# Modelling exchange rate volatility dynamics: Empirical evidence from South Africa

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# Modelling exchange rate volatility dynamics: Empirical evidence from South Africa

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**Abstract**

In this paper, we extend the literature on modelling exchange rate volatility in South Africa by estimating a range of models, including some that attempt to account for structural breaks and long memory. We examine the key nominal exchange rates of the South African rand and replicate common findings in the literature; particularly that volatility is 'persistent'. We investigate whether this 'persistence' is due to structural breaks or long memory, and the extent of asymmetric responses of the rand to 'good news' and 'bad news'. Our results show that while long memory is evident in the actual processes, a structural break analysis reveals that this feature is partially explained by unaccounted shifts in volatility regime; the most striking finding is the remarkable fall in the estimates of volatility persistence when considerably more structural breaks than those identified in recent studies are detected and integrated into the generalised autoregressive conditional heteroscedasticity (GARCH) framework. Furthermore, the asymmetrical GARCH model results provide evidence of leverage effects, indicating that negative shocks imply a higher next period volatility than positive shocks. The empirical results also shed light on the timing and likely triggers of volatility regime switching.

**Keywords**: Asymmetry, GARCH, long memory, modelling volatility, structural change, volatility persistence

**JEL classification**: C13, C32, F31, G10 and G15

## 1 Introduction

Since the collapse of the Bretton Woods international monetary system, elevated volatility is a conspicuous attribute of exchange rates; emerging market currencies, in particular. With the demise of the dual exchange rate system on 10 March 1995, the ensuing gradual relaxation of exchange controls and the current 'noninterventionist' exchange rate policy of the South African authorities,

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rand volatility is perhaps not surprising.[1,2] However, the frequent and often persistent gyrations of the rand in the short-term (and the medium- to long-term swings in the currency) are of concern and require rigorous investigation.

Heightened exchange rate instability can have serious adverse and pervasive ramifications. In the absence of well-developed derivatives markets, unpredictable variations of exchange rates could mean large private and public sector losses or profits.[3] Even though exchange rate volatility – a short-run phenomenon – can have undesirable effects, its impact is lessened substantially by the availability of foreign exchange derivatives in the well-developed global foreign exchange market. However, the limited amount of long-term hedging instruments compared with their short-term counterparts has cost implications for international investors, importers and exporters; persistent medium- and long-run exchange rate misalignments can have depressing effects on the volume of trade, mainly exports. Price instability also impacts on the real sector of the economy: it can negatively affect fixed investment, economic growth and employment. In South Africa, currency volatility is an important element of exchange rate, monetary and macroeconomic policy decisions and there is thus a strong need for modelling and forecasting volatility. Understanding the sources of currency volatility can also go a long way in trying to contain this (largely but not entirely undesirable) phenomenon, and in turn curtailing its effects.[4]

A number of studies – largely in developed countries – employ GARCH models to characterize time-varying volatility of financial asset returns. Recent research builds on the earlier foundational work by Engle (1982), Bollerslev (1986), Nelson (1991), Ding *et al* (1993), Glosten, *et al* (1993), Inclan and Tiao (1994), Baillie *et al* (1996), Bollerslev and Mikkelsen (1996), Ding and Granger (1996), and Andersen and Bollerslev (1997[a] and 1998). Generally, failure to account for structural breaks and long memory can lead to spurious results. Furthermore, empirical evidence shows that exchange rates tend to be more sensitive to bad news than good news.

In this paper, we estimate the volatility – conditional and unconditional variances – of the key nominal bilateral and effective exchange rates of the rand. In

[1] Noninterventionist policy in this context means that the central bank does not intervene in the foreign exchange market to influence the rand, but instead occasionally accumulates foreign currency reserves, *albeit* nonaggressively and when market conditions are conducive (during spells of rand strength), to diminish exchange rate risk arising from external liquidity. The latter economic rationale for central bank intervention is a contentious issue though.

[2] Although the impact of structural features of the foreign exchange market such as exchange controls is also a controversial issue, a cross-section study by Canales-Kriljenko and Habermeier (2004) uncovers lower nominal effective exchange rate (NEER) volatility in countries where trade in domestic currency by non-residents is restricted; the limitation of banks' foreign exchange positions also tends to dampen NEER instability.

[3] National Treasury formulates exchange rate policy, although the central bank is mandated to implement the policy. So profits and losses incurred by the central bank related to its operations in the foreign exchange market are largely absorbed by National Treasury as expenditure in its budget.

[4] Exchange rate volatility may be desirable though for speculators and currency derivative sellers. Currency volatility also acts as a signal of uncertainty to market participants and policymakers.

summary, our key findings are: i) statistical test results indicate strong and widespread instability in unconditional volatility – between 20 and 44 breakpoints are detected; ii) volatility persistence falls markedly when fractional integration and a larger set of sudden and gradual structural shifts are accounted for; iii) results suggest an asymmetric response to news – negative shocks raise volatility more than positive ones of equal magnitude; and, iv) the timing of changes in volatility regimes, and thus their likely causes, are more or less consistent with the exchange rate level shifts detected in a recent study by May ($2015^a$).

The paper unfolds as follows. Section 2 reviews the related literature. Section 3 provides a motivation for the GARCH-type modelling approach, followed by a discussion of the volatility models that we include in this analysis. Data description and preliminary tests on the returns series are presented in section 4. Regression analysis estimates are reported and interpreted in section 5 – preceded by a detection of the structural break points. Section 6 presents a descriptive analysis of the volatility structural change points identified in the preceding section. Concluding remarks and some directions for future research are proposed in the epilogue.

## 2 Literature review

Measuring the extent of exchange rate volatility is useful, and a necessary precursor for prognosis of plausible sources of exchange rate instability and establishing the (direct and indirect) effects of such volatility, which can have economy-wide repercussions. The extensive research on exchange rate volatility undertaken over the past two decades or so, time-varying volatility in particular, reflects its importance in a host of financial areas such as investment, portfolio diversification, security valuation, risk management and derivative pricing.

A subset of these studies focuses specifically on the international currency prices of developed countries and some emerging markets to explore the presence of long-range dependence in the volatility of exchange rates. The common conclusion in a number of studies is that the volatility process of exchange rate returns is well described by a long-memory process implying that shocks to the volatility process tend to be persistent, having long-lasting effects. Andersen *et al* (2001), Andersen and Bollerslev ($1997^b$), Baillie *et al.* (1996), Ding and Granger (1996), and Dacorogna *et al.* (1993) are some cases in point.

Another group of studies explores the significance and nature of structural shifts in exchange rate volatility and its impact on estimation results. In this second set of studies, strong persistent conditional exchange rate volatility is to some extent a spurious feature due to unaccounted structural change. Structural shift adapted models change the dynamics of volatility – volatility persistence is much weaker suggesting that both structural breaks and long memory are important characteristics of the currency returns data. The magnitude of decline in estimated persistence varies across different exchange rates and samples. Thupayagale and Jefferis (2011), Duncan and Liu (2009), Morana and Beltratti (2004), Malik (2003), Nakatsuma (2000), and Beine and Laurent (2000) fall into the latter category of studies.

The asymmetrical response of volatility to positive and negative shocks is an additional property of exchange rate returns. The occurrence (or non-occurrence) and degree of asymmetry though is exchange rate series specific. Estimates from a Panel of nineteen Arab countries provide evidence of leverage effects for the majority of currencies,[5] indicating that negative shocks imply a higher next period volatility than positive shocks (Abdalla, 2012). Furthermore, some empirical results also tend to be asymmetric model specific; for example, exponential generalised autoregressive conditional heteroscedasticity (EGARCH) results suggest the existence of asymmetric behaviour in volatility, while the threshold generalised autoregressive conditional heteroscedasticity (TARCH) model remains insignificant in evidence from Pakistan (Mughal and Kamal, 2009).

# 3 Econometric methodology and motivation

GARCH-type models, employed in this study, are popular not only for their simplicity and parsimony, but also because of their generalisation of other measures of volatilities. We present a brief overview of the broad set of conditional volatility measures estimated in this study, to facilitate the interpretation of the estimation results in section 5. More detailed presentations are provided in the literature we cite. Stylised facts about volatility note several salient characteristics about financial time series. These include fat tail (leptokurtic) distributions of risky asset returns, asymmetry, time-varying volatility, volatility clustering, (pronounced) persistence, mean reversion, and comovements of volatilities across assets and financial markets. The GARCH class of conditional volatility models – probably the most extensively applied family of volatility models – are designed to deal with just this set of issues.

In the basic (G)ARCH models, returns, $r_t$, are assumed to have the following process defined as conditionally normally distributed with time-depending variance:

$$r_t = \gamma + \chi + \phi + \tau + \varepsilon_t \tag{1}$$

$$\varepsilon_t = h_t z_t \tag{2}$$

$$z_t \sim N(0, 1) \tag{3}$$

where the parameter $\gamma$ is the mean return, $\chi$ is the 1-lag autoregressive (AR(1)) parameter, $\phi$ is the 1-lag moving average (MA(1)) parameter, $\tau$ is the ARCH-in-mean (ARCH-M) parameter,[6] $\varepsilon_t (= h_t z_t)$ is the disturbance term and $z_t$ is purely random or white noise.[7] By assumption, $z_t$ is serially uncorrelated with

[5] Asymmetric models in conditional variance have been applied mainly to stock returns. A 'leverage effect' – when stock prices change, in response to news shocks, induces an inverse change in its volatility – is an additional property of financial time series (Black, 1976)

[6] Parameter $\tau$ captures the conditional variance (or standard deviation) impact (Engle *et al.* (1987)).

[7] Alternative specifications of the mean equation also include exogenous factors.

a mean equal to zero but the conditional variance, $h_t^2$, is time varying. The conditional variance denoted by $h_t^2$ is specified according to one of the competing GARCH class models presented below. The standardised residuals (or shocks) are measured as $z_t = \frac{\varepsilon_t}{h_t}$.[8]

### 3.1 Standard linear symmetrical ARCH and GARCH models

This group of models assumes that positive and negative shocks have a symmetric impact on conditional volatility:

$$ARCH(p): h_t^2 = \omega + \sum_{k=1}^{p} \alpha_k \varepsilon_{t-k}^2 \ ; \alpha_1 > \alpha_2 > ... > \alpha_p; \omega > 0; \alpha_k \geq 0 \tag{4}$$

$$\text{and } 0 \leq \sum_{k=1}^{p} \alpha_k < 1$$

$$GARCH(p,q): h_t^2 = \omega + \sum_{k=1}^{p} \alpha_k \varepsilon_{t-k}^2 + \sum_{j=1}^{q} \beta_j h_{t-j}^2 \ ; (p, \omega > 0); (q, \alpha_k, \beta_j \geq 0), \tag{5}$$

$$\text{and } \left[ \sum_{k=1}^{p} \alpha_k + \sum_{j=1}^{q} \beta_j \right] < 1$$

$$IGARCH(p,q): h_t^2 = \frac{\omega}{[1-\beta(L)]} + \left\{ 1 - \varphi(L)(1-L)[1-\beta(L)]^{-1} \right\} \varepsilon_t^2; \tag{6}$$

$$\sum_{k=1}^{p} \alpha_k + \sum_{j=1}^{q} \beta_j = 1$$

where $p$ refers to the lag on the disturbance term, $\varepsilon_t^2$, and $q$ to the lag on the conditional variance, $h_t^2$.[9] The $ARCH(p)$ model (equation 4) proposed by Engle (1982) is the simplest case in the ARCH family. The original GARCH$(p,q)$ model introduced by Bollerslev (1986), an extension of the basic ARCH model which includes lags of $h_t^2$ to avoid long lag lengths on $\varepsilon_t^2$, is given by specification (5). To account for persistence of volatility, a limitation of the standard GARCH model, Bollerslev and Mikkelsen (1996) developed the IGARCH model

[8] The estimated residuals, $\hat{\varepsilon}_t$, and estimated conditional standard deviation, $\hat{h}_t$, are measured in the units in which the regressand is measured. The values of the standardised estimated residuals, $\hat{z}_t$, will therefore be pure numbers (devoid of units of measurement) and can be compared to the standardised residuals of other regressions.

[9] Note that for $h_t^2$ to be interpreted as a (conditional) variance, it must always be nonnegative.

in equation (6).[10] If $\sum_{k=1}^{p}\alpha_k+\sum_{j=1}^{q}\beta_j=1$,[11] then shocks to conditional variance are persistent and the shock is always present, in contrast to where the shock dies out when $\sum_{k=1}^{p}\alpha_k+\sum_{j=1}^{q}\beta_j<1$.[12] In the IGARCH model, conditional variance is now a hyperbolic function representing a gradual decay in the effects of shocks. An integrated model has been shown to be powerful for prediction over a short horizon, as it is not conditioned on a mean level volatility, and as a result it adjusts to changes in unconditional volatility quickly (Poon and Granger, 2003).

## 3.2 Standard nonlinear asymmetrical GARCH models

In the symmetric GARCH models, positive and negative shocks of the same magnitude have the same effect on conditional volatility. The nonlinear extensions of the GARCH model presented below were designed to allow for asymmetries; for example, different effects of 'good news' (positive shocks) and 'bad news' (negative shocks):

$$EGARCH(p,q):\ln h_t^2=\omega+\sum_{k=1}^{p}\alpha_k g(z_{t-k})+\sum_{j=1}^{q}\beta_j\ln h_{t-j}^2;\ g(z_t)\equiv\theta_1 z_t+\theta_2\left[|z_t|-E|z_t|\right] \tag{7}$$

$$GJR-GARCH(p,q):h_t^2=\omega+\sum_{k=1}^{p}\left(\alpha_k\varepsilon_{t-k}^2+\alpha_k^* S_{t-k}^-\varepsilon_{t-k}^2\right)+\sum_{j=1}^{q}\beta_j h_{t-j}^2 \tag{8}$$

$$APARCH(p,q):h_t^\delta=\omega+\sum_{k=1}^{p}\alpha_k\left(|\varepsilon_{t-k}|-\alpha_k^*\varepsilon_{t-k}\right)^\delta+\sum_{j=1}^{q}\beta_j h_{t-j}^\delta \tag{9}$$

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[10] Using the lag or backshift operator, $\alpha(L)=\alpha_1L^1+...+\alpha_pL^p$ and $\beta(L)=\beta_1L^1+...+\beta_qL^q$ and manipulating equation (5), this GARCH$(p,q)$ process mayalso be expressed as an ARMA $(m,p)$ process in $\varepsilon_t^2$ : $[1-\alpha(L)-\beta(L)]\,\varepsilon_t^2=\omega+[1-\beta(L)]\left(\varepsilon_t^2-h_t^2\right)$. When the $[1-\alpha(L)-\beta(L)]\,\varepsilon_t^2$ polynomial contains a unit root, that is, the sum of all the $\alpha_k$ and the $\beta_j$ equals unity, the IGARCH model in equation (6) is derived, where $\varphi(L)=[1-\alpha(L)-\beta(L)]\,(1-L)^{-1}$. (Mathematical derivation in May (2015$^b$).

[11] This condition can also be written as an approximation: $\sum_{k=1}^{p}\alpha_k+\sum_{j=1}^{q}\beta_j\approx 1$.

[12] Conditional variance in equation (5) is stationary but nonstationary in equation (6).

In the conditional variance equation (7) of the exponential GARCH (EGARCH) model, proposed by Nelson (1991), the asymmetry effect is introduced by the nonlinear function $g(z_t)$: the term $\theta_2 [|z_t| - E|z_t|]$ represents the *magnitude effect* (deviations between realised and expected standardised shock) and the term $\theta_1 z_t$ is the *sign effect* (negative shock or positive shock). The EGARCH model allows financial markets to respond asymmetrically to 'bad news', a negative shock ($z_t < 0$), and 'good news', a positive innovation ($z_t > 0$), even though the observed shocks are of same magnitude or absolute value. The GJR-GARCH model (equation (8)), proposed by Glosten, Jagannathan, and Runkle (GJR) (1993) presents an alternative where $S^-_{t-k}$ is a dummy variable that takes the value unity when $\alpha^*_k$ is negative and zero when it is positive.[13] In the asymmetric power ARCH (APARCH) model (specification (9)), introduced by Ding, Granger, and Engle (1993), the parameter $\delta$ plays the role of a Box-Cox power transformation of the conditional standard deviation process and the asymmetric absolute residuals,[14] while $\alpha^*_k$ reflects the so-called 'leverage effect'. A benefit of this model is that it combines the flexibility of a varying exponent with the asymmetry coefficient to account for the 'leverage effect'. The APARCH model nests several of the most popular univariate parameterizations such as the standard linear GARCH and GJR-GARCH models.

### 3.3 Modelling short and long memory: Fractionally-integrated GARCH (FIGARCH) models

In some financial time series, volatility tends to die off quite slowly thus making the razor's edge distinction between stationary, $I(0)$, and unit root, $I(1)$, processes too restrictive. The FIGARCH model proposed by Baillie *et al.* (1996) allows for non-integer orders of integration $I(d)$, $(0 < d < 1)$, and thus for more subtle mean reverting behaviour in time series. This class of GARCH structure is based on a ARFIMA-type representation model so that the ARMA parameters capture the short-run behaviour of the time series while the fractional parameter allows for modelling the long-run dependence. Muller *et al.* (1997) provide economic justifications for long memory behaviour in a heterogeneous market with diverse agents: "Short-term traders evaluate the market at a higher frequency and have a shorter memory than long-term traders" while "long-term traders may look at the market only once a day or less frequently". The FIGARCH model is an extension of the IGARCH model,

$$FIGARCH(p,d,q) : h_t^2 {=} \frac{\omega}{[1-\beta(L)]} + \left\{1 - \varphi(L)(1-L)^d 1 - \beta(L)^{-1}\right\} \varepsilon_t^2 \quad (10)$$

[13] Engle and Ng (1993) find that the EGARCH model leads to a conditional variance that is too high and more volatile than the GJR-GARCH, although it captures most of the asymmetry. Thus, the EGARCH model is more appropriate for capturing heightened short-term volatility during a crisis.

[14] Statisticians Box and Cox's (1964) method is one particular way of parameterising a power transform – a suitable power transformation is automatically identified for the data which can make big improvements in model fit.

where the fractional differencing parameter, $d$, indicates the rate of decay; that is, the speed at which shocks die out over time. Although mean reverting, shocks to $h_t^2$ will die out at a slow hyperbolic rate of decay determined by $d$ in the variance equation, while the short-run dynamics are modelled by the conventional AR(1) and MA(1) parameters in the GARCH model variance equation. The IGARCH has $d = 1$, and the standard GARCH has $d = 0$.

Bollerslev and Mikkelson (1996) extend the fractional integration idea to the EGARCH model,

$$FIEGARCH(p, d, q) : \ln h_t^2 = \omega + \varphi(L)^{-1}(1-L)^d [1 + \alpha(L)] g(z_{t-1}). \quad (11)$$

Tse's (1998) FIAPARCH($p, d, q$) model is represented by:

$$FIAPARCH(p, d, q) : h_t^\delta = \omega + \left\{1 - [1 - \beta(L)]^{-1} \varphi(L)(1-L)^d\right\} (|\varepsilon_t| - \alpha^* \varepsilon_t)^\delta . \quad (12)$$

## 3.4 Modelling structural change

In some instances, there may be obvious points at which a break in structure might have taken place – a war, geopolitical tensions, a piece of legislation, an oil shock, a policy framework shift, financial market liberalisation, a change in investors' behaviour, *etcetera*. Ignoring structural changes in estimations is a misspecification of the conditional variance which may produce estimates incorrectly suggesting persistence; that is, a stationery process being misinterpreted as a non-stationery one. A second consequence is that forecasting may be undermined.

**Sudden Structural Change GARCH (SSC-GARCH) Class Models**

The simplest way to account for structural breaks involves the use of dummy variables – SSC-GARCH models account for known and unknown breaks in the data using dummy variables. Inclan and Tiao (IT) (1994) propose a procedure based on an iterated cumulative sums of squares (ICSS) to detect multiple change points in the variance of a sequence of independent observations or stochastic process. Sanso *et al.* (2004) note drawbacks in the IT test. The IT test assumes that the disturbances are independent and Gaussian, while preliminary tests in section 4 show that the distributions are leptokurtic and asymmetric conditional volatility is persistent. Thus the IT test is strictly appropriate only when the stochastic process is mesokurtic and the conditional variance is constant. The test may exhibit large size distortions for leptokurtic and platykurtic innovations, possibly (but not certainly) invalidating its use in the time series of floating exchange rates, and financial time series in general. If the distribution is leptokurtic or heavy tailed, one can expect many rejections of the constant variance null hypothesis, implying that some (but not necessarily all) of the structural breaks detected by IT tests may be spurious. To overcome these problems, Sanso *et al.* (2004) propose new tests that take the fourth order moment properties of disturbances and conditional heteroscedasticity into explicit account. The $ICSS(\kappa_1)$ statistic controls for the kurtosis of the series, while the $ICSS\left(\kappa_2\right)$ statistic controls for kurtosis and conditional heteroskedasticity (Sanso *et al*, 2004).

Following Vyrost *et al.* (2011), the structural breaks that are detected are used to partition the observations into groups corresponding to regimes, during which the variance is considered to be constant. For example, the simple GARCH(1,1) model with breaks is expressed as:

$$h_t^2 = \omega + \sum_{k=1}^{p} \alpha_k \varepsilon_{t-k}^2 + \sum_{j=1}^{q} \beta_j h_{t-j}^2 + \sum_{i=0}^{N_T} \gamma_i D_i\left(t\right) \tag{13}$$

where $\sum_{i=0}^{N_T} \gamma_i D_i\left(t\right)$ is the additional explanatory variable in the variance equation of a standard GARCH model, and the persistence of volatility is still given by $(\alpha + \beta)$. An advantage of all the above breakpoint tests – IT tests and modifications of IT tests, $\kappa_1$ and $\kappa_2$ tests – is that once the 'potential' breakpoints have been identified and verified, one can further explore the possible and likely causes of each of the structural shifts.

**Adaptive-GARCH (A-GARCH) Class Models**

To account for the persistence of the conditional variance process, Ding and Granger (1996) and Baillie *et al.* (1996), amongst others, proposed the adaptive-GARCH (A-GARCH) class models, an alternative to the SSC-GARCH approach. Simpler ARCH and GARCH models allow the conditional variance to change over time leaving the unconditional variance constant. A-GARCH models allow for time variance in both the conditional and unconditional variance. Baillie and Morana (2009) introduced the long memory volatility adaptive-FIGARCH (A-FIGARCH) model to account for both long memory and structural change in the unconditional variance. Allowing for the intercept $\omega$ in the conditional variance equation to be time-varying according to Andersen and Bollerslev's (1997[a] and 1998) flexible functional form, Baillie and Morana's (2009) A-FIGARCH conditional variance equation may be written in a form analogous to the FIGARCH model as:

$$\begin{aligned} h_t^2 &= \omega_0 + \sum_{j=1}^{k} \left[\psi_j \sin\left(2\pi jt/T\right) + \rho_j \cos\left(2\pi jt/T\right)\right] \\ &\quad + \left[1 - \varphi\left(L\right)\left(1-L\right)^d \left[1 - \beta\left(L\right)\right]^{-1}\right] \varepsilon_t^2. \end{aligned} \tag{14}$$

In order for the conditional variance to be positive almost certainly at each point in time requires

$$\omega_t = \omega_0 + \sum_{j=1}^{k} \left[\psi_j \sin\left(2\pi jt/T\right) + \rho_j \cos\left(2\pi jt/T\right)\right] > 0 \tag{15}$$

$$\text{and } \left[1 - \varphi\left(L\right)\left(1-L\right)^d \left[1 - \beta\left(L\right)\right]^{-1}\right] \varepsilon_t^2 \geq 0.$$

In this study, the flexible functional form or time-varying unconditional variance is extended to the other GARCH models; A-FIEGARCH and A-FIAPARCH models are two innovations in this empirical research. A great advantage of the A-GARCH type models approach over the ICSS procedure is that structural shifts can be incorporated in the variance equation without identifying the breaks, potentially a more efficient approach. An obvious drawback of A-GARCH type models is that one cannot identify structural breakpoints, inhibiting an investigation of their likely causes, an advantage of the ICSS procedure. Here, we estimate both types of models. This is a valuable exercise – the regression results allow one to compare and contrast the effectiveness of each model in capturing time-varying unconditional variance.

# 4 Data

## 4.1 Data issues

The sample period covers 13 March 1995 to 31 August 2010.[15] This empirical study analyses the key daily nominal bilateral exchange rate levels of the rand – US dollar/rand (USD/ZAR), euro/rand (EUR/ZAR), British pound (sterling)/rand (GBP/ZAR), and Japanese yen/rand (JPY/ZAR). To receive aggregated information, the returns of the 15-currency nominal effective exchange rate (NEER) of the rand are also examined.[16] Daily exchange rate data were kindly provided by the South African Reserve Bank (SARB).

The continuously compounded return is defined as $r_t = \ln\left(e_t/e_{t-1}\right) * 100$ where $e_t$ is the spot rate on day $t$ for the daily series. After filtering the data for calendar effects – weekends and local public holidays – the full daily sample of returns consists of 3864 observations for each exchange rate.[17]

## 4.2 Preliminary test results

As we can see from Table A1, Panel A (Appendix), all the returns show evidence of non-normality with negative skewness, which means that the left tail is particularly extreme, and the kurtosis statistics suggest that probability distribution functions are peaked or leptokurtic. Because shocks to the US dol-

[15] Our analysis is a revised extract from the PhD in Economics dissertation (May, 2015$^b$). The start point of this sample horizon is motivated by the South African authorities' reversion to a single exchange rate mechanism on March 13, 1995.

[16] The currencies in the trade basket and their weights, expressed as percentages in descending order of importance, are: Euro (34.82), US dollar (14.88), Chinese yuan (12.49), British pound (10.71), Japanese yen (10.12), Swiss franc (2.83), Australian dollar (2.04), Indian rupee (2.01), Swedish krona (1.99), South Korean won (1.96), Hong Kong dollar (1.48), Singapore dollar (1.40), Brazilian real (1.37), Israeli shekel (1.11), and Zambian kwacha (0.80). The indirect foreign exchange rates of the rand (foreign currency per unit of rands) are used to ensure that the bilateral rate quotations are consistent with the NEER quotation – the SARB compiles the indirect NEER of the rand.

[17] Although the cuts do not capture all the holiday market slowdowns such as holidays of the US, UK, Germany and Japan (G4 economies), they do succeed in eliminating the most important such daily calendar effects.

lar are typically transmitted to other bilateral (floating) exchange rates, the rand crosses show relatively weaker non-normality; modest asymmetry in the yen/rand exchange rates, in particular, may be due to greater foreign exchange market interventions by the Bank of Japan (Bank of Japan, 2000). Pronounced non-normality in the NEER is perhaps not surprising as its level is determined by continuous random changes in all its 15 components, including high-risk emerging market currencies.

The presence of unit roots in the returns series is formally tested by applying a battery of tests.[18] All the returns series variables appear stationary at the 1% level of significance; in stark contrast to the inability to reject the unit root null in exchange rate levels, even in the presence of structural shift (May, 2015[a]).

All the graphs in Figure A1 (Appendix) indicate that the foreign currency returns exhibit volatility clustering – periods of low volatility tend to follow periods of high volatility, with high volatility periods concentrated in the vicinity of global crises and domestic financial markets upheavals.

The *Ljung Box* or modified $Q$-statistics reported in Panel B of Table 1 (Appendix) give no strong evidence of serial correlation in any of the exchange rate returns. With respect to the squared returns, in Panel C (Table 1, Appendix), the *Ljung Box* $Q$-statistics give clear indication of serial correlation, and the Engle (1982) Lagrange multiplier statistics in Panel D (Table 1, Appendix) offer significant evidence of ARCH effects, an indication that the data are candidates for GARCH-type modelling.

# 5 Empirical results of GARCH (1,1) models

A particularly appropriate non-Gaussian (or non-normal) error distribution for financial time series is the asymmetric Student's $t$-distribution to capture both skewness and excess kurtosis in the standardised residuals (Fernandez and Steel, 1998).

## 5.1 Basic symmetric and asymmetric GARCH models estimation results

Detailed estimation results are presented in the Appendix. The mean equation results for the basic symmetric and asymmetric GARCH models are in Panel A of Tables B1-B5 (Appendix). To remove serial correlation in the standardised residuals, the EUR/ZAR and JPY/ZAR are modelled with the augmented mean equation – the explanatory variables are the mean returns, AR(1) and MA(1). Overall, the results for the augmented mean equation, an improved specification, are still inadequate to remove autocorrelation for up to 20 lags in the

[18] The augmented Dickey-Fuller (ADF) test (1979), the Phillips-Perron (PP) test (1988), Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) test (1992) and the Dickey-Fuller Generalised Least Squares (DF-GLS) test proposed by Elliott, Rothenberg and Stock (ERS) (1996). The comprehensive set of unit root test results for both the exchange rate levels and returns are available from the authors upon request.

raw standardised residuals for the USD/ZAR, GBP/ZAR and NEER returns – persistence of autocorrelation in these series implies that currency returns are (additionally) being driven by exogenous factors. The specifications of the mean equation for the latter three returns were improved by adding two exogenous explanatory variables – percentage change in the USD gold price (with parameter $\kappa$) and percentage change in EUR/USD exchange rate (with parameter $\nu$) – and omitting the AR and MA explanatory variables. For the EUR/ZAR and JPY/ZAR series, the AR(1) and MA(1) coefficients, $\chi$ and $\phi$ respectively, are statistically significant at the 1% level. A positive mean reversion parameter, $\chi$, suggests that there is a tendency for a high (low) currency return in one period to be followed by a higher (lower) return in the next period; that is, the presence of volatility clustering, highly persistent returns and a short memory process in the level of the returns with a tendency to revert to their long-run average or time-varying mean after an extended period. Ubiquitous negatively signed MA(1) parameters, $\phi$, imply that the effect of the shock in period $t-1$ on the return in period $t$ dissipates weakening the combined effect of the current and immediate past shocks on the current return. For the USD/ZAR, GBP/ZAR and NEER returns, both the $\kappa$ and $\nu$ parameter estimates are statistically different from zero and correctly signed. For example, an increase in the US dollar gold price causes rand appreciation against the US dollar,[19] and euro appreciation against the US dollar translates into a higher dollar price of rands (the rand generally tracks the euro due to strong economic ties, trade and finance, in particular, between the euro zone and South Africa). The ARCH-M model is often used in financial applications where the expected return on an asset, rand holdings by foreigners in this instance, is related to the expected asset risk – the estimated coefficient on the expected risk, $\tau$, is a measure of the risk-return trade-off. Although all positively signed, suggesting that the increased risk of converting foreign currency denominated assets into rand holdings is associated with an excess return, none of these coefficients are statistically significant at the 1% level; a weak risk-return relationship is nevertheless evident in the pound/rand exchange rate.

The conditional variance results from the standard GARCH-type models show that all the estimated ARCH($\alpha_1$) and GARCH($\beta_1$) coefficients are significant at the 99% level of confidence (Panel B in Tables B1-B5, Appendix). This provides evidence of volatility clustering.

The signs and magnitudes of the symmetric GARCH and IGARCH point estimates ( $\alpha_1 = \underline{+}0.12$ and $\beta_1 = \underline{+}0.88$ ) are generally consistent with their respective values reported in the empirical finance and financial economics literature reviewed. The positive signed $\alpha_1^*$ for all currency returns series when the GJR-GARCH model is implemented makes sense because the impact of negative shocks on volatility is measured by the size of $\alpha_1+\alpha_1^*$, and $\alpha_1$ captures the effect of positive shocks; $\alpha_1 > 0$ and $\alpha_1^* > 0$, evident in all GJR model estimation results ensures that $\alpha_1+\alpha_1^* > \alpha_1$ so that negative shocks have a greater impact

[19] However, the relative importance of gold in SA exports declined to around 10% in 2010 from 22% some-odd in 1995.

on volatility than positive ones. The less robust asymmetry in the USD/ZAR – measured by the difference between positive and negative shocks – suggests that its news impact curve (NICs) is much closer to a symmetric news impact curve than that of its counterparts.[20] A negative signed and statistically significant $\theta_1$, across all data sets using EGARCH, is evidence of a leverage effect; in line with expectations since positive shocks tend to have smaller impacts. The absolute value of the parameters $\theta_1 + \theta_2$ in the EGARCH model reflects the magnitude of the positive shocks $\left(\varepsilon_{t-1}^2 > 0\right)$ and the absolute value of the parameters $\theta_1 - \theta_2$ reflects the magnitude of the negative shocks $\left(\varepsilon_{t-1}^2 < 0\right)$. Indeed, $|\theta_1 - \theta_2| > |\theta_1 + \theta_2|$ for all estimated coefficients.

If $\alpha_1 + \beta_1 < 1$, the process $\varepsilon_t^2$ is second order stationary, and a shock to the conditional variance, $h_t^2$ (or its variants) has a decaying impact on $h_{t+k}^2$ and is asymptotically negligible. A closer look at the variance equation parameters in Panel B (Tables B1-B5, Appendix) reveals that $\alpha_1 + \beta_1 \approx 1$ and $\alpha_1 + \alpha_1^* + \beta_1 \approx 1$ for the GARCH(1,1) and GJR-GARCH(1,1) model results for almost all currency return volatilities; that is the conditional variance of currency returns are approximately nonstationary indicating that volatility shocks are highly persistent. This result is often observed in high frequency data when structural breaks are not accounted for. However, $\varepsilon^+ : \alpha_1 + \beta_1 = 0.9282$ for positive shocks to JPY/ZAR returns in the GJR model suggests its variance is mean reverting but the rate of decay of shocks is very slow. When structural shifts are ignored, overall, the results are consistent with some of the empirical work; that is, currency return volatility is also highly persistent when the symmetric GARCH(1,1) model and the (simpler) asymmetric GJR-GARCH(1,1) models are applied to financial asset and currency returns data. Also, the much higher EGARCH values for $\alpha_1 + \beta_1$, significantly above unity, corroborates Engle and Ng (1993) findings that the EGARCH model is found to lead to a conditional variance that is too high and more volatile than the GJR-GARCH, although it captures most of the asymmetry. The APARCH model estimates lie between the latter two sets of estimates – the statistically significant power transformation parameter, $\delta$, in APARCH suggests that the power transformation identified by APARCH is suitable for the data; but not necessarily the best. 'Best fit' model ranking is explored in a subsequent study only after taking into account structural shift.

Fat tails and asymmetry are evident in the error distributions regardless of the model applied or time series estimated (Panel C in Tables B1-B5, Appendix).

## 5.2 Long memory and structural change: GARCH model estimation results

Beine and Laurent (2000) integrate the long memory and structural breaks approaches and empirically show that there is evidence of a strong interaction between volatility persistence and structural change but find that these two

[20] NICs, introduced by Pagan and Schwert (1990) and popularised by Engle and Ng (1993), measure how new information is incorporated into volatility estimates.

salient features in time series exchange rate data are imperfect substitutes in the sense that both characteristics are necessitated to capture all of the observed persistence in volatility.

One purpose of estimating the mean, $\omega$, in the variance equation is to calculate the constant unconditional variance or volatility, ${}_u\sigma^2$. In Tables B1 to B5 (Appendix), the null hypothesis that the unconditional variance is constant at the 1% level significance for some exchange rate returns, and at the 5% level of significance for other currency returns, is rejected in all instances; this provides a justification for modelling volatility with structural shift parameters. The joint Nyblom-Hansen (NH) test can be used to verify the constancy of the mean and variance equation parameters, and the error distributions (Nyblom, 1989; Hansen, 1990). The test is approximately the Lagrange multiplier test (or locally most powerful test) of the null of constant parameters – that is, there is no structural change – against the alternative that the parameters follow a martingale. The alternative incorporates simple structural breaks of unknown timing as well as random walk parameters.[21] We do not reject the null of parameter stability if the Nyblom statistic for a parameter is less than the critical value; the asymptotic 1% and 5% critical values are reported in Panel F of Tables B1 to B5 (Appendix). All the joint *NH* statistics test statistics obtained from the basic symmetric and asymmetric GARCH models unambiguously indicate joint instability in all the model parameters and justify extending the basic models to incorporate structural shifts.

The strong and widespread evidence of both long memory and instability in the variance equation parameters motivates the combined fitting of fractional integration and structural change to the GARCH models. Tables B6 to B10 (Appendix) report the estimation results for the A-FIGARCH-type models – accounting for both long memory and smooth transitional structural change. The mean equation estimation results are uniform to those produced by the simple GARCH models in Tables B1 to B5 (Appendix) – the signs of the parameters remain the same whilst the sizes of the coefficients, standard errors and $p$-values are only marginally different. The long memory parameter (*d-FIGARCH*) is statistically significant at the 99% level of confidence across the board – confirming long-run dependence behaviour evident in financial assets nominal prices. The most appropriate flexible functional form (trigonometric function) used to capture smooth structural changes varies across both currency returns and models. Here, only the results for the significant ones are reported. Perhaps the most crucial findings are that the unconditional variance (or long-run variance), ${}_u\hat{\sigma}^2$ (reflected in $p-$values for $\omega$ in the variance equation results), is no longer nonstationary when long memory and smooth structural change are accounted for in the simple GARCH framework.

The stationarity or nonstationarity of the conditional variance is captured by the volatility persistence statistics, $(\alpha_1 + \beta_1)$ in the symmetric models, and $\alpha_1 + \alpha_1^* + \beta_1$, $|\theta_1 + \theta_2| + \beta_1$ and $|\theta_1 - \theta_2| + \beta_1$ in the asymmetric models (in

[21] See Hansen (1990) for a description of the test statistics and discussion of their interpretation.

Tables B1-B10, Appendix). The conditional variance of shocks in the symmetric GARCH (1,1) model and positive shocks when less extreme assymetric APARCH is applied now appear stationary, but conditional variance remains nonstationary for negative shocks in the APARCH model and in the extreme EGARCH model, regardless of the shock sign (except for yen/rand series); *albeit* lower.

Although the Nyblom test can be informative about the type of structural change (detect whether the structural change is in the mean and/or variance equations parameters), and the A-GARCH-type models flexible functional form captures smooth structural change, neither one identify the actual break points as required by the SSC-GARCH models. Estimation of the SSC-GARCH models is a four-step procedure. First, the breakpoints of the different volatility regimes are identified in the residuals of the mean equations using the standard ICSS, ICSS ($\kappa_1$), and ICSS ($\kappa_2$) tests. The variance equations are then extended with dummy variables regressors to capture the latter breakpoints. The SSC-GARCH model is then estimated with all the breaks identified in the latter set of tests, and then re-estimated with only the statistically significant breaks that influence conditional variance.

Table A2 (Appendix) reports the number of change points identified by the ICSS procedure. The $\kappa_1$ and $\kappa_2$ structural break point tests detect a substantially lower number of breaks (not reported here), but to begin from a more general situation, we employed the significant ICSS shifts in the conditional variance equation. The largest number of breaks, in absolute terms, are identified in the US dollar/rand series with the yen/rand returns detecting the least – less than 50% of those in the US dollar/rand data. Relatively speaking, for each data series, statistically significant variance equation breakpoints range between 73% to 83% of the total change points identified - the euro/rand and US dollar/rand are the extrema – suggesting that the ICSS tests are still quite robust in the presence of non-normality in the disturbances.

The SSC-GARCH models results are presented in Tables B11 and B12 (Appendix). When attempting to estimate the SSC-GARCH models incorporating fractional integration, no convergence is achieved using numerical derivatives. Algorithms often encounter problems in locating the maximum likelihood estimates which is unsurprising in this instance given the large number of structural shifts – 16 to 44 breaks. The problem of no convergence also arises in the more complex and demanding asymmetric EGARCH and APARCH models. An extreme difficulty in convergence may be an indication that the model chosen is too complex and does not describe the data well and hence the most effective way of avoiding convergence problems is to select a simpler model that adequately describes the data. Silva and Tenreyro (2011) argue that although in some cases it is not possible to bypass this problem using some sort of data transformation, using different optimisation methods or specifications can address the problem. Even when estimating the SSC-GARCH and SSC-GJR-GACH models without fractional integration, in some cases, the inclusion of endogenous and exogenous variables in the mean equation also lead to nonconvergence. Using both comparative frameworks, $(\alpha_1 + \beta_1)$ for positive shocks and $\alpha_1 + \alpha_1^* + \beta_1$ in the case of negative shocks (in Tables B11 and B12, Appendix) are much lower for the SSC-GARCH models than those produced by the adaptive-GARCH models (in Tables B6 to B10, Appendix) suggesting that models with the shifts observed in the data are better approximated by abrupt structural change as opposed to smooth structural change.

The findings in this study can be compared with those of other studies. For example, this study's US dollar/rand results applying the SSC-GARCH model can be compared with those of Duncan and Liu (2009) for the same model and frequency, and a fairly similar sample period 3 January 1994 to 31 March 2009 (3794 observations) – the sample period in this study covers the period 14 March 1995 to 31 August 2010 (3864 observations). Duncan and Liu (2009) detect 19 significant shifts in the volatility of the rand/US dollar exchange rate with 16 of these having a statistically significant effect on the variance in contrast to 44 breaks and 36 significant ones identified in this study. Consequently, and in line with expectations, the volatility persistence value $(\alpha_1+\beta_1) = 0.4835$ in this paper is substantially lower than the comparative value of 0.6903 estimated by Duncan and Liu (2009). The differences in the volatility persistence outcomes can be linked to a number of factors. The main suspect is the divergence in the level of statistical significance and consequently the critical $t$-statistic values yardsticks used in the ICSS tests of the two comparative studies – we apply an asymptotic critical value of $D^*_{0.05} = 1.358$ (a confidence level of 95%) compared with $D^*_{0.01} = 1.628$ (99% confidence level) in Duncan and Liu (2009). Other possible minor influences are the slightly different sample periods, different specifications of the mean equations resulting in different sized regressor shocks $(\varepsilon_t^2)$ in the variance equation and an application of the skewed Student-$t$ distribution in our analysis which may differ from the one employed by Duncan and Liu (2009). Additionally, their data was sourced from the I-net Bridge databank – here, the data was obtained from the SARB database. Also, this paper analyses the US dollars per rand returns whilst Duncan and Liu (2009) investigate the rands per US dollar returns. A number of other differences cannot be ruled out.

In a recent study, Thupayagale and Jefferis' (2011) surprisingly uncover only four to six volatility regime shifts in the nominal exchange rates of the rand against the G4 currencies for a much larger sample period (January 1990 to November 2010) that encompasses both Duncan and Liu's (2009) and our sample. Both the latter studies employ the methodology of Bai and Perron (1998, $2003^{a,b}$). Very briefly, Bai and Perron ($2003^a$) use an efficient algorithm to obtain global minimisers of the sum of squared residuals based on the principle of dynamic programming which requires at most least-squares operations of order $O(T^2)$ for any number of breaks. However, they caution that care must be taken when using particular specifications; for example, the tests can miss the true break values too often which perhaps helps to explain the detection gap in their study and Duncan and Liu (2009) and our empirical analysis.

In the remaining section of the empirical results (section 6), the timing and potential causes of structural shift are explored and compared with those in the unit root AR processes of the raw returns in May ($2015^a$).

## 6 Descriptive analysis of structural breakpoints

From Table A2 (Appendix) in the preceding section, volatility regime switching is less frequent in the yen/rand but more frequent in the US dollar/rand. Table B13 (Appendix) presents the timing of each change point identified by the ICSS test that has a significant bearing on variance at the 95% level of confidence. To explore the number of breaks that coincide across series, a maximum interval lag of 5 business days is allowed for. Initially focusing on the four bilateral rates only, there is not a single common breakpoint across the four bilateral exchange rates, 10 common change points in three bilateral rates, and 14 in two bilateral rates. Twenty shifts in the weighted exchange rate coincide with one or more breaks in the bilateral rates. Overlapping breakpoints are more prominent in the US dollar and the two European currencies' bilateral exchange rates of the rand.

The duration of the volatility regimes ranges between 3 and 777 business days, and not surprisingly, the US dollar/rand records the shortest regime and the yen/rand the longest – the latter may be explained by the Bank of Japan's interventions aimed, in part, at dampening the effects of shocks on yen volatility. Trailing the yen/rand, there is also relatively greater tranquillity in the euro/rand – the currency of South Africa's major trading partner in both goods and financial assets.

Reverting to the 10 change points which are pervasive – occur across three bilateral exchange rates – Table A3 (Appendix) ties up these break points with important economic and non-economic events. The timing of these particular changes in volatility regimes is more or less consistent with the structural shifts detected in the changes in the levels of the exchange rates in May ($2015^a$).[22] The coincidence of structural shifts in both the levels and volatility of returns implies that a sharp movement in the exchange rate is usually or often accompanied by volatility as well – that is, large movements in exchange rates when their exact timing is unanticipated causes uncertainty and thus nervousness in the market. Bidirectional causality is not only plausible but likely as investors and speculators offload foreign assets whose prices suddenly become erratic leading to a plunge in the foreign currency's international price.

## 7 Concluding remarks and discussion

Exchange rate volatility – a manifestation of uncertainty – and its causes and effects is arguably the most topical issue in international finance in the post-Bretton Woods era. The analysis undertaken in this study motivates the use of GARCH-type volatility models for the rand exchange rates, estimates the standard models for these rates and replicates common findings in the liter-

[22] The number of change points discovered in the levels is significantly less due to the limitations of the estimation models applied in May ($2015^a$) as opposed to the ICSS tests applied in this study – individual structural break adapted unit root tests in May are modelled to detect a maximum of only two break points in each series.

ature that volatility is 'persistent'. It investigates whether this 'persistence' is due to structural breaks or long memory. The data sample spans a more flexible exchange rate regime in South Africa. The descriptive statistics in the preliminary analysis confirm some of the stylized facts about nominal financial time series such as leptokurtic distributions, ARCH effects and volatility clustering of risky assets returns, indicating that the data are candidates for GARCH-type modelling. Furthermore: i) the extent of asymmetric responses of the rand to 'good news' and bad news' are considerable – negative shocks have a greater effect on volatility than their positive counterparts; ii) the results indicate strong and widespread instability in unconditional volatility – between 20 and 44 breakpoints are detected, iii) volatility persistence falls markedly when fractional integration and a larger set of structural shifts are accounted for; iv) the approximating models across the board reflect the importance of long memory, asymmetry and structural change, both abrupt and smooth, in exchange rate volatility modelling; v) a consequence of accounting for the latter phenomena is that the unconditional variance is stationary in contrast to the most of the simpler models estimated which suggest nonstationarity, supporting the view that results that find that the volatility process is not mean reverting may not have adequately accounted for such breaks; vi) the timing of changes in volatility regimes, and thus their likely causes, are more or less consistent with the exchange rate level shifts detected in other studies; and, vii) the pricing of risk varies across exchange rates – only the GBP/ZAR ARCH-M parameter is statistically significant and at the same time correctly signed (+) at the 5% level, suggesting that the increased risk of converting pound denominated assets into rand holdings is weakly associated with an excess return.

Therefore, accounting for long memory, asymmetric responses to shocks, and in particular, structural change, in the variance of the currency returns of the rand has produced some novel and striking evidence that advances work undertaken over the past decade or so on the nominal exchange rates of the rand. The question of whether rand volatility is excessive remains a perennial issue that also requires rigorous investigation. The rand's asymmetric response to news – negative shocks raise volatility more than positive ones of equal magnitude – also prompts an inspection of the effect of macroeconomic announcements on the foreign exchange rates of the rand around the time of the announcement. The response of the rand to monetary policy pronouncements, under an inflation targeting monetary policy framework and floating exchange rate regime, is explored using high-frequency minute-by-minute exchange rate data in May *et al* (2016).

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## Appendix

**Table A1:** Summary statistics

| | USD/ZAR | EUR/ZAR | GBP/ZAR | JPY/ZAR | NEER |
|---|---|---|---|---|---|
| ***A.*** *Exchange rate returns,* $\boldsymbol{r_t}$ | | | | | |
| Minimum | -10.552 | -9.584 | -9.349 | -11.409 | -9.665 |
| Maximum | 7.403 | 5.890 | 5.731 | 8.691 | 5.516 |
| Mean | -0.018 | -0.018 | -0.018 | -0.020 | -0.018 |
| Standard deviation | 1.077 | 1.037 | 1.039 | 1.307 | 0.999 |
| Skewness | -0.657 (0.000) | -0.621 (0.000) | -0.582 (0.000) | -0.519 (0.000) | -0.693 (0.000) |
| Kurtosis | 6.676 (0.000) | 5.705 (0.000) | 5.605 (0.000) | 5.657 (0.000) | 6.878 (0.000) |
| JB (*prob*) | 7454 (0.000) | 5489 (0.000) | 5276 (0.000) | 5326 (0.000) | 7927 (0.000) |
| ***B.*** *Exchange rate returns residuals,* $\boldsymbol{\varepsilon_t}$ | | | | | |
| Ljung-Box *Q* statistics $(p = 2)$ | 4.720 (0.94) | 1.779 (0.411) | 2.599 (0.273) | 5.184 (0.075) | 4.362 (0.113) |
| $(p = 20)$ | 33.32 (0.031) | 23.53 (0.263) | 34.40 (0.024) | 30.77(0.058) | 28.86 (0.091) |
| ***C.*** *Exchange rate returns squared residuals,* $\boldsymbol{\varepsilon_t^2}$ | | | | | |
| Ljung-Box *Q* statistics $(p = 2)$ | 398.8 (0.000) | 222.2 (0.000) | 245.9 (0.000) | 737.8 (0.000) | 306.1 (0.000) |
| $(p = 20)$ | 2227 (0.000) | 1111 (0.000) | 1442 (0.000) | 3773 (0.000) | 1603 (0.000) |
| ***D.*** *Exchange rate squared returns,* $\boldsymbol{r_t^2}$ | | | | | |
| ARCH LM statistics $(p = 2)$ | 175.1 (0.000) | 98.7 (0.000) | 109.5 (0.000) | 324.7 (0.000) | 134.9 (0.000) |
| $(p = 20)$ | 39.80 (0.000) | 22.39 (0.000) | 29.98 (0.000) | 60.50 (0.000) | 30.05 (0.000) |

**Table A2:** ICSS test breakpoints

| Structural breaks | USD/ZAR | EUR/ZAR | GBP/ZAR | JPY/ZAR | NEER |
|---|---|---|---|---|---|
| Identified | 44 | 37 | 38 | 20 | 37 |
| Statistically significant* | 36 | 27 | 29 | 16 | 28 |

* At 95% or more levels of confidence (in GARCH variance equations).

**Table A3:** Common structural shifts in rand volatility – timing and potential triggers

| *Dates* | *Shocks* |
|---|---|
| 12-14 Feb 1996 | #Rand suffered a speculative currency attack – followed by a shift in SARB's intervention policy in foreign exchange market |
| 13-14 May 1996 | # Moderation in volatility as relative market stability returns following currency crisis in February 1996 |
| 23 Oct 1996 | # Rumours of an imminent relaxation of exchange controls triggers another speculative attack on the rand following a brief interlude of relative stability |
| 22-27 Oct 1997 | # Adverse effects of Southeast-Asian financial markets contagion which erupted in July 1997 in Thailand |
| 10-11 Jun 1998 | #Nervousness about prospects for emerging markets – Southeast-Asian financial markets contagion continued to spread to other emerging markets in April and May 1998 |
| 21-22 Jul 1998 | # Rand instability elevated further as concerns about financial troubles in Russia surface – exacerbated by a build-up in SA's net open forward position (NOFP) |
| 04-09 Feb 1999 | # Markets settle somewhat after Brazilian real crisis in January 1999 |
| 24-28 Jan 2002 | # Tranquillity in foreign exchange market following a string of events that unnerved the currency in 2001 – concerns about domestic fundamentals, anticipated policy shifts, rumours, declining commodity prices, and global financial market turmoil due to terrorist attacks on the U.S. in September |
| 12-13 Jul 2005 | # Positive international credit rating agencies' upgrades and outlooks for South Africa reduce rand volatility |
| 02-03 Oct 2008 | # 2007-2008 US financial market crisis spillover effects on rand |

**Figure A1:** Daily bilateral exchange rate returns, $r_t$ (expressed as percent)

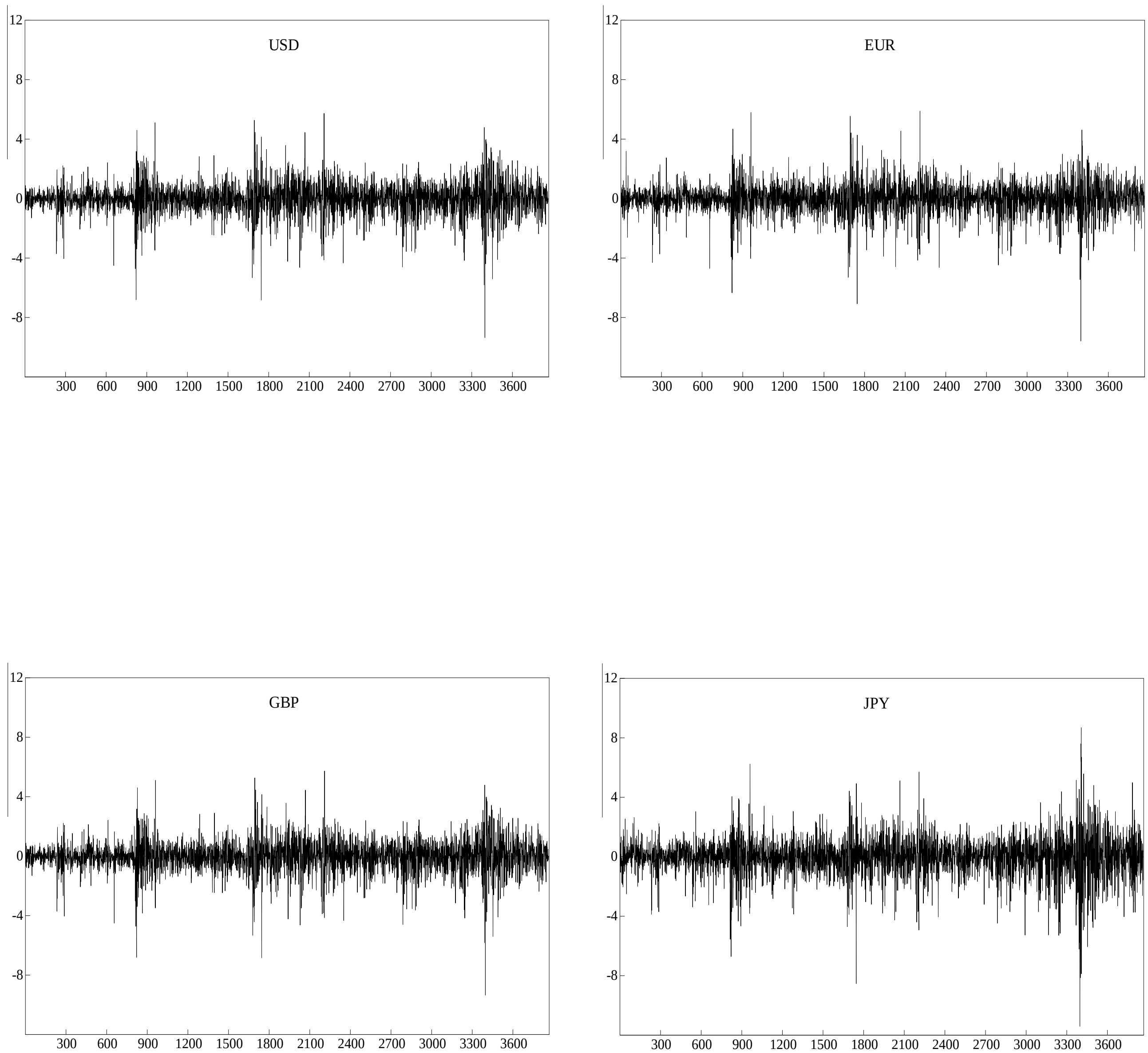

**Table B1:** Comparative USD/ZAR basic GARCH models estimation results

| Parameter | GARCH | IGARCH | GJR-GARCH | EGARCH | APARCH |
|---|---|---|---|---|---|
| **A.** Mean equation results | | | | | |
| $\gamma$ | -0.015 (0.021) | -0.013 (0.045) | -0.017 (0.011) | -0.022 (0.000) | -0.019 (0.005) |
| $\kappa$ | 0.103 (0.000) | 0.105 (0.000) | 0.104 (0.000) | 0.101 (0.000) | 0.102 (0.000) |
| $\nu$ | -0.225 (0.000) | -0.229 (0.000) | -0.226 (0.000) | -0.226 (0.000) | -0.224 (0.000) |
| ***B.*** Variance equation results | | | | | |
| $\omega$ | 0.001 (0.038) | 0.001 (0.011) | 0.001 (0.032) | -7.156 (0.002) | 0.002 (0.049) |
| $\delta$ | - | - | - | - | 1.436 (0.000) |
| $\alpha_1$ | 0.149 (0.000) | 0.112 (0.000) | 0.127 (0.000) | -0.298 (0.002) | 0.141 (0.000) |
| $\alpha_1^*$ | - | - | 0.037 (0.059) | - | 0.116 (0.012) |
| $\alpha_1 + \alpha_1^*$ | - | - | 0.1638 | - | 0.2573 |
| $\theta_1$ | - | - | - | -0.056 (0.002) | - |
| $\theta_2$ | - | - | - | 0.333 (0.000) | - |
| $\lvert\theta_1 + \theta_2\rvert$ | - | - | - | 0.2775 | - |
| $\lvert\theta_1 - \theta_2\rvert$ | - | - | - | 0.3885 | - |
| $\beta_1$ | 0.875 (0.000) | 0.888 | 0.877 (0.000) | 0.993 (0.000) | 0.892 (0.000) |
| $\alpha_1 + \beta_1$ | 1.023 | 1.000 | - | - | - |
| $\varepsilon^+ : \alpha_1 + \beta_1$ | - | - | 1.003 | - | 1.033 |
| $\varepsilon^- : \alpha_1 + \alpha_1^* + \beta_1$ | - | - | 1.040 | - | 1.149 |
| $\varepsilon^+ : \lvert\theta_1 + \theta_2\rvert + \beta_1$ | - | - | - | 1.271 | - |
| $\varepsilon^- : \lvert\theta_1 - \theta_2\rvert + \beta_1$ | - | - | - | 1.382 | - |
| **C.** Skewed Student-*t* distribution statistic for residuals, $\varepsilon_t$ | | | | | |
| Asymmetry | -0.135 (0.000) | -0.131 (0.000) | -0.135 (0.000) | -0.145 (0.000) | -0.138 (0.000) |
| Tail | 5.4680 (0.00) | 6.486 (0.000) | 5.488 (0.000) | 5.5801 (0.000) | 5.478 (0.000) |
| **D.** Mean equation standardised residuals serial correlation statistics, $Q_{LB(z_t)}$ | | | | | |
| Lag =10 | 9.394 (0.495) | 9.457 (0.489) | 9.393 (0.495) | 9.579 (0.478) | 9.588 (0.477) |
| Lag =20 | 23.127 (0.283) | 23.585 (0.261) | 22.795 (0.299) | 24.761 (0.211) | 23.599 (0.260) |
| Lag =50 | 50.418 (0.457) | 50.589 (0.450) | 49.145 (0.508) | 49.816 (0.481) | 49.292 (0.502) |
| **E.** Mean equation <u>squared</u> standardised residuals serial correlation statistics, $Q_{LB(z_t^2)}$ | | | | | |
| Lag =10 | 18.205 (0.021) | 21.775 (0.005) | 15.955 (0.043) | 13.793 (0.087) | 41.541 (0.000) |
| Lag =20 | 25.408 (0.114) | 29.220 (0.046) | 24.066 (0.153) | 26.286 (0.093) | 50.458 (0.000) |
| Lag =50 | 54.307 (0.247) | 58.605 (0.140) | 51.166 (0.351) | 52.121 (0.317) | 78.893 (0.003) |
| **F.** Joint Nyblom stability test statistics | | | | | |
| $\omega$ | 19.633 | 19.922 | 19.950 | 20.919 | 20.026 |

*Joint statistic of the Nyblom test of stability* - $H_0$: Parameter is constant and $H_1$: Parameter is unstable. The asymptotic 1% and 5% critical values for joint Nyblom statistics are 0.75 and 0.47, respectively.

**Table B2:** Comparative EUR/ZAR basic GARCH models estimation results

| Parameter | GARCH | IGARCH | GJR-GARCH | EGARCH | APARCH |
|---|---|---|---|---|---|
| ***A.*** Mean equation results | | | | | |
| $\gamma$ | -0.005 (0.643) | -0.007 (0.529) | -0.009 (0.423) | -0.025 (0.214) | -0.051 (0.197) |
| $\chi$ | 0.619 (0.000) | 0.618 (0.000) | 0.581 (0.000) | 0.522 (0.000) | 0.520 (0.001) |
| $\phi$ | -0.665 (0.000) | -0.664 (0.000) | -0.621 (0.000) | -0.566 (0.000) | -0.564 (0.000) |
| **B.** Variance equation results | | | | | |
| $\omega$ | 0.187 (0.004) | 0.014 (0.004) | 0.021 (0.008) | -1.760 (0.000) | 0.022 (0.004) |
| $\delta$ | - | - | - | - | 1.221 (0.000) |
| $\alpha_1$ | 0.102 (0.000) | 0.112 (0.000) | 0.073 (0.000) | -0.390 (0.002) | 0.107 (0.000) |
| $\alpha_1^*$ | - | - | 0.049 (0.023) | - | 0.116 (0.012) |
| $\alpha_1 + \alpha_1^*$ | - | - | 0.1219 | - | 0.2573 |
| $\theta_1$ | - | - | - | -0.073 (0.001) | - |
| $\theta_2$ | - | - | - | 0.273 (0.000) | - |
| $\lvert\theta_1 + \theta_2\rvert$ | - | - | - | 0.2006 | - |
| $\lvert\theta_1 - \theta_2\rvert$ | - | - | - | 0.3456 | - |
| $\beta_1$ | 0.886 (0.000) | 0.888 | 0.886 (0.000) | 0.980 (0.000) | 0.896 (0.000) |
| $\alpha_1 + \beta_1$ | 0.988 | 1.000 | - | - | - |
| $\varepsilon^+ : \alpha_1 + \beta_1$ | - | - | 0.959 | - | 1.002 |
| $\varepsilon^- : \alpha_1 + \alpha_1^* + \beta_1$ | - | - | 1.008 | - | 1.118 |
| $\varepsilon^+ : \lvert\theta_1 + \theta_2\rvert + \beta_1$ | - | - | - | 1.181 | - |
| $\varepsilon^- : \lvert\theta_1 - \theta_2\rvert + \beta_1$ | - | - | - | 1.326 | - |
| **C.** Skewed Student-*t* distribution statistic for residuals, $\varepsilon_t$ | | | | | |
| Asymmetry | -0.136 (0.000) | -0.138 (0.000) | -0.134 (0.000) | -0.144 (0.000) | -0.140 (0.000) |
| Tail | 5.541 (0.000) | 5.183 (0.000) | 5.541 (0.000) | 5.664 (0.000) | 5.603 (0.000) |
| **D.** Mean equation standardised residuals serial correlation statistics, $Q_{LB(z_t)}$ | | | | | |
| Lag =10 | 14.430 (0.071) | 14.946 (0.060) | 10.629 (0.224) | 12.137 (0.145) | 11.439 (0.178) |
| Lag =20 | 20.018 (0.332) | 20.721 (0.294) | 16.221 (0.577) | 18.632 (0.415) | 17.539 (0.487) |
| Lag =50 | 45.573 (0.614) | 44.912 (0.600) | 40.255 (0.779) | 42.513 (0.696) | 41.575 (0.732) |
| **E.** Mean equation squared standardised residuals serial correlation statistics, $Q_{LB(z_t^2)}$ | | | | | |
| Lag =10 | 3.133 (0.926) | 3.359 (0.910) | 2.825 (0.945) | 8.341 (0.410) | 5.873 (0.661) |
| Lag =20 | 8.934 (0.961) | 10.139 (0.927) | 8.868 (0.963) | 14.500 (0.696) | 11.971 (0.849) |
| Lag =50 | 26.772 (0.994) | 28.020 (0.991) | 28.359 (0.989) | 38.264 (0.842) | 33.938 (0.938) |
| **F.** Joint Nyblom stability test statistics | | | | | |
| $\omega$ | 4.594 | 3.563 | 5.123 | 4.6112 | 4.949 |

*Joint statistic of the Nyblom test of stability* - $H_0$: Parameter is constant and $H_1$: Parameter is unstable. The asymptotic 1% and 5% critical values for joint Nyblom statistics are 0.75 and 0.47, respectively.

**Table B3:** Comparative GBP/ZAR GARCH basic models estimation results

| Parameter | GARCH | IGARCH | GJR-GARCH | EGARCH | APARCH |
|---|---|---|---|---|---|
| **A.** Mean equation results | | | | | |
| $\gamma$ | -0.075 (0.007) | -0.075 (0.006) | -0.071(0.009) | -0.089 (0.002) | -0.079 (0.006) |
| $\kappa$ | 0.1067 (0.000) | 0.107 (0.000) | 0.109 (0.000) | 0.110 (0.000) | 0.109 (0.000) |
| $\nu$ | 0.186 (0.000) | 0.185 (0.000) | 0.186 (0.000) | 0.186 (0.000) | 0.186 (0.000) |
| $\tau$ | 0.074 (0.047) | 0.074 (0.041) | 0.058 (0.037) | 0.071 (0.070) | 0.062 (0.103) |
| **B.** Variance equation results | | | | | |
| $\omega$ | 0.009 (0.009) | 0.009 (0.003) | 0.009 (0.010) | -1.923 (0.001) | 0.011 (0.007) |
| $\delta$ | - | - | - | - | 1.494 (0.000) |
| $\alpha_1$ | 0.113 (0.000) | 0.112 (0.018) | 0.081 (0.018) | -0.422 (0.00) | 0.110 (0.000) |
| $\alpha_1^*$ | - | - | 0.052 (0.005) | - | 0.173 (0.001) |
| $\alpha_1 + \alpha_1^*$ | - | - | 0.1326 | - | 0.2836 |
| $\theta_1$ | - | - | - | -0.078 (0.000) | - |
| $\theta_2$ | - | - | - | 0.305 (0.000) | - |
| $\lvert\theta_1 + \theta_2\rvert$ | - | - | - | 0.2274 | - |
| $\lvert\theta_1 - \theta_2\rvert$ | - | - | - | 0.3834 | - |
| $\beta_1$ | 0.888 (0.000) | 0.8878 | 0.892 (0.000) | 0.986 (0.000) | 0.899 (0.000) |
| $\alpha_1 + \beta_1$ | 1.000 | 1.000 | - | - | - |
| $\varepsilon^+ : \alpha_1 + \beta_1$ | - | - | 0.973 | - | 1.010 |
| $\varepsilon^- : \alpha_1 + \alpha_1^* + \beta_1$ | - | - | 1.024 | - | 1.183 |
| $\varepsilon^+ : \lvert\theta_1 + \theta_2\rvert + \beta_1$ | - | - | - | 1.213 | - |
| $\varepsilon^- : \lvert\theta_1 - \theta_2\rvert + \beta_1$ | - | - | - | 1.369 | - |
| **C.** Skewed Student-*t* distribution statistic for residuals, $\varepsilon_t$ | | | | | |
| Asymmetry | -0.081 (0.000) | -0.081 (0.000) | -0.085 (0.000) | -0.100 (0.000) | -0.089 (0.000) |
| Tail | 6.087 (0.000) | 6.102 (0.000) | 6.142 (0.000) | 6.243 (0.000) | 6.159 (0.000) |
| **D.** Mean equation standardised residuals serial correlation statistics, $Q_{LB(z_t)}$ | | | | | |
| Lag =10 | 10.060 (0.435) | 10.057 (0.436) | 10.621 (0.388) | 10.662 (0.308) | 11.123 (0.348) |
| Lag =20 | 23.948 (0.245) | 23.940 (0.245) | 24.124 (0.237) | 25.674 (0.177) | 24.961 (0.203) |
| Lag =50 | 53.196 (0.352) | 53.172 (0.353) | 43.741 (0.333) | 54.468 (0.309) | 54.124 (0.319) |
| **E.** Mean equation <u>squared</u> standardised residuals serial correlation statistics, $Q_{LB(z_t^2)}$ | | | | | |
| Lag =10 | 11.916 (0.155) | 11.888 (0.156) | 11.375 (0.181) | 19.458 (0.013) | 14.773 (0.063) |
| Lag =20 | 19.946 (0.336) | 19.881 (0.340) | 20.327 (0.315) | 25.784 (0.105) | 22.954 (0.192) |
| Lag =50 | 57.421 (0.497) | 47.361 (0.499) | 47.621 (0.488) | 47.087 (0.510) | 48.181 (0.466) |
| **F.** Joint Nyblom stability test statistics | | | | | |
| $\omega$ | 11.796 | 11.594 | 12.400 | 11.837 | 12.458 |

*Joint statistic of the Nyblom test of stability* - $H_0$: Parameter is constant and $H_1$: Parameter is unstable. The asymptotic 1% and 5% critical values for joint Nyblom statistics are 0.75 and 0.47, respectively.

**Table B4**: Comparative JPY/ZAR GARCH basic models estimation results

| Parameter | GARCH | IGARCH | GJR-GARCH | EGARCH | APARCH |
|---|---|---|---|---|---|
| **A.** Mean equation results | | | | | |
| $\gamma$ | 0.003 (0.827) | 0.001 (0.968) | -0.009 (0.574) | -0.007 (0.637) | -0.010 (0.542) |
| $\chi$ | 0.555 (0.000) | 0.545 (0.000) | 0.443 (0.008) | 0.462 (0.001) | 0.420 (0.011) |
| $\phi$ | -0.602 (0.000) | -0.592 (0.000) | -0.480 (0.004) | -0.503 (0.000) | -0.458 (0.006) |
| **B.** Variance equation results | | | | | |
| $\omega$ | 0.037 (0.001) | 0.024 (0.001) | 0.045 (0.001) | -1.072 (0.002) | 0.042 (0.000) |
| $\delta$ | - | - | - | - | 1.406 (0.000) |
| $\alpha_1$ | 0.106 (0.000) | 0.118 (0.000) | 0.059 (0.000) | -0.334 (0.006) | 0.108 (0.000) |
| $\alpha_1^*$ | - | - | 0.085 (0.001) | - | 0.290 (0.000) |
| $\alpha_1 + \alpha_1^*$ | - | - | 0.1435 | - | 0.3974 |
| $\theta_1$ | - | - | - | -0.081 (0.000) | - |
| $\theta_2$ | - | - | - | 0.260 (0.000) | - |
| $\lvert\theta_1 + \theta_2\rvert$ | - | - | - | 0.1789 | - |
| $\lvert\theta_1 - \theta_2\rvert$ | - | - | - | 0.3415 | - |
| $\beta_1$ | 0.874 (0.000) | 0.882 | 0.869 (0.000) | 0.975 (0.000) | 0.878 (0.000) |
| $\alpha_1 + \beta_1$ | 0.979 | 1.000 | - | - | - |
| $\varepsilon^+ : \alpha_1 + \beta_1$ | - | - | 0.928 | - | 0.986 |
| $\varepsilon^- : \alpha_1 + \alpha_1^* + \beta_1$ | - | - | 1.012 | - | 1.275 |
| $\varepsilon^+ : \lvert\theta_1 + \theta_2\rvert + \beta_1$ | - | - | - | 1.154 | - |
| $\varepsilon^- : \lvert\theta_1 - \theta_2\rvert + \beta_1$ | - | - | - | 1.316 | - |
| **C.** Skewed Student-*t* distribution statistic for residuals, $\varepsilon_t$ | | | | | |
| Asymmetry | -0.139 (0.000) | -0.137 (0.000) | -0.139 (0.000) | -0.140 (0.000) | -0.140 (0.000) |
| Tail | 6.594 (0.000) | 5.809 (0.000) | 6.723 (0.000) | 6.619 (0.000) | 6.717 (0.000) |
| **D.** Mean equation standardised residuals serial correlation statistics, $Q_{LB(z_t)}$ | | | | | |
| Lag =10 | 10.040 (0.262) | 11.232 (0.189) | 5.929 (0.655) | 8.345 (0.401) | 6.301 (0.614) |
| Lag =20 | 19.892 (0.339) | 21.586 (0.251) | 15.955 (0.596) | 17.739 (0.473) | 16.140 (0.583) |
| Lag =50 | 41.621 (0.730) | 43.376 (0.663) | 37.914 (0.851) | 38.961 (0.821) | 37.642 (0.859) |
| **E.** Mean equation squared standardised residuals serial correlation statistics, $Q_{LB(z_t^2)}$ | | | | | |
| Lag =10 | 4.543 (0.805) | 3.848 (0.871) | 2.812 (0.946) | 9.209 (0.325) | 4.523 (0.807) |
| Lag =20 | 15.041 (0.659) | 18.861 (0.401) | 11.947 (0.850) | 17.147 (0.513) | 13.772 (0.744) |
| Lag =50 | 38.215 (0.843) | 42.118 (0.712) | 43.156 (0.671) | 44.369 (0.622) | 45.852 (0.561) |
| **F.** Joint Nyblom stability test statistics | | | | | |
| $\omega$ | 5.194 | 4.142 | 5.649 | 5.121 | 5.337 |

*Joint statistic of the Nyblom test of stability* - $H_0$: Parameter is constant and $H_1$: Parameter is unstable. The asymptotic 1% and 5% critical values for joint Nyblom statistics are 0.75 and 0.47, respectively.

**Table B5:** Comparative NEER basic GARCH models estimation results

| Parameter | GARCH | IGARCH | GJR-GARCH | EGARCH | APARCH |
|---|---|---|---|---|---|
| **A.** Mean equation results | | | | | |
| $\gamma$ | -0.017 (0.024) | -0.016 (0.044) | -0.021 (0.006) | -0.026 (0.003) | -0.024 (0.003) |
| $\kappa$ | 0.340 (0.000) | 0.333 (0.000) | 0.339 (0.000) | 0.340 (0.000) | 0.343 (0.000) |
| $\nu$ | 0.098 (0.000) | 0.100 (0.000) | 0.099 (0.000) | 0.094 (0.000) | 0.097 (0.000) |
| **B.** Variance equation results | | | | | |
| $\omega$ | 0.002 (0.047) | 0.002 (0.012) | 0.002 (0.042) | -6.585 (0.009) | 0.003 (0.031) |
| $\delta$ | - | - | - | - | 1.247 (0.000) |
| $\alpha_1$ | 0.160 (0.000) | 0.121 (0.000) | 0.120 (0.000) | -0.443 (0.000) | 0.137 (0.000) |
| $\alpha_1^*$ | - | - | 0.066 (0.002) | - | 0.192 (0.000) |
| $\alpha_1 + \alpha_1^*$ | - | - | 0.1858 | - | 0.3288 |
| $\theta_1$ | - | - | - | -0.083 (0.000) | - |
| $\theta_2$ | - | - | - | 0.368 (0.000) | - |
| $\lvert\theta_1 + \theta_2\rvert$ | - | - | - | 0.2858 | - |
| $\lvert\theta_1 - \theta_2\rvert$ | - | - | - | 0.4508 | - |
| $\beta_1$ | 0.867 (0.000) | 0.8788 | 0.872 (0.000) | 0.994 (0.000) | 0.897 (0.000) |
| $\alpha_1 + \beta_1$ | 1.027 | 1.000 | - | - | - |
| $\varepsilon^+ : \alpha_1 + \beta_1$ | - | - | 0.991 | - | 1.034 |
| $\varepsilon^- : \alpha_1 + \alpha_1^* + \beta_1$ | - | - | 1.058 | - | 1.226 |
| $\varepsilon^+ : \lvert\theta_1 + \theta_2\rvert + \beta_1$ | - | - | - | 1.280 | - |
| $\varepsilon^- : \lvert\theta_1 - \theta_2\rvert + \beta_1$ | - | - | - | 1.445 | - |
| **C.** Skewed Student-*t* distribution statistic for residuals, $\varepsilon_t$ | | | | | |
| Asymmetry | -0.122 (0.000) | -0.119 (0.000) | -0.124 (0.000) | -0.143 (0.000) | -0.130 (0.000) |
| Tail | 4.930 (0.000) | 5.867 (0.000) | 4.951 (0.000) | 5.101 (0.000) | 4.941 (0.000) |
| **D.** Mean equation standardised residuals serial correlation statistics, $Q_{LB(z_t)}$ | | | | | |
| Lag =10 | 6.177 (0.800) | 6.637 (0.759) | 6.269 (0.792) | 6.555 (0.767) | 7.949 (0.634) |
| Lag =20 | 15.744 (0.732) | 16.768 (0.668) | 15.313 (0.758) | 17.711 (0.607) | 17.294 (0.634) |
| Lag =50 | 40.792 (0.820) | 41.121 (0.810) | 40.143 (0.839) | 41.434 (0.801) | 42.023 (0.781) |
| **E.** Mean equation squared standardised residuals serial correlation statistics, $Q_{LB(z_t^2)}$ | | | | | |
| Lag =10 | 15.246 (0.055) | 14.103 (0.079) | 13.339 (0.101) | 23.893 (0.002) | 31.764 (0.000) |
| Lag =20 | 22.746 (0.201) | 21.382 (0.261) | 21.314 (0.264) | 30.302 (0.035) | 38.515 (0.003) |
| Lag =50 | 43.494 (0.658) | 41.880 (0.721) | 42.609 (0.693) | 54.557 (0.239) | 61.117 (0.097) |
| **F.** Joint Nyblom stability test statistics | | | | | |
| $\omega$ | 27.304 | 27.548 | 27.568 | 27.393 | 27.635 |

*Joint statistic of the Nyblom test of stability* - $H_0$: Parameter is constant and $H_1$: Parameter is unstable. The asymptotic 1% and 5% critical values for joint Nyblom statistics are 0.75 and 0.47, respectively.

**Table B6:** Comparative USD/ZAR adaptive-FIGARCH models estimation results

| Parameter | A-FIGARCH | A-FIEGARCH | A-FIAPARCH |
|---|---|---|---|
| **A.** Mean equation results | | | |
| $\gamma$ | -0.014 (0.032) | -0.024 (0.000) | -0.022 (0.001) |
| $\kappa$ | 0.101 (0.000) | 0.101 (0.000) | 0.101 ( 0.000) |
| $\nu$ | -0.225 (0.000) | -0.221 (0.000) | -0.227 (0.000) |
| **B.** Variance equation results | | | |
| $\omega$ | 0.481 (0.002) | -4.372 (0.000) | 0.302 (0.007) |
| $\psi_1$ | -0.617 (0.000) | -0.487 (0.041) | -0.483 (0.000) |
| $\psi_2$ | 0.262 (0.000) | * | 0.211 (0.001) |
| $\rho_1$ | -0.432 (0.005) | -0.328 (0.100) | -0.248 (0.022) |
| $\rho_2$ | * | 0.313 (0.060) | * |
| $d - FIGARCH$ | 0.401 (0.000) | 0.345 (0.000) | 0.289 (0.000) |
| $\delta$ | - | - | 1.901 (0.000) |
| $\alpha_1$ | 0.226 (0.004) | -0.543 (0.000) | 0.336 (0.001) |
| $\alpha_1^*$ | - | - | 0.369 (0.001) |
| $\alpha_1 + \alpha_1^*$ | - | - | 0.7056 |
| $\theta_1$ | - | -0.094 (0.000) | - |
| $\theta_2$ | - | 0.307 (0.000) | - |
| $\lvert\theta_1 + \theta_2\rvert$ | - | 0.2124 | - |
| $\lvert\theta_1 - \theta_2\rvert$ | - | 0.4010 | - |
| $\beta_1$ | 0.492 (0.000) | 0.895 (0.000) | 0.489 (0.000) |
| $\alpha_1 + \beta_1$ | 0.719 | - | - |
| $\varepsilon^+ : \alpha_1 + \beta_1$ | - | - | 0.825 |
| $\varepsilon^- : \alpha_1 + \alpha_1^* + \beta_1$ | - | - | 1.042 |
| $\varepsilon^+ : \lvert\theta_1 + \theta_2\rvert + \beta_1$ | - | 1.108 | - |
| $\varepsilon^- : \lvert\theta_1 - \theta_2\rvert + \beta_1$ | - | 1.296 | - |
| **C.** Skewed Student-*t* distribution statistic for residuals, $\varepsilon_t$ | | | |
| Asymmetry | -0.145 (0.000) | -0.140 (0.000) | -0.156 (0.000) |
| Tail | 6.436 (0.000) | 5.783 (0.000) | 6.158 (0.000) |
| **D.** Mean equation standardised residuals serial correlation statistics, $Q_{LB(z_t)}$ | | | |
| Lag =10 | 10.106 (0.431) | 10.703 (0.381) | 10.618 (0.388) |
| Lag =20 | 26.412 (0.153) | 26.815 (0.141) | 28.005 (0.109) |
| Lag =50 | 55.596 (0.272) | 56.300 (0.251) | 55.791 (0.266) |

* Parameters are statistically insignificant at 1%, 5% and 10% levels – model is estimated without these trigonometric structural change variables.

**Table B7**: Comparative EUR/ZAR adaptive-FIGARCH models estimation results

| Parameter | A-FIGARCH | A-FIEGARCH | A-FIAPARCH |
|---|---|---|---|
| **A.** Mean equation results | | | |
| $\gamma$ | -0.004 (0.752) | -0.032 (0.114) | -0.015 (0.210) |
| $\chi$ | 0.621 (0.000) | 0.545 (0.001) | 0.566 (0.001) |
| $\phi$ | -0.670 (0.000) | -0.586 (0.000) | -0.599 (0.000) |
| **B.** Variance equation results | | | |
| $\omega$ | 0.615 (0.000) | -4.644 (0.000) | 0.454 (0.000) |
| $\psi_1$ | * | * | * |
| $\psi_2$ | -0.284 (0.008) | * | * |
| $\rho_1$ | * | * | -0.1928 (0.043) |
| $\rho_2$ | * | 0.334 (0.002) | 0.166 (0.043) |
| $d-FIGARCH$ | 0.305 (0.000) | 0.173 (0.000) | 0.197 (0.000) |
| $\delta$ | - | - | 1.826 (0.000) |
| $\alpha_1$ | 0.270 (0.005) | -0.480 (0.000) | 0.305 (0.023) |
| $\alpha_1^*$ | - | - | 0.488 (0.003) |
| $\alpha_1+\alpha_1^*$ | - | - | 0.7928 |
| $\theta_1$ | - | -0.097 (0.000) | - |
| $\theta_2$ | - | 0.257 (0.000) | - |
| $\lvert\theta_1+\theta_2\rvert$ | - | 0.1599 | - |
| $\lvert\theta_1-\theta_2\rvert$ | - | 0.3531 | - |
| $\beta_1$ | 0.456 (0.000) | 0.923 (0.000) | 0.401 (0.004) |
| $\alpha_1+\beta_1$ | 0.726 | - | - |
| $\varepsilon^+:\alpha_1+\beta_1$ | - | - | 0.706 |
| $\varepsilon^-:\alpha_1+\alpha_1^*+\beta_1$ | - | - | 1.194 |
| $\varepsilon^+:\lvert\theta_1+\theta_2\rvert+\beta_1$ | - | 1.083 | - |
| $\varepsilon^-:\lvert\theta_1-\theta_2\rvert+\beta_1$ | - | 1.277 | - |
| **C.** Skewed Student-*t* distribution statistic for residuals, $\varepsilon_t$ | | | |
| Asymmetry | -0.143 (0.000) | -0.147 (0.000) | -0.147 (0.000) |
| Tail | 6.174 (0.000) | 5.812 (0.000) | 6.088 (0.000) |
| **D.** Mean equation standardised residuals serial correlation statistics, $Q_{LB(z_t)}$ | | | |
| Lag =10 | 10.106 (0.017) | 11.023 (0.200) | 9.3863 (0.311) |
| Lag =20 | 24.536 (0.138) | 18.742 (0.408) | 16.854 (0.533) |
| Lag =50 | 48.151 (0.467) | 44.498 (0.617) | 39.482 (0.805) |

* Parameters are statistically insignificant at 1%, 5% and 10% levels – model is estimated without these trigonometric structural change variables.

**Table B8:** Comparative GBP/ZAR adaptive-FIGARCH models estimation results

| Parameter | A-FIGARCH | A-FIEGARCH | A-FIAPARCH |
|---|---|---|---|
| **A.** Mean equation results | | | |
| $\gamma$ | -0.053 (0.002) | -0.067 (0.000) | -0.062 (0.000) |
| $\kappa$ | 0.103 (0.000) | 0.103 (0.000) | 0.104 (0.000) |
| $\nu$ | 0.181 (0.000) | 0.184 (0.000) | 0.178 (0.000) |
| $\tau$ | 0.060* (0.013) | -0.053 (0.019)* | -0.052 (0.017)* |
| **B.** Variance equation results | | | |
| $\omega$ | 0.636 (0.000) | -5.671 (0.000) | 0.193 (0.001) |
| $\psi_1$ | -0.360 (0.027) | -0.381 0.004) | ** |
| $\psi_2$ | ** | ** | ** |
| $\rho_1$ | -0.304 (0.037) | ** | ** |
| $\rho_2$ | ** | ** | ** |
| $d-FIGARCH$ | 0.357 (0.000) | 0.381 (0.004) | 0.244 (0.000) |
| $\delta$ | - | - | 1.9827 (0.000) |
| $\alpha_1$ | 0.338 (0.000) | -0.502 (0.000) | 0.438 (0.000) |
| $\alpha_1^*$ | - | - | 0.337 (0.002) |
| $\alpha_1+\alpha_1^*$ | - | - | 0.776 |
| $\theta_1$ | - | -0.091 (0.000) | - |
| $\theta_2$ | - | 0.293 (0.000) | - |
| $\lvert\theta_1+\theta_2\rvert$ | - | 0.202 | - |
| $\lvert\theta_1-\theta_2\rvert$ | - | 0.384 | - |
| $\beta_1$ | 0.545 (0.000) | 0.946 (0.000) | 0.540 (0.000) |
| $\alpha_1+\beta_1$ | 0.883 | - | - |
| $\varepsilon^+:\alpha_1+\beta_1$ | - | - | 0.978 |
| $\varepsilon^-:\alpha_1+\alpha_1^*+\beta_1$ | - | - | 1.315 |
| $\varepsilon^+:\lvert\theta_1+\theta_2\rvert+\beta_1$ | - | 1.148 | - |
| $\varepsilon^-:\lvert\theta_1-\theta_2\rvert+\beta_1$ | - | 1.330 | - |
| **C.** Skewed Student-*t* distribution statistic for residuals, $\varepsilon_t$ | | | |
| Asymmetry | -0.087 (0.000) | -0.092 (0.000) | -0.098 (0.000) |
| Tail | 6.885 (0.000) | 6.453 (0.000) | 6.805 (0.000) |
| **D.** Mean equation standardised residuals serial correlation statistics, $Q_{LB(z_t)}$ | | | |
| Lag =10 | 11.920 (0.291) | 12.294 (0.266) | 13.596 (0.192) |
| Lag =20 | 27.167 (0.131) | 26.902 (0.138) | 28.031 (0.109) |
| Lag =50 | 55.909 (0.263) | 56.203 (0.254) | 55.313 (0.281) |

* Conditional variance

** Parameters are statistically insignificant at 1%, 5% and 10% levels – model is estimated without trigonometric structural change variables.

**Table B9:** Comparative JPY/ZAR adaptive-FIGARCH models estimation results

| Parameter | A-FIGARCH | A-FIEGARCH | A-FIAPARCH |
|---|---|---|---|
| **A.** Mean equation results | | | |
| $\gamma$ | 0.008 (0.579) | -0.007 (0.590) | -0.009 (0.560) |
| $\chi$ | 0.559 (0.000) | 0.468 (0.001) | 0.446 (0.005) |
| $\phi$ | -0.608 (0.000) | -0.506 (0.000) | -0.479 (0.003) |
| **B.** Variance equation results | | | |
| $\omega$ | 1.099 (0.000) | -1.379 (0.000) | 0.754 (0.000) |
| $\psi_1$ | * | -0.335 (0.033) | * |
| $\psi_2$ | -0.317 (0.066) | * | * |
| $\rho_1$ | * | * | * |
| $\rho_2$ | * | 0.246 (0.036) | * |
| $d-FIGARCH$ | 0.327 (0.000) | 0.353 (0.000) | 0.226 (0.000) |
| $\delta$ | - | - | 1.787 (0.000) |
| $\alpha_1$ | 0.288 (0.000) | -0.341 (0.011) | 0.325 (0.000) |
| $\alpha_1^*$ | - | - | 0.461 (0.000) |
| $\alpha_1+\alpha_1^*$ | - | - | 0.7855 |
| $\theta_1$ | - | -0.101 (0.000) | - |
| $\theta_2$ | - | 0.230 (0.000) | - |
| $\lvert\theta_1+\theta_2\rvert$ | - | 0.1295 | - |
| $\lvert\theta_1-\theta_2\rvert$ | - | 0.3311 | - |
| $\beta_1$ | 0.494 (0.000) | 0.794 (0.000) | 0.440 (0.000) |
| $\alpha_1+\beta_1$ | 0.782 | - | - |
| $\varepsilon^+:\alpha_1+\beta_1$ | - | - | 0.764 |
| $\varepsilon^-:\alpha_1+\alpha_1^*+\beta_1$ | - | - | 1.225 |
| $\varepsilon^+:\lvert\theta_1+\theta_2\rvert+\beta_1$ | - | 0.924 | - |
| $\varepsilon^-:\lvert\theta_1-\theta_2\rvert+\beta_1$ | - | 1.125 | - |
| **C.** Skewed Student-*t* distribution statistic for residuals, $\varepsilon_t$ | | | |
| Asymmetry | -0.139 (0.000) | -0.133 (0.000) | -0.141 (0.000) |
| Tail | 7.197 (0.000) | 6.803 (0.000) | 7.244 (0.000) |
| **D.** Mean equation standardised residuals serial correlation statistics, $Q_{LB(z_t)}$ | | | |
| Lag =10 | 12.829 (0.118) | 9.252 (0.322) | 6.392 (0.603) |
| Lag =20 | 22.895 (0.195) | 18.419 (0.428) | 16.974 (0.525) |
| Lag =50 | 45.593 (0.572) | 41.004 (0.753) | 39.433 (0.806) |

* Parameters are statistically insignificant at 1%, 5% and 10% levels – model is estimated without these trigonometric structural change variables.

**Table B10:** Comparative NEER/ZAR adaptive-FIGARCH models estimation results

| Parameter | A-FIGARCH | A-FIEGARCH | A-FIAPARCH |
|---|---|---|---|
| **A.** Mean equation results | | | |
| $\gamma$ | -0.016 (0.034) | -0.028 (0.000) | -0.027 (0.001) |
| $\kappa$ | 0.330 (0.000) | 0.346 ( 0.000) | 0.333 (0.000) |
| $\nu$ | 0.095 (0.000) | 0.093 (0.000) | 0.093 (0.000) |
| **B.** Variance equation results | | | |
| $\omega$ | 0.397 (0.004) | -5.073 (0.000) | 0.278 (0.010) |
| $\psi_1$ | -0.399 (0.003) | * | -0.376 (0.001) |
| $\psi_2$ | * | * | 0.148 (0.008) |
| $\rho_1$ | -0.267 (0.014) | * | -0.222 (0.024) |
| $\rho_2$ | * | 0.473 (0.059) | * |
| $d - FIGARCH$ | 0.328 (0.000) | 0.502 (0.000) | 0.241 (0.000) |
| $\delta$ | - | - | 1.847 (0.000) |
| $\alpha_1$ | 0.266 (0.000) | -0.741 (0.000) | 0.4325 (0.000) |
| $\alpha_1^*$ | - | - | 0.520 (0.000) |
| $\alpha_1 + \alpha_1^*$ | - | - | 0.952 |
| $\theta_1$ | - | -0.115 (0.000) | - |
| $\theta_2$ | - | 0.337 (0.000) | - |
| $\lvert\theta_1 + \theta_2\rvert$ | - | 0.222 | - |
| $\lvert\theta_1 - \theta_2\rvert$ | - | 0.451 | - |
| $\beta_1$ | 0.497 (0.000) | 0.881 (0.000) | 0.535 (0.000) |
| $\alpha_1 + \beta_1$ | 0.763 | - | - |
| $\varepsilon^+ : \alpha_1 + \beta_1$ | - | - | 0.967 |
| $\varepsilon^- : \alpha_1 + \alpha_1^* + \beta_1$ | - | - | 1.487 |
| $\varepsilon^+ : \lvert\theta_1 + \theta_2\rvert + \beta_1$ | - | 1.103 | - |
| $\varepsilon^- : \lvert\theta_1 - \theta_2\rvert + \beta_1$ | - | 1.332 | - |
| **C.** Skewed Student-*t* distribution statistic for residuals, $\varepsilon_t$ | | | |
| Asymmetry | -0.131 (0.000) | -0.139 (0.000) | -0.150 (0.000) |
| Tail | 5.865 (0.000) | 5.386 (0.000) | 5.610 (0.000) |
| **D.** Mean equation standardised residuals serial correlation statistics, $Q_{LB(z_t)}$ | | | |
| Lag =10 | 6.857 (0.739) | 6.421 (0.779) | 7.860 (0.643) |
| Lag =20 | 17.623 (0.612) | 16.209 (0.704) | 18.959 (0.525) |
| Lag =50 | 42.825 (0.754) | 40.878 (0.818) | 46.017 (0.634) |

* Parameters are statistically insignificant at 1%, 5% and 10% levels – model is estimated without these trigonometric structural change variables.

**Table B11:** Comparative SSC-GARCH models estimation results

| Parameter | USD/ZAR | EUR/ZAR | GBP/ZAR | JPY/ZAR | NEER |
|---|---|---|---|---|---|
| **A.** Mean equation results | | | | | |
| $\gamma$ | -0.018 (0.006) | -0.011 (0.591) | -0.019 (0.159) | -0.015 (0.324) | -0.003 (0.753) |
| $\chi$ | - | 0.519 (0.000) | - | - | - |
| $\phi$ | - | -0.639 (0.000) | - | - | - |
| **B.** Variance equation results | | | | | |
| $\omega$ | 1.036 (0.000) | 1.151 (0.000) | 0.904 (0.000) | 1.269 (0.000) | 1.167 (0.000) |
| Structural breaks* | 36 | 27 | 29 | 16 | 28 |
| $D_1 - D_{XX}$ | -0.982 : 9.638<br>(0.000 : 0.062) | -0.977 : 1.903<br>(0.000 : 0.063) | -0.932 : 4.525<br>(0.000 : 0.042) | -1.226 : 8.961<br>(0.000 : 0.002) | -1.143 : 5.013<br>(0.000 : 0.095) |
| $\alpha_1$ | 0.059 (0.000) | 0.045 (0.001) | 0.077 (0.000) | 0.075 (0.000) | 0.060 (0.000) |
| $\beta_1$ | 0.424 (0.000) | 0.591 (0.000) | 0.551 (0.000) | 0.692 (0.000) | 0.550 (0.000) |
| $\alpha_1 + \beta_1$ | 0.4835 | 0.6362 | 0.6275 | 0.7672 | 0.6099 |
| **C.** Skewed Student-*t* distribution statistic for residuals, $\varepsilon_t$ | | | | | |
| Asymmetry | -0.112 (0.000) | -0.112 (0.000) | -0.101 (0.000) | -0.110 (0.000) | -0.115 (0.000) |
| Tail | 13.272 (0.000) | 10.871 (0.000) | 14.472 (0.000) | 8.914 (0.000) | 10.510 (0.000) |
| **D.** Mean equation standardised residuals serial correlation statistics | | | | | |
| Lag =10 | 10.442 (0.403) | 4.623 (0.797) | 10.027 (0.438) | 7.294 (0.697) | 113.977 (0.174) |
| Lag =20 | 24.331 (0.228) | 9.491 (0.947) | 26.855 (0.139) | 11.957 (0.590) | 21.326 (0.378) |
| Lag =50 | 48.546 (0.532) | 29.633 (0.983) | 57.087 (0.229) | 43.867 (0.717) | 47.541 (0.573) |

* To conserve space, $D_{XX}$ is the number of significant change points plus unity. The information in parenthesis is the range for the relevant statistics for each dummy variable. The individual break point results may be requested from the author.

**Table B12:** Comparative SSC-GJR-GARCH models estimation results

| Parameter | USD/ZAR | EUR/ZAR | GBP/ZAR | JPY/ZAR | NEER |
|---|---|---|---|---|---|
| **A.** Mean equation results | | | | | |
| $\gamma$ | -0.023 (0.001) | -0.000 (0.992) | -0.028 (0.012) | -0.005 (0.753) | -0.012 (0.200) |
| $\chi$ | - | 0.596 (0.000) | - | - | - |
| $\phi$ | - | -0.636 (0.000) | - | - | - |
| $\kappa$ | - | - | 0.096 (0.000) | - | - |
| $\nu$ | - | - | 0.172 (0.000) | - | - |
| **B.** Variance equation results | | | | | |
| $\omega$ | 0.954 (0.000) | 0.719 (0.000) | 0.954 (0.000) | 1.185 (0.000) | 0.914 (0.000) |
| Structural breaks* | 36 | 24 | 29 | 16 | 29 |
| $D_1 - D_{XX}$ | -0.943 : 9.748<br>(0.000 : 0.040) | -0.663 : 2.438<br>(0.000 : 0.051) | -0.918 : 4.0329<br>(0.000 : 0.099) | -1.131 : 8.978<br>(0.000 : 0.001) | -0.900 : 5.395<br>(0.000 : 0.079) |
| $\alpha_1$ | -0.021 (0.203) | -0.019 (0.111) | 0.007 (0.666) | -0.021 (0.047) | -0.035 (0.000) |
| $\alpha_1^*$ | 0.143 (0.000) | 0.117 (0.000) | 0.127 (0.000) | 0.164 (0.000) | 0.160 (0.000) |
| $\alpha_1 + \alpha_1^*$ | 0.1429** | 0.1166** | 0.1266** | 0.1434 | 0.1249 |
| $\beta_1$ | 0.460 (0.000) | 0.593 (0.000) | 0.601 (0.000) | 0.708 (0.000) | 0.594 (0.000) |
| $\varepsilon^+ : \alpha_1 + \beta_1$ | 0.4601** | 0.5929** | 0.6009** | 0.6868 | 0.5587 |
| $\varepsilon^- : \alpha_1 + \alpha_1^* + \beta_1$ | 0.6030** | 0.7095** | 0.7275 | 0.8511 | 0.7185 |
| **C.** Skewed Student-*t* distribution statistic for residuals, $\varepsilon_t$ | | | | | |
| Asymmetry | -0.136 (0.000) | -0.121 (0.000) | -0.090 (0.000) | -0.129 (0.000) | -0.134 (0.000) |
| Tail | 13.422 (0.000) | 10.672 (0.000) | 14.867 (0.000) | 9.271 (0.000) | 11.157 (0.000) |
| **D.** Mean equation standardised residuals serial correlation statistics | | | | | |
| Lag =10 | 9.425 (0.492) | 3.339 (0.911) | 13.006 (0.223) | 6.279 (0.791) | 12.582 (0.248) |
| Lag =20 | 23.550 (0.263) | 8.398 (0.972) | 27.580 (0.120) | 16.826 (0.664) | 20.171 (0.447) |
| Lag =50 | 46.704 (0.606) | 28.646 (0.988) | 57.835 (0.209) | 42.257 (0.774) | 47.119 (0.590) |

* See Table B11. ** $\alpha_1$is statistically insignificant (or indifferent from zero) implying $\alpha_1 + \alpha_1^* = \alpha_1^*$, and $\alpha_1 + \beta_1 = \beta_1$.

**Table B13**: Timing of structural shifts in exchange rate returns variance*

| *Break points* | USD/ZAR | EUR/ZAR | GBP/ZAR | JPY/ZAR | NEER |
|---|---|---|---|---|---|
| 06-Jan-1995 | N | N | Y | N | N |
| 24-Mar-1995 | Y | N | N | N | N |
| 09-May-1995 | N | Y | N | N | Y |
| 31-May-1995 | N | Y | N | N | Y |
| 20-Jul-1995 | N | N | N | N | Y |
| 16-Aug-1995 | Y | N | N | N | N |
| 03-Oct-1995 | N | N | N | N | Y |
| 21-Nov-1995 | N | N | N | Y | N |
| 12-Feb-1996 | N | N | N | Y | N |
| 13-Feb-1996 | N | Y | N | N | N |
| 14-Feb-1996 | Y | N | N | N | N |
| 21-Feb-1996 | N | N | Y | N | Y |
| 22-Feb-1996 | Y | N | N | N | N |
| 22-Apr-1996 | N | Y | N | N | N |
| 23-Apr-1996 | Y | N | N | N | N |
| 13-May-1996 | N | N | Y | Y | Y |
| 14-May-1996 | Y | N | N | N | N |
| 14-Jun-1996 | Y | N | N | N | N |
| 09-Jul-1996 | N | Y | Y | N | N |
| 31-Jul-1996 | N | Y | N | N | N |
| 07-Aug-1996 | N | N | Y | N | N |
| 23-Oct-1996 | N | Y | Y | Y | Y |
| 04-Feb-1997 | Y | N | N | N | N |
| 21-Feb-1997 | N | Y | N | N | N |
| 24-Feb-1997 | N | N | N | N | Y |
| 14-Mar-1997 | N | N | Y | N | N |
| 20-Mar-1997 | Y | N | N | N | N |
| 21-Jul-1997 | Y | N | Y | N | N |
| 19-Aug-1997 | Y | N | N | N | N |
| 22-Aug-1997 | Y | N | Y | N | N |
| 22-Oct-1997 | N | Y | N | N | N |
| 24-Oct-1997 | Y | N | N | N | N |
| 27-Oct-1997 | N | N | Y | N | N |
| 30-Oct-1997 | N | N | N | N | Y |
| 04-Nov-1997 | Y | N | N | N | N |
| 02-Jan-1998 | Y | N | N | N | N |
| 22-Jan-1998 | Y | N | N | N | N |
| 23-Jan-1998 | N | N | N | N | Y |
| 18-May-1998 | N | N | N | N | Y |
| 10-Jun-1998 | N | Y | Y | N | Y |
| 11-Jun-1998 | Y | N | N | N | N |
| 21-Jul-1998 | N | Y | N | Y | Y |
| 22-Jul-1998 | Y | N | N | N | N |
| 31-Dec-1998 | N | Y | N | N | N |
| 04-Feb-1999 | N | N | N | Y | N |
| 08-Feb-1999 | N | N | N | N | Y |
| 09-Feb-1999 | Y | N | Y | N | N |
| 12-Jul-1999 | Y | N | N | N | N |
| 26-Jan-2000 | Y | N | N | N | N |
| 28-Mar-2000 | N | N | N | N | Y |
| 11-Apr-2000 | N | N | Y | N | N |
| 13-Jun-2000 | N | N | Y | N | N |

* 'Y' denotes a statistically significant volatility break point; 'N' denotes no-break.

| *Break points* | USD/ZAR | EUR/ZAR | GBP/ZAR | JPY/ZAR | NEER |
|---|---|---|---|---|---|
| 14-Sep-2000 | N | N | Y | N | N |
| 04-Jan-2001 | Y | N | N | N | N |
| 26-Apr-2001 | Y | N | N | N | N |
| 20-Sep-2001 | N | N | N | N | Y |
| 21-Sep-2001 | Y | N | N | N | N |
| 13-Nov-2001 | N | N | Y | N | N |
| 27-Nov-2001 | N | Y | N | N | Y |
| 24-Jan-2002 | N | Y | Y | N | Y |
| 28-Jan-2002 | Y | N | N | N | N |
| 28-Feb-2002 | N | Y | N | N | N |
| 18-Mar-2002 | N | N | Y | Y | Y |
| 20-Mar-2002 | Y | N | N | N | N |
| 13-Dec-2002 | Y | N | N | N | N |
| 26-Jun-2003 | N | N | Y | N | N |
| 12-Dec-2003 | N | Y | N | N | N |
| 15-Dec-2003 | Y | N | N | N | N |
| 15-Jan-2004 | N | N | Y | N | N |
| 19-Jan-2004 | N | N | N | Y | Y |
| 28-Apr-2004 | N | N | N | Y | N |
| 11-May-2004 | N | Y | N | N | N |
| 13-May-2004 | N | N | Y | N | Y |
| 12-Aug-2004 | Y | N | N | N | N |
| 12-Jul-2005 | N | Y | N | N | N |
| 13-Jul-2005 | N | N | Y | Y | Y |
| 20-Sep-2005 | N | N | Y | N | N |
| 23-Sep-2005 | N | Y | N | N | N |
| 12-Dec-2005 | N | N | N | Y | N |
| 18-Apr-2006 | N | N | N | Y | N |
| 20-Apr-2006 | N | Y | N | N | Y |
| 24-Apr-2006 | Y | N | N | N | N |
| 23-Jun-2006 | N | N | Y | N | N |
| 16-Aug-2006 | N | Y | N | N | N |
| 02-Nov-2006 | N | Y | N | N | Y |
| 07-Nov-2006 | Y | N | N | N | N |
| 24-Nov-2006 | N | N | Y | N | N |
| 13-Mar-2007 | N | N | N | Y | N |
| 24-Jul-2007 | N | N | N | Y | N |
| 09-Oct-2007 | N | N | Y | N | N |
| 11-Jan-2008 | N | Y | N | N | N |
| 14-Jan-2008 | Y | N | N | N | N |
| 09-Apr-2008 | N | N | N | N | Y |
| 03-Sep-2008 | Y | N | N | N | N |
| 15-Sep-2008 | N | Y | N | N | N |
| 02-Oct-2008 | N | N | N | Y | Y |
| 03-Oct-2008 | Y | N | Y | N | N |
| 30-Oct-2008 | N | Y | N | N | Y |
| 03-Nov-2008 | Y | N | N | N | N |
| 11-Dec-2008 | N | Y | N | N | N |
| 19-Jan-2009 | N | N | Y | N | N |
| 15-May-2009 | N | N | N | Y | N |
| 02-Oct-2009 | N | Y | N | N | N |
| 26-Oct-2009 | N | N | Y | N | N |
| 05-Nov-2009 | N | N | N | N | Y |

* 'Y' denotes a statistically significant volatility break point; 'N' denotes no-break.