A Large Factor Model for Forecasting Macroeconomic Variables in South Africa

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Abstract

This paper uses large Factor Models (FMs) which accommodates a large cross-section of macroeconomic time series for forecasting per capita growth rate, inflation, and the nominal short-term interest rate for the South African economy. The FMs used in this study contains 267 quarterly series observed over the period of 1980Q1-2006Q4. The results, based on the RMSEs of one- to four-quarters-ahead out of sample forecasts over 2001Q1 to 2006Q4, indicate that the FMs tend to outperform alternative models such as an unrestricted VAR, Bayesian VARs (BVARs) and a typical New Keynesian Dynamic Stochastic General Equilibrium (NKDSGE) model in forecasting the three variables under consideration, hence, indicating the blessings of dimensionality.

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1 Introduction

For a long time policy makers, the general public, and academics have been interested in producing accurate forecasts of economic variables for various reasons. Model builders have exploited the recent development in computation to write simple and complex models that simulate reality with high degree of accuracy. There is an increasing need of large information to mimic economic relationships. However, traditional economic models, such univariate time series and multivariate VAR, are limited, in that they cannot accommodate large number of time series. Although the VAR is popular, compared to the traditional macroeconometric models, for its forecasting ability, it has serious limitations - the most conspicuous being that it cannot accommodate a large panel of series without the risk of running short of degrees of freedom. Thus, given, the issue of overparametrization, the main problem of small-scale models lie in the decision regarding the choice of correct variables to include. In practice, however, forecasters and policymakers often extract information from many series than the ones that can be included in a VAR.

As Bernanke and Boivin (2003) argue eloquently, central banks monitor and analyze literally thousands of data from various sources. Since central banks pay the costs of analyzing a wide range data to improve their decisions, econometric models should considerably take into account the marginal benefits that increasing information brings into forecasting. Progress has been made in last decades to accommodate large panel of time series in forecasting through the use of factor models. The initial contribution in this era comes from the works of Sargent and Sims (1977) and Geweke (1977), who introduced the dynamic factor approach to macroeconomics. They exploited the dynamic interrelationship of variables and then reduced the number of common factors even further. However, the approach followed by Sargent and Sims (1977) and Geweke (1977) is too restrictive, in that, it imposes orthogonality of idiosyncratic components. Chamberlain (1983) and Chamberlain and Rothschild (1983) admit the possibility of weakly cross-sectional correlation of idiosyncratic components.

Recently, though, these large factor models have been improved through advances in estimation techniques proposed by Stock and Watson (2002b), Kapetanios and Marcellino (2004) and Forni et al. (2005), by accounting for serial correlation and weakly cross-sectional correlation of idiosyncratic components. This progress, in turn, has generated an increasing interest in academia, international organizations, central banks, and the governments in the usage of these models, simply because they can accommodate a large panel of time series in forecasting economic variables.

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However, there is still quite a bit of divergence in opinion as to whether factor models with large cross-section of time series tend to outperform traditional econometric models with limited number of variables. While on one hand, studies such as Giannone and Matheson (2007), Van Nieuwenhuyze (2005), Cristadoro et al. (2005), Forni et al. (2005), Schneider and Spitzer (2004), Kabundi (2004), Forni et al. (2001) and Stock and Waston (2002a, 2002b, 1999, 1991, and 1989) provide evidence of improvement in forecasting performance of macroeconomic variables using factor analysis, Schumacher (2006), Schumacher and Dreger (2004), Gosselin and Tkacz (2001) and Angelini et al. (2001) found no or only minor improvements in forecasting ability. These conflicting results have led to a fascinating debate as to whether the victory proclaimed by proponents of large models were not too early. Some attribute the success of large models to the different circumstances. Banerjee et al. (2005), for example, find that small models forecast macroeconomic variables better than factor models. In addition, they also find that the performance of factor models differs based on countries. Factor models are relatively good at forecasting real variables in the US compared to the euro area, while, euro area nominal variables are easier to predict than US nominal variables, using factor models. Furthermore, Boivin and Ng (2006) claim that the composition of the dataset and the size of the cross-section dimension matter in producing better forecasts with factor models.

In such a backdrop, this paper exploits the information contained in a large-dimensional factor model framework, developed by Stock and Watson (2002b) (henceforth SW) and Forni Hallin, Lippi and Reichlin (2005) (henceforth FHLR), to forecast per capita growth (percentage change in real per capita GDP), inflation (percentage change in the implicit GDP deflator) and a measure of short-term nominal interest rate (91-days Treasury bill rate) for South Africa, over the out-of-sample horizon spanning the period of 2001Q1 to 2006Q4, with an in-sample period of 1980Q1 to 2000Q4. The forecasting performance of the FMs, estimated under alternative assumptions regarding the interaction between the factors and the variables of interest, are evaluated in comparison to three other alternative models, namely, an unrestricted classical VAR, optimal Bayesian VARs[1] (BVARs) and a New-Keynesian Dynamic Stochastic General Equilibrium (NKDSGE), on the basis of the Root Mean Squared Error (RMSE) of the out-of-sample forecasts. Although Kabundi (2007) used the DFM to assess the synchronization of South Africa and the US, and the channels through which the US supply and demand shocks are transmitted, to our knowledge, this is the first attempt to use a large FM to forecast key macroeconomic variables in South Africa. Moreover, it must be indicated that for

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[1] See Section 5 for further details regarding the issue of optimality of BVARs.

the exception of Wang (2008), the comparison of a FM and a DSGE model is rare and, hence, worth a discussion, especially in the context of a developing economy like South Africa. Note, allowing for a NKDSGE model as an alternative forecasting framework, helps us to compare between the athoretical models, like the FM, VAR and the BVARs with a microfounded theoretical model. Besides, the introduction and the conclusions, the remainder of the paper is organized as follows: Section 2 lays out the large FM, while Section 3 discusses the data used to estimate the FM. Section 4 outlines the basics of the VAR, BVAR and the NKDSGE models, and Section 5 presents the results from the forecasting exercise.

2 The Model

This study uses the large FM to extract common components between macroeconomic series, and then these common components are used to forecast output growth, inflation rate, and nominal interest rates. In the VAR models, since all variables are used in forecasting, the number of parameters to estimate depend on the number of variables n. With such a large information set, n, the estimation of a large number of parameters leads to a curse of dimensionality. In the FM, the information set is accounted by few factors $q << n$, which transforms the curse of dimensionality into a blessing of dimensionality.

The FM expresses individual times series as the sum of two unobserved components: a common component driven by a small number of common factors and an idiosyncratic component, which are specific to each variable. The relevance of the method is that the FM is able to extract the few factors that explain the comovement of all South African macroeconomic variables. SW and FHLR demonstrate that when the number of factors is small relative to the number of variables and the panel is heterogeneous, the factors can be recovered from the present and past observations.

Consider a $n \times 1$ covariance stationary process $Y_t = (y_{1t}, ..., y_{nt})'$. Suppose that X_t is the standardized version of Y_t, i.e. X_t has a mean zero and a variance equal to one. Under FM X_t is described by a factor model, it can be written as the sum of two orthogonal components:

$$x_{it} = b_i(L)f_t + \xi_{it} = \lambda_i F_t + \xi_{it} \tag{1}$$

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or, in vector notation:

$$X_{it} = B(L)f_t + \xi_{it} = \Lambda F_t + \xi_{it} \tag{2}$$

where f_t is a $q \times 1$ vector of dynamic factors, $B(L) = B_0 + B_1 L + ... + B_s L^S$ is an $n \times q$ matrix of factor loadings of order s, ξ_{it} is a $n \times 1$ vector of idiosyncratic components, F_t is $r \times 1$ vector of factors, with $r = q(s + 1)$. However, in more general framework $r \geq q$, instead of the more restrictive $r = q(s + 1)$. In a DFM, f_t and ξ_{it} are mutually orthogonal stationary process, while $\chi_{it} = B(L)f_t$ is the common component.

In factor analysis jargon, $X_t = B(L)f_t + \xi_{it}$ is referred to as dynamic factor model, and $X_t = \Lambda F_t + \xi_{it}$ is the static factor model. Similarly, f_t is regarded as vector of dynamic factors while F_t is the vector of static factors. Since dynamic common factors are latent, they need to be estimated. It is important to point out that the estimation technique used matters for factor forecasts. This paper uses two leading methods in the literature of large FM, namely, those proposed by SW and FHLR. SW uses the static principal component approach (PCA) on X_t. The factor estimates are therefore the first principal components of X_t, i.e. $\hat{F}_t = \hat{\Lambda}' X_t$, where $\hat{\Lambda}$ is the $N \times r$ matrix of the eigenvectors corresponding to the r largest eigenvalues of the sample covariance matrix $\hat{\Sigma}$.

On the other hand FHLR is a weighted version of the principal components estimator by SW based on dynamic PCA, which exploits information of leads and lags of variables where time series are converted to the frequency domain. However dynamic PC is a two-sided filter. This causes a problem at the end of the sample making it difficult to estimate and forecast the common component since no future observations are available. FHLR solves this problem by proposing a two-step approach. In the first step, it relies on the dynamic approach in the estimation of the covariance matrices of the common and idiosyncratic component (at all leads and lags) through an inverse Fourier transform of the spectral density matrices. It involves the estimating the eigenvalues and eigenvectors decomposition of spectral density matrix of X_t, $\hat{\Sigma}(\theta)$ which has rank q, corresponding to q largest eigenvalues. For each frequency, θ, the spectral density matrix of X_t, which is estimated using the frequency $-\pi < \theta < \pi$, can be decomposed into the spectral densities of the common and the idiosyncratic component, $\Sigma(\theta) = \Sigma_\chi(\theta) + \Sigma_\xi(\theta)$. Hence, the spectral density matrix of common component $\hat{\Sigma}_\chi(\theta)$ is estimated. In the second step, this information is used to compute the factor space by r linear combinations of X_t that maximizes the contemporaneous covariance matrices estimated explained by the common factors, estimated in the first step. These r linear combinations are the solutions from a dynamic principal component

problem and have the efficient property of reducing the idiosyncratic noise in the common factor space to a minimum, by selecting the variables with the highest common/idiosyncratic variance ratio. Importantly, this one-sided approach is only used to estimate and forecast the common component.

The rationale of using both the SW and FHLR is motivated by conflicting results in the literature. For example, Eickmeier and Ziegler (2008) find that the FHLR tends to outperform the SW in forecasting output. However, Schumacher (2007) finds minor improvements of FHLR over SW in predicting out. While for inflation, Eickmeier and Ziegler (2008) find the advantages of SW over FHLR. In contrast, Forni et al. (2003) find the FHLR does better for both output and inflation. Finally, Stock and Watson (2005), Boivin and Ng (2005), and D'Agostion and Giannone (2006) only find modest difference between the two methods.

For forecasting purposes, we use a small VAR containing variable(s) of interest augmented by extracted common factors using Stock and Watson (2002a) approach. This approach is similar to the univariate Static and Unrestricted (SU) approach of Bovin and Ng (2005). Therefore, the forecasting equation to predict y_t is given by

$$\begin{pmatrix} \hat{y}_{t+h} \\ \hat{F}_{t+h} \end{pmatrix} = \hat{\Phi}(L) \begin{pmatrix} y_t \\ F_t \end{pmatrix} \tag{3}$$

In addition, we consider another forecasting approach, the Bayesian estimation of the Equation (3), called BFAVAR. As Boivin and Ng clearly put it, AR and VAR are special cases of equation 3. When the factors and the parameters are known, the FAVAR approach should produce smaller mean squared errors. However, in practice one does not observe the factors, they should be estimated. And the forecasting equation should be correctly specified.

Finally, for FHLR we adopt the Dynamic-Nonparametric approach (DN) as discussed by Boivin and Ng (2005). The forecasting equation is as follows

$$\hat{y}_{t+h} = \hat{\chi}_{t+h} + \hat{\varphi}(L)\hat{\xi}_t \tag{4}$$

where $\hat{\chi}_{t+h}$ is obtained by artificially projecting χ_{t+h} on $\hat{F}_t^{FHLR}$, such that $\hat{\chi}_{t+h} = \hat{\Gamma}_\chi(k)Z(Z'\hat{\Sigma}Z)^{-1}Z'X_t$. Z is the r generalized eigenvectors of $\hat{\Gamma}_\chi(k)$ with respect to $\hat{\Gamma}_\xi(k)$ under normalization $Z'\hat{\Gamma}_\xi(0)Z = 1$, and $\hat{\Gamma}_\chi(k)$ and $\hat{\Gamma}_\xi(k)$ are covariance matrices of common and idiosyncratic components at different leads and lags. Since $\hat{F}_t^{FHLR} = Z'X_t$, then $\hat{\chi}_{t+h} = \hat{\Gamma}_\chi(k)Z(Z'\hat{\Sigma}Z)^{-1}\hat{F}_t^{FHLR}$.

3 Data

It is imperative in an approximate factor analysis to extract common components from a data rich environment. After extracting common components of output growth, inflation rate, and nominal interest rates, we make out-of-sample forecast for one, two, three, and four quarters ahead. Furthermore, we estimate the idiosyncratic component with AR(p) processes as suggested by Boivin and Ng (2005).

The data set contains 267 quarterly series of South Africa, ranging from real, nominal, and financial sectors. We also have intangible variables, such as confidence indices. In addition to national variables, the paper uses a set of global variables such as commodity industrial inputs price index and crude oil prices. The data also comprises series of major trading partners such as Germany (GE), the United Kingdom (UK), and the United States (US) of America. The in-sample period contains data from 1980Q1 to 2000Q4. All series are seasonally adjusted and covariance stationary. The more powerful DFGLS test of Elliott, Rothenberg, and Stock (1996), instead of the most popular ADF test, is used to assess the degree of integration of all series. All nonstationary series are made stationary through differencing. The Schwarz information criterion is used in the selection of appropriate lag length in such a way that no serial correction is left in the stochastic error term. Where there were doubts about the presence of a unit root, the KPSS test proposed by Kwiatowski, Phillips, Schmidt, and Shin (1992), with the null hypothesis of stationarity, was applied. All series are standardized to have a mean of zero and a constant variance. The details about the statistical treatment of all data are available upon request. The in-sample period contains data from 1980Q1 to 2000Q4, while the out-of-sample set is 2001Q1-2006Q4.

We consider the following FM specifications:

- FHLR: this is FM based on FHLR specification, including two dynamic common factors and five static common factors;

- UFAVAR: this is a SW FM which includes one of the variables of interest and five common static factors;

- MFAVAR: this is a SW FM which includes all the three variables of interest and five common static factors;

- UBFAVAR: this is a SW FM using one of the variables of interest and five common static factors, which, in turn, are estimated based on Bayesian restrictions discussed below in Section 4;

- MBFAVAR: this is a SW FM, with a specification similar to the MFAVAR, except that the current model applies Bayesian restrictions on lag of the variables, discussed in the next section.

4 Alternative Forecasting Models

In this study, the FMs, in its various forms, are our benchmark models. However, to evaluate the forecasting performance of the FMs, we require alternative models. In our case, these are namely, the unrestricted classical VAR, BVARs, and a NKDSGE model, developed recently by Liu *et al.* (2008) for forecasting the South African economy, besides the naive random walk (RW) model with a drift[2] This section outlines the basics of the above-mentioned competing models.

An unrestricted VAR model, as suggested by Sims (1980), can be written as follows:

$$Y_t = C + \sum_{i=1}^{p} A_i Y_{t-i} + \varepsilon_t \tag{5}$$

where Y_t is a $(n \times 1)$ vector of variables being forecasted; A_i, $i = 1.....p$ are $(n \times n)$ autoregressive matrices; C is a $(n \times 1)$ vector of constant terms; and ε is a $(n \times 1)$ vector of white-noise error terms.

One drawback of VAR models is overparameterization, which, in turn, leads to large out-of-sample forecasting errors. A popular alternative is to use a Bayesian VAR (BVAR) model. Instead of eliminating longer lags, the Bayesian method imposes restrictions on these coefficients. As described in Litterman (1981,1986a, 1986b), Doan *et al.* (1984), Todd (1984), and Spencer (1993), the "Minnesota" prior[3] means on variable j in equation i at lag m take the following form:

$$E(A_{ijm}) = \begin{cases} 1 & \text{if } i = j, k{=}1 \\ 0 & \text{otherwise} \end{cases}$$

The specification of the standard deviation of the distribution of the prior imposed on variable j in equation i at lag m, for all i, j and m, $S(A_{ijm})$, is given as follows:

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[2]Formally, $y_t = c + y_{t-1} + \epsilon_t$, with ϵ_t being a white-noise.

[3]The outlined prior is referred to as the "Minnesota prior" due to its development at the University of Minnesota and the Federal Reserve Bank at Minneapolis.

$$S(A_{ijm}) = [w \times g(m) \times f(i,j)]\frac{\hat{\sigma}_i}{\hat{\sigma}_j} \tag{6}$$

where:

$$f(i,j) = \begin{cases} 1 & \text{if } i = j \\ \\ k_{ij} & \text{otherwise, } 0 \leq k_{ij} \leq 1 \end{cases}$$

$$g(m) = m^{-d}, \qquad d > 0$$

The term w measures the standard deviation on the first own lag, and also indicates the overall tightness. A decrease in the value of w results in a tighter prior. The function $g(m)$ measures the tightness on lag m relative to lag 1, and is assumed to have a harmonic shape with a decay of d. An increase in d, tightens the prior as the number of lag increases. [4] $f(i,j)$ represents the tightness of variable j in equation i relative to variable i, thus, reducing the interaction parameter k_{ij} tightens the prior. $\hat{\sigma}_i$ and $\hat{\sigma}_j$ are the estimated standard errors of the univariate autoregression for variable i and j, respectively. In the case of $i \neq j$, the standard deviations of the coefficients on lags are not scale invariant (Litterman, 1986b: 30). The ratio, $\frac{\hat{\sigma}_i}{\hat{\sigma}_j}$ in (??), scales the variables so as to account for differences in the units of magnitudes of the variables.

Note the apriori assumptions on the coefficients $A_1......A_p$ are that they are independent and normally distributed. In addition, the covariance matrix of the residual is diagonal and known. Finally, the prior on C is a diffuse one.

Finally, the motivation to also use a NKDSGE model, besides the VAR and the BVARs, as a competing forecasting model to the FMs, emanates from a recent study by Liu *et al.* (2008). In this paper, the authors used a NKDSGE model[5], along the lines of Ireland (2004), and forecasted the growth rate, inflation, and the 91-days Treasury bill rate for the South African economy over the period of 2001Q1 to 2006Q4. The results indicated that, in terms of out-of-sample forecasting, the NKDSGE model outperformed both the unrestricted VAR and the BVARs for inflation, but not for per capita growth and the nominal short-term interest rate. However, the differences in the RMSEs were

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[4]In this paper, we set the overall tightness parameter (w) equal to 0.3, 0.2, and 0.1, and the harmonic lag decay parameter (d) equal to 0.5, 1 and 2. These parameter values are chosen so that they are consistent with the ones used by Liu *et al.* (2008).

[5]See also Liu and Gupta (2007) and Liu *et al.* (2007) for usage of calibrated and estimated versions of the Hansen (1985) Real Business Cycle model in forecasting the South African economy.

not significantly different across the models. Given, that South Africa has moved to an inflation targeting framework since February 2000, the ability of the NKDSGE model to outperform the VAR and the BVARs in terms of forecasting inflation, gave the model tremendous economic importance. Given this, we decided to incorporate the same NKDSGE model used by Liu *et al.* (2008), as one of the alternative models to the large FM. However, unlike the authors, we use a different period of estimation. While Liu *et al.* (2008) used 1970Q1 to 2000Q4, we started from the period of 1980Q1, which is the same as that for the FM.

Formally, the NKDSGE model is described by the following eight equations[6]:

$$\hat{x}_t = E_t\hat{x}_{t+1} - (\hat{r}_t - E_t\hat{\pi}_{t+1}) + \left(1 - \frac{1}{\eta}\right)(1 - \rho_a)\hat{a}_t \tag{7}$$

$$\hat{\pi} = \beta E_t\hat{\pi}_{t+1} + \psi\hat{x}_t - \hat{\theta}_t/\phi, \qquad \psi = \eta\left(\frac{\theta - 1}{\phi}\right) \tag{8}$$

$$\hat{r}_t = \rho_r\hat{r}_{t-1} + \rho_\pi\hat{\pi}_t + \rho_g\hat{g}_t + \rho_x\hat{x}_t + \varepsilon_{rt} \qquad \varepsilon_{rt} \sim i.i.d.(0, \sigma_r^2) \tag{9}$$

$$\hat{x}_t = \hat{y}_t - \frac{1}{\eta}\hat{a}_t \tag{10}$$

$$\hat{g}_t = \hat{y}_t - \hat{y}_{t-1} + \hat{z}_t \tag{11}$$

$$\hat{a}_t = \rho_a\hat{a}_{t-1} + \varepsilon_{at} \qquad 0 \leq \rho_a < 1, \ \varepsilon_{at} \sim i.i.d.(0, \sigma_a^2) \tag{12}$$

$$\hat{\theta}_t = \rho_\theta\hat{\theta}_{t-1} + \varepsilon_{\theta t} \qquad 0 \leq \rho_\theta < 1, \ \varepsilon_{\theta t} \sim i.i.d.(0, \sigma_\theta^2) \tag{13}$$

$$\hat{z}_t = \varepsilon_{zt} \qquad \varepsilon_{zt} \sim i.i.d.(0, \sigma_z^2) \tag{14}$$

Equations (**??**) and (**??**) models the expectational IS curve and the New Keynesian Phillips curve, respectively, while (**??**) presents the interest rate rule pursued by the monetary authority. Following Ireland (2004) and Liu *et al.* (2008), the terms of the NKDSGE model is defined as follows[7]: y_t measures the output; x_t is the output gap; r_t is the nominal short-term interest rate; π_t is the inflation rate; g_t output growth; η ($\geq$1) captures the degree of marginal disutility from labor; a_t is the preference shock; $0 < \beta < 1$ is the discount factor; θ_t is the cost-push shock; ϕ governs the magnitude of the cost of price adjustment; $\psi = \eta\left(\frac{\theta - 1}{\phi}\right)$; ε_{rt} captures the monetary policy shock; ρ_i, $i = a$, r, x, π, g, and θ, captures the persistence parameters, and; z_t is the technology shock. Given this, equations (10) and (11) defines stochastic process governing the deviations of the output-gap and growth rate, while, equations (12), (13) and (14) outlines the processes for the preference, cost-push and technology shocks, respectively.

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[6]See Ireland (2004) and Liu *et al.* (2007b) for details regarding the microfoundations of the model.

[7]A letter with a hat above indicates its deviation from its steady-state.

As far as estimation is concerned, the BVAR models are estimated using Theil's (1971) mixed estimation technique, which involves supplementing the data with prior information on the distribution of the coefficients. For each restriction imposed on the parameter estimated, the number of observations and degrees of freedom are increased by one in an artificial way. Therefore, the loss of degrees of freedom associated with the unrestricted VAR is not a concern in the BVAR. On the other hand, the NKDSGE model is in a state-pace form and can be estimated via the maximum likelihood approach.[8]

5 Results

In this section, we compare the one- to four-quarters-ahead RMSEs of the alternative models in relation to the large FMs for the out-of-sample forecast horizon of 2001Q1 to 2006Q4. However, before we proceed to the forecasting results, a discussion is due as to how the number of factors in the FMs were chosen. There are various statistical approaches in determining the number of factors in the FM. The most used in the literature of factor models are two, namely the Bai and Ng (2002) and the Forni et al. (2000) approach. On one hand, the number of static factors r is determined using the Bai and Ng (2002) selection criteria. And on the other hand, we estimate the number of dynamic factors q using the method proposed by Forni et al. (2000).[9] The second criteria suggests the choice of q be based on the variance explained by the ith eigenvalue. Furthermore, there should be a substantial gap between the variance explained by the qth eigenvalue and the $(q+1)^{th}$. Forni et al. (2000) propose to include factors as long as they explain a certain percentage of total variance, such as 5 percent. As indicated in Table 1 the Bai and Ng (2002) approach proposes five static factors based on IC_{p1} and IC_{p2} criteria, while the PC_{p1} and PC_{p2} criteria suggest seven factors. Following Bai and Ng (2002) we adopt the five factors based on IC_p over PC_p criteria, since they are more desirable in practice and they do not depend on the maximum number of factors included. Moreover, Bai and Ng (2007) also suggests five static factors and two primitive or dynamic factors. Similar to the latter method, the dynamic principal

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[8]For further details, please refer to Ireland (2004) and Liu *et al*. (2007, 2008). It must, however, be pointed out that The maximum likelihood technique used in this paper as developed by Ireland (2004) is subject to two problems: First,the estimation is quite sensitive to the starting values and; second, due to the relatively large number of coefficients, convergence can often be a problem. Hence ideally, one would want to use the recently developed Bayesian Markov Chian Monte Carlo methods to estimate the DSGE models.

[9]Hallin and Liska (2007) have recently proposed a more robust method for selecting the number of dynamic factors when the latter are estimated by dynamic principal components.

component technique, as proposed by Forni et al. (2000) suggests two dynamic factors as indicated in the last column of Table 1. The first two dynamic principal components explain approximately 99 percent of variation, while the eigenvalue of the third component is $0.005 < 0.05$.[10]

Table 1. Determining r and q:

k	PC_{p1}	PC_{p2}	PC_{p3}	IC_{p1}	IC_{p2}	IC_{p3}	PCA
1	0.838	0.840	0.831	-0.153	-0.149	-0.166	0.908
2	0.795	0.799	0.783	-0.186	-0.177	-0.212	0.087
3	0.770	0.776	0.751	-0.202	-0.189	-0.241	0.005
4	0.756	0.764	0.731	-0.206	-0.189	-0.259	-
5	0.744	0.755	0.713	***-0.211***	***-0.189***	-0.277	-
6	0.739	0.752	0.702	-0.208	-0.182	-0.287	-
7	***0.736***	***0.751***	0.693	-0.204	-0.173	-0.296	-
8	0.737	0.754	0.688	-0.196	-0.160	-0.300	-
9	0.739	0.758	0.684	-0.188	-0.148	-0.306	-
10	0.743	0.764	0.681	-0.179	-0.135	-0.310	-

Note: Denotes the minimum based on Bai and Ng (2002) criteria.

Given that we now know, how we determine the number of factors, four points must be emphasized regarding the forecasting exercise: First, unlike the FMs, the small-scale VAR, the small-scale BVAR and the NKDSGE are estimated using data[11] on only the three variables of interest, with all the data obtained from the Quarterly Bulletins of the South African Reserve Bank (SARB), except for the population size, which is obtained from the World Development Indicators of the World Bank. Note the RW model is estimated separately for each of the three variables; Second, even though the FMs incorporate global variables, given that the NKDSGE model is based on closed-economy, we, as

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[10]The cumulated variance of 99 percent is rare in the factor analysis literature. However, a considerable number of papers in the literature have found 2 dynamic factors to be optimal (Forni and Reichlin, 1998; Forni, Hallin, Lippi, and Reichlin, 2001; van Nieuwenhuyze, 2005).

[11]Note to be consistent with the NKDSGE model, we estimate the VAR and the BVARs using demeaned data for the three variables of interest, and, hence, stationarity is not an issue. The same was also vindicated by the Augmented Dickey-Fuller (ADF), Phillips-Perron (PP), Dickey-Fuller with GLS detrending (DF-GLS) and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) tests of stationarity. Besides, given that the Bayesian approach is entirely based on the likelihood function, the associated inference does not need to take special account of nonstationarity (Sims *et al.* (1990)).

in Liu *et al.* (2008), use the percentage change in the GDP deflator as an appropriate measure of inflation rather than the CPI, simply to ensure consistency of comparison between the alternative models; Third, the stable[12] (FA)VAR and the B(FA)VARs were estimated with four lags, as determined by the unanimity of the sequential modified LR test statistic, Final Prediction Error (FPE), Akaike Information Criterion (AIC), Schwarz Information Criterion (SIC), and the Hannan-Quinn Information (HQIC) criterion. While, the M(B)FAVAR were estimated with 8 lags determined by the LR crireia, FPE criteria, AIC and HQIC. Further, the U(B)FAVAR for growth and inflation were estimated with 8 lags as well, decided by the LR criteria, FPE criteria and AIC criterion for the former and the LR criteria and AIC by the latter. Finally the U(B)FAVAR for the Treasury bill rate was based on one lag as suggested by the SC and HQIC[13] and; Fourth, the optimality of the B(FA)VARs are based on the minimum average RMSEs[14] for the one- to four-quarters-ahead forecasts, produced by the combination of the values of the hyperparameters defining the overall weight (w) and tightness (d). The main results, as reported in Tables 2 to 4, can be summarized as follows:

- Per Capita Growth Rate: The MBFAVAR ($w = 0.2$, $d = 1$) outperforms all other models producing the lowest minimum average RMSEs. The "optimal" MBFAVAR is followed by the FHLR, the "optimal" BVAR, the "optimal" UBFAVAR, the unrestricted VAR, the FAVAR, the RW model, the NKDSGE and ultimately the MFAVAR;

- Inflation: As with per capita growth rate, the MBFAVAR ($w = 0.2$, $d = 1$) outperforms all the other models, followed by the FHLR. Contrary to the case of the per capita growth rate, where the "optimal" BVAR scores well among small-scale models, the classical VAR is the best performer in this case, with the RW model ranking as the overall last;

- 91-days Treasury bill rate: Unlike the above cases, the FHLR stands out in forecasting the Treasury bill rate,

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[12]Stability was ensured since no roots were found to lie outside the unit circle.

[13]Note, in order to decide on the "optimal" lag length, we required at least two of the lag-selection tests to agree on a particular lag-length.

[14]Zellner (1986: 494) pointed out that "the optimal Bayesian forecasts will differ depending upon the loss function employed and the form of predictive probability density function". In other words, Bayesian forecasts are sensitive to the choice of the measure used to evaluate the out-of-sample forecast errors. However, Zellner (1986) also indicated that the use of the mean of the predictive probability density function for a series, is optimal relative to a squared error loss function, and the Mean Squared Error (MSE), and, hence, the RMSE is an appropriate measure to evaluate performance of forecasts, when the mean of the predictive probability density function is used. Thus, the paper uses RMSEs to evaluate out-of-sample forecasting performances of alternative models.

when compared to other alternative models. The second and third best performers are, respectively, the "optimal" UBFAVAR (w = 0.1, d = 1) and the "optimal" MBFAVAR (w = 0.1, d = 2). The small-scale "optimal" BVAR performs the best amongst the four small-scale models. Note the RW model beats the VAR, BVAR, NKDSGE, UFAVAR and the MFAVAR.

Table 2. RMSE (2001Q1-2006Q4): Per Capita Growth

QA	1	2	3	4	Average
RW	0.995	0.998	1.019	1.046	1.015
VAR (4)	0.479	0.541	0.619	0.735	0.593
BVAR (w = 0.1, d = 2.0)	0.385	0.403	0.495	0.641	0.481
NKDSGE	0.9894	1.1259	1.7950	1.2081	1.2796
FHLR	0.489	0.431	0.438	0.443	0.450
UFAVAR	1.077	1.099	1.151	1.122	1.112
MFAVAR	1.325	1.295	1.400	1.375	1.349
UBFAVAR (w=0.2, d=1)	0.547	0.517	0.507	0.508	0.520
MBFAVAR (w=0.2, d=1)	0.467	0.416	0.401	0.432	***0.429***

QA: Quarter Ahead; RMSE: Root Mean Squared Error in %.

Table 3. RMSE (2001Q1-2006Q4): Inflation

QA	1	2	3	4	Average
RW	3.828	5.408	6.265	6.670	5.543
VAR (4)	0.262	0.313	0.370	0.434	0.345
BVAR (w=0.1, d=1)	0.261	0.311	0.375	0.465	0.353
NKDSGE	0.297	0.421	0.577	0.695	0.498
FHLR	0.303	0.269	0.275	0.272	0.280
UFAVAR	0.812	0.931	0.836	0.809	0.847
MFAVAR	1.077	1.291	1.329	1.417	1.278
UBFAVAR (w=0.2, d=1)	0.277	0.307	0.297	0.289	0.293
MBFAVAR (w=0.2, d=1)	0.264	0.287	0.266	0.257	***0.268***

QA: Quarter Ahead; RMSE: Root Mean Squared Error in %.

Table 4. RMSE (2001Q1-2006Q4): Treasury Bill Rate

QA	1	2	3	4	Average
RW	0.935	1.683	2.274	2.746	1.909
VAR (4)	0.921	1.769	2.505	3.113	2.077
BVAR (w=0.2, d=1)	0.918	1.699	2.343	2.865	1.956
NKDSGE	1.130	1.979	2.622	3.991	2.430
FHLR	1.691	1.205	1.160	1.035	***1.273***
UFAVAR	1.320	1.925	2.503	2.837	2.146
MFAVAR	3.257	4.864	6.286	7.996	5.601
UBFAVAR (w=0.1, d=2)	1.161	1.587	2.008	2.344	1.775
MBFAVAR (w=0.1, d=2)	1.142	1.581	2.023	2.366	1.778

QA: Quarter Ahead; RMSE: Root Mean Squared Error in %.

In order to evaluate the models' forecast accuracy, we perform the across-model test between the "optimal" FMs with that of the RW model, the VAR, "optimal" BVARs and the NKDSGE model. The across-model test is based

on the statistic proposed by Diebold and Mariano (1995). The test statistic is defined as the following. For instance, let $\{e_t^i\}_{t=1}^T$, with i = RW, VAR, BVARs, and NKDSGE, denote the associated forecast errors from the alternative models and $\{e_t^d\}_{t=1}^T$ denote the forecast errors from the alternative forms of the FM. The test statistic is then defined as $s = \frac{l}{\sigma_l}$, where l is the sample mean of the "loss differentials" with $\{l_t\}_{t=1}^T$ obtained by using $l_t = (e_t^i)^2 - (e_t^d)^2$ for all $t = 1, 2, 3, ..., T$, and where σ_l is the standard error of l. The s statistic is asymptotically distributed as a standard normal random variable and can be estimated under the null hypothesis of equal forecast accuracy, i.e. $l = 0$. Therefore, in this case, a positive value of s would suggest that the particular form of a FM outperforms the specific alternative model the comparison is made against in terms of out-of-sample forecasting. Results are reported in Table 5.

Table 5. Across-Model Test Statistics

Quarters Ahead	1	2	3	4
(A) Per capita growth				
FHLR vs. RW	1.9511**	1.9532**	2.1231**	2.1561**
MBFAVAR vs. VAR	-1.4474	1.6752*	1.9752**	2.4474**
MBFAVAR vs. BVAR	-1.5995	-1.4465	1.6917*	1.9251*
MBFAVAR vs. NKDSGE	1.9388*	1.7348*	3.3206***	2.5360**
(B) Inflation				
FHLR vs. RW	3.1193***	3.2515***	3.7511***	3.9919***
MBFAVAR vs. VAR	-1.4372	1.5061	1.6567*	1.7021*
MBFAVAR vs. BVAR	-1.4375	1.4969	1.6597*	1.7177*
MBFAVAR vs. NKDSGE	1.3574	1.5564	1.7487*	1.8448*
(C) Treasury bill				
FHLR vs. RW	-1.6512*	1.6501*	2.1552**	2.8061***
FHLR vs. VAR	-1.6721*	1.6866*	2.2401**	2.9112***
FHLR vs. BVAR	-1.6813*	1.6504*	2.2151**	2.8568***
FHLR vs. NKDSGE	-1.6068	1.6774*	2.4960**	2.9322***

Note:***(**)[*] indicates significance at 1%(5%)[10%] level of significance.

In general, and at least at the 10 percent level of significance, the "optimal" FMs tend perform better than the alternative models in predicting the three variables of our concern, for each of the one- to four-steps ahead forecasts.[15] In other words, based on the average RMSEs and, more importantly, the Diebold and Mariano (1995) test statistic, we have relatively strong evidence of the fact that there is significant statistical gain in using the "optimal" FMs over other atheoretical and theoretical alternatives in forecasting key macroeconomic variables in South Africa for majority

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[15]The exception to this being the one-quarter-ahead forecast for the short-term interest rate, relative to the RW model, the unrestricted VAR and the "optimal" BVAR.

of the one- to four-quarters-ahead forecast horizons.

Purely from an economic point of view, this paper in conjunction with the work of Kabundi (2007) highlights that the FM can not only be used to analyze the degree of synchronization of South Africa with the US with success, but the framework also has tremendous potential of being used as a forecasting tool when compared to small-scale models, given its ability to handle large amount of information on a wide set of variables that tends to affect a small open developing economy like South Africa, and in our context surely the three key macroeconomic variables, namely, per capita growth rate, inflation, and short-term rate of interest. The fact that there exist a FM that tends to outperform the naive RW model as well as small-scale models that only account for the role of a particular variable or the interaction amongst the variables of interest, clearly highlights the possible misspecification, in the sense that the latter set of models fail to utilize the effect of a large number of other variables not used in their estimation. Given that South Africa targets inflation since the February of 2000, the importance of having a model that can forecast inflation and the instrument used to achieve it is of paramount importance. Moreover, more accurate forecasts for nominal interest rate and the inflation rate, implictly implies better forecasts for the real interest rate - a key monetary policy variable. It is important to point out that the South African government has well-defined growth targets based on the Accelerated and Shared Growth Initiative for South Africa (ASGISA) program simply because per capita growth rate is, perhaps, one of the easiest measures of economic performance of a country. Besides, as suggested by Naraidoo and Gupta (2009), there exists strong evidence in South Africa of the role played by per-capita growth rate in the interest rate rule. Given this, the fact that FMs are well-suited for forecasting the per capita growth rate, over above interest rate and inflation, relative to alternative models, makes them an attractive forecasting tool.

At this stage, it is, perhaps, important to provide some possible economic intuition behind the results obtained. First, given that the FMs, whether it is the MBFAVAR (w=0.1, d=2) or the FHLR, are the best performing models, the results are a clear indication of the importance of information contained in the factors, which, in turn, are derived from 267 quarterly series. Second, the fact that the MBFAVAR with the most tight priors tends to perform better relative to all other models for growth and inflation, is an indication of the role that persistence plays in determining the future path of these variables. On the other hand, with the FHLR, which exploits information of leads and lags of variables, being the stand out performer for forecasting the short-term interest rate, the role of past information as well as expected values of the factors cannot be denied. This is not surprising, especially given that South Africa targets

inflation since February 2000, and, hence, one would expect the short-run interest rate to be determined by past as well as future values of important factors explaining most of the variation in the economy.

6   Conclusions

This paper assesses the forecasting performance of a large-scale FMs, accommodating 267 quarterly series for South Africa, in comparison to the RW model, the VAR, BVARs and a typical NKDSGE model. The model extracts five static factors and two dynamic factors that explains most of the variation in the entire panel. These factors are then used to forecast output growth, inflation rate and the nominal interest rate, based on large FMs estimated under classical and Bayesian assumptions over the period of 1980Q1 to 2000Q4.

The alternative models are evaluated based on the minimum average RMSEs for the one- to four-quarters-ahead forecasts over an out-of-sample horizon of 2001Q1 to 2006Q4. Overall, the results show that there exits a specific form of a FM, whether based on Bayesian assumptions or that incorporates both static and dynamic factors, which tends to outperform all other model in forecasting the three variables of interest, indicating the blessing of dimensionality.

At this stage, it is important to stress the following facts, and in the process identify future areas of research: (i) Practically speaking, a central bank, or for that matter any forecaster, would ideally want to include a large number of variables, into the forecasting model, to obtain forecasts for the variables of key interest. In this regard, the VAR estimation is disadvantaged due to the curse of dimensionality. The BVAR though, can be considered a valid alternative to FM as it can equally accommodate large number of variables, given that its estimation is based on the Theil's (1971) mixed estimation technique, which amounts to supplementing the data with prior information on the distribution of the coefficients, therefore, the loss of degrees of freedom associated with the unrestricted VAR is no longer a concern. (ii) In addition, due to fact that the problem associated with the degrees of freedom is no longer an issue for the FM and the BVAR, these models are also capable of forecasting simultaneously a large number of time series, beyond the possible key variables of interest; (iii) There are, however, limitations to using the Bayesian approach. Firstly, the forecast accuracy depends critically on the specification of the prior, and secondly, the selection of the prior based on some objective function for the out-of-sample forecasts may not be "optimal" for the time period beyond the period chosen to produce the out-of-sample forecasts, and; (iv) Finally, general to any traditional statistically

estimated models, for example the large FM, the RW model, the VAR, and BVARs used for forecasting at the business cycle frequencies, there are couple of other concerns. Such procedures perform reasonably well as long there are no structural changes experienced in the economy, but changes of this nature, whether in or out of the sample, would then render the models inappropriate. Alternatively, these models are not immune to the 'Lucas Critique'. Furthermore, the estimation procedures used here are linear in nature, and, hence, they fail to take into account of nonlinearities in the data. In this regard, the role of microfounded DSGE models cannot be disregarded. The fact that the NKDSGE, based on the sample period used, is outperformed by all the other models, mainly calls for a better DSGE model of the South African economy, by extending the current model to incorporate facts like habit persistence and wage rigidities, role of capital in the production process, but, perhaps, more importantly, the role of external shocks, given South Africa's small open economy structure.

But, whatever the limitations of the large FM, as we show in this paper, one cannot gainsay the importance of this kind of modeling strategy in forecasting three key variables, namely, per capita growth rate, inflation and the short-term interest rate for South Africa over the period of 2001Q1 to 2006Q4. Clearly, the FM tends to perform well, relative to alternative popular forecasting methods, in predicting the South African economy.

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