



Should Central Banks of Small Open Economies Respond to Exchange Rate Fluctuations? The Case of South Africa¹

Sami Alpanda², Kevin Kotze³ and Geoffrey Woglom⁴

Working Paper Number 174

¹ Correspondence: Sami Alpanda, Department of Economics, Amherst College. We thank Stan du Plessis, Alberto Ortiz, Ben Smit, Pao-Lin Tien, as well as seminar participants at the 5th Annual Workshop in Macroeconomic Research at Liberal Arts Colleges and the South African National Treasury for suggestions and comments. All remaining errors are our own.

² Department of Economics, Amherst College

³ Department of Economics, University of Stellenbosch

⁴ Department of Economics, Ameherst College

Should Central Banks of Small Open Economies Respond to Exchange Rate Fluctuations? The Case of South Africa^{*}

Sami AlpandaKevin KotzéGeoffrey WoglomAmherst CollegeUniversity of StellenboschAmherst College

March 2010

Abstract

We estimate a New Keynesian small open economy DSGE model for South Africa, using Bayesian techniques. The model features imperfect competition, incomplete asset markets, partial exchange rate pass-through, and other commonly used nominal and real rigidities, such as sticky prices, price indexation and habit formation. We study the effects of various shocks on macroeconomic variables, and calculate the optimal Taylor rule coefficients using a loss function for the central bank. We find that the optimal Taylor rule places a heavier weight on inflation and output than the estimated Taylor rule, but almost no weight on the depreciation of currency.

Keywords: optimal monetary policy, small open economy, Bayesian estimation

JEL Classification: F41, E52

^{*}Correspondance: Sami Alpanda, Department of Economics, Amherst College, P.O. Box 5000, Amherst, MA 01002; e-mail: salpanda@amherst.edu; tel: 1-(413) 542-2271; fax: 1-(413) 542-2090. We thank Stan du Plessis, Alberto Ortiz, Ben Smit, Pao-Lin Tien, as well as seminar participants at the 5th Annual Workshop in Macroeconomic Research at Liberal Arts Colleges and the South African National Treasury for suggestions and comments. All remaining errors are our own.

1 Introduction

Dynamic Stochastic General Equilibrium (DSGE) models provide economists with an organized framework that can be used to analyze economic phenomena. When these models include a rich structure (with nominal and real rigidities, and a relatively large number of shocks), they have been able to forecast macroeconomic variables fairly well; and in some cases they outperform competing models, such as Vector Autoregressions (Smets and Wouters, 2003). Over the last few years, these models have been extended to include open economy features, and they are now regarded as commonplace tools that are used by central banks around the world for monetary policy analysis and forecasting purposes.¹

In this paper, we build a small open economy DSGE model, and analyze its implications for optimal monetary policy in South Africa. In particular, we investigate whether the central bank should condition on exchange rate movements when it sets its interest rate policy. Nominal currency depreciation feeds into domestic inflation directly by increasing the foreign component of CPI, and indirectly through its effect on the marginal costs of domestic producers. This warrants a contractionary response through an increase in the interest rate, but that in turn would increase the variability of output, especially when currency depreciation rates are volatile. The optimal response of the central bank to exchange rate fluctuations depends on the quantitative importance of these effects. Current currency depreciation could also provide additional information to the central bank regarding current CPI inflation, when the central bank can observe inflation rates only with a lag. This informational aspect supports the use of an interest rate rule that also responds to currency depreciation.

The structure of the model we use in this paper is similar to that of Justiniano and Preston (forthcoming), Ortiz and Sturzenegger (2007) and Gali and Monacelli (2005).² We consider a small open economy with imperfect competition, incomplete asset markets, partial exchange rate pass-through, and other commonly used nominal and real rigidities, such as sticky prices, price indexation and habit formation. The model includes a relatively large set of disturbances, nine in total.³ Domestic shocks to demand, productivity, the interest rate and mark-up (i.e. domestic cost-

 $^{^{1}}$ See for example Gali and Monacelli (2005), and Justinano and Preston (forthcoming) for recent examples of DSGE models with a small open economy. Tovar (2008) provides a recent summary of the DSGE models utilized by different central banks.

²Justiniano and Preston (forthcoming) construct a similar model for Australia, Canada and New Zealand; Ortiz and Sturzenegger (2007) construct a similar (but smaller) model for South Africa to consider changes to monetary policy; and Gali and Monacelli (2005) construct a calibrated model for small open economies.

 $^{{}^{3}}$ The use of a large set of shocks, as well as features such as price indexation to generate inflation persistence,

push shocks) are commonly used in closed economy New Keynesian DSGE models (c.f. Rabanal and Rubio, 2005). We also add a country risk-premium shock to break the interest-parity condition, and a foreign cost-push shock to break the law-of-one-price for imported goods (as per Justiniano and Preston, forthcoming). The other three shocks also relate to the foreign sector; namely foreign output, inflation and interest rates. We estimate the model using Bayesian methods and recent South African macroeconomic data. We then discuss the implications of the model using impulse responses generated by each of the shocks. The simulated moments generated by the model match the moments of their data counterparts fairly well. The variance decomposition exercise indicates that most of the volatility of output is due to demand shocks, and most of the volatility in CPIinflation is due to foreign cost-push shocks.

The model is then used to investigate the degree to which monetary policy should be conducted in response to changes in past interest rates, inflation, output gap, and the exchange rate, when monetary policy follows a generalized Taylor rule. Due to the presence of cost-push shocks, the Central Bank faces a tradeoff in reducing the volatility of inflation and output. Optimal estimates for the policy rule coefficients are obtained by minimizing a loss function that includes the variance of inflation, output and the interest rate. We find that the optimal Taylor rule places more emphasis on inflation and output than the estimated Taylor rule coefficients, and almost no weight on the nominal exchange rate. These results are similar to the findings of Justiniano and Preston (forthcoming) who study optimal monetary policy in the context of Australia, Canada and New Zealand, but in contradiction to the findings of Smets and Wouters (2002) who study this issue for the Euro area.

In the following section, we describe the model economy. Sections 3 and 4 discuss the estimation of the parameters and the implications of the model. Optimal monetary policy is presented in Section 5 and Section 6 concludes.

2 The Model Economy

We formulate a New Keynesian, small open economy DSGE model that is similar to those of Monacelli (2003), Gali and Monacelli (2005), and Justiniano and Preston (forthcoming). Nominal rigidities are introduced in the form of quadratic price adjustment costs for monopolistically competitive intermediate goods producers (Rotemberg, 1982). We abstract from secular real growth

has been advocated by, among others, Christiano et. al (2005) and Smets and Wouters (2003, 2007). Canova and Sala (2006) and Chari et al. (2008) suggest that the inclusion of these elements may result in identification problems during estimation, and that some of the shocks and features commonly used in this literature, such as mark-up shocks and price indexation, may not be structural or consistent with microeconomic evidence.

and assume zero inflation at the steady-state of the model.⁴ We also abstract from money, since monetary policy is conducted through an interest rate rule, and money demand plays no specific role in the analysis (Woodford, 2003).⁵

2.1 Households

The economy is populated by a unit measure of identical and infinitely-lived households. The households' preferences over consumption, c, and labor, n, are described by the following utility function:

$$E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[\Theta_\tau \left\{ \frac{\left(c_\tau - \zeta C_{\tau-1}\right)^{1-\sigma}}{1-\sigma} - \frac{n_\tau^{1+\gamma}}{1+\gamma} \right\} \right]$$
(1)

where t indexes time, β is the time-discount factor, $1/\sigma$ is the intertemporal-elasticity of substitution, and $1/\gamma$ is the Frisch-elasticity of labor supply. ζ is a consumption habit parameter. The habit level of consumption depends on past aggregate consumption, C, and is treated as an externality by the households (Abel, 1990).⁶ Θ is an exogenous demand shock, whose natural logarithm follows an AR(1) process⁷

$$\log \Theta_t = \rho_\Theta \log \Theta_{t-1} + \varepsilon_{\Theta,t}.$$
 (2)

The consumption index, c, is a composite of a home good, c_h , and a foreign good, c_f . It is described by

$$c_{t} = \left[(1-\alpha)^{\frac{1}{\eta}} c_{h,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} c_{f,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$
(3)

where α is a level parameter determining the importance of foreign goods in overall consumption, and $\eta > 0$ is the elasticity of substitution between home and foreign goods.

The home good, c_h , is purchased at a price of p from domestic final goods producers, and the foreign good, c_f , is directly imported from foreigners at a price of p_f , which is denominated in local currency. Note that the consumption price index (CPI), p_c , is related to the domestic and foreign goods prices according to

$$p_{c,t} = \left[(1-\alpha) p_t^{1-\eta} + \alpha p_{f,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}$$
(4)

 $^{^{4}}$ Both of these features could be easily incorporated without affecting the results, provided that the real growth rates and the rate of inflation are moderate.

 $^{^{5}}$ This is equivalent to including money in the utility function with a separable specification.

⁶In equilibrium, $C_t = c_t$.

 $^{^7 {\}rm The}~\rho$'s in the shock processes are the persistence parameters, and the innovations designated by ε are assumed to be Gaussian.

and CPI-inflation factor is defined as $\pi_t^c = p_{c,t}/p_{c,t-1}$.⁸

Denoting the price of the composite consumption good, c, as p_c , the consumption aggregate in real terms can be written as

$$c_t = \frac{p_t}{p_{c,t}} c_{h,t} + \frac{p_f}{p_{c,t}} c_{f,t}.$$
 (5)

The aggregate consumption index implies that in equilibrium the share of home goods and foreign goods in overall consumption are given respectively by

$$\frac{c_{h,t}}{c_t} = (1-\alpha) \left(\frac{p_t}{p_{c,t}}\right)^{-\eta} \tag{6}$$

$$\frac{c_{f,t}}{c_t} = \alpha \left(\frac{p_{f,t}}{p_{c,t}}\right)^{-\eta}.$$
(7)

The households' period t budget constraint is

$$\frac{p_{c,t}}{p_t}c_t + \frac{b_t}{p_t} + \frac{e_t b_t^*}{p_t} \le \frac{w_t}{p_t}n_t + i_{t-1}\frac{b_{t-1}}{p_t} + i_t^*\phi_{t-1}\frac{e_t b_{t-1}^*}{p_t} + \frac{\Pi_t}{p_t}$$
(8)

where w is the nominal wage rate, e is the nominal exchange rate, and Π is profits received from the domestic intermediate goods producers. As explained later, these intermediate firms are monopolistically competitive and generate pure profits in equilibrium.

Households hold domestic bonds, b, which pay a gross nominal interest of i. They also hold foreign bonds, b^* , which pay a gross nominal interest of $i^*\phi$ where, i^* is the foreign gross nominal interest rate and ϕ is a risk-premium factor. The foreign nominal interest rate, i^* , is exogenouslydetermined and follows an AR(2) process:⁹

$$\log i_t^* = (1 - \rho_{1,i^*} - \rho_{2,i^*}) \log \overline{i^*} + \rho_{1,i^*} \log i_{t-1}^* + \rho_{2,i^*} \log i_{t-2}^* + \varepsilon_{i^*,t}$$
(9)

where $\overline{i^*}$ is the mean of i_t^* . The risk-premium factor, ϕ , is given by

$$\phi_{t-1} = \exp\left(\Phi_{t-1} - \chi a_{t-1}\right) \tag{10}$$

where Φ is an exogenous risk-premium shock, and $\chi > 0$ regulates the sensitivity of the risk-premium

⁸The theoretical CPI, p_c , is a *perfect price index* which takes into account the elasticity of substitution between home and foreign goods. We will ignore this in our estimation which uses the empirical counterpart of CPI.

 $^{^{9}}$ Justiniano and Preston (forthcoming) also consider using a VAR specification for the three foreign variables, namely, foreign interest rates, inflation and output. They report that the results are very similar to the case where they use separate AR(2) processes for these foreign variables. We do not follow the VAR route in this paper since we already have a large number of parameters to estimate.

to changes in the ratio of foreign bond holdings to trend-GDP, a, which is defined as

$$a_{t-1} = \frac{e_{t-1}b_{t-1}^*}{p_{t-1}\overline{y}}$$
(11)

where \overline{y} is the steady-state value of real GDP.¹⁰ The exogenous risk premium shock is assumed to follow an AR(1) process given by

$$\Phi_t = (1 - \rho_{\Phi}) \overline{\Phi} + \rho_{\Phi} \Phi_{t-1} + \varepsilon_{\Phi,t}$$
(12)

where $\overline{\Phi}$ is the mean of Φ_t .

The households' problem is to maximize utility subject to its budget constraint and appropriate No-Ponzi conditions. In equilibrium, the marginal utility of increasing current consumption is equated to the marginal disutility from reducing current income:

$$\Theta_t \left(c_t - \zeta c_{t-1} \right)^{-\sigma} = \lambda_t \tag{13}$$

where λ is the Lagrange multiplier on the household's budget constraint. Similarly, the households determine their labor supply by equating the marginal disutility from work to the marginal utility gain from increasing wage income:

$$\Theta_t n_t^{\gamma} = \lambda_t \left(w_t / p_t \right). \tag{14}$$

The first-order-condition with respect to b yields the asset pricing equation for domestic bonds

$$1 = E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{i_t}{\pi_{t+1}} \right) \right] \tag{15}$$

where $\beta \lambda_{t+1}/\lambda_t$ is the stochastic discount factor, and π is the inflation factor (derived from the GDP-deflator), defined by $\pi_t = p_t/p_{t-1}$.

Similarly, the first-order-condition with respect to b^* yields the asset pricing equation for foreign bonds

$$1 = E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{d_{t+1} i_t^* \phi_t}{\pi_{t+1}} \right) \right]$$
(16)

where d is the depreciation factor of domestic currency, defined by $d_t = e_t/e_{t-1}$.

¹⁰To obtain some intuition for the risk-premium specification, consider the case where foreign bond-holdings, and therefore a, is negative. Then, an increase in foreign debt or a positive risk-premium shock increases the risk-premium ϕ . We need debt-elastic interest rates to ensure that the stochastic discount factor in the model is stationary. See Schmitt-Grohe and Uribe (2003) for more on this issue.

2.2 Production

Domestic production is undertaken by two types of firms: final goods producers and intermediate goods producers. The intermediate goods producers are monopolistically competitive, and they hire labor to produce differentiated products. These products are then aggregated by the final goods firms into a homogenous product that can be used for home consumption, c_h , or for exports, c_h^* . The final goods firms are perfectly competitive; they are introduced into the model for tractability only, as is customary in many New Keynesian models.

2.2.1 Final Goods Producers

Final goods producers are perfectly competitive. They purchase differentiated home-goods, y(j), from the intermediate goods producers indexed by j. They aggregate these differentiated goods into a final good, y, using the following production function:

$$y_t = \left[\int_0^1 y_t\left(j\right)^{\frac{\theta_t - 1}{\theta_t}} dj\right]^{\frac{\theta_t}{\theta_t - 1}} \tag{17}$$

where θ_t is the elasticity of substitution between the intermediate goods. Let θ be the steadystate value of θ_t . Then as we show later, at the steady-state, $\theta/(\theta - 1)$ is the gross mark-up over marginal cost that monopolistically competitive intermediate firms charge when they make their pricing decisions. We follow Rabanal and Rubio (2005) and Smets and Wouters (2003), and let $\mu_t = \theta_t/(\theta_t - 1)$ which is specified as an exogenous and i.i.d. mark-up shock; hence,

$$\log \mu_t = \log \overline{\mu} + \varepsilon_{\mu,t} \tag{18}$$

where $\overline{\mu} = \theta / (\theta - 1)$.¹¹

The final output good is either consumed domestically, designated by c_h , or exported abroad, designated by c_h^* ; hence,

$$y_t = c_{h,t} + c_{h,t}^*. (19)$$

Since the final goods producers are perfectly competitive, their profit maximization problem is static and is given by

$$\max p_t c_{h,t} + e p_{h,t}^* c_{h,t}^* - \int_0^1 p_t(j) y_t(j) dj$$
(20)

 $^{^{11}\}mbox{Note}$ that assuming i.i.d. mark-up shocks also facilitates easier identification of the price indexation parameters in the estimation.

where p(j) is the price of the intermediate good j. p_h^* is the export price of the home-origin good in units of the foreign currency, and e is the nominal exchange rate (in units of domestic currency per unit of foreign currency). We assume that the final goods producers do not price discrimate when they export; hence,

$$e_t p_{h,t}^* = p_t. (21)$$

The final goods producers' maximization problem yields the following demand function for the intermediate goods

$$y_t(j) = \left(\frac{p_t(j)}{p_t}\right)^{-\theta_t} y_t.$$
(22)

The foreign demand for home-goods is determined by the following relationship

$$c_{h,t}^* = \left(c_{h,t-1}^*\right)^{\delta} \left[\alpha^* y_t^* \left(\frac{p_t}{e_t p_t^*}\right)^{-\eta}\right]^{1-\delta}$$
(23)

where δ is a persistence parameter determining the extent to which current level of exports are determined by past exports.¹² α^* is a level parameter regulating the share of the home-produced consumption goods in the overall expenditure of foreigners. The foreign aggregate output level, y_t^* , is determined exogenously and follows an AR(2) process:

$$\log y_t^* = (1 - \rho_{1,y*} - \rho_{2,y*}) \log \overline{y^*} + \rho_{1,y*} \log y_{t-1}^* + \rho_{2,y*} \log y_{t-2}^* + \varepsilon_{y*,t}.$$
(24)

The foreign aggregate price level, p_t^* , is also exogenous, where the foreign inflation factor $\pi_t^* = p_t^*/p_{t-1}^*$ follows an AR(2) process:

$$\log \pi_t^* = \rho_{1,\pi^*} \log \pi_{t-1}^* + \rho_{2,\pi^*} \log \pi_{t-2}^* + \varepsilon_{\pi^*,t}.$$
(25)

2.2.2 Intermediate Goods Producers

There is a unit measure of monopolistically competitive intermediate goods producers indexed by j. Their technology is described by the following production function:

$$y_t(j) = z_t n_t(j) \tag{26}$$

¹²This persistence can be motivated by a habit specification in the utility of foreigners (Lim and McNelis, 2008).

where z is the aggregate productivity shock, and n(j) is the amount of (homogenous) labor input used in the production of intermediate good j. The aggregate productivity shock follows an AR(1) process:

$$\log z_t = \rho_z \log z_{t-1} + \varepsilon_{z,t}.$$
(27)

The intermediate goods firms take the demand function of the final goods producers as given and set prices to maximize the present value of profits. The firms discount future earnings at the same rate as households and their objective function is given by

$$\max E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \frac{\lambda_{\tau}}{\lambda_t} \left[\frac{p_{\tau}(j)}{p_{\tau}} y_{\tau}(j) - \frac{w_{\tau}}{p_{\tau}} n_{\tau}(j) - \frac{\kappa}{2} \left(\frac{p_{\tau}(j)/p_{\tau-1}(j)}{\pi_{\tau-1}^{\varphi}} - 1 \right)^2 y_{\tau} \right]$$
(28)

where the last term is the quadratic cost of price adjustment. The parameter κ regulates the magnitude of the price adjustment costs, which is also scaled by the aggregate domestic output. The price-adjustment cost is incurred when the increase in the firm's own price deviates from the past inflation rate, where the parameter φ regulates the extent to which current price changes are indexed to past inflation.¹³

We assume that the price adjustment costs do not affect the actual cash-flow of the firms, but only affect their objective function (c.f. De Paoli et. al, 2007).¹⁴ The intermediate firms distribute all profits back to households after paying for wages; hence, the real distributions of firm j is given by

$$\frac{\Pi_t(j)}{p_t} = \frac{p_t(j)}{p_t} y_t(j) - \frac{w_t}{p_t} n_t(j).$$
(29)

The firm's maximization with respect to its own price, after imposing a symmetric equilibrium and algebraic manipulation, yields the following expression

$$\left(\frac{\pi_t}{\pi_{t-1}^{\varphi}} - 1\right) \frac{\pi_t}{\pi_{t-1}^{\varphi}} = E_t \left\{ \beta \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\pi_{t+1}}{\pi_t^{\varphi}} - 1\right) \frac{\pi_{t+1}}{\pi_t^{\varphi}} \frac{y_{t+1}}{y_t} \right\} + \frac{1}{\kappa} \left[(1 - \theta_t) + \theta_t \frac{w_t/p_t}{z_t} \right].$$
(30)

When log-linearized, the above expression is the familiar New Keynesian Phillips curve with indexation,

$$\widehat{\pi}_t - \varphi \widehat{\pi}_{t-1} = \beta E_t \left[\widehat{\pi}_{t+1} - \varphi \widehat{\pi}_t \right] + \frac{\theta - 1}{\kappa} \left[\widehat{\mu}_t + \widehat{w}_t - \widehat{p}_t - \widehat{z}_t \right]$$
(31)

 $^{^{13}}$ Price adjustment costs in New Keynesian models were introduced by Rotemberg (1982). The specification here is similar to Ireland (2001), except that in his model, current price changes are indexed to the past price changes of the individual firm instead of the aggregate inflation rate.

 $^{^{14}}$ Alternatively, one could assume these price adjustment costs are real costs, and therefore affect the feasibility condition of the model (c.f. Chugh, 2007). Note however, this alternative specification yields equivalent results when the model is solved by log-linearization, since the adjustment costs are of higher order than linear.

where deviations in current inflation from the steady-state is determined by deviations in past inflation, expected future inflation, mark-up shocks and the marginal costs of firms.¹⁵

2.2.3 Foreign Producers

Foreign producers can also be modeled in a similar fashion, although here we opt to simply consider their final pricing equation in log-linearized form, instead of describing their pricing problem fully. We assume that foreign goods are imported directly from foreigners who engage in monopolistic competition themselves, and that they price-to-market when they sell their goods to the domestic market.¹⁶ Their pricing equation is given by

$$\widehat{\pi}_{f,t} - \varphi^* \widehat{\pi}_{f,t-1} = \beta E_t \left[\widehat{\pi}_{f,t+1} - \varphi^* \widehat{\pi}_{f,t} \right] + \frac{\theta - 1}{\kappa^*} \left(\widehat{q}_t - \widehat{s}_t + \widehat{\Psi}_t \right)$$
(32)

where π_f is the inflation in the foreign goods price; $\pi_{f,t} = p_{f,t}/p_{f,t-1}$. φ^* is an indexation parameter, and κ^* is the foreign cost of price-adjustment. Analogous to the domestic mark-up shock, Ψ_t is an exogenous cost-push shock whose logarithm is assumed to be distributed i.i.d.:

$$\log \Psi_t = \varepsilon_{\Psi, t}.\tag{33}$$

The real exchange rate, q, is defined as $q_t = e_t p_t^* / p_t$.¹⁷ The terms-of-trade, s, is defined as p_f / p . Their difference is analogous to the marginal cost of foreign producers (actually intermediaries) who buy the product at $e_t p_t^*$ and sell it at $p_{f,t}$. This difference can also be thought as the deviation from the law-of-one-price, $\hat{\psi}_{f,t}$ (Monacelli, 2005):

$$\widehat{\psi}_{f,t} = \widehat{e}_t + \widehat{p}_t^* - \widehat{p}_{f,t} = \widehat{q}_t - \widehat{s}_t.$$
(34)

2.3 The Central Bank

The central bank targets the nominal interest rate target using a Taylor rule,

$$\log i_{t} = \rho_{i} \log i_{t-1} + (1 - \rho_{i}) \left[a_{\pi} \log \pi_{t-1}^{c} + a_{y} \log \frac{y_{t-1}}{\overline{y}} + a_{d} \log d_{t} + \log \overline{i} \right] + \varepsilon_{i,t}, \quad (35)$$

 $^{^{15}\}mathrm{A}$ hat over a variable is the log-deviation of the variable from its steady-state.

 $^{^{16}}$ A related approach would be to assume that foreign goods are intermediated by *domestic importers* which mark-up the foreign price in a staggered fashion (c.f. Justiniano and Preston, forthcoming).

¹⁷We assume that the law of 1-price holds at the steady-state; hence, $\overline{q} = 1$, where a bar over a variable indicates its steady-state value.

where \overline{i} is the steady-state value of the nominal interest rate. ρ_i determines the extent of interest rate smoothing, and the parameters a_{π} , a_y , a_d determine the importance of CPI inflation, detrended output and the nominal depreciation of the exchange rate in the Taylor rule. The last term, ε_i , is an i.i.d. interest rate shock which is assumed to be Gaussian.

Note that the central bank conditions its interest rate rule on *lagged* output and CPI inflation, but *current* depreciation of currency. We use a lagged specification for inflation and output gap to capture the delays in data dissemination for these variables, but since current data on exchange rates are readily available, we use the current depreciation rates.¹⁸

We follow Justiniano and Preston (forthcoming) and let the central bank condition on detrended output as opposed to the model-implied output gap. The model-implied output gap is defined as the percent difference between actual output and the natural rate of output, where the natural rate is the output level that would be achieved if prices were flexible. The natural rate of output is unobservable in practice, and therefore detrended output is more commonly used as a measure of the output gap (Neiss and Nelson, 2005).

2.4 Market Clearing Conditions

All goods, labor and asset markets clear. We assume that domestic bonds are inside bonds; hence, they are in zero-supply:

$$b_t = 0 \tag{36}$$

for all t. The profits received by households are equal to the distributions of the domestic intermediate firms:¹⁹

$$\Pi_t = \int_0^1 \Pi_t \left(j \right) di. \tag{37}$$

The budget constraint of the households, coupled with the definitions of the pure profits received by households, yield the expression for the balance of payments:

$$\frac{e_t}{p_t} \left[b_t^* - i_t^* \phi_{t-1} b_{t-1}^* \right] = n x_t = c_{h,t}^* - \frac{p_{f,t}}{p_t} c_{f,t}$$
(38)

 $^{^{18}}$ We also estimated our model using current inflation and output in the Taylor rule, but this alternative specification generated very similar results. This is in line with the findings in Taylor (1999).

¹⁹Unlike Justiniano and Preston (forthcoming), the profits resulting from the imported goods do not accrue to domestic consumers in our model, since foreign goods are imported directly from foreigners who price-to-market when they sell their goods to the domestic market.

where nx stands for net exports. Note that the national income-expenditure-output identity in the model is described by

$$\frac{w_t}{p_t}n_t + \frac{\Pi_t}{p_t} = \frac{p_{c,t}}{p_t}c_t + nx_t = z_t n_t = y_t.$$
(39)

The model's equilibrium is defined as prices and allocations such that households maximize utility subject to their budget constraint, the final goods producers maximize profits, the intermediate home-goods producers maximize the present value of distributions paid out to households and all markets clear. We only consider symmetric equilibria where each variable indexed by j is equal across all intermediate-goods firms.

2.5 Log-linear Approximation to the Model

We log-linearize the variables around their steady-state to obtain a linear system of equations that characterize the equilibrium of the above model (see the Appendix for the list of equations).²⁰

The South African trade balance as a percent of GDP averaged -1.2% between 1994-2008, but was positive between 1998 and 2003. With these considerations, we decided to set the steady-state trade balance to zero, i.e. (nx/y) = 0. The implied steady-state ratio of foreign debt to GDP, \bar{a} , is also zero since

$$\overline{a} = -\frac{\overline{(nx/y)}}{\overline{i} - 1}.$$
(40)

Setting the risk-premium shock's mean value, $\overline{\Phi}$, equal to $-\log(\beta i^*)$ ensures that $\overline{(nx/y)} = \overline{a} = 0.^{21}$

The core relationships of the model are the IS curve, the Phillips curves, the Taylor rule, the interest-parity condition and the balance of payments condition.

Using the households' first-order conditions with respect to consumption and bond holdings, the demand side of the model can be reduced to the New Keynesian IS curve:

$$\widehat{c}_{t} = \frac{1}{1+\zeta} E_{t} \left[\widehat{c}_{t+1} \right] + \frac{\zeta}{1+\zeta} \widehat{c}_{t-1} - \frac{1-\zeta}{\sigma \left(1+\zeta\right)} \left(\widehat{i}_{t} - E_{t} \left[\widehat{\pi}_{t+1}^{c} \right] \right) + \widetilde{\Theta}_{t}.$$

$$\tag{41}$$

²⁰A bar over a variable indicates its steady-state value, and a hat indicates its log-deviation from its steady-state (i.e. $\hat{x}_t = \log x_t - \log \bar{x}$). For variables that can become negative, namely Φ_t and a_t , we use level-deviations instead (i.e. $\hat{x}_t = x_t - \bar{x}$).

²¹Note, however, that we do not need to set specific values for Φ and i^* , since they do not enter any of the log-linearized equilibrium conditions.

where we have redefined the demand shock as^{22}

$$\widetilde{\Theta}_{t} = \frac{\left(1 - \rho_{\Theta}\right)\left(1 - \zeta\right)}{\sigma\left(1 + \zeta\right)}\widehat{\Theta}_{t}.$$

Current consumption demand depends on a weighted average of past and expected future consumption, is inversely related to the real interest rate (where the relevant inflation is the expected CPI-inflation), and is positively related to the demand shocks.

Using the national accounting identity and the definition of net exports, we can relate consumption to output as

$$\widehat{y}_t = \alpha \widehat{s}_t + \widehat{c}_t + \alpha \left(\widehat{c}_{h,t}^* - \widehat{m}_t \right) \tag{42}$$

where exports, $\widehat{c}^*_{h,t},$ are determined by past exports, the real exchange rate and foreign output:

$$\widehat{c}_{h,t}^* = \delta \widehat{c}_{h,t-1}^* + (1-\delta) \left(\eta \widehat{q}_t + \widehat{y}_t^* \right).$$
(43)

Imports, $\hat{m}_t = \hat{s}_t + \hat{c}_{f,t}$, can be expressed as

$$\widehat{m}_t = \widehat{c}_t + \left[1 - \eta \left(1 - \alpha\right)\right] \widehat{s}_t \tag{44}$$

whereby increases in overall consumption result in an increase in imports. Note that with $\eta (1 - \alpha) < 1$, imports would increase at impact, following an increase in the terms-of-trade (i.e. the price of imported goods, p_f , becomes more expensive relative to the home-good prices, p).²³

The relationships between the real exhange rate, terms-of-trade and inflation rates are defined by

$$\widehat{q}_t - \widehat{q}_{t-1} = \widehat{d}_t + \widehat{\pi}_t^* - \widehat{\pi}_t \tag{45}$$

$$\widehat{s}_t - \widehat{s}_{t-1} = \widehat{\pi}_{f,t} - \widehat{\pi}_t. \tag{46}$$

The intermediate goods producers' pricing decision yields the domestic New Keynesian Phillips curve

$$\widehat{\pi}_{t} = \frac{\beta}{1+\beta\varphi} E_{t}\left[\widehat{\pi}_{t+1}\right] + \frac{\varphi}{1+\beta\varphi} \widehat{\pi}_{t-1} + \frac{\theta-1}{\kappa\left(1+\beta\varphi\right)} \widehat{mc}_{t} + \widetilde{\mu}_{t}$$

$$\tag{47}$$

 $^{^{22}}$ This is without loss of generality since the demand shock has zero mean, and the redefined demand shock follows an AR(1) process similar to the original demand shock. This transformation helps in the identification of parameters in the estimation.

²³This would help generate a J-curve effect in our model.

where we have redefined the mark-up shock as^{24}

$$\widetilde{\mu}_t = \frac{\theta - 1}{\kappa \left(1 + \beta \varphi \right)} \widehat{\mu}_t$$

Current inflation is affected by lagged inflation (due to indexation), expected future inflation, markup shocks and the marginal costs of firms, $\widehat{mc}_t = \widehat{w}_t - \widehat{p}_t - \widehat{z}_t$, which in turn can be expressed as

$$\widehat{mc}_t = \gamma \widehat{y}_t - (1+\gamma) \,\widehat{z}_t + \alpha \widehat{s}_t + \frac{\sigma}{1-\zeta} \,(\widehat{c}_t - \zeta \widehat{c}_{t-1})$$

which indicates that increases in the terms-of-trade, s, result in an increase in the marginal cost of domestic firms.

Similarly, the foreign New Keynesian Phillips curve is given by

$$\widehat{\pi}_{f,t} = \frac{\beta}{1+\beta\varphi^*} E_t \left[\widehat{\pi}_{f,t+1}\right] + \frac{\varphi^*}{1+\beta\varphi^*} \widehat{\pi}_{f,t-1} + \frac{\theta-1}{\kappa^* \left(1+\beta\varphi^*\right)} \widehat{\psi}_{f,t} + \widetilde{\Psi}_t \tag{48}$$

where, again, the foreign cost-push shocks are redefined as

$$\widetilde{\Psi}_t = \frac{\theta - 1}{\kappa^* \left(1 + \beta \varphi^* \right)} \widehat{\Psi}_t.$$

In this foreign-goods Phillips curve, $\hat{\psi}_f$ is the change in the deviations from the law-of-one-price, and is defined as

$$\widehat{\psi}_{f,t} - \widehat{\psi}_{f,t-1} = \widehat{d}_t + \widehat{\pi}_t^* - \widehat{\pi}_{f,t}.$$

Note that CPI inflation, $\hat{\pi}^c$, is related to GDP-deflator inflation, $\hat{\pi}$, and to imported-goods price inflation, $\hat{\pi}_f$, with

$$\widehat{\pi}_t^c = (1 - \alpha)\,\widehat{\pi}_{t-1} + \alpha\widehat{\pi}_{f,t-1}.\tag{49}$$

The Taylor rule of the central bank, in log-linearized form, is given by

$$\hat{i}_t = \rho_i \hat{i}_{t-1} + (1 - \rho_i) \left[a_\pi \hat{\pi}_{t-1}^c + a_y \hat{y}_{t-1} + a_d \hat{d}_t \right] + \varepsilon_{i,t}.$$
(50)

The first-order conditions of households with respect to domestic and foreign bonds yield the interest-parity condition:

$$\hat{i}_t - \hat{i}_t^* = E_t \left[\hat{d}_{t+1} + \left(\hat{\Phi}_t - \chi \hat{a}_t \right) \right].$$
(51)

²⁴Similar to the demand shock, this transformation is without loss of generality.

An increase in the interest rate differential strengthens the currency today, but causes expected depreciation tomorrow to rise. Similarly, an increase in the country risk premium depreciates the currency today, but reduces expected depreciation in the future.

The balance of payments equation equates the flow of assets with the flow of goods across borders:

$$\widehat{a}_t - \frac{1}{\beta} \widehat{a}_{t-1} = \alpha \left(\widehat{c}_{h,t}^* - \widehat{m}_t \right).$$
(52)

3 Estimation

In this section we describe the estimation procedure using Bayesian techniques, the data used in the estimation, and the prior distributions of the parameters.

3.1 Bayesian Estimation

The dynamic linear system of equations characterizing equilibrium can be summarized as

$$E_t \left[f \left(\xi_{t+1}, \xi_t, \xi_{t-1}, u_t; \Xi \right) \right] = 0, \quad u_t \sim NID \left[0, V \left(\Xi \right) \right]$$
(53)

where ξ_t is the vector of variables, u_t is the vector containing the orthogonal Gaussian shocks whose variance-covariance matrix is given by the diagonal matrix V, and Ξ is the vector of parameters.^{25,26} Given the parameter values, Ξ , the Blanchard-Kahn method can be used to find the *policy functions* that describe how the variables, ξ_t , evolve over time as a function of their past values, ξ_{t-1} , and the current realization of shocks, u_t , under rational expectations. These policy functions, g, are linear in the variables, and can be written as:²⁷

$$\xi_t = g\left(\xi_{t-1}, u_t; \Xi\right) = g_{\xi}\left(\Xi\right) \xi_{t-1} + g_u\left(\Xi\right) u_t.$$
(54)

The above solution can be thought of as the *transition equation* of a state-space representation, describing the evolution of all variables in the model, including the unobservables. The *measurement*

²⁵Since the foreign shock processes are specified as AR(2) processes, the vector of current variables ξ_t includes the first lags of the foreign variables.

²⁶Note that we set $\overline{\Phi} = -\log(\beta \overline{i^*})$ to ensure that $\overline{(nx/y)} = \overline{a} = 0$, but we do not need to set specific values for $\overline{\Phi}$ and $\overline{i^*}$ since they do not enter any of the log-linearized equilibrium conditions. Similarly, the export parameter, α^* , does not enter any of the log-linearized equations; hence, is ignored in the estimation.

 $^{^{27}}$ See Blanchard and Kahn (1980), Uhlig (1999), and the Dynare manual for more on this.

equation describes how the full set of variables are related to the observed variables, ξ_t^* . Since we assume no measurement error, our measurement equation is given by

$$\xi_t^* = M\xi_t \tag{55}$$

where M is a matrix that picks the elements of ξ_t that are observable.

Given a prior density for the parameters, $\Upsilon(\Xi)$, and the observable series $\xi^* = \{\xi^*_t\}_{t=1}^T$, Bayes' rule implies that the posterior distribution of the parameters is proportional to the product of the prior and the likelihood function

$$\Upsilon\left(\Xi|\xi^*\right) \propto L\left(\xi^*|\Xi\right)\Upsilon\left(\Xi\right) \tag{56}$$

where the likelihood function, $L(\xi^*|\Xi)$, is evaluated using the Kalman filter (Hamilton, 1994, and Ireland, 2001). To construct the entire posterior distribution and identify its corresponding moments, Markov Chain Monte Carlo (McMc) simulation methods are employed (An and Schorfheide, 2007, and Fernandez-Villaverde and Rubio-Ramirez, 2004). We use the estimation software *Dynare* to estimate the parameters and compute the policy functions.²⁸

3.2 The Data

To estimate the model, we use eight observable variables that are measured at a quarterly frequency for the period 1994Q1 - 2008Q4: detrended output, \hat{y} , GDP-deflator inflation, $\hat{\pi}$, CPI-inflation, $\hat{\pi}^c$, nominal interest rate, \hat{i} , nominal currency depreciation, \hat{d} , foreign detrended output, \hat{y}^* , foreign GDP-deflator inflation, $\hat{\pi}^*$, and foreign nominal interest rate, \hat{i}^* .²⁹ The South African Reserve Bank (SARB) Quarterly Bulletins are the source of the South African data, with the exception of CPI-inflation which was sourced from Statistics South Africa. The foreign variables are proxied by measures of the U.S. economy, and the sources of data are the Bureau of Economic Analysis and the Federal Reserve Board.

Since the model is quarterly, the measures of inflation rates and interest rates are expressed as

 $^{^{28}}$ Dynare uses Christopher Sim's *csminwel* optimization algorithm to find the mode, and the *Metropolis-Hastings* algorithm to find the moments of the posterior distribution. The estimated means of the posterior distributions are then used in constructing the policy functions using the *Blanchard-Kahn* method. See the Dynare manual for more details.

 $^{^{29}}$ There was a break in the South African data series in 1993, so we chose not to include data prior to 1994.

non-annualized quarterly rates.³⁰ The measures of detrended output are expressed as the percentage deviation of quarterly real GDP from their linear trends.³¹ The domestic and foreign interest rates, \hat{i} and \hat{i}^* , are represented respectively by the South African three-month treasury-bill rate, and the U.S. Federal Funds rate. The depreciation rate of currency, \hat{d} , is the quarterly percentage change in the South African rand (per unit of foreign currency calculated by a trade-weighted measure). Since the model assumes zero inflation at the steady state, the data for the inflation rates and the model counterparts are demeaned.

3.3 The Prior Distributions

The prior distributions for the parameters are given in Table 1, and are very similar to the ones used in Justiniano and Preston (forthcoming). We calibrate (i.e. specify dogmatic priors for) the time-discount parameter, β , and the import share parameter, α , to make sure that the model's steady-state will be able to match the observed mean interest rate and trade ratio in the data (Ireland, 2001). We set $\beta = 0.99$, reflecting a 4% annual real interest rate at the steady-state. The ratio of South African imports to GDP averaged about 28% over the sample period; hence we set $\alpha = 0.28$. The estimation had trouble identifying the elasticity of risk-premium with respect to the ratio of foreign debt to gdp parameter, χ ; hence we follow Justiniano and Preston (forthcoming) and set it equal to 0.01.

The prior for the habit parameter, ζ , is fairly uninformative with a beta-distribution that has a mean of 0.5 and standard deviation of 0.25. To generate hump-shaped impulse responses for output, the model requires a high level of persistence in export demand. Hence, we assume that the δ parameter has a beta prior with a mean of 0.8 and a standard deviation of 0.1.

For σ , the inverse of the elasticity of intertemporal substitution, we specify a gamma prior with a mean of 1.2 and a standard deviation of 0.4. The parameter η has a gamma prior with a mean of 1.5 and a standard deviation of 0.75, reflecting *a priori* expectation of a high elasticity of substitution between home and foreign goods. The parameter γ also has a gamma prior with a mean of 1.5 (reflecting a Frisch-elasticity of labor supply of 2/3) and a standard deviation of 0.75.

The use of uninformative priors for the price indexation parameters φ and φ^* generated low posterior estimates; which results in the model failing to generate inflation persistence, and hump-

 $^{^{30}}$ The annualized quarterly expressions for the deviations of inflation and interest rate from their steady-states are 4 times their non-annualized quarterly figures.

³¹Estimation results using HP-filtered output were very similar to those using linear detrending.

shaped impulse responses for inflation. Therefore, we consider beta distributions with a mean of 0.7 and a standard deviation of 0.1 for these parameters.³²

The price-adjustment cost parameter, κ , and the elasticity of substitution between the intermediate goods, θ , appear only in the Phillips curve expression and cannot be separately identified. We therefore calibrate the θ parameter to 6 which implies $\overline{\mu} = 1.2$; i.e intermediate firms charge a 20% mark-up in price over marginal cost (Rabanal and Rubio-Ramirez, 2005, and Smets and Wouters, 2003). Note that with a Calvo-pricing setting for the intermediate goods producers (Calvo, 1983), the Phillips curve expression in the model would be given by

$$\widehat{\pi}_{t} = \frac{\beta}{1+\beta\varphi} E_{t} \left[\widehat{\pi}_{t+1}\right] + \frac{\varphi}{1+\beta\varphi} \widehat{\pi}_{t-1} + \frac{(1-\tau)\left(1-\tau\beta\right)}{\tau\left(1+\beta\varphi\right)} \widehat{mc}_{t} + \widetilde{\mu}_{t}$$
(57)

where τ is the fraction of firms that keep their prices constant in any given period, and $1/(1-\tau)$ is the average duration of price stickiness (Justiniano and Preston, forthcoming). This is equivalent to the Phillips curve with Rotemberg-type costs of price adjustment given in (47) when

$$\frac{\theta - 1}{\kappa} = \frac{(1 - \tau)\left(1 - \tau\beta\right)}{\tau}.$$
(58)

With price stickiness that extends over five quarters, τ is equal to 4/5, and the right hand side of the above expression is close to 1/20. This would imply that with $\theta = 6$, κ is about 100. Taking these factors into consideration, the price-adjustment cost parameters for domestic and foreign goods, κ and κ^* , are both assumed to have a gamma prior with a mean of 100 and a standard deviation of 20. The relatively high and informative priors that are used for the price-adjustment cost parameters help match the magnitudes of the impulses generated from our model, especially for inflation and output, to the corresponding impulses generated by the forecasting model of the SARB (Smal et. al, 2007).

For the Taylor rule parameters, we again follow Justiniano and Preston (forthcoming), and assume that the priors for a_{π} , a_y , and a_d all have a gamma distribution with means of 1.5, 0.25 and 0.25 respectively. Note that these are long-run response coefficients in a Taylor rule that uses *non-annualized* quarterly data. If we had utilized annualized figures for the interest rate, inflation and currency depreciation, the corresponding coefficient for the output gap would have been four times higher than the one we use here. The prior for the interest rate smoothing parameter, ρ_i , has

 $^{^{32}}$ These priors are slightly more informative than Justiniano and Preston (forthcoming), but is in line with Smets and Wouters (2003).

a beta distribution with a mean of 0.5 and a standard deviation of 0.25.³³

The prior distributions used for the other domestic shocks reflect moderately high persistence with mean 0.8 and standard deviation $0.1.^{34}$ The priors for the standard deviations of all the domestic shocks are fairly uninformative, with an inverse-gamma distribution with a mean of 0.5% and infinite variance.

The priors for the standard deviation of the foreign shocks are the same as those for the domestic shocks. For the persistence parameters of the foreign variables, we considered normal-distributed priors. The priors for the first lags of the AR(2) processes are distributed by $N(0.59, 0.2^2)$, $N(0.9, 0.1^2)$ and $N(0.9, 0.1^2)$ for foreign inflation, output and interest rate. The priors for the second lags were assumed to be $N(0, 0.25^2)$.

4 Results

In this section, we first present the estimates for the posterior distributions of the parameters. We then report some of the key implications of our model; including the impulse responses of the model variables to innovations in each of the shocks, the simulated moments of the model variables, and their forecast error variance decomposition.

4.1 The Posterior Moments

The estimates for the mean and the 10%-90% marks of the posterior distributions of the parameters are reported in Table 1. The density functions for the posteriors are plotted along with the priors in Figures (1)-(4).³⁵

The mean estimates for the Taylor rule parameters are, by and large, standard. The Taylor rule is fairly persistent with mean ρ_i equal to 0.92, and the mean estimates for a_{π} , a_y , and a_d are 1.42, 0.29 and 0.25 respectively, implying that the SARB does condition partially on the depreciation rate of its currency. The estimates for a_{π} and a_y are heavily influenced by the choice of the prior, as can be seen from Figure (3). The Taylor rule coefficients are consistent with previous estimates

³³Our results were very similar when we used slightly different priors for the Taylor rule parameters. In particular, we tried a gamma distribution with a mean of 0.125 and a standard deviation of 0.125 for the output coefficient a_y , and a beta prior for ρ_i with a mean of 0.7 and a standard deviation of 0.1, as in Rabanal and Rubio-Ramirez (2005).

³⁴This is except for the mark-up and the external cost-push shocks, which were assumed to be i.i.d..

 $^{^{35}}$ For the Metropolis-Hastings algorithm in Dynare, we use five chains of 100,000 draws each with a 45% initial burn-in phase. The acceptance rate for each chain is about 28%.

in the literature regarding monetary policy in South Africa, possibly with the exception of the coefficient on inflation (Woglom, 2003).

The shocks are also fairly persistent, partly due to the prior distributions assumed for these parameters. The innovations to the risk premium and the external cost-push shocks have fairly large standard deviations, while the innovation to the Taylor rule has a standard deviation of 0.24% (i.e. about 1% annualized), similar to the estimates for the U.S. and the European Union (Smets and Wouters, 2003). The persistence and the standard deviation parameters of the productivity shock are not well identified by the data, as the prior and the posterior distributions for these parameters are almost identical.³⁶

The habit parameter, ζ , has a mean equal to 0.83, which is fairly high despite the uninformative prior that was imposed in the estimation. The indexation parameters in the Phillips curves, φ and φ^* , have estimated means of 0.47 and 0.61. When we initially estimated these parameters with uninformative priors, we found that the mean of their posteriors had very low values; this did not generate hump-shaped impulse responses for output and inflation, which is more in line with previous VAR evidence. Smets and Wouters (2003) find that habits and price indexation play an important role in generating intrinsic persistence in the model; this led us to employ more informative priors for these parameters.

The mean of the posterior distribution for γ is 1.59, corresponding to a labor supply elasticity of 0.68, which is within the range of values typically obtained in the literature. The elasticity of substitution between home and foreign goods, η , is estimated as 0.57, which is similar to the results of Justiniano and Preston (forthcoming), despite the high prior mean. The mean estimates for the price-adjustment cost parameters, κ and κ^* , are 96.6 and 112.8, which, according to equation (58), implies an average duration of price stickiness of about five quarters for both home and imported goods prices. The data does not appear to identify these parameters particularly well (especially the domestic price-adjustment cost parameter), and the resulting estimates are driven mainly by their prior values. The estimates for price stickiness are somewhat higher than those found in other studies (Smets and Wouters, 2003). Lower adjustment costs, however, generate unreasonably high impulse responses to output and inflation at the impact period of a monetary shock. The SARB forecasting model, for example, suggests that 100 basis points increase in the annualized interest rate leads to about a 40 basis points decline in annualized inflation and a 20 basis points decline

 $^{^{36}}$ Both the i.i.d. mark-up shock and the productivity shock affect marginal costs in a similar fashion, so separate identification of these shocks requires more data than was used in the estimation. We choose not to do this here since we have abstracted from capital in the production function.

in detrended output (Smal et. al, 2007). Generating similar magnitudes in our model requires the price stickiness to last about 5-6 quarters.

4.2 Impulse Responses

In Figures (5)-(10), we plot the impulse responses of the key variables in the model to a one standard-deviation innovation in each shock. The impulse responses obtained are, by and large, standard.

Following a positive innovation in the Taylor rule (i.e. a positive ε_i), output, consumption, and inflation all decline, while the currency strengthens at impact period. The trade balance to GDP ratio improves as a result of declining output, but this is reversed over time. The magnitudes of the impulses are in line with other studies on monetary policy in South Africa (Smal et. al, 2007, and Harjes and Ricci, 2008). A near 80 basis points increase in the annualized interest rate, reduces annualized CPI inflation by about 90 basis points and output by about 35 basis points.

A positive innovation to productivity, ε_z , increases output in a hump-shaped manner. The shock also eases inflationary pressure, which leads to a reduction in the interest rate (since the Taylor rule coefficient on inflation is stronger than the coefficients on output and depreciation).

The impulse responses also move in the expected directions following an innovation to the demand shock, $\varepsilon_{\tilde{\Theta}}$. A positive demand shock increases consumption and output, which fuels inflation and causes the interest rate to rise. In addition, the currency depreciates along with a deterioration in the trade-balance-to-GDP ratio.

A positive innovation to the mark-up shock, $\varepsilon_{\tilde{\mu}}$, increases inflation and lowers output (and consumption and labor). As such, a positive mark-up shock acts as a cost-push shock, which shifts the Phillips curve and presents a less favorable tradeoff between inflation and output to the SARB. The currency depreciates after the impact period while interest rates rise, since the Taylor rule places more emphasis on rising inflation.

A shock to the risk premium through a positive innovation to ε_{Φ} raises the domestic interest rate, causing a depreciation of the currency and higher inflation. The net-exports-to-GDP ratio decreases at impact, but eventually becomes positive due to the increase in exports. Output increases accordingly even when consumption declines (not shown).

The innovation to the foreign cost-push shock, $\varepsilon_{\tilde{\Psi}}$, increases the price of imported goods, which causes the trade balance to deteriorate. The currency depreciates, and eventually net exports become positive through the J-curve effect as agents eventually start to move away from more costly imported goods. CPI inflation increases due to import prices, and consumption declines. Output declines at impact as both consumption and net exports decline. Interest rates rise because inflation and currency depreciation rates increase.

The impulses to the foreign shocks also behave as expected (not shown). The foreign interest rate shocks generate similar impulse responses to those of the risk premium shock, and foreign inflation shocks generate similar impulse responses to a negative external cost-push shock (i.e. exchange rate shock).³⁷ A positive innovation in the foreign output shock, ε_{y^*} , increases exports at impact, appreciates the domestic currency, lowers the terms-of-trade and increases inflation. However, the magnitudes of these impulses are rather small.

4.3 Moments of Model Variables and Variance Decomposition

In Table 2, we report the theoretical moments of the model's key variables along with their data counterparts. In particular, we look at the standard deviation of variables, sd, their cross-correlation with output, c(., y), and their autocorrelation coefficients at different lags, ac(.).

The standard deviations generated by the model match their data counterparts fairly well. The model generates moderately volatile output with a standard deviation of 2.39%, close to the 2.23% of the data. The model slightly overestimates the volatility of inflation numbers in the data; we get standard deviations of 1.52% and 1.86% for GDP-deflator and CPI-inflation respectively, as opposed to their data counterparts which are about 1.1% each. The model matches the volatility of nominal currency depreciation fairly well, but slightly overestimates the volatility of the interest rate.

The model matches the cross-correlations of output with inflation and the currency depreciation rates very well. In addition, the model generates a high level of persistence in output, similar to what is observed in the data. Interest rates also exhibit high persistence due to the fairly high Taylor rule persistence parameter, ρ_i .

The GDP-deflator inflation and CPI inflation are both moderately persistent in the model, although it is surprising to note that the GDP-deflator inflation is not at all persistent in the data. Currency depreciation also has a low but positive persistence in the data, which could be the result

 $^{^{37}}$ The fact that these shocks produce similar impulse responses also points to the difficulty in identifying them separately if foreign data is not explicitly used in the estimation.

of a managed-floating policy conducted by the SARB.³⁸ This feature is not replicated by the model, which produces close to zero persistence in the nominal depreciation rate.

Table 3 reports the unconditional forecast-error variance decomposition of model variables with respect to each of the shock innovations. Innovations to the Taylor rule seem to play only a small role in accounting for the volatility in output and inflation, suggesting that the monetary policy conducted by the SARB may have reduced economic volatility, and has not directly contributed to it. Productivity shocks do not explain much of the variation in the key variables of the model, although as pointed out before, the domestic mark-up shocks affect the economy in similar ways, and the estimates attribute a much larger role for these domestic cost-push shocks, especially for GDP-deflator inflation volatility. Similarly, demand shocks are important; they account for more than a half of the variation in output, and about a third of the variation in GDP-deflator inflation.³⁹ The foreign shocks seem to play a negligible role in the economy, except for foreign output which explains about a quarter of the variation in the real exchange rates and terms of trade. Part of the reason for the small contribution of foreign shocks could be due to proxying the rest of the world with the U.S..

About two thirds of the volatility in CPI inflation is explained by risk premium and external cost-push shocks. These two *open economy shocks* also explain about a fifth of the variation in output. The unconditional variance in the currency depreciation rate, the real exchange rate, and the terms of trade are, to a large extent, due to these open economy shocks as well.

5 Optimal Monetary Policy

In this section, we posit an objective function for monetary policy makers based on the variance of output, inflation, and the interest rate. We then investigate the implied Taylor rule coefficients that are optimal from the perspective of this objective function. We restrict attention to optimal monetary policy in the context of Taylor rules of the form used in the model specified in equation (50), where the central bank adjusts the current nominal interest rate in response to lagged values

³⁸Central banks of small open economies may choose to manage exchange rates to reduce the level of uncertainty that is encountered in the export sector, thereby promoting international trade. In addition, a stable exchange rate may reduce uncertainty in financial markets, credibility and liability dollarization concerns, volatility in prices (through high pass-through effects), business cycle volatility, and the disruptive speculative activities of currency traders.

³⁹Although previous studies of small open economies suggest that demand shocks affect the variation in measures of inflation to a similar degree, they have not always found that they play such an important role in the variation of output (Enders and Hurn, 2006).

of the interest rate, CPI inflation, detrended output, and the current currency depreciation.

It is an open question in the literature as to whether a central bank of an open economy should condition on exchange rate movements when it sets its interest rate policy (Monacelli, 2005, and Justiniano and Preston, 2008). Exchange rate movements directly affect the foreign component of CPI inflation, and indirectly affect the domestic component of CPI inflation through their effect on the marginal cost of domestic producers. Hence, higher currency depreciation warrants a contractionary response by the central bank through an increase in the interest rate. In the presence of volatile exchange rates, however, this would cause frequent changes in interest rates and increase the variability of output. The optimal response of a central bank is therefore ambiguous and depends on the quantitative importance of these effects.

With our specification of the Taylor rule, the central bank cannot condition on current inflation rates (perhaps proxying for lags in information gathering), but can condition on current currency depreciation. Since current nominal currency depreciation feeds into current inflation, there may also be an informational gain on the part of policy makers to warrant conditioning on exchange rate movements.

5.1 The Loss Function of the Central Bank

We do not use the utility function of the representative household as the policy makers' objective function since the consumption variance generated from the model is unrealistically high.⁴⁰ Instead, we posit a loss function for policy makers that depends on the variation in CPI inflation, detrended output, and the nominal interest rate:⁴¹

$$L_t(a_{\pi}, a_y, a_d, \rho_i) = \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[\left(\widehat{\pi}_t^c\right)^2 + \lambda_y \left(\widehat{y}_t\right)^2 + \lambda_i \left(\widehat{i}_t\right)^2 \right]$$
(59)

where $\lambda_y, \lambda_i > 0$ are the weights on variation in output and interest rates relative to the variation in inflation.

The loss function (59) can be evaluated for each set of policy parameters in the Taylor rule, a_{π} , a_{y} , a_{d} , and ρ_{i} . Since all the other parameters are structural, we assume that they cannot be affected by monetary policy makers. We also ignore parameter uncertainty, and keep the values of

 $^{^{40}}$ See Ambler et. al (2004) for an optimal open economy Taylor rule generated using the utility function of the representative household. They also consider the impact of money in the utility function since their Taylor rule responds to money supply growth as well.

 $^{^{41}}$ Note that from a utility maximizing perspective, minimizing the variation in output may not be optimal if most of the variation in detrended output is due to changes in productivity (i.e. changes in the natural rate of output).

the structural parameters at the estimated means of their posterior distribution.

Considering the limiting case with $\beta = 1$, the objective function of the policy makers is analogous to minimizing a weighted sum of the unconditional variances:

$$L(a_{\pi}, a_{y}, a_{d}, \rho_{i}) = var(\widehat{\pi}^{c}) + \lambda_{y} var(\widehat{y}) + \lambda_{i} var(\widehat{i}).$$
(60)

For given values for the weight parameters, λ_y and λ_i , we calculate the set of policy parameters that minimizes the loss function above. We restrict attention to policy parameters which are consistent with long-run stability; hence $a_{\pi} > 1$ and $0 \le \rho < 1$. Since the choice of weights is somewhat arbitrary, we repeat this procedure for different values of λ_y and λ_i .

5.2 Optimal Taylor Rule Coefficients

As a preliminary exercise, we first fix three of the four policy parameters to the estimated values of their posterior mean, and then compute the loss function for all possible values of the remaining policy parameter. The results are given in Figure (11), where we set $\lambda_y = 0.5$ and $\lambda_i = 1$. The partially-optimal policy (keeping three of the four coefficients equal to their estimated values) prefers higher long-run response coefficients for inflation and output, but similar coefficients for the currency depreciation rate and the lagged interest rate. These partially-optimal coefficients are 3.42, 1.21, 0.35 and 0.89 on inflation, output, currency depreciation, and smoothing respectively.

Next, we let all of the four policy parameters to vary simultaneously and report the results in Table 4. For reference, the estimated values from the model are given in column (1). In column (2), we allow all of the coefficients in the Taylor rule to vary, and find that the optimal policy requires nearly a unit root in interest rates.⁴² Consequently, in the remaining columns we set the coefficient on the smoothing parameter, ρ_i , equal to its estimated value of 0.916, and optimize over the remaining parameters. With $\lambda_y = 0.5$ and $\lambda_i = 1$, the optimal long-run response coefficients for inflation, output and currency depreciation are found as 2.71, 1.20, and 0.01 respectively. The coefficients on inflation and output are higher than the estimated Taylor rule coefficients, and the coefficient on the currency depreciation is almost zero - see column (3). Columns (4)-(5) illustrate how the optimal policy varies with the relative weight on the output variance. Column (6) illustrates the sensitivity of the results to the relative weight on interest rate smoothing, λ_i .

⁴²Note that the long-run coefficients on inflation and output are very high in this case, but these need to be multiplied by $1 - \rho_i$ to get the short-term responses. These are of the same order of magnitude as the short-run responses obtained from the estimated coefficients.

The results in columns (3)-(6) tell a fairly consistent story; an optimal policy rule: i) will only have a modest impact on the standard deviation of inflation - at most a decrease of 16.7% calculated from columns (1) and (5); ii) will have somewhat more of an impact on the standard deviation of output - at most a decrease of 43.9% calculated from columns (1) and (4); iii) the reduction in inflation and output volatility comes at the cost of greater interest rate volatility; iv) requires making the interest rate more sensitive to past inflation rates and output gaps - with the exception of column (5); v) involves a modest tradeoff between output volatility and inflation volatility - see columns (4) and (5) for example; vi) requires little or no feedback from the currency depreciation rate.⁴³

This last result is important because it provides more evidence in support of Justiniano and Preston's (forthcoming) finding that optimal policy in small open economies places no weight on the exchange rate. We would expect to find that the model's optimal Taylor rule coefficient values are larger than the estimated coefficients, since the SARB would be more cautious in its monetary policy in the presence of data, model and parameter uncertainty (which we do not account for in the model). We indeed find larger optimal response coefficients for inflation and output, but surprisingly the optimal coefficient for currency depreciation is much smaller than the estimated value, and close to zero. This result is partially due to the fact that the risk premium shock accounts for a large portion of the variation in currency depreciation, but a much smaller portion of the variation in output, CPI inflation and interest rates. This implies that, if the SARB were to respond to variations in the rate of currency depreciation, then it would pass the effects of the risk premium shock onto the domestic economic variables. The close to zero optimal coefficient on currency depreciation also suggests that the informational gain from observing the currency depreciation rate in a timely fashion is not quantitatively important.

These findings are in contrast to that of Smets and Wouters (2002) who suggest that optimal policy for the Euro area is responsive to the exchange rate. Our model is very similar to that in Justiniano and Preston (forthcoming), and the countries for which they estimate the model (Australia, Canada, and New Zealand) are relatively small and open, as is South Africa. The major difference between these group of countries and South Africa is the greater volatility of the inflation and the exchange rates in South Africa.

While we do get similar results for the exchange rate, our results on optimal policy in general

 $^{^{43}}$ One might view column (5) as an exception, but recall that these are the long-run coefficients. The short-run coefficient on the currency depreciation rate for column (5) is 0.016, so that a 1 standard deviation change in currency depreciation would call for an immediate 11.6 basis points increase in the interest rate.

are quite different from those of Justiniano and Preston (forthcoming). For example, they find that the optimal policy for the countries they studies has much more feedback from inflation and output than we have.⁴⁴ They also find a much greater tradeoff between output and inflation volatility, and that policy can have a much greater influence on inflation and output volatility.⁴⁵ And, finally they do not find a tradeoff between output and inflation volatility and interest rate volatility. As opposed to our findings in Table 4, the optimal Taylor rule in Justiniano and Preston's (forthcoming) in all cases implies a lower standard deviation in interest rates than the standard deviation in the corresponding estimated model. The difference in results would suggest that South African monetary policy has been more systematic, and shocks to the Taylor rule have been less important in determining output and inflation volatility.

Unfortunately, the optimal policy results also have discouraging implications for the inflation targeting rule currently in place in South Africa. Even with a zero weight on output, the standard deviation of the quarterly inflation rate is 1.55 percent, which corresponds to an annualized rate of over 6 percent. Note that our model overestimates the volatility of CPI inflation, so adjusting for this implies about 4% standard deviation in annualized inflation. Such volatility suggests that it will be difficult to maintain annualized inflation within the current 3 percentage point target range, even over a 12-month horizon. The relatively small impact that optimal policy has on inflation volatility suggests that the Reserve Bank will have to continue the *flexible* inflation targeting regime proposed by Svensson (2007).

We also tried different specifications for the Taylor rule; in particular we considered a case where the SARB conditions on the *current* values of inflation and output, and the results were essentially unchanged.⁴⁶ We also considered a case where the SARB conditions on *lagged* values of currency depreciation as well as lagged values of inflation and output. The optimal long-run coefficients were largely unchanged in this case as well.

 $^{^{44}}$ Justiniano and Preston's (forthcoming) optimal policies always imply a unit root in the interest rate. Consequently, the coefficients in their Table 3 are short-run coefficients, whereas those in our Table 4 are long-run coefficients. For example, the smallest optimal coefficient on inflation they report is 0.8 and the largest is 2.27. The largest short-run coefficient on inflation in our Table 4 - excluding column (2) - is 0.316 in column (6). They report small coefficients on detrended output, but they include output growth with relatively large coefficients.

 $^{^{45}}$ For example, in their Table 3, optimal policy can reduce the standard deviation of inflation by over 54% in Australia; by over 71% in Canada, and by over 55% in New Zealand relative to the corresponding models' standard deviations.

 $^{^{46}\}mathrm{All}$ the other parameters are kept the same as per the benchmark case.

6 Conclusion

In this paper, we build a small open economy DSGE model, estimate it using Bayesian methods, and analyze its implications for optimal monetary policy in South Africa. In particular, we investigate whether the central bank should condition on exchange rate movements when it sets its interest rate policy via a Taylor rule. The optimal coefficients for the policy rule are obtained by minimizing a loss function that includes the variance of inflation, output, and the interest rate. We find that the optimal policy places a heavier weight on inflation and output than the estimated Taylor rule for South Africa, but a zero coefficient on the depreciation rate of currency.

The observed emphasis central banks place on exchange rate variations could also be due to other motives not captured in our model, such as reduction of uncertainty encountered in the export sector, or in financial markets. The investigation of these issues is left for future research.

References

- [1]Abel, A. B. (1990): "Asset Pricing under Habit Formation and Catching up with the Joneses," AEA Papers and Proceedings, 80, 38-42.
- [2]Ambler, S., A. Dib and N. Rebei (2004): "Optimal Taylor Rules in an Estimated Model of a Small Open Economy," Bank of Canada Working Paper, No. 2004-36.
- [3]An, S. and F. Schorfheide (2007): "Bayesian Analysis of DSGE Models," *Econometric Reviews*, 26, pp. 211-219.
- [4]Blanchard, O.J. and Kahn, C.M. (1980): "The Solution of Linear Difference Models under Rational Expectations," *Econometrica*, 48, pp. 1305–1311.
- [5]Calvo, G. (1983): "Staggered Prices in a Utility Maximizing Framework," Journal of Monetary Economics, 12, pp. 383-398.
- [6]Canova, F. and Sala, L. (2006): "Back to square one: Identification issues in DSGE models," European Central Bank, Working Paper Series No. 583.
- [7]Chari V. V., P. J. Kehoe, and E. R. McGrattan (2008): "New Keynesian Models: Not Yet Useful for Policy Analysis," Federal Reserve Bank of Minneapolis, Staff Report 409.
- [8]Christiano, L. J., M. Eichenbaum and C. L. Evans (2005): "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy", *Journal of Political Economy*, 113, pp. 1-45.
- [9]Chugh, S. K. (2006): "Optimal Fiscal and Monetary Policy with Sticky Wages and Sticky Prices," *Review of Economic Dynamics*, 9, pp. 683-714.
- [10]Clarida, R., J. Gali and M. Gertler (1999): "The Science of Monetary Policy: A New Keynesian Perspective," *Journal of Economic Literature*, 37, pp. 1661-1707.
- [11]De Paoli, B., A. Scott and O. Weeken (2007): "Asset Pricing Implications of a New Keynesian Model," Bank of England Working Paper, No. 326.
- [12]Enders, W. and S. Hurn (2006): "Identifying aggregate demand and supply shocks in a small open economy," Oxford Economic Papers, 59, pp. 411-429.
- [13]Erceg, C. J., D. W. Henderson and A. T. Levin (2000): "Optimal Monetary Policy with Staggered Wage and Price Contracts," *Journal of Monetary Economics*, 46, pp. 281-313.
- [14]Fernandez-Villaverde, J. and J. F. Rubio-Ramirez (2004): "Comparing Dynamic Equilibrium Economies to Data: A Bayesian Approach," *Journal of Econometrics*, 123, pp. 153-187.
- [15]Fuhrer, J. C. and G. R. Moore (1995): "Inflation Persistence," The Quarterly Journal of Economics, 110, pp. 127-159.
- [16]Gali, J. (2008): Monetary Policy, Inflation, and the Business Cycle: An Introduction to the

New Keynesian Framework. Princeton University Press.

- [17]Gali, J. and M. Gertler (1999): "Inflation Dynamics: A Structural Econometrics Analysis," Journal of Monetary Economics, 44, pp. 195-222.
- [18]Gali, J. and T. Monacelli (2005): "Monetary Policy and Exchange Rate Volatility in a Small Open Economy," *Review of Economic Studies*, 72, 707-734.
- [19]Harjes T. and L. A. Ricci (2008): "A Bayesian-Estimated Model of Inflation Targeting in South Africa," International Monetary Fund, Working Paper No. 48.
- [20]Ireland, P. N. (2001): "Sticky Price Models of the Business Cycle: Specification and Stability," Journal of Monetary Economics, 47, pp. 3-18.
- [21]Justiniano, A. and B. Preston (forthcoming): "Monetary Policy and Uncertainty in an Empirical Small Open Economy Model," *Journal of Applied Econometrics*.
- [22]Lim G. C. and P. D. McNelis (2008): Computational Macroeconomics for the Open Economy. MIT Press: Cambridge, Massachusetts.
- [23]Lubik, T. and F. Schorfheide (2007): "Do Central Banks Respond to Exchange Rates? A Structural Investigation," *Journal of Monetary Economics*, 54(4), pp. 1069-1087.
- [24]Mankiw, N. G. (2001): "The Inexorable and Mysterious Tradeoff between Inflation and Unemployment," *The Economic Journal*, 111, C45-C61.
- [25]Melick, W. and G. Galati (2006): "The Evolving Inflation Process: an Overview," Bank of International Settlements, Working Paper No. 196.
- [26]Monacelli, T. (2003): "Monetary Policy in a Low Pass-through Environment," European Central Bank, Working Paper No. 227.
- [27]Neiss K. and E. Nelson (2005): "Inflation Dynamics, Marginal Cost, and the Output Gap: Evidence from Three Countries," Journal of Money, Credit and Banking, 37, pp. 1019-1045.
- [28]Olaffson, T. T. (2006): "The New Keynesian Phillips Curve: In Search of Improvements and Adaptation to the Open Economy," Central Bank of Iceland, Working Paper No. 31.
- [29]Ortiz, A. and F. Sturzenegger (2007): "Estimating SARB's Policy Reaction Function," South African Journal of Economics, 75(4), pp. 659-680.
- [30]Rabanal, P. and J. F. Rubio-Ramirez (2005): "Comparing New Keynesian Models in the Euro Area: A Bayesian Approach," Federal Reserve Bank of Atlanta, Working Paper No. 2003-30a.
- [31]Rotemberg, J. J. (1982): "Monopolistic Price Adjustment and Aggregate Output," Review of Economic Studies, 49, pp. 517-31.
- [32]Schmitt-Grohe, S. and M. Uribe (2003): "Closing Small Open Economy Models," Journal of International Economics, 61, pp. 163-195.

- [33]Smal D., C. Pretorius, and N. Ehlers (2007): "The Core Forecasting Model of the South African Reserve Bank," South African Reserve Bank Working Paper, WP/07/02.
- [34]Smets, F. and R. Wouters (2002): "Openness, Imperfect Exchange Rate Pass-through and Monetary Policy," *Journal of Monetary Policy*, 49, 947-981.
- [35] (2003): "An Estimated Stochastic Dynamic General Equilibrium Model for the Euro Area," *Journal of the European Economic Association*, 1, pp. 1123-1175.
- [36]Svensson, Lars E. O. (2007): "Optimal Inflation Targeting: Further Developments of Inflation Targeting," in Mishkin, Frederic, and Klaus Schmidt-Hebbel (eds.), Monetary Policy under Inflation Targeting, Banco Central de Chile, pp. 187-225.
- [37]Tovar, C. E. (2008): "DSGE Models and Central Banks," Bank for International Settlements Working Papers, No. 258.
- [38]Taylor, John B. (1999): Monetary Policy Rules, edited by John B. Taylor. Chicago: The University of Chicago Press.
- [39]Uhlig, H. (1999): "A Toolkit for Analyzing Nonlinear Dynamic Stochastic Models Easily," in
 R. Marimon and A. Scott (eds.), Computational Methods for the Study of Dynamic Economies,
 Oxford University Press, pp. 30-61.
- [40]Woglom, G. (2003): "How has Inflation Targeting Affected Monetary Policy in South Africa," South African Journal of Economics, 71(2), pp. 198-210.
- [41]Woodford, M. (2003): Interest and Prices: Foundations of a Theory of Monetary Policy. Princeton University Press.

A Log-Linearized Equilibrium Conditions⁴⁷

(A1) New-Keynesian IS-curve:

$$\widehat{c}_{t} = \frac{1}{1+\zeta} E_{t} \left[\widehat{c}_{t+1} \right] + \frac{\zeta}{1+\zeta} \widehat{c}_{t-1} - \frac{1-\zeta}{\sigma \left(1+\zeta\right)} \left(\widehat{i}_{t} - E_{t} \left[\widehat{\pi}_{t+1}^{c} \right] \right) - \widetilde{\Theta}_{t}$$

(A2) National accounting identity:

$$\widehat{y}_t = \alpha \widehat{s}_t + \widehat{c}_t + \alpha \left(\widehat{c}_{h,t}^* - \widehat{m}_t \right)$$

(A3) Exports:

$$\widehat{c}_{h,t}^* = \delta \widehat{c}_{h,t-1}^* + (1-\delta) \left(\eta \widehat{q}_t + \widehat{y}_t^* \right)$$

(A4) Imports:

$$\widehat{m}_t = \widehat{c}_t + \left[1 - \eta \left(1 - \alpha\right)\right] \widehat{s}_t$$

(A5) Domestic-good New-Keynesian Phillips curve:

$$\widehat{\pi}_{t} = \frac{\beta}{1+\beta\varphi} E_{t}\left[\widehat{\pi}_{t+1}\right] + \frac{\varphi}{1+\beta\varphi} \widehat{\pi}_{t-1} + \frac{\theta-1}{\kappa\left(1+\beta\varphi\right)} \widehat{mc}_{t} + \widetilde{\mu}_{t}$$

⁴⁷Note that a bar over a variable indicates its steady-state value, and a hat over a variable indicates its logdeviation from its steady-state (i.e. $\hat{x}_t = \log x_t - \log \overline{x}$). For variables that can become negative, namely, Φ_t , a_t and $(nx/y)_t$, we used level-deviations instead (i.e. $\hat{x}_t = x_t - \overline{x}$).

(A6) Marginal cost of domestic firms:

$$\widehat{mc}_{t} = \gamma \widehat{y}_{t} - (1+\gamma) \,\widehat{z}_{t} + \alpha \widehat{s}_{t} + \frac{\sigma}{1-\zeta} \left(\widehat{c}_{t} - \zeta \widehat{c}_{t-1}\right)$$

 $\left(A7\right)$ For eign-good New-Keynesian Phillips curve:

$$\widehat{\pi}_{f,t} = \frac{\beta}{1+\beta\varphi^*} E_t \left[\widehat{\pi}_{f,t+1}\right] + \frac{\varphi^*}{1+\beta\varphi^*} \widehat{\pi}_{f,t-1} + \frac{\theta-1}{\kappa^* \left(1+\beta\varphi^*\right)} \widehat{\psi}_{f,t} + \widetilde{\Psi}_t$$

(A8) CPI-inflation:

$$\widehat{\pi}_t^c = (1 - \alpha)\,\widehat{\pi}_{t-1} + \alpha\widehat{\pi}_{f,t-1}$$

 $\left(A9\right)$ Real exchange rate:

$$\widehat{q}_t - \widehat{q}_{t-1} = \widehat{d}_t + \widehat{\pi}_t^* - \widehat{\pi}_t$$

(A10) Terms of trade:

$$\widehat{s}_t - \widehat{s}_{t-1} = \widehat{\pi}_{f,t} - \widehat{\pi}_t$$

 $\left(A11\right)$ Deviations from the law-of-one-price:

$$\widehat{\psi}_{f,t} - \widehat{\psi}_{f,t-1} = \widehat{d}_t + \widehat{\pi}_t^* - \widehat{\pi}_{f,t}$$

(A12) Taylor rule:

$$\widehat{i}_t = \rho_i \widehat{i}_{t-1} + (1 - \rho_i) \left[a_\pi \widehat{\pi}_{t-1}^c + a_y \widehat{y}_{t-1} + a_d \widehat{d}_t \right] + \varepsilon_{i,t}$$

(A14) Uncovered interest-parity condition:

$$\widehat{i}_t - \widehat{i}_t^* = E_t \left[\widehat{d}_{t+1} + \left(\widehat{\Phi}_t - \chi \widehat{a}_t \right) \right]$$

(A15) Balance of payments:⁴⁸

$$\widehat{a}_t - \frac{1}{\beta}\widehat{a}_{t-1} = \alpha \left(\widehat{c}_{h,t}^* - \widehat{m}_t\right)$$

(A16) Real interest rate:

$$\widehat{r}_t = \widehat{i}_t - E_t \left[\widehat{\pi}_{t+1} \right]$$

 $\left(A17\right)$ Productivity shocks:

$$\widehat{z}_t = \rho_z \widehat{z}_{t-1} + \varepsilon_{z,t}$$

(A18) Demand shocks:

$$\widetilde{\Theta}_t = \rho_{\widetilde{\Theta}} \widetilde{\Theta}_{t-1} + \varepsilon_{\widetilde{\Theta},t}$$

 $^{^{48}{\}rm The}$ left-hand side of (A15) is the capital account to (trend) GDP ratio, whereas the right-hand side is the net-exports to (trend) GDP ratio.

(A19) Mark-up shocks:

$$\widetilde{\mu}_t = \varepsilon_{\widetilde{\mu},t}$$

(A20) Risk premium shocks:

$$\widehat{\Phi}_t = \rho_{\Phi} \widehat{\Phi}_{t-1} + \varepsilon_{\Phi,t}$$

(A21) Foreign cost-push shocks:

$$\widetilde{\Psi}_t = \varepsilon_{\widetilde{\Psi},t}$$

 $\left(A22\right)$ For eign interest rate:

$$\widehat{i}_t^* = \rho_{1,i^*} \widehat{i}_{t-1}^* + \rho_{2,i^*} \widehat{i}_{t-2}^* + \varepsilon_{i^*,t}$$

 $\left(A23\right)$ For eign output:

$$\widehat{y}_t^* = \rho_{1,y*} \widehat{y}_{t-1}^* + \rho_{2,y*} \widehat{y}_{t-2}^* + \varepsilon_{y*,t}$$

(A24) Foreign inflation:

$$\widehat{\pi}_t^* = \rho_{1,\pi^*} \widehat{\pi}_{t-1}^* + \rho_{2,\pi^*} \widehat{\pi}_{t-2}^* + \varepsilon_{\pi^*,t}$$

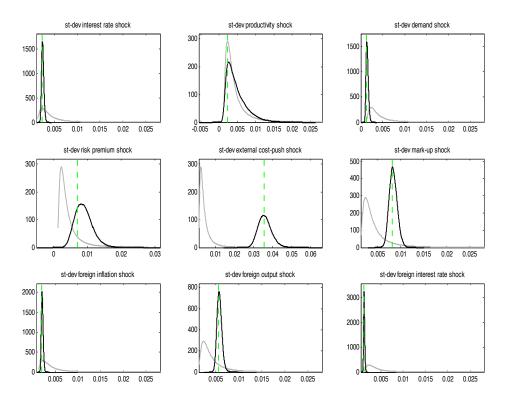


Figure 1: Prior distrib. (grey), posterior distrib. (black) and its mode (green) -1

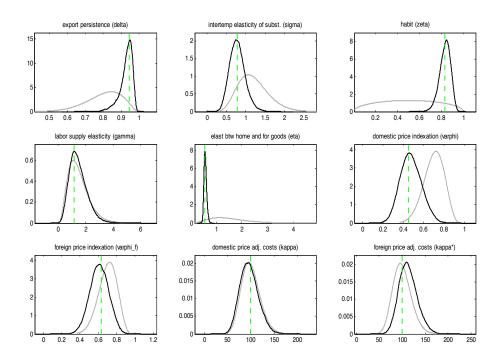


Figure 2: Prior distrib. (grey), posterior distrib. (black) and its mode (green) -2

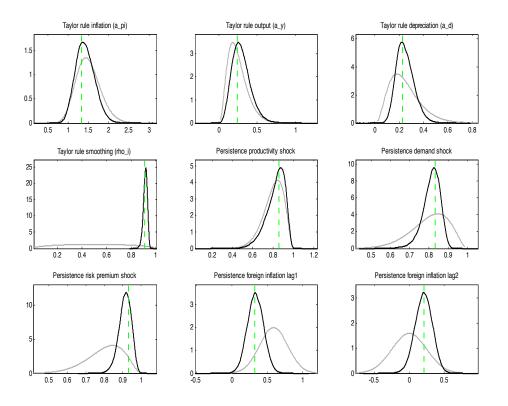


Figure 3: Prior distrib. (grey), posterior distrib. (black) and its mode (green) -3

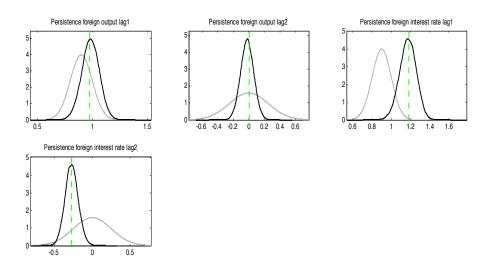


Figure 4: Prior distrib. (grey), posterior distrib. (black) and its mode (green) -4

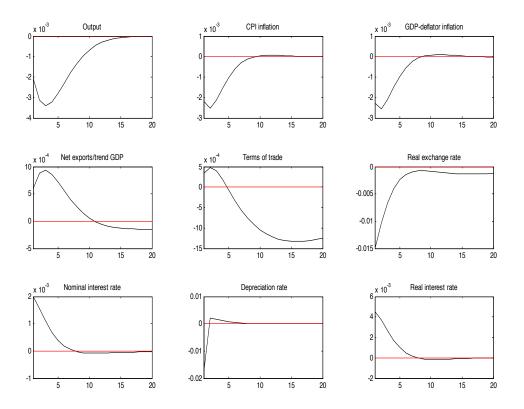


Figure 5: Impulse responses to 1 st. dev. innovation in the Taylor rule, i.

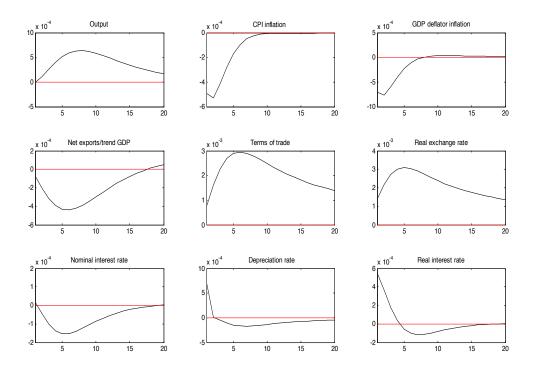


Figure 6: Impulse responses to 1 st. dev. innovation in the productivity shock, $\boldsymbol{z}.$

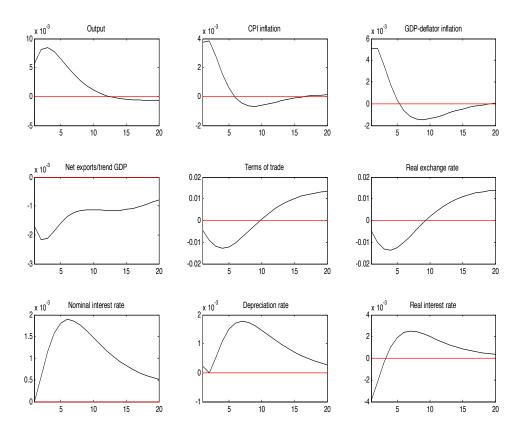


Figure 7: Impulse responses to 1 st. dev. innovation in the demand shock, $\widetilde{\Theta}.$

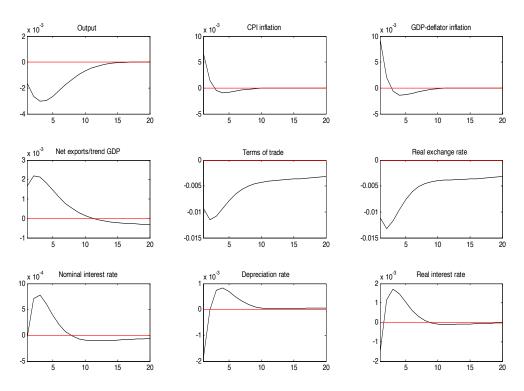


Figure 8: Impulse responses to 1 st. dev. innovation in the mark-up shock, $\widetilde{\mu}.$

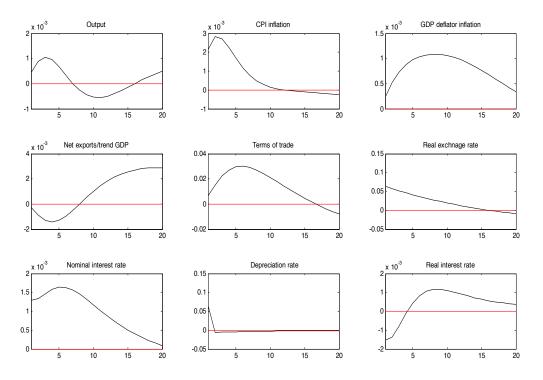


Figure 9: Impulse responses to 1 st. dev. innovation in the risk premium shock, $\Phi.$

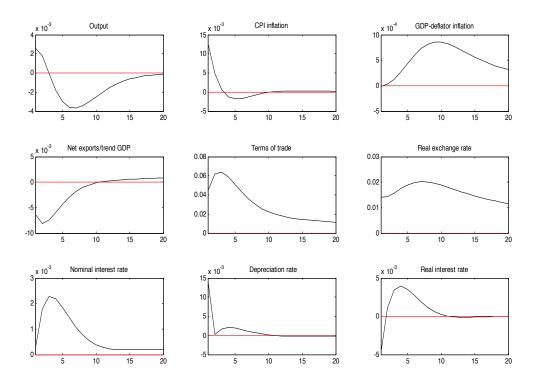


Figure 10: Impulse responses to 1 st.dev. innovation in the external cost-push shock, $\widetilde{\Psi}$.

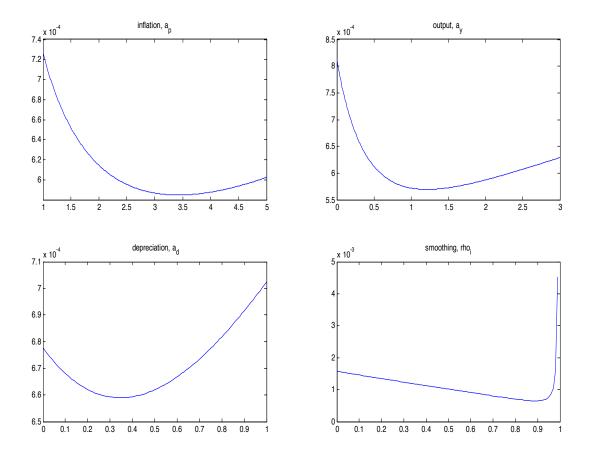


Figure 11: Loss functions when each policy parameter is varied in turn ($\lambda_y = 0.5, \lambda_i = 1$)

TADE 1. I HOI AND I OSCINI DISCUBLICIS OF I ATAMENED	e Ionollin					
Parameter	Symbol		Prior			Posterior
		Distribution	Mean	St. Dev.	Mean	$[10\% \ , \ 90\%]$
Time discount factor	β	calibrated	0.99	ı	1	1
Habit formation in consumption	Ç	Beta	0.5	0.25	0.831	$[0.753\ ,\ 0.915]$
Inverse of intertemporal elasticity of substitution	α	Gamma	1.2	0.4	0.780	[0.441, 1.088]
Inverse of labor supply elasticity	Z	Gamma	1.5	0.75	1.478	$[1.472\ ,\ 2.447]$
Elasticity of substitution btw. home and foreign goods	μ	Gamma	1.5	0.75	0.572	$[0.485 \ , \ 0.654]$
Share of imported good in consumption	σ	$\operatorname{calibrated}$	0.28	ı	ı	ı
Sensitivity of risk-premium to foreign bond holdings	X	$\operatorname{calibrated}$	0.01	ı		ı
Elasticity of substitution btw. intermediate goods	θ	$\operatorname{calibrated}$	9	ı	ı	ı
Persistence of export demand	δ	Beta	0.8	0.1	0.928	[0.877, $0.979]$
Price indexation for home-produced goods	Э	Beta	0.7	0.1	0.474	[0.306, 0.643]
Price indexation for foreign-produced goods	e*	Beta	0.7	0.1	0.608	[0.443, 0.777]
Price-adjustment cost for home-produced goods	Я	Gamma	100	20	96.560	[64.233, 127.979]
Price-adjustment cost for foreign-produced goods	κ^*	Gamma	100	20	112.796	[80.678, 145.242]
Taylor rule coefficient for CPI inflation	a_{π}	Gamma	1.5	0.3	1.422	$[1.031\ ,\ 1.793]$
Taylor rule coefficient for output	a_y	Gamma	0.25	0.13	0.294	$[0.100 \ , \ 0.478]$
Taylor rule coefficient for currency depreciation	a_d	Gamma	0.25	0.13	0.249	$[0.135\;,0.365]$
Taylor rule coefficient for interest rate smoothing	ρ_i	Beta	0.5	0.25	0.916	[0.891, 0.944]

Table 1: Prior and Posterior Distributions of Parameters (cont.)	of Paramet	cers (cont.)				
Parameter	\mathbf{Symbol}		Prior		P	Posterior
		Distribution	Mean	St. Dev.	Mean	$[10\% \ , \ 90\%]$
Persistence for productivity shock	ρ_z	Beta	0.8	0.1	0.823	[0.684, $0.960]$
for demand shock	$ ho_{\widetilde{\Theta}}$	Beta	0.8	0.1	0.812	[0.744, $0.885]$
for risk-premium shock	ρ_{Φ}	Beta	0.8	0.1	0.912	[0.859 , $0.966]$
for 1st lag of foreign interest rate	$ ho_{1,i^*}$	Normal	0.9	0.1	1.171	[1.030, 1.312]
for 2nd lag of foreign interest rate	$ ho_{2,i^*}$	Normal	0	0.25	-0.272	[-0.415, -0.131]
for 1st lag of foreign output	$ ho_{1,y*}$	Normal	0.9	0.1	0.983	$[0.853 \ , \ 1.116]$
for 2nd lag of foreign output	$ ho_{2,y*}$	Normal	0	0.25	-0.022	$[-0.158 \ , \ 0.120]$
for 1st lag of foreign inflation	$ ho_{1,\pi^*}$	Normal	0.59	0.2	0.332	$[0.149\;,\;0.521]$
for 2nd lag of foreign inflation	$ ho_{2,\pi^*}$	Normal	0	0.25	0.204	[0.008, $0.409]$
St. dev. for interest rate shock	σ_i	Inv. Gamma	0.5%	8	0.24%	[0.20% , 0.28%]
for productivity shock	σ_z	Inv. Gamma	0.5%	8	0.48%	[0.13%~,~0.87%]
for demand shock	$\sigma_{\widetilde{\Theta}}$	Inv. Gamma	0.5%	8	0.14%	$[0.10\% \ , \ 0.18\%]$
for mark-up shock	$\sigma_{\widetilde{\mu}}$	Inv. Gamma	0.5%	8	0.84%	$[0.69\% \ , \ 0.97\%]$
for risk premium shock	σ_{Φ}	Inv. Gamma	0.5%	8	0.89%	$[0.46\% \ , \ 1.29\%]$
for external cost-push shock	$\sigma_{\widetilde{\Psi}}$	Inv. Gamma	0.5%	8	3.59%	$[3.02\%\ ,\ 4.16\%]$
for foreign interest rate shock	σ_{i^*}	Inv. Gamma	0.5%	8	0.12%	$[0.10\% \ , \ 0.14\%]$
for foreign output shock	σ_{y^*}	Inv. Gamma	0.5%	8	0.58%	$[0.49\% \ , \ 0.66\%]$
for foreign inflation shock	σ_{π^*}	Inv. Gamma	0.5%	8	0.21%	$[0.18\%\ ,\ 0.25\%]$

Variable	Symbol		Da	Data			Model	lel	
		sd(%)	c(.,y)	ac(1)	ac(2)	sd(%)	c(.,y)	ac(1)	ac(2)
Output	y	2.23		0.97	0.92	2.39	1	0.93	0.80
GDP-deflator inflation	π	1.10	0.19	-0.05	-0.14	1.52	0.25	0.59	0.34
CPI inflation	π^{c}	1.08	0.38	0.47	0.11	1.86	0.36	0.50	0.20
Nominal interest rate	i	0.79	0.12	0.95	0.85	1.00	0.00	0.94	0.86
Currency depreciation	d	6.90	0.12	0.22	0.13	7.29	0.03	-0.03	-0.02
Real exchange rate	d	ı	ı	I	ı	26.57	-0.25	0.96	0.92
Terms of trade	s	ı	ı	I	ı	27.13	-0.31	0.98	0.94

Variable	Variable	Error va	st Error Variance Decomposition (%)) nonisoqi		Shocks				
		Interest	Productivity	Demand	Mark-up	Risk-Prem.	For. cost-push	Fore	Foreign shocks	ocks
	Symbol	ε_i	εz	υ ω	εũ	ε_{Φ}	εĩ	ε_{i^*}	ε_{u^*}	ε#*
Output	y	9.6	0.7	58.5	8.1	3.9	15.1	0.1	3.7	0.0
GDP-def. inflation	π	8.5	0.7	34.9	41.3	8.0	3.1	0.1	3.4	0.0
CPI inflation	π^{c}	5.7	0.2	12.1	14.1	10.1	55.7	0.2	1.9	0.0
Nom. interest rate	i	8.2	0.1	31.8	1.8	28.3	21.6	0.3	7.8	0.0
Currency depreciation	d	5.7	0.0	0.5	0.1	82.3	4.0	1.7	5.4	0.2
Real exch. rate	d	0.6	0.2	8.1	1.3	50.6	9.5	0.7	29.0	0.0
Terms of trade	s	0.1	0.2	7.4	1.2	31.6	33.6	0.3	25.7	0.0

Table 4: Optimal Taylor Rule Coefficients	or H	Sule Coeffici	ients				
		(1)	(2)	(3)	(4)	(2)	(9)
		Estimated	Optir	$\mathbf{Optimal} \ \mathbf{Rule} \ \mathbf{with} \ \mathbf{weights} \ (\lambda_y,\lambda_i)$	with w	veights	(λ_y,λ_i)
Taylor rule coefficient			(0.5, 1)	(0.5, 1)	(1, 1)	(0,1)	(0.5, 0.5)
Inflation a	a_{π}	1.42	18.73	2.71	2.46	3.54	3.76
Output a	a_{u}	0.29	19.76	1.20	1.96	0.07	1.62
Currency depreciation a	a_d	0.25	4.22	0.01	-0.05	0.19	-0.03
Smoothing ρ	ρ_i	0.916	0.994	ı	I	ı	ı
lue $(\times 10^{-4})$			5.15	5.42	6.47	3.73	4.64
standard deviation (%)							
CPI inflation π	π^{c}	1.86	1.77	1.67	1.77	1.55	1.59
Output 3	ĥ	2.39	1.49	1.59	1.34	2.17	1.53
Interest rate	i	1.00	0.95	1.17	1.25	1.15	1.35
Currency depreciation 6	q	7.29	6.89	7.09	7.30	6.62	7.03

e Coefficients	(1) (2) (3) (4) (5)	Estimated Optimal Rule with weights $(\lambda$	$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	1.42 18.73 2.71 2.46 3.54	$0.29 \qquad 19.76 \qquad 1.20 \qquad 1.96 \qquad 0.07$	0.25 4.22 0.01 -0.05 0.19	0.916 0.994 - $-$	5.15 5.42 6.47 3.73		$1.86 \qquad 1.77 \qquad 1.67 \qquad 1.77 \qquad 1.55$	2.39 1.49 1.59 1.34 2.17	
Table 4: Optimal Taylor Rule Coefficients		Ă	Taylor rule coefficient	Inflation a_{π}	Output a_y	Currency depreciation $a_{\vec{d}}$	Smoothing ρ_i	Loss fnc. value $(\times 10^{-4})$	standard deviation (%)	CPI inflation $\hat{\pi}^c$	Output y	