 \\ 0 \end{bmatrix} m2_t + \begin{bmatrix} \kappa_4 \\ 0 \\ 0 \end{bmatrix} sp_t + \begin{bmatrix} \kappa_5 \\ 0 \\ 0 \end{bmatrix} ctax_t + \begin{bmatrix} 0 \\ \Delta_1 \\ 0 \end{bmatrix} y_t$$

$$+ \begin{bmatrix} 0 \\ \Delta_2 \\ 0 \end{bmatrix} iz_t + \begin{bmatrix} 0 \\ \Delta_3 \\ 0 \end{bmatrix} h_t + \begin{bmatrix} \varepsilon_t \\ \mu_t \\ v_t \end{bmatrix} \quad (39)$$

In order to obtain the transfer equations, multiply both sides by the adjoint matrix $A^*(A^* = \det A.A^{-1})$, with:

$$[\lambda(L) - \gamma(L)] \cdot \begin{bmatrix} s_t \\ p_t \\ n_t \end{bmatrix} = \begin{bmatrix} -\gamma(L)\delta_0 \\ -\delta_0 \\ \delta_0[\lambda(L) - \gamma(L)] \end{bmatrix} + \begin{bmatrix} -\gamma(L)\delta_1 \\ -\delta_1 \\ \delta_1[\lambda(L) - \gamma(L)] \end{bmatrix} S_{t-1} + \begin{bmatrix} -\gamma(L)\kappa_1 \\ -\kappa_1 \\ 0 \end{bmatrix} w_t$$

$$+ \begin{bmatrix} -\gamma(L)\kappa_2 \\ -\kappa_2 \\ 0 \end{bmatrix} r_t + \begin{bmatrix} -\gamma(L)\kappa_3 \\ -\kappa_3 \\ 0 \end{bmatrix} m2_t + \begin{bmatrix} -\gamma(L)\kappa_4 \\ -\kappa_4 \\ 0 \end{bmatrix} sp_t + \begin{bmatrix} -\gamma(L)\kappa_5 \\ -\kappa_5 \\ 0 \end{bmatrix} ctax + \begin{bmatrix} \lambda(L)\Delta_1 \\ \Delta_1 \\ 0 \end{bmatrix} y_t +$$

$$\begin{bmatrix} \lambda(L)\Delta_2 \\ \Delta_2 \\ 0 \end{bmatrix} iz_t + \begin{bmatrix} \lambda(L)\Delta_3 \\ \Delta_3 \\ 0 \end{bmatrix} h_t + \begin{bmatrix} -\gamma(L)\varepsilon_t + \lambda(L)\mu_t - \gamma(L)v_t \\ -\varepsilon_t + \mu_t - v_t \\ [\lambda(L) - \gamma(L)]v_t \end{bmatrix} \quad (40)$$

This equation can be transformed into a system of linear equations for both price and sales supply.

$$[\lambda(L) - \gamma(L)]s_t = -\gamma(L)\delta_0 - \gamma(L)\delta_1 S_{t-1} - \gamma(L)\kappa_1 w_t - \gamma(L)\kappa_2 r_t - \gamma(L)\kappa_3 m2_t - \gamma(L)\kappa_4 sp_t$$

$$-\gamma(L)\kappa_5 ctax_t + \lambda(L)\Delta_1 y_t + \lambda(L)\Delta_2 iz_t + \lambda(L)\Delta_3 h_t - \gamma(L)\varepsilon_t + \lambda(L)\mu_t - \gamma(L)v_t$$

$$S_t = \frac{1}{\lambda(L)-\gamma(L)}[-\gamma(L)\delta_0 - \gamma(L)\delta_1 S_{t-1} - \gamma(L)\kappa_1 w_t - \gamma(L)\kappa_2 r_t - \gamma(L)\kappa_3 m2_t - \gamma(L)\kappa_4 sp_t$$
$$-\gamma(L)\kappa_5 ctax_t + \lambda(L)\Delta_1 y_t + \lambda(L)\Delta_2 iz_t + \lambda(L)\Delta_3 h_t - \gamma(L)\varepsilon_t + \lambda(L)\mu_t - \gamma(L)v_t] \quad (41)$$

where:
- $\lambda(L)$and$\gamma(L)$:lag operators
- SP (Stock Prices) and M (Money Supply: M2);
- iz: the world import growth;
- $\ln\left(\frac{S_t}{S_{t-1}}\right) = s_t; \ln\left(\frac{N_t}{N_{t-1}}\right) = n_t; \ln\left(\frac{W_t}{W_{t-1}}\right) = w_t; \ln\left(\frac{r_t}{r_{t-1}}\right) = r_t$

3 RESULTS

We introduce this section with results pertaining to the government expenditures series. Figure 1 presents a cyclical and non-cyclical trend of SA government expenditures as well as the derived frequency response function. The cyclical component in the series is strong and heavily influences the entire trend. Also, in Figure 2 we observe that the innovation component in our DGP (Data Generating Process) is *iid*.

Using equations 7, 8 and 9 the figures below describe both labour and capital market distortions as a result of taxation under the optimal framework discussed in the methodology section.

3.1 *Labour and capital markets distortions*

3.1.1 Expectation of the state-contingent distortions

Figures 3 and 4 represent, respectively, the level of distortion that taxation under an optimal framework has on labour and capital income. The general understanding has long been that taxing capital income should move parallel with taxing labour income even though this unified perspective does not always translate into fostering long run economic growth. Simply illustrated, developing countries will rather tend to foster their economic growth by increasing capital stock which leads to a more productive labour force. Conversely, taxation of capital income most likely encumbers capital deepening. However, it is relevant to mention the existence of neutral forms of capital income taxation (Abel, 2007). These will be types of capital income taxes with no effect on firms' capital investment decisions (ibid) but rather providing elusive lump-sum taxes that help corrode tax distortions.

It transpires from these two figures that capital income is more distorted than labour income which seems to suggest that firm owners have faster ways to assuage tax effects on labour rather than effects on capital. From basic macroeconomic theories, we know that labour can be subject to several transformations and is therefore considered variable in the short-run while capital is considered fixed in the short-run. Remember that we deal with a relatively short lag structure – one year - which explains these results.

3.1.2 Social Cost of Taxation - $\phi\{E\{\tilde{\pi}\tilde{K}\}\}^2$

In order to compute the social cost of taxation, we obtained ϕ through the optimization process described as follows. First, we had to find η from ηL^2 .

Thereafter, once we obtained η, we could compute the ψ which represents the dependency of investment decisions on the expected net returns and the responsiveness of savings to anticipated tax liabilities.

From the optimization described above, the following identity is obtained.

$$\psi = \eta \frac{L}{K}$$

Once ϕ has been obtained we could obtain the entire social cost of taxation - $\psi\{E\{\tilde{\pi}\tilde{K}\}\}^2$ and the entire loss function.

$$\Xi_t = E_{t-1}\left\{\frac{1}{2}\tilde{\pi}\tilde{L}\right\} + E_{t-1}\left\{\frac{1}{2}\tilde{\mu}\ \tilde{K}_t^2\right\} + \frac{1}{2}\phi\{E_{t-1}\left\{\tilde{\pi}\tilde{K}_t\right\}\}^2$$

Considering Fig. 7, it seems rather clear that the loss encountered under optimal taxation, as developed by Judd (1989), is out of hand during major outbreaks, especially the 2009 recession. For most of the time the loss has been minimized except for the 2009 recession as well as other periods of major reforms such as 1) 1998 when some major tax collection reforms were introduced at the South African Revenue Services, and 2) 1994 when South Africa acceded to democracy and experienced drastic changes at all levels.

Something else that is worth mentioning regarding Figure 7 is the ability of economic and business entities operating in the country to shift the taxation burden from labour to capital and vice-versa. When the burden of taxation becomes heavier on one side, firms have their way to restructure and shift the burden in order to lessen the loss.

4 CONCLUSION

Overall, findings of this study are backed by theoretical expectations. Firstly, optimal tax rates are highly persistent. Secondly, in an uncertain environment, optimal tax policy affects labour income and capital income differently raising the need to dissociate the two effects unlike what has been claimed by some researchers.

In this paper we have examined optimal taxation under a dynamic stochastic environment and assess its functioning and its time-consistency during major economic shocks with data on the South African economy. We designed a tax structure with labour and capital income in order to minimize any form of distortions and marginal deadweight losses using a quadratic loss function. From our empirical results, it emerges that capital income is more distorted than labour income which seems to suggest that firm owners have faster ways to assuage tax effects on labour rather than effects on capital. Subsequently, this study highlights the ability of economic and business entities operating in the country to shift the tax burden from labour to capital and vice-versa. When the burden of taxation becomes heavier on one side, firms have their way to restructure and shift the burden in order to lessen the loss. However, it appears that the loss encountered under optimal taxation is out of hand during major outbreaks, especially the 2009 recession. For most of the time the loss has been minimized except for the 2009 recession as well as other periods of major reforms such as 1994 and 1998 as explained in the previous section.

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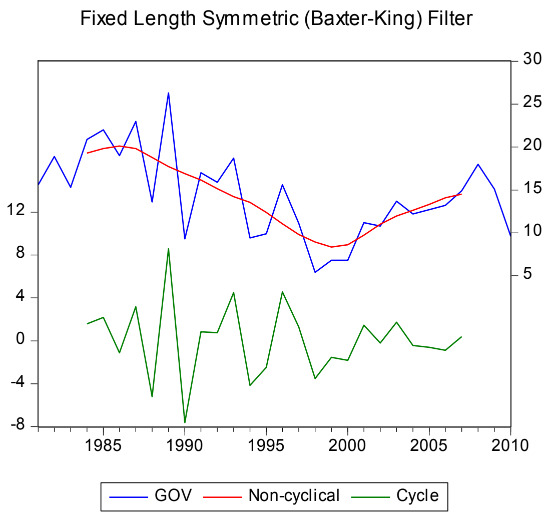
Fig. 1 - Cyclical effects of SA Gov expenditures



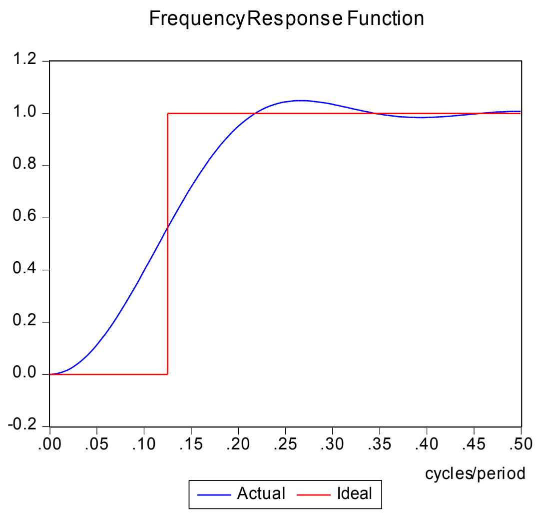


Fig 2 - Plot of ε_t

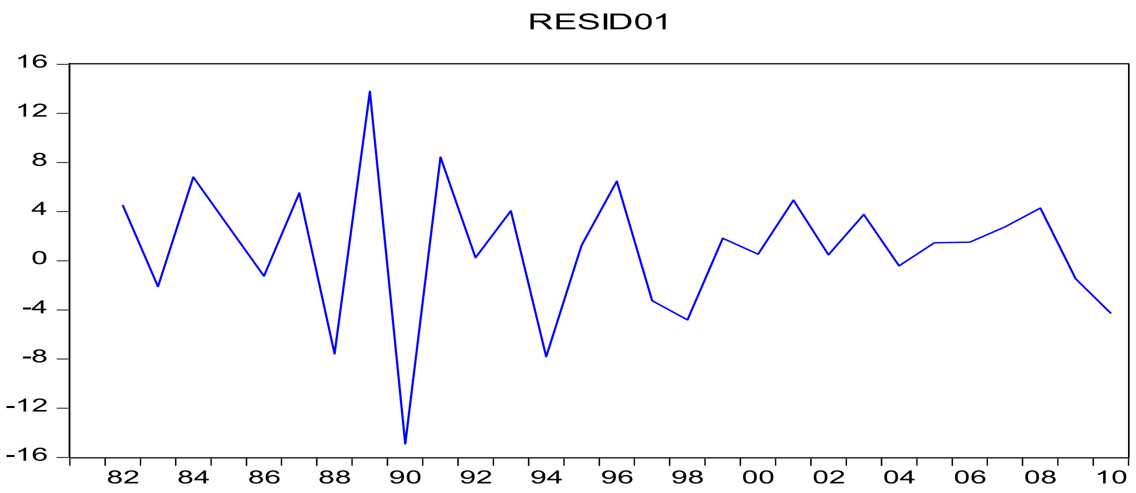


Fig. 3 - Quantile normal of labour income distortions

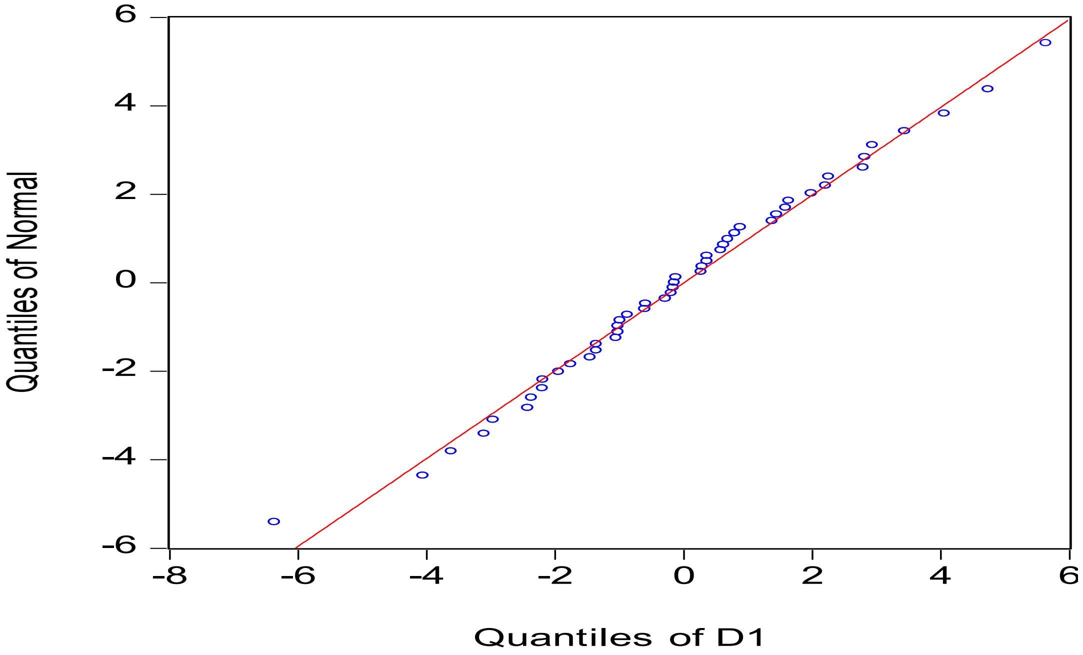


Fig. 4 – Quantile normal of capital income distortions

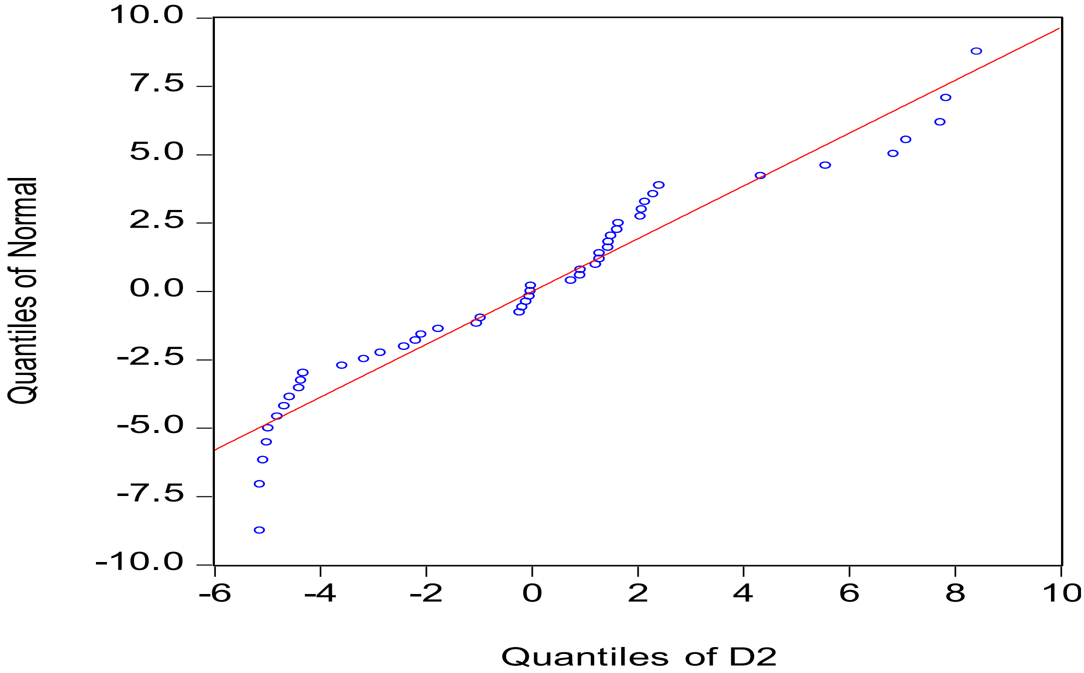


Fig. 5 - Kernel density of ψ

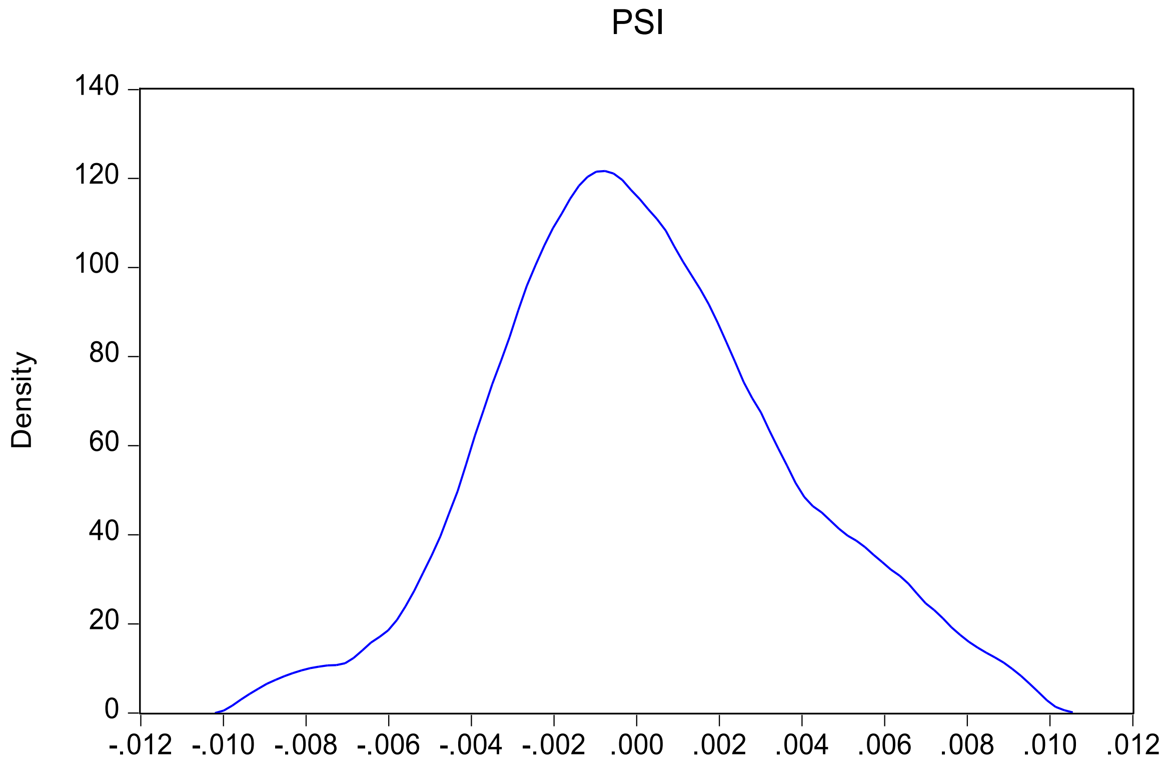


Fig. 6 - Q-Q plot for ψ

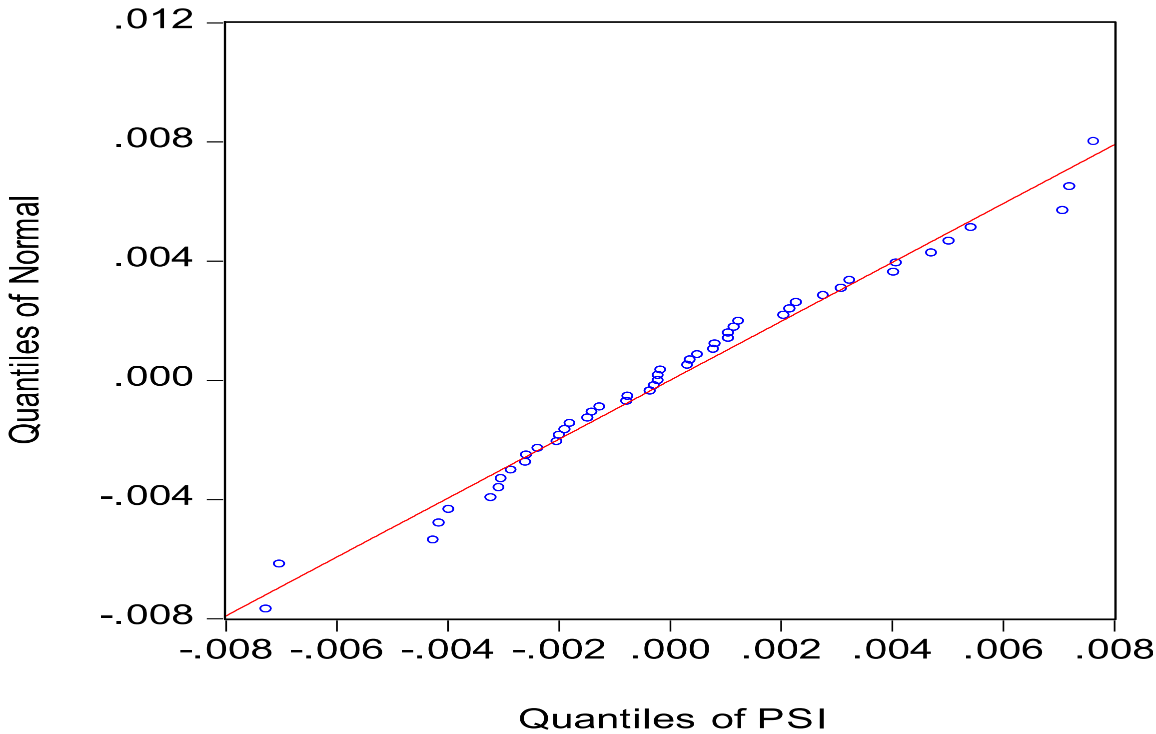


Fig. 7 – Loss function

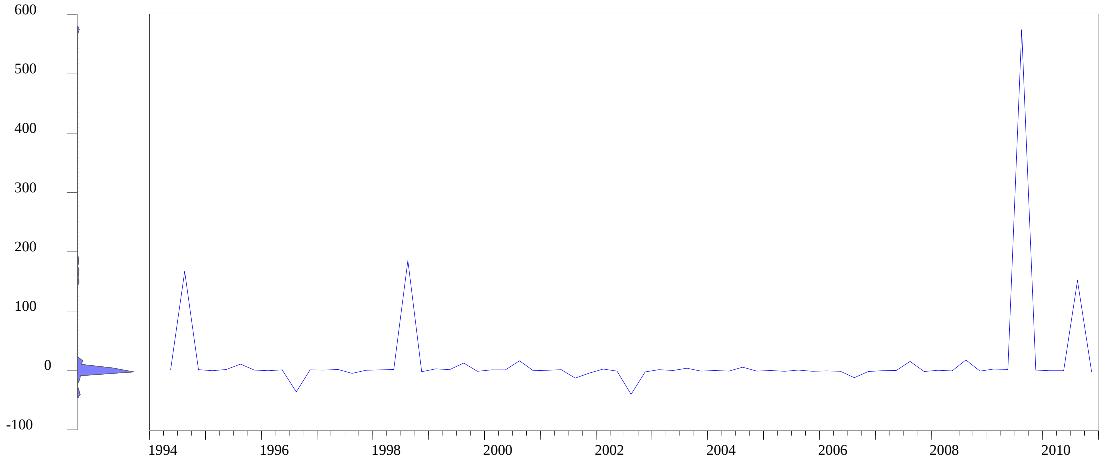


Fig. 8 - Kernel density of the Loss function