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The Determinants of Exchange Rate Volatility in South Africa^{*}

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Abstract

This paper investigates the determinants of exchange rate volatility in South Africa for the period 1986–2013 using the New Open Economy Macroeconomics model by Obstfeld & Rogoff (1996) and Hau (2002). The main focus of the paper is to test the hypothesis that economic openness decreases Rand (ZAR) volatility. This follows South Africa's liberalisation of its capital account in the mid-1990s and the mixed results in the literature on the relationship between exchange rate volatility and economic openness. Employing monthly time series data, GARCH models are estimated. The study finds that switching to a floating exchange rate regime has a significant positive effect on ZAR volatility. The results also indicate that trade openness significantly reduces ZAR volatility only when bilateral exchange rates are used, but finds the opposite when multilateral exchange rates are used. The study also finds that volatility of output, commodity prices, money supply and foreign reserves significantly influence ZAR volatility.

Keywords: Exchange Rate Volatility, GARCH. JEL Classification: F31, C22

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1 Introduction

Increasing financial liberalisation since the collapse of the Bretton Woods system in the 1970s has rendered exchange rates volatile in both developed and developing countries. As a result, the causes and effects of exchange rate volatility have become of particular interest to both researchers and policymakers. South Africa liberalised its capital account in March 1995 following the abolishment of the dual exchange rate system which had been in place since the mid-1980s. The South African currency (the Rand, henceforth ZAR) has subsequently been more volatile (Arezki, Dumitrescu, Freytag & Quintyn 2014, Ricci 2005). But, one might ask, why study the performance of the ZAR? The answer is that the ZAR is one of the most important emerging market currencies according to the 2013 survey by the Bank for International Settlements (BIS) (see table 1).

< Insert Table 1 Here>

Given the above, the question that follows is, did economic openness in March 1995 increase ZAR volatility? This follows findings by empirical Some researchers find that economic openness reduces exchange studies. rate volatility (Hau 2002, Calderón 2004, Bleaney 2008), while others find the opposite or no relationship (Amor & Sarkar 2008, Caporale, Amor & Rault 2009, Grydaki & Fountas 2010, Chipili 2012, Jabeen & Khan 2014). Due to conflicting results in empirical studies, only an empirical analysis can show the relationship between exchange rate volatility and economic openness in a country which has experienced an institutional change in its exchange rate regime. As such, this paper follows the modified version of the New Open Economy Macroeconomics model of Obstfeld & Rogoff (1996) by Hau (2002). This theoretical model asserts that there should be a negative relationship between exchange rate volatility and economic openness. That is, more open economies should have less exchange rate volatility. This study also tests the hypothesis that economic openness decreases ZAR volatility.

Few studies investigate the determinants of ZAR volatility. Arezki et al. (2014) examine the relationship between ZAR volatility and gold price volatility. Farrell (2001) analyses whether the imposition of capital controls in the mid-1980s affected commercial ZAR variability differently to financial ZAR variability between 1985 and 1995. This paper contributes to the literature by finding the sources of ZAR volatility using output volatility, money supply volatility, foreign reserves' volatility, commodity price volatility, openness, and a dummy for capital account liberalisation, as explanatory variables.

Several factors motivate this study. Firstly, many variables influence the level of the ZAR (Aron, Elbadawi & Kahn 1997, MacDonald & Ricci 2004, Frankel 2007, Saayman 2007, Faulkner & Makrelov 2008). Many variables might also cause large swings in the exchange rates. Secondly, exchange rate volatility is important in macroeconomics literature. In South Africa there is evidence of exchange rate volatility having significant effect on macroeconomic factors such as employment and trade (Todani & Munyama 2005, Mpofu 2013, Aye, Gupta, Moyo & Pillay 2014). Finding the sources of exchange rate volatility is relevant to policymakers and researchers to assist them to investigate how to tackle some of the effects of exchange rate volatility.

Thirdly, studies by Hau (2002) and Calderón (2004) attempt to find the sources of exchange rate volatility in South Africa¹. However, these studies use cross-country data and find aggregate results which do not isolate country specific effects. Besides, Hau (2002) states that the theoretical linkage between openness and real exchange rate volatility depends on the magnitude of the monetary and real shocks of each country. This suggests that analysing the sources of exchange rate volatility at a country level will likely be better for formulation of the correct type of policy response(s). Furthermore, they measure exchange rate volatility using very low frequency data (i.e. yearly data) yet exchange rate volatility will be best measured using either very high frequency data (i.e. intraday or daily data) or low frequency data (i.e. monthly or quarterly data).

Fourthly, the ZAR is indeed volatile. Using simple standard deviations of log real exchange rate of the domestic currency per United States dollar, table 2 shows the volatility of the ZAR compared to selected emerging markets. This table indicates that the ZAR is on average more volatile than the Indian, South Korean and Russian currencies but less volatile than Turkish, Brazilian and Malaysian currencies for the period $1992 - 2013^2$.

<Insert Table 2 Here>

Using GARCH models for the period 1986 to 2013 and employing monthly data, the study finds that switching to a floating exchange rate regime increases exchange rate volatility, trade openness reduces exchange rate volatility using the bilateral exchange rate of ZAR/US dollar while the opposite is found when using effective exchange rate. The results suggest that trade with some of South Africa's trading partners is less open. The results also show that volatility of output, commodity prices, money supply, and foreign reserves significantly influence ZAR volatility.

The structure of the paper is as follows: section 2 presents the literature

¹This follows the fact that South Africa is included in their sample of countries analysed.

²This shorter period is chosen due to lack of data for some variables used when calculating the real exchange rate prior to 1992M7. Here Real Exchange Rate is calculated as Nominal exchange rate * $\frac{CPI^*}{CPI}$ where CPI* is the foreign price and CPI is the domestic price.

review. Section 3 presents the theoretical model of exchange rate volatility. Section 4 reports the data and the descriptive statistics of the data used. Section 5 presents the econometric approach used, while section 6 reports empirical results. Section 7 provides the conclusion.

2 Literature Review

There is no general consensus on the macroeconomic determinants of exchange rate volatility in the literature. This is due to different approaches used based on different theoretical models of exchange rate level determination. Some studies examine the sources of exchange rate volatility using a specific exchange rate level model, whilst others are based on a synthesis of exchange rate level models.

Examples of specific models are as follows. First are studies based on the monetary models of exchange rate level determination (Morana 2009, Grydaki & Fountas 2009, Grydaki & Fountas 2010). These studies emphasise monetary variables as the determinants of exchange rate volatility. Second is the Optimum Currency Areas' model (Bayoumi & Eichengreen 1998, Devereux & Lane 2003). These studies put emphasis on trade linkages; asymmetry or similarity of economic shocks to output, country size and geographic factors, as the determinants of exchange rate volatility. Third is the New Open Economy Macroeconomics (Hau 2002, Calderón 2004, Amor & Sarkar 2008, Caporale et al. 2009). These studies stress that monetary variables and non-monetary factors are important in explaining exchange rate volatility.

Research which synthesises exchange rate models include studies by Chipili (2012) and Jabeen & Khan (2014) to mention a few. These studies use variables from different specific models considered important in explaining exchange rate movements in the countries of their studies. However, other studies find no link between macroeconomic fundamentals and exchange rate volatility (Flood & Rose 1995). Such studies support the role of nonmacroeconomic determinants of exchange rate volatility. For example, microstructure factors like the aggregation of a large number of news sources (Morana 2009 who cites Andersen & Bollerslev 1997).

Of the different models used for finding the determinants of exchange rate volatility above, this study uses the New Open Economy Macroeconomics model. This is due to the opening up of the financial system in South Africa to the rest of the world in March 1995. Prior to March 1995, South Africa had followed a dual exchange rate system from September 1985 to March 1995. During this period, the foreign exchange transactions of non-resident

portfolio investors on the capital account was separate from all other foreign exchange transactions³. This was the result of the increased volatility of the South African ZAR during the period 1982 to 1985 because of political pressure from the international community, which imposed trade sanctions because of Apartheid. The unification of the financial and commercial ZAR systems of capital controls in March 1995 make the use of New Open Economy Macroeconomics model appropriate to investigate the impact of such a change in institutional settings on the relationship between exchange rate volatility and its fundamentals. Subsequent studies discussed in the empirical literature find the following:

Arezki et al. (2014) employs a Vector Error Correction Model (VECM) to examine the relationship between the South African ZAR and gold price volatility for the period 1980 — 2010, using monthly data. Their results indicate that gold price volatility is vital in explaining the excessive exchange rate volatility of the ZAR. However, their paper only used commodity prices which do not capture a larger set of fundamental relative price movements. This paper contributes to this literature by using more explanatory variables for the determinants of South African ZAR volatility. In addition, this paper contributes to the debate about exchange rates in South Africa by focusing on the determinants of exchange rate volatility (i.e. the second moment of the relationship between the exchange rate and its determinants) given that most studies in South Africa have analysed the determinants of the level of the exchange rate (i.e. the first moment of the relationship between the exchange rate and its determinants were the exchange rate and its determinants of the level of the exchange rate (i.e. the first moment of the relationship between the exchange rate and its determinants) given that Makrelov 2004, Frankel 2007, Saayman 2007, Faulkner & Makrelov 2008).

Hau (2002) employs cross-sectional analysis on 48 (23 OECD and 25 non-OECD) countries over the period 1980 - 1998. He uses annual data on real effective exchange rate (REER) volatility measured as the moving sample standard deviation of REER percentage changes over three-year period. With control variables of per capita GDP, dummies for revolutions and coups, central bank independence, and exchange rate commitments, Hau finds a negative relationship between real exchange rate volatility and trade openness. That is, more open economies will have less real exchange rate volatility. The theoretical linkage between real exchange rate volatility and openness depends on the magnitude of monetary and real shocks of each country. Hau (2002) therefore re-estimates the regression equation using only OECD countries, given that they are more homogeneous⁴. He still finds

³The financial ZAR system of capital controls was imposed on non-resident portfolio investors while the other was the commercial ZAR system.

⁴This is based on the notion that these countries experience similar shocks which are relatively of the same magnitude.

a negative relationship between real exchange rate volatility and trade openness, but the results are more pronounced (they have higher explanatory power) than the results using 48 countries.

Calderón (2004) uses a GMM method on 77 industrial and developing countries over the period 1974 - 2003. Calderon uses annual data on REER volatility measured as the standard deviation of changes in the REER over a 5-year period, as well as the volatility of real exchange rate fundamentals. Calderón (2004) finds that there is a negative relationship between real exchange rate volatility and economic openness. He also finds a negative relationship between real exchange rate volatility and government spending volatility. He, however, finds a positive relationship between real exchange rate volatility and output, money supply, and terms of trade volatilities respectively. Using the same GMM method, Amor & Sarkar (2008) also find a negative relationship between exchange rate volatility and trade openness for ten South and South East Asia economies. Bleaney (2008) also finds similar results for real exchange rate volatility and trade openness.

Caporale et al. (2009) find a similar negative relationship between real exchange rate volatility and trade openness for the period 1979 - 2004. Their results show that there is a positive relationship between real exchange rate volatility and financial openness for the entire sample, which comprises 39 developing countries (20 from Latin America, ten from Asia, and nine from $MENA^5$). These results are similar to Amor & Sarkar (2008). However, the regressions for the three separate regions find different results. For the Asian region, they find that financial openness causes real exchange rates to be more volatile, but REER volatility is mainly due to domestic real shocks, while external shocks play a small role. For the MENA region, they find that financial openness causes the real exchange rate to be less volatile, but REER volatility is mainly caused by monetary and real shocks. For the Latin American region, they find that external and monetary shocks are the main sources of real exchange rate volatility. The results by Hau (2002) for OECD countries only and the analysis by Caporale et al. (2009) suggest that finding the sources of exchange rate volatility for a single country is more appropriate for policymakers because the results are not generalised. This study also improves on studies that use standard deviation as the proxy for volatility, because GARCH models are able to describe the time-varying volatility directly which the standard deviation models are unable to do.

Using daily data from 1 January 1999 to 31 December 2004, (Stancık 2006, Stancık 2007) investigates the determinants of real exchange rate volatility

⁵The countries in the MENA region include: Algeria, Egypt, Iran, Israel, Jordan, Morocco, Syria, Tunisia and Turkey.

for six Central and Eastern European countries. The study focuses on trade openness, news factors, and exchange rate regimes as explanatory variables. Real exchange rate used is the bilateral between the Euro and the U.S. dollar. Real exchange rate volatility is measured using the threshold autoregressive conditional heteroskedasticity (TARCH) model. The final model is estimated using Ordinary Least Squares (OLS) and the results for each country indicate that there is a negative relationship between real exchange rate volatility and trade openness for the four countries. The other two countries show insignificant coefficients between real exchange rate volatility and trade openness. The news factor presents mixed results for different countries.

Chipili (2012) examines the sources of volatility of the Zambian kwacha exchange rate (real and nominal) using the GARCH models (GARCH, TARCH and EGARCH). He finds that both monetary factors (money supply, inflation, short-term domestic interest rate, and foreign reserves), and real factors (terms of trade, openness, and output) affect exchange rate volatility. The results indicate that real factors have smaller effects on exchange rate volatility than monetary factors. This suggest that monetary policy has an important role in mitigating the volatility of the exchange rate. His results show that using the GARCH(1,1) and TARCH(1,1) models, the relationship between exchange rate volatility and openness is insignificant. Using an EGARCH model, he finds a positive and significant relationship between exchange rate volatility and openness for the kwacha and 19 other currencies, except for Zimbabwean dollar, which is negative and significant. He asserts that the different findings regarding openness for some exchange rate volatility suggest that the degree of openness, that is, the extent of trade linkages between Zambia and her trading partners, is low relative to what is implied by theory.

Jabeen & Khan (2014) also use various macroeconomic factors to find the determinants of exchange rate volatility in Pakistan using GARCH(1,1) and TARCH(1,1) models. Their study finds that real output volatility, foreign exchange reserves' volatility, inflation volatility, productivity, and terms of trade volatility are important determinants of exchange rate volatility. Their study uses trade restrictions measured by the reciprocal of trade openness and the results find positive and insignificant coefficients for this variable. Morana (2009) also finds support for macroeconomic fundamentals in influencing exchange rate volatility. Morana (2009) argues that the exchange rate is an important determinant of aggregate demand, and therefore conducts the Granger-Causality test to establish the direction of causality. The results show that causality is bi-directional, but is stronger from macroeconomic factors to exchange rate volatility than vice-versa. This suggest that stability in the macroeconomic variables is recommended to reduce exchange rate volatility, which contradicts the findings of Flood & Rose (1995) in their

study for G-7 countries.

3 Theoretical Model

This paper uses the New Open Economy Macroeconomics (NOEM) model as the theoretical framework linking exchange rate volatility, economic openness and the volatility of exchange rate fundamentals. The NOEM model is based on the work of Obstfeld & Rogoff (1996) which formalises exchange rate determination in the context of dynamic general equilibrium models with explicit microfoundation, imperfect competition, and nominal rigidities.

The NOEM model is ideal for measuring economic openness and the volatility of exchange rate fundamentals of South Africa for the following reasons: First, South Africa is a good case study following the liberalisation of the capital account in March 1995. Such institutional change in the exchange rate regime enables one to test the hypothesis that economic openness leads to a reduction in exchange rate volatility, as asserted by Hau (2002), and investigate the relationship between exchange rate volatility and its fundamentals.

Secondly, this model emphasises that both monetary and non-monetary variables are important in explaining exchange rate volatility. This is unlike models based only on exchange rate determination using monetary or optimum currency areas. Monetary variables might matter in South Africa in trying to explain the ZAR volatility because the ZAR is volatile while at the same time the monetary authorities have not been able to stabilise inflation rates at low levels. This follows arguments by Dornbusch (1976) and Rogoff (1999). Dornbusch's (1976) overshooting model shows that monetary policy shocks might lead to disproportionately large fluctuations in exchange rates. Dornbusch's (1976) model asserts that monetary instability can lead to excessive exchange rate instability, thus putting the blame for exchange rate volatility on monetary authorities.

However, Rogoff (1999) argues that monetary authorities cannot take the blame for causing exchange rate volatility, as most industrial economies have stabilised inflation rates⁶ and their exchange rates are still significantly volatile. Although South Africa is categorised as an emerging market economy, its financial sector is well developed. At the same time, its inflation rate has not fallen and stabilised at low levels. This supports the use of monetary variables as explanatory variables for exchange rate volatility in South Africa.

⁶The countries in question are the U.S.A, Japan and European countries where the inflation rates stabilised at rates below 3 percent.

Other studies in South Africa (Aron et al. 1997, MacDonald & Ricci 2004, Frankel 2007, Saayman 2007, Faulkner & Makrelov 2008) show that nonmonetary factors significantly influence the level of the ZAR. This finding, together with arguments by Meese & Rogoff (1983) that monetary models are unable to replicate and forecast exchange rate swings, imply that monetary variables are only one of several factors driving exchange rate volatility. Thus the literature that uses the NOEM model argues that non-monetary factors have gained importance in explaining exchange rate volatility.

To show the link between exchange rate volatility, economic openness, and the volatility of exchange rate fundamentals, this study follows the work of Obstfeld & Rogoff (1996) and Hau $(2002)^7$. Using the first order conditions derived from the basic set up of the model, trade openness is defined as

$$Openness = \frac{P_T C_T}{P_N C_N + P_T C_T} = \varphi \tag{1}$$

Following this definition, the dynamics of the model are analysed taking the log-linear approximation from the initial steady state. The temporary percentage change from the initial steady state is denoted by $X = \frac{(X_1 - \overline{X_0})}{\overline{X_0}}$ while the permanent percentage change from the initial steady state is denoted by $\overline{X} = \frac{(\overline{X} - \overline{X_0})}{\overline{X_0}}$. The model first analyses monetary shocks. The model assumes that the

The model first analyses monetary shocks. The model assumes that the economy encounters an unanticipated permanent monetary shock, that is, $M^S = \overline{M^S}$. This assumption and log-linearizing equation 29 around the steady state results in the following equation:

$$\varepsilon (m-p) = p_T - p + \frac{\beta}{1-\beta} (p_T - p)$$
(2)

Given that the prices of nontradables are fixed in the short-run, that is, $P_N = 0$ and the long-run neutrality of money, $\overline{P_T} = \overline{M^S}$ leads to:

$$P_T = \frac{\beta + (1 - \beta)\varepsilon}{\beta + (1 - \beta)(1 - \varphi + \varphi\varepsilon)}\overline{M^S}$$
(3)

Since the law of one price holds for tradables, the short-run percentage price change is proportional to money supply and exchange rate. That is, $P_T = M^S = E$. Given that consumption smoothing implies a constant consumption of tradables, it then means $C_T = 0$. Following this and the nominal rigid nontradables' prices implies that the real price of nontradables decreases

 $^{^7\}mathrm{See}$ the appendix for the detailed explanation of the basic set up and steady state analysis of the model.

and their demand increases. Hence log linearising equation 27 depicts that consumption expansion in nontradables is proportional to the tradable price increase. That is, $C_N = P_T$. So using equation 1, Hau (2002) shows that the percentage real exchange rate change is given by:

$$E - P = P_T - P = (1 - \varphi) P_T = (1 - Openness) M^S$$
(4)

Thus

$$Vol = \left[\varepsilon \left(E - P\right)^2\right]^{\frac{1}{2}} = \left(1 - Openness\right)\sigma_M^2 \tag{5}$$

Meaning more open economies are expected to have less real exchange rate volatility, holding all other things constant. This is the hypothesis to be tested in this paper, as mentioned in section 1.

Second, the model analyses real shocks. The model assumes the nontraded sector faces an unanticipated permanent increase in marginal disutility and log linearising equation 28 gives the following equation:

$$-P_T - C_T = \kappa + y_N \tag{6}$$

Given that the model assumes constant endowment of tradables, $\overline{y_T}$, constant net foreign assets and the consumption-smoothing motive, means that $C_T = 0$. If equation 27 is log linearised, we get $C_T = P_T$ given rigid nontradable prices ($P_N = 0$). Market clearing conditions for nontradables, $C_N = y_N$, then determines the fluctuations in the prices of tradables to get, $P_T = \frac{1}{2}\kappa$. Since the price of tradables is linked to the world price level, the volatility of the real exchange rate is given by:

$$E - P = P_T - P = (1 - \varphi) P_T = (1 - Openness) \frac{1}{2} \kappa$$
(7)

Thus

$$Vol = (1 - Openness)\sigma_{\kappa}^2 \tag{8}$$

Meaning an unanticipated real shock also generates the negative relationship between economic openness and real exchange rate volatility as monetary shock.

Lastly the model analyses fiscal shocks, and assumes the economy faces an unanticipated permanent fiscal shock. Using this information and log linearising equation 32, as well as other equations from the model, leads to the following relationship between real exchange rate volatility and government spending:

$$Vol = (1 - Openness)\sigma_G^2 \tag{9}$$

Controlling for various explanatory variables, the model states that there should be a negative relationship between exchange rate volatility and economic openness. The next section defines all the variables used in this paper.

4 Data

This paper uses monthly time series data for South Africa from 1986M2 — 2013M11 obtained from the South African Reserve Bank (SARB), Datastream, and International Monetary Fund (IMF)'s International Financial Statistics (IFS). All indices used have the base year of 2010. All the variables are seasonally adjusted using TRAMO/SEATS⁸ ARIMA tools. This is done to remove cyclical seasonal movements that are common in time series data observed at monthly and quarterly frequency. The variables are defined as follows:

The dependent variable is the real exchange rate volatility measured using the conditional variance from a GARCH(1,1) process based on the following equation:

$$x_t = \alpha_0 + \alpha_1 x_{t-1} + \varepsilon_t$$

$$h_t^2 = \lambda_0 + \lambda_1 \varepsilon_{t-1}^2 + \lambda_2 h_{t-1}^2$$
(10)

where $x_t = \text{dlog}(\text{real exchange rate})$ and $h_t^2 = \text{conditional variance of } \varepsilon_t$. The real exchange rate is calculated as follows:

$$RER = E * \frac{P^*}{P} \tag{11}$$

where RER refers to real exchange rate, E refers to nominal exchange rate using South African ZAR per United States dollar, P^{*} refers to foreign price index and P refers to domestic price index. Which price indices to use remains a practical problem. The literature suggests the use of consumer price index (CPI), wholesale price index / producer price index, GDP deflators and unit labour costs. Because these are the data available, two different measures of the real exchange rate are applied in this study. One based on consumer prices (equation 12 below) and, the second on the relative prices of tradables and non-tradables (equation 13 below) as follows:

$$RERCPI = E * \frac{CPI^*}{CPI} \tag{12}$$

$$RERWPI = E * \frac{P_T^*}{P_N} = E * \frac{WPI^*}{CPI}$$
(13)

In addition to bilateral RER, multilateral RER is also used. Real Effective Exchange Rate (REER) refers to the trade weighted real exchange rate.

⁸TRAMO stands for Time Series Regression with ARIMA noise, missing values and outliers. SEATS stands for Signal Extration in ARIMA Time Series.

This data is for the 20 trading partners of South Africa, and based on manufacturing goods. Both bilateral and multilateral nominal exchange rates are used. Nominal Exchange Rate (RUSNOM) refers to the nominal exchange rate for the ZAR per US dollar. Nominal Effective Exchange rate (NEER) refers to the trade weighted nominal exchange rate for the 20 trading partners of South Africa.

Independent variables include, firstly *Output* volatility, where output is measured using real GDP. This variable is used to proxy real productivity shocks. However, RGDP is not available in monthly frequency. As a result, monthly RGDP is interpolated using the cubic spline method from quarterly RGDP. Secondly *Money Supply volatility*, which uses the narrow definition of money supply, that is, M1. *Openness* is the third variable, in which trade openness (to) is measured as the ratio of exports of goods and services and imports of goods and services to nominal GDP. The values of the three variables are all expressed in domestic currency. However, due to the nonexistence of monthly GDP data, monthly GDP data is interpolated from quarterly nominal series using the cubic spline method. The cubic spline method is common in the literature for converting either annual or quarterly GDP data to monthly data (Chipili 2012)⁹.

Foreign Reserves volatility is the fourth variable, and for this gross reserves are used. This variable is used for economic openness given that through openness, central banks are able to accumulate foreign reserves. Commodity Prices volatility is the fifth variable and real gold price in domestic currency based on the pricing in London is used to proxy commodity prices. This study uses commodity price volatility as one of the determinants of exchange rate volatility, unlike other studies that have used Terms of Trade (TOT) volatility. This follows the finding by other researchers (Cashin, Cespedes & Sahay 2002, MacDonald & Ricci 2004, Frankel 2007) that TOT tends not to be significant in most countries that are commodity exporters as one of the determinant of exchange rate, whilst commodity prices tend to be significant. MacDonald & Ricci (2004) assert two reasons for this. First, commodity prices are more accurate in terms of measurements, unlike TOT, which are based on arbitrary construction of country-specific export and import deflators. Second, commodity prices data are frequently available for analysis.

Using real gold price volatility as an independent variable might cause one to argue that there is reverse causality (i.e. endogeneity problem). For

⁹Chipili (2012) converts annual to monthly while Schneider et al.(2007), "Yemen: Exchange Rate Policy in the Face of Dwindling Oil Exports" International Monetary Fund Working Paper No.0705, converts to quarterly.

example, Arezki et al. (2014) find that between 1979 and 1995 causality runs from ZAR volatility to gold price volatility, but between 1995 and 2010, causality runs from gold price volatility to ZAR volatility. Accordingly, I run Granger causality tests between ZAR volatility and gold price volatility. The results for this test are shown in table 11 and indicate that for the study period for this paper, causality runs from gold price volatility to ZAR volatility only¹⁰. That is, gold price volatility causes ZAR volatility. There is therefore no issue of possible reverse causality. The sixth and last variable is *Exchange Rate Regime*, represented by a dummy variable. The dummy for this variable takes the value of one from 1995M4 onwards and zero otherwise. Following the definition of the variables to be used in section 5 when econometric analysis is undertaken, first I present the preliminary tests for the variables in the next section.

<Insert Table 11 Here>

4.1 Descriptive Statistics

Estimating empirical models using time series data requires the variables be stationary, implying unit root tests should be done before carrying out any analysis. Accordingly, I apply the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests to find the order of integration of the variables. Table 3 shows that all the variables are integrated of order one $\{I(1)\}$ while table 4 indicates that all the variables except trade openness are integrated of order one $\{I(1)\}$.

<Insert Table 3 and 4 Here>

After finding the stationarity properties of all the variables, I find the summary statistics of all the stationary variables to show some key stylised facts. Table 5 indicates that the variables exhibit similarities to the behaviour of financial time series. That is, having excess kurtosis, and not following a normal distribution. For example, eight out of ten variables indicate excess kurtosis. The kurtosis of the standard normal distribution is three. The skewness of the variables is not equal to zero, which implies the variables do not follow a standard normal distribution. Using the Jarque-Bera statistic, table 5 shows that nine out of ten variables are not normally distributed, given that they have significant coefficients. Furthermore, table 5 shows that money supply and output varies less than the exchange rate (using the bilateral ZAR/US dollar and nominal effective exchange rates) based on the

¹⁰The table reported in the appendix shows that one lag is used. However, I also try using lags from two up to 12 given that the data is monthly. The results confirm that causality runs from gold price volatility to ZAR volatility only.

standard deviation measure of variability. This is similar to findings by Flood & Rose (1995), Hviding, Nowak & Ricci (2004) and Chipili (2012).

<Insert Table 5 Here>

Having removed the unit root from the variables, I examine if all the variables, with the exception of trade openness, have volatility clustering, that is, the presence of ARCH effects — meaning there is heteroskedasticity in these variables. Accordingly, I apply the ARCH Lagrange Multiplier (ARCH-LM) test and the White (1980) test in cases where ARCH-LM is not adequate to detect heteroskedasticity. The estimated mean equation for each variable includes a constant and the lags of the corresponding variable only. Table 6 indicates the presence of volatility clustering in all nine variables, with significance at 1% level for output, gold price, money supply, real and nominal effective exchange rates, and 5% level for real and nominal bilateral exchange rates for ZAR/US dollar, while foreign reserves are significant at 10%. Having volatility clustering implies that it is appropriate to use GARCH models. Figures 1 to 5 show the estimated conditional variance for the exchange rates and confirms that there is volatility clustering. That is, large changes tend to be followed by large changes and small changes tend to be followed by small changes, and periods of tranquility interchange with periods of high volatility.

<Insert Table 6 and Figures 1–5 Here>

Table 7 shows the correlation matrix for exchange rates volatility, economic openness, and volatility of exchange rate fundamentals. The table indicates that there is a negative correlation between exchange rate volatility and trade openness. The negative correlation between exchange rates volatility and trade openness implies that the higher the degree of trade openness in goods and services, the lower the volatility of exchange rates. This is a preliminary confirmation of the hypothesis mentioned in section 1 and asserted in section 3.

<Insert Table 7 Here>

5 Econometric Approach

The empirical literature mostly proxies volatility of the variable(s) in question by either conditional variance or standard deviation models. Standard deviation methods includes both predictable and unpredictable components of volatility, whilst the conditional variance method is a better proxy for uncertainty because it contains an unpredictable component of volatility. Conditional variance models include ARCH-type, stochastic volatility, and implied volatility. This study focuses on ARCH-type models, following their introduction by Engle (1982) and their extension by Bollerslev (1986).

The exchange rates exhibit volatility clustering whereby large changes tend to be followed by large changes, and small changes by small changes. Periods of tranquility interchange with periods of high volatility, making successive exchange rate changes dependent on each other (Kwek & Koay 2006, Chipili 2012). The empirical literature confirms that exchange rates, like other financial time series, show non-linear behaviour (Chipili 2012). Such behaviour can be estimated using GARCH models, given that GARCH models are able to model and forecast time-varying variance.

This study therefore utilises GARCH(1,1) and EGARCH(1,1) models to examine the sources of exchange rate volatility in South Africa. A GARCH(1,1)model is adopted, following the literature, which shows that such a model is parsimonious, even though higher order models do exist. The estimated empirical equations are:

Mean equation for exchange rate volatility

$$\Delta x_{t} = \lambda_{0} + \sum_{i=1}^{q} \lambda_{i} \Delta x_{t-1} + \phi_{1} \Delta Output_{t} + \phi_{2} \Delta MS1_{t} + \phi_{3} \Delta Open_{t} (14) + \phi_{4} \Delta Fxres_{t} + \phi_{5} \Delta Rgoldp_{t} + \phi_{6} Exrate \operatorname{Re} gime_{t} + \varepsilon_{t}$$

Variance equation for exchange rate volatility using a GARCH(1,1) method

$$h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta h_{t-1}^2 + \varphi Exrate \operatorname{Re} gime_t + \zeta_1 \Delta Output_t + (15)$$

$$\zeta_2 \Delta MS1_t + \zeta_3 \Delta Open_t + \zeta_4 \Delta Fxres_t + \zeta_5 \Delta Rgoldp_t + v_t$$

Variance equation for exchange rate volatility using an $\mathrm{EGARCH}(1,1)$ method

$$\ln(h_t^2) = \alpha_0 + \alpha_1(\varepsilon_{t-1}/h_{t-1}^{0.5}) + \lambda_1(\varepsilon_{t-1}/h_{t-1}^{0.5}) + \beta \ln(h_{t-1}^2) + (16)$$

$$\varphi Exrate \operatorname{Re} gime_t + \zeta_1 \Delta Output_t + \zeta_2 \Delta MS1_t + \zeta_3 \Delta Open_t + \zeta_4 \Delta Fxres_t + \zeta_5 \Delta Rgoldp_t + v_t$$

where Δx_t is the logarithmic first difference in the exchange rate; ε_t is residuals that are used to test for the presence of ARCH effects in the exchange rate; q is the lag length; h_t^2 is conditional variance of x_t derived from GARCH(1,1); $\lambda_0, \lambda_i, \phi_{1,\dots,6}, \lambda_1, \alpha_{0,1}, \beta, \varphi$ and ζ_1, \dots, ζ_5 are parameter coefficients to be estimated. Even though the objective of the study is to find macroeconomic factors that drive exchange rate volatility, the explanatory variables are also included in the mean equation. This is done because exchange rate volatility is uncertain and as such the impact of exchange rate levels should be controlled for. This is also because at monthly frequency, fundamentals matter for exchange rate movements. This differs from the study by Fidrmuc & Horváth (2008), who do not include explanatory variables in the mean equation (they only include lagged values of the exchange rate) because at daily frequency, fundamentals do not matter much in explaining the movements of the exchange rate. An EGARCH model is also estimated because the literature shows that asset prices react differently to bad and good news. This indicates that it is also appropriate to estimate GARCH models with asymmetry effects. There are two models with asymmetry effects namely, threshold GARCH (TGARCH) and exponential GARCH (EGARCH) models. However, this study uses an EGARCH model only because, according to Enders (2010), this model has advantages over the TGARCH model. An advantage of the EGARCH model is that it does not require restriction of non-negativity of coefficients, as in a GARCH model.

6 Results

In estimating the GARCH models, various AR(p) model specifications for the mean equation are used together with the variance equation. That is, estimating equations 14 and 15 for a GARCH(1,1) model, and equations 14 and 16 for an EGARCH model. The best model is chosen, based on the diagnostic tests of standardised residuals, which show the absence of serial correlation and no remaining ARCH effects. When both GARCH(1,1)and EGARCH(1,1) are significant for a specific exchange rate series, the best model is also based on the model with the larger value of log likelihood, and the smallest values for Akaike Information Criteria (AIC) and Schwartz Information Criteria (SIC). The exchange rate series estimated include real effective exchange rate (REER), nominal effective exchange rate (NEER), real bilateral exchange rate for the ZAR/US dollar, measured using consumer price indices for both countries (RERCPI), and one using the wholesale price index for the foreign country and consumer price index for the domestic country (RERWPI), and nominal bilateral ZAR/US dollar (RUS-NOM). Accordingly, the Q-statistic for standardised residuals, the Q-statistic for squared standardised residuals, and the ARCH-LM in table 10 indicate that there is no serial autocorrelation and no ARCH effects remaining, given the insignificant p-values. The results show that conditional volatility is persistent and mean reverting in all exchange rates, given that h_{t-1}^2 coefficient is significant and less than one, as shown in tables 8 and 9.

<Insert Tables 8, 9 and 10 Here>

The results for the EGARCH models show that the asymmetric term is insignificant for RERCPI, RERWPI, RUSNOM and NEER series, while it is significant and negative for the REER series at 10% level. This suggests that there is no impact of news effect on the RERCPI, RERWPI, RUSNOM and NEER series at monthly level. This is in line with the efficient market hypothesis which states that the effect of news on asset prices like exchange rates clears fast and is immediately reflected in the changes of the asset price in question. At monthly frequency therefore news might have less effect. These results are similar to other studies that do not find significant effects of asymmetric GARCH models at monthly frequency, like those of Jabeen & Khan (2014) and Chipili (2012). The significance of EGARCH, using REER suggests that negative news leads to a higher increase in exchange rate volatility, compared to positive news.

In addition, the results about insignificance of the asymmetric term which captures the impact of news, suggests that the behaviour of the exchange rate should also be analysed using short-term periods, for example, daily or intraday data. This follows some researchers (see e.g. Flood & Taylor 1996, MacDonald 1999, Morana 2009) who argue that exchange rate movements cannot always be explained by flow demand and supply components, but by using market microstructure models.

Conditional volatility persists for about a month on average following a shock in REER, NEER and RERCPI based on the half-life (HL) measure. But conditional volatility persists for about six months and 12 months on average following a shock in RUSNOM and RERWPI series respectively. The persistence of past shocks on conditional volatility measured by HL is calculated as $\log(0.5) / \log(h_{t-1}^2)$. HL then captures the period it takes for a shock to volatility to decrease to half its original size and h_{t-1}^2 is the speed of convergence to the steady state level. Furthermore, the results show that REER and RUSNOM are GARCH(1,1) models, given the significance of both ε_{t-1}^2 and h_{t-1}^2 terms. However, for RERCPI, RERWPI and NEER series, the results indicate the presence of strong GARCH effects, given the significance of h_{t-1}^2 only, a result which is similar to the study by Singh (2002).

Given the study's objective of finding the determinants of exchange rate volatility, only the parameters in the variance equation(s) are analysed. The results show that the exchange rate regime dummy is positive and significant. This means that switching to a floating exchange rate system leads to significantly more exchange rate volatility, which is consistent with most findings in the literature (see e.g. Canales-Kriljenko & Habermeier 2004, Stancık 2007, Chipili 2012) and the hypothesis of the ZAR behaviour as mentioned in section 1 by some researchers (see e.g. Arezki et al. 2014, Ricci 2005).

Using the exchange rate series of REER, NEER, RERWPI and RUSNOM, the results show that real gold price volatility has significant and positive effects on ZAR volatility. This implies that, as gold price volatility increases so does ZAR volatility. The significance of real gold price volatility in influencing ZAR volatility is similar to findings of the study by Arezki et al. (2014) who use a different method. The positive effect is similar to studies that use terms of trade volatility (see e.g. Calderón 2004, Caporale et al. 2009, Jabeen & Khan 2014).

The results indicate negative and significant coefficients for trade openness using the RERWPI and RUSNOM series. This means that as trade openness increases, exchange rate volatility decreases. These results are in line with the theoretical model explained in section 3 and the results found by other studies (Hau 2002, Calderón 2004, Caporale et al. 2009). However, using REER series, the results are positive and significant, which is contrary to what theory indicates as mentioned in section 3. The positive and significant value suggests that the degree of openness is low relative to what the theory says. This implies that South Africa needs to increase its trading with the 20 countries (or some of them) used in the construction of the REER by the South African Reserve Bank. The results may also be affected by the use of aggregate as opposed to bilateral trade data, as proposed by OCA theory (Hau 2002).

Foreign reserves' volatility has a negative and significant value for NEER. This implies that changes in foreign reserves create confidence in foreign markets, as argued by Hviding et al. (2004). This follows the argument that high and adequate international reserves are important for the prevention of a currency crisis, given that it signals the ability of the central bank to intervene in the foreign exchange market to stabilise the currency as well as boost confidence for credit ratings.

The results are negative and significant for money supply volatility using the RERCPI series. This result is similar to Morana (2009) who finds a negative value in one country and Grydaki & Fountas (2010), who find negative money supply volatility for Argentina and Chile. Carrera & Vuletin (2002) assert that the negative effect is associated with increased interest rates, which lead to a decrease in money supply, and therefore a decline in exchange rate volatility. This suggests that the higher interest rates in South Africa lead to more short-term capital inflows, with the expectation of higher returns, and thus increases exchange rate volatility.

Output volatility has a positive and significant effect on RERWPI volatility. This is with the perspective of Friedman (1953) that exchange rate volatility might be caused by macroeconomic instability. This means that, as instability increases, exchange rate volatility also increases. The coefficients are insignificant when using RERCPI, RUSNOM and NEER series but negative and significant when using REER series. The negative value is also similar to arguments presented by Friedman (1953) that it is possible to have high output volatility leading to lower exchange rate volatility. This means that there are some traders who are not concerned about instability in a country and want to invest regardless, as long as they will ultimately benefit. This phenomenon is widely seen in countries with many natural resources, for example, gold, or diamonds, or oil. Jabeen & Khan (2014) also find a negative and significant relationship between output volatility and exchange rate volatility for Pakistan, using US dollar currency. The insignificant output value confirms the claims of Flood & Rose (1995) that macroeconomic volatility is not an important source of exchange rate volatility.

7 Conclusion

This paper investigates the determinants of real and nominal exchange rate volatility using both bilateral (ZAR/US dollar) and effective exchange rates over the period 1986M2 — 2013M11 for South Africa. Using GARCH(1,1) and EGARCH(1,1) models, the study has two objectives: First, it tests the hypothesis that economic openness decreases exchange rate volatility in South Africa. Second, it examines other macroeconomic factors that cause exchange rate volatility. The results show that switching to a floating exchange rate system leads to an increase in ZAR volatility, as hypothesised by some researchers (see e.g. Arezki et al. 2014, Ricci 2005). This is informed by evidence of a positive and significant effect of a dummy variable post March 1995 when South Africa liberalised its capital account. The results also show that trade openness decreases ZAR volatility in South Africa using, the real and nominal bilateral ZAR/US dollar, and that other macroeconomic factors also influence ZAR volatility.

The results for macroeconomic factors are summarised as follows: Real gold price volatility increases ZAR volatility. The significance of this variable in influencing exchange rate volatility is similar to findings in the study by Arezki et al. (2014), who use a different method. Foreign reserves' changes reduce exchange rate volatility, which is in line with the finding by Hviding et al. (2004). Money supply volatility influences exchange rate volatility negatively, which suggest that increases in the interest rate leads to higher exchange rate volatility. The results also indicate that output volatility increases exchange rate volatility, using bilateral exchange rate. However, when using the real effective exchange rate, the results between output volatility and exchange rate volatility are the opposite. This is in line with the argu-

ments by Friedman (1953) and findings by Jabeen & Khan (2014).

However, the results indicate that real factors (commodity prices volatility, output volatility, and openness) have higher magnitudes of influence, compared to monetary factors. An increase in exchange rate volatility might hurt the economy via adverse effects on employment growth and trade. This suggests that the South Africa government should focus more on real factors if they aim to reduce exchange rate volatility. For example, it is necessary to evaluate the costs of increasing openness and understand the relationship between exchange rate volatility and fundamentals, rather than just focusing on exchange rate levels. This follows the recent debate on whether capital controls are appropriate, in view of surges in capital inflows into emerging markets. The fact that monetary factors also influence exchange rate volatility implies that monetary authorities also have a part to play in reducing exchange rate volatility.

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8 Appendix

Table 1: Selected Developed and Emerging Market Currency Distribution of global exchange market (percentage shares of average daily turnover in April)-1998 to 2013

Currency	1998	2001	2004	2007	2010	2013
United States dollar	86.8(1)	89.9(1)	88.0(1)	85.6(1)	84.9(1)	87.0(1)
European euro	(32)	37.9(2)	37.4(2)	37.0(2)	39.1(2)	33.4(2)
Japanese yen	21.7(2)	23.5(3)	20.8(3)	17.2(3)	19.0(3)	23.0(3)
British pound	11.0(3)	13.0(4)	16.5(4)	14.9(4)	12.9(4)	11.8(4)
Australian dollar	3.0(6)	4.3(7)	6.0(6)	6.6(6)	7.6(5)	8.6(5)
Canadian dollar	3.5(5)	4.5(6)	4.2(7)	4.3(7)	5.3(7)	4.6(7)
Mexican peso	0.5(9)	0.8(14)	1.1(12)	1.3(12)	1.3(14)	2.5(8)
Chinese renminbi	0.0(30)	0.0(35)	0.1(29)	0.5(20)	0.9(17)	2.2(9)
Russian rouble	0.3(12)	0.3(19)	0.6(17)	0.7(18)	0.9(16)	1.6(12)
Turkish lira	(33)	0.0(30)	0.1(28)	0.2(26)	0.7(19)	1.3(16)
Korean won	0.2(18)	0.8(15)	1.1(11)	1.2(14)	1.5(11)	1.2(17)
South African rand	0.4(10)	0.9(13)	0.7(16)	0.9(15)	0.7(20)	1.1(18)
Brazilian real	0.2(16)	0.5(17)	0.3(21)	0.4(21)	0.7(21)	1.1(19)
Indian rupee	0.1(22)	0.2(21)	0.3(20)	0.7(19)	1.0(15)	1.0(20)
Polish zloty	0.1(26)	0.5(18)	0.4(19)	0.8(17)	0.8(18)	0.7(22)
Malaysian ringgit	0.0(27)	0.1(26)	0.1(30)	0.1(28)	0.3(25)	0.4(25)
Chilean peso	0.1(24)	0.2(23)	0.1(25)	0.1(30)	0.2(29)	0.3(28)

Note: the number outside the brackets represents the share of the currency while the number in brackets represents the rank of the currency.

Source: Bank for International Settlements, Triennial Central Bank Survey (2013).

Table 2:	Standard Deviations of Re	al domestic	currency per	US dollar, 1992-
2013				

2010								
Year	Brazil	India	SA	SK	Malaysia	Mexico	Russia	Turkey
1992	2.65[2.72]	0.15[0.15]	0.98[1]	0.07[0.07]	1.24[1.27]	0.34[0.35]	3.67[3.75]	1.56[1.60]
1993	2.30[2.92]	1.36[1.73]	0.79[1]	0.06[0.08]	1.64[2.08]	0.33[0.42]	1.81[2.29]	2.40[3.05]
1994	4.12[4.89]	0.16[0.19]	0.84[1]	0.06[0.07]	1.63[1.94]	1.55[1.84]	1.35[1.61]	37.86[45]
1995	3.32[7.37]	0.52[1.15]	0.45[1]	0.18[0.40]	1.14[2.54]	4.21[9.35]	1.06[2.35]	9.54[21.2
1996	0.89[0.66]	0.60[0.44]	1.35[1]	0.15[0.11]	0.84[0.63]	0.72[0.54]	0.36[0.27]	6.97[5.18]
1997	0.68[0.70]	0.340.35]	0.98[1]	1.50[1.53]	4.23[4.34]	0.81[0.83]	0.17[0.18]	3.67[3.76
1998	0.63[0.27]	0.47[0.20]	2.29[1]	1.06[0.46]	6.34[2.77]	1.26[0.55]	3.34[1.46]	3.27[1.43]
1999	12.54[21]	0.28[0.48]	0.59[1]	0.39[0.66]	0.37[0.63]	0.63[1.07]	0.47[0.80]	1.75[2.97
2000	2.01[1.86]	0.21[0.19]	1.08[1]	0.36[0.33]	0.29[0.27]	0.73[0.67]	0.40[0.37]	1.87[1.73
2001	3.78[2.10]	0.25[0.14]	1.80[1]	0.32[0.18]	0.32[0.18]	0.75[0.42]	0.13[0.07]	10.16[5.6
2002	4.17[2.76]	0.21[0.14]	1.51[1]	0.28[0.18]	0.22[0.15]	0.54[0.36]	0.19[0.13]	6.67[4.42]
2003	3.18[3.01]	0.22[0.21]	1.06[1]	0.34[0.32]	0.33[0.31]	0.99[0.94]	0.29[0.28]	5.54[5.23]
2004	2.53[1.22]	0.31[0.15]	2.07[1]	0.31[0.15]	0.27[0.13]	0.70[0.34]	0.39[0.19]	6.16[2.98]
2005	2.71[1.72]	0.21[0.13]	1.57[1]	0.24[0.15]	0.51[0.32]	0.50[0.32]	0.27[0.17]	4.67[2.96]
2006	2.50[1.22]	0.36[0.17]	2.05[1]	0.23[0.11]	0.83[0.40]	0.83[0.40]	0.26[0.13]	9.62[4.69]
2007	3.00[2.31]	0.43[0.33]	1.30[1]	0.22[0.17]	0.98[0.75]	0.43[0.33]	0.29[0.22]	5.10[3.93]
2008	9.48[3.45]	0.65[0.24]	2.75[1]	1.01[0.37]	1.91[0.69]	1.88[0.68]	0.63[0.23]	20.72[7.5]
2009	3.29[2.11]	0.61[0.39]	1.56[1]	0.87[0.55]	1.60[1.03]	1.38[0.88]	1.58[1.02]	6.79[4.35]
2010	3.67[3.25]	0.51[0.45]	1.13[1]	0.51[0.45]	1.47[1.30]	0.85[0.75]	0.57[0.51]	9.37[8.28]
2011	7.23[4.48]	0.59[0.37]	1.61[1]	0.56[0.35]	2.46[1.52]	0.99[0.61]	0.92[0.57]	8.05[4.98]
2012	5.67[4.18]	0.73[0.54]	1.35[1]	0.29[0.21]	1.96[1.45]	1.12[0.83]	0.86[0.64]	5.91[4.36
2013	5.29[4.35]	0.71[0.58]	1.22[1]	0.27[0.22]	1.76[1.44]	0.89[0.73]	0.49[0.40]	6.17[5.07
Ave	3.89[3.58]	0.45[0.40]	1.38[1]	0.42[0.32]	1.47[1.19]	1.02[1.05]	0.89[0.80]	7.90[6.83

Note: the number not in the bracket represents the standard deviation for that year calculated using the real exchange rate for the 12 months from 1993 to 2012. 1992 is used only from July to December and 2013 is used only from January to November. The number in the square bracket refers to how the volatility of another currency is relative to South Africa's ZAR. A value less than one implies that the currency in question has less volatility when compared to South Africa's ZAR. SA = South Africa. SK = South Korea. Source: Author's own calculations.

Table 3: Unit Root Tests using Augmented Dickey-Fuller method

Variable	ADF-statistic	ADF-statistic	Crit	ical Valu	ıes	
	levels	first difference	1%	5%	10%	Prob
LRERCPI	-2.429	-13.356	-2.572	-1.942	-1.616	0.0000***
LRERWPI	-2.564	-12.680	-2.572	-1.942	-1.616	0.0000^{***}
LRUSNOM	-2.172	-12.950	-2.572	-1.942	-1.616	0.0000^{***}
LREER	-3.329	-13.404	-2.572	-1.942	-1.616	0.0000^{***}
LNEER	-2.942	-13.295	-2.572	-1.942	-1.616	0.0000^{***}
LFXRES	-3.224	-5.414	-2.572	-1.942	-1.616	0.0000^{***}
LM1	-1.450	-6.524	-2.572	-1.942	-1.616	0.0000^{***}
LOUTPUT	-1.546	-2.911	-2.572	-1.942	-1.616	0.0036^{***}
LTO	-3.053	-24.371	-2.572	-1.942	-1.616	0.0000^{***}
LRGOLDP	-1.648	-15.421	-2.572	-1.942	-1.616	0.0000^{***}

Notes: Variables are defined as in section 4. *** indicates significant at 1%. The values in levels include the constant and trend.

Source: Output using Eviews 8.

 Table 4: Unit Root Tests using Phillips-Perron method

Variable	PP-statistic	PP-statistic	Critica	l Values		
	levels	first difference	1%	5%	10%	Prob
LRERCPI	-2.189	-13.172	-2.572	-1.942	-1.616	0.0000***
LRERWPI	-3.064	-13.785	-2.572	-1.942	-1.616	0.0000^{***}
LRUSNOM	-1.746	-12.898	-2.572	-1.942	-1.616	0.0000^{***}
LREER	-3.000	-12.999	-2.572	-1.942	-1.616	0.0000^{***}
LNEER	-2.340	-13.185	-2.572	-1.942	-1.616	0.0000***
LFXRES	-2.623	-18.391	-2.572	-1.942	-1.616	0.0000^{***}
LM1	-1.070	-20.653	-2.572	-1.942	-1.616	0.0000***
LOUTPUT	-1.678	-6.152	-2.572	-1.942	-1.616	0.0000***
LTO	-7.624		-3.986	-3.423	-3.135	0.0000***
LRGOLDP	-1.816	-15.443	-2.572	-1.942	-1.616	0.0000***

Notes: Variables are defined as in section 4. *** indicates significant at 1%. The values in levels include a constant and trend.

Source: Output using Eviews 8.

Variables	Obs	Mean	Std.Dev	Skewness	Kurtosis	Jarque-Bera
DRERCPI	334	-5.24E-05	0.0339	0.6254	8.4403	433.6542***
DRERWPI	334	-0.000262	0.0343	0.2981	6.6296	188.2813^{***}
DRUSNOM	334	0.004391	0.0348	0.7678	8.2241	412.6215***
DREER	334	-0.000243	0.0263	-1.8816	15.266	2291.028***
DNEER	334	-0.005165	0.0300	-1.1637	10.6334	8886.2750***
DFXRES	334	0.018038	0.0707	2.3148	21.7178	5174.093***
DM1	334	0.011719	0.0278	0.5286	4.3713	41.7200***
DOUTPUT	334	0.002266	0.0091	-0.4648	2.4613	16.06644^{***}
DTO	334	-0.000512	0.0886	-0.1780	2.9423	1.8094
DRGOLDP	334	0.006050	0.0428	0.7261	5.9301	148.8278^{***}

Notes: *** indicates significant at 1%. Obs = number of observation. Std.Dev = standard deviation Source: Output using Eviews 8.

Table 6: Heteroskedasticity test

			j	
Variable	F-statistic	Prob. F	Obs*R-squared	Prob
DRERCPI	4.0457	0.0185**	7.9696	$0.0186^{**}(w)$
DRERWPI	4.3226	0.0140^{**}	8.5010	$0.0143^{**}(w)$
DRUSNOM	4.3829	0.0132^{**}	8.6167	$0.0135^{**}(w)$
DREER	14.1441	0.0002^{***}	13.6450	$0.0002^{***}(lm)$
DNEER	6.3059	0.0021^{***}	12.2579	$0.0022^{***}(w)$
DFXRES	2.3356	0.0554^{*}	9.2209	$0.0558^{*}(lm)$
DM1	9.4531	0.0023^{***}	9.2455	$0.0024^{***}(lm)$
DOUTPUT	27.2892	0.0000^{***}	25.3576	$0.0000^{***}(lm)$
DRGOLDP	3.6182	0.0279^{**}	7.1450	$0.0281^{***}(lm)$

Notes:***,**,* indicates significant at 1%,5% and 10% respectively. (w) indicates that the white test is used and (lm) indicates that the ARCH-LM test is used. Source: Output using Eviews 8.

	Dto										1.000	
	rgoldp									1.000	-0.024	
iables	output								1.000	0.088	-0.018	
l the vari	m1							1.000	-0.241	-0.071	-0.032	
rix for al	fxres						1.000	0.021	-0.006	-0.056	-0.010	
ion mati	neer					1.000	0.013	0.052	0.024	0.655	-0.023	
Correlat	reer				1.000	0.938	0.018	0.076	0.040	0.649	-0.008	
Table 7:	rusnom			1.000	0.866	0.909	-0.042	0.054	0.012	0.694	-0.070	
	rerwpi		1.000	0.920	0.764	0.865	-0.071	0.170	-0.042	0.630	-0.064	
	rercpi	1.000	0.913	0.980	0.873	0.928	-0.015	0.073	0.028	0.664	-0.072	
		rercpi	rerwpi	rusnom	reer	neer	fxres	m1	output	rgoldp	Dto	

defined as volatility.	
(Dto) are	
lotes: all the variables except trade openness (ource: Output using Eviews 8.



Figure 1: Estimated Conditional Variance for RERCPI

Figure 2: Estimated Conditional Variance for RERWPI





Figure 3: Estimated Conditional Variance for RUSNOM

Figure 4: Estimated Conditional Variance for REER



Figure 5: Estimated Conditional Variance for NEER



		Table o: GF	ARCH(1,1) resu	ILS	
Parameters	RERCPI	RERWPI	RUSNOM	REER	NEER
Mean eq.					
constant	-0.0031	-0.0031*	0.0034^{**}	0.0002	-0.0056***
OUTPUT	-0.0221	0.2601^{**}	0.1507	-0.1238	0.0972
MS1	-0.0243	0.0268	0.0393	0.00644	0.0098
TO	-0.0143	0.0057	0.0050	-0.0058	0.0093
FXRES	0.0352	0.0229	0.0296^{**}	-0.0352^{***}	-0.0274**
RGOLDP	0.4236^{***}	0.4102^{***}	0.3828^{***}	-0.2848***	-0.3683***
DUM95	0.0011	0.0012	-0.0026	0.0018	0.0056^{***}
$\mathrm{AR}(1)$	0.2969^{***}	0.2763^{***}	0.3260^{***}	0.4518^{***}	0.2369^{***}
AR(2)	-0.1018^{**}	-0.1402^{***}	-0.0874**	-0.1456^{**}	-0.1144***
AR(3)				-0.0546	0.0584
AR(4)					0.0120
Variance eq.					
constant	0.000425^{***}	$1.82E-05^{***}$	$5.69E-06^{***}$	$9.35E-05^{***}$	0.000138^{***}
$arepsilon^2_{t-1}$	0.051049	-0.008295	0.042193^{*}	0.151845^{**}	-0.034893
h_{t-1}^2	0.456813^{***}	0.945597^{***}	0.897797^{***}	0.418091^{***}	0.629204^{***}
OUTPUT	0.003226	0.003028^{**}	0.002112	-0.005413^{***}	-0.001911
MS1	-0.005038^{***}	-0.000583	0.000151	-0.000181	-0.001079
TO	0.000839	-0.001149^{***}	-0.001027^{***}	0.000388^{*}	-9.12E-05
FXRES	-0.000196	-2.57E-05	7.87E-05	4.62E-05	-0.000392^{***}
RGOLDP	0.002519	0.001221^{**}	0.001453^{*}	0.001101^{***}	0.002243^{***}
DUM95	-5.58E-05	$1.64E-05^{***}$	$1.95E-05^{**}$	$4.58E-05^{**}$	7.57E-05
$\varepsilon_{t-1}^2 + h_{t-1}^2$	0.5078	0.9373	0.9401	0.5699	0.5943
Notes:***,**,*	indicates significan	it at $1\%, 5\%$ and 1	0% respectively. ϵ	d stands for equa	tion. The variables are
defined in section	on 4. The p-values	s are based on Bol	lerslev-Wooldridge	e robust standard	errors and covariance.
Source: Output	using Eviews 8.				

1+0 Table 8. CARCH/1 1)

	та	DIE 9. EQUINOI	$\mathbf{T}(\mathbf{T},\mathbf{T})$		
Parameters	RERCPI	RERWPI	RUSNOM	REER	NEER
Mean eq.					
constant	-0.0012	-0.0036**	0.0028^{*}	5.41E-05	-0.0048***
OUTPUT	0.0990	0.2899^{**}	0.2521^{**}	-0.1589^{**}	0.1201^{*}
MS1	-0.0059	0.0565	0.0308	-0.0012	-0.0175
TO	-0.0076	0.0081	-0.0041	0.0048	0.0238^{***}
FXRES	0.0275^{*}	0.0321^{***}	0.0240^{*}	-0.0174^{***}	-0.0134^{*}
RGOLDP	0.3721^{***}	0.390450^{***}	0.3702^{***}	-0.2194^{***}	-0.3588***
DUM95	-0.0015	0.000129	-0.0029	0.0023	0.0058^{***}
$\operatorname{AR}(1)$	0.3293^{***}	0.311881^{***}	0.3664^{***}	0.4983^{***}	0.3068^{***}
AR(2)	-0.1092^{**}	-0.155444^{***}	-0.1072**	-0.1506^{***}	-0.1592^{***}
AR(3)		0.110265^{***}		-0.0502	
AR(4)		-0.106173^{**}		0.1089^{***}	
AR(5)		-0.006562		-0.1083^{***}	
AR(6)		-0.085971**		0.0542^{*}	
Variance eq.					
constant	-0.327496^{***}	-0.241081^{***}	-0.163864^{***}	-2.052698^{**}	-0.898405^{***}
$\alpha_1\{\varepsilon_{t-1}/(h_{t-1})^{0.5}\}$	-0.153281^{**}	-0.082110	-0.086716	0.151501	-0.349279^{***}
$\lambda_1 \{arepsilon_{t-1}/(h_{t-1})^{0.5}\}$	0.015563	0.049909	0.041082	-0.127942^{*}	-0.008927
$\ln(h_{t-1})$	0.945410^{***}	0.966386^{***}	0.978000^{***}	0.792796^{***}	0.866341^{***}
OUTPUT	10.02419^{**}	4.311940	7.945157^{**}	-17.36371^{***}	1.786508
MS1	-1.453856	0.135521	1.161004	1.6748	1.814034
TO	-0.448686	-0.979166	-0.390136	0.1495	-0.632399^{*}
FXRES	0.490851	-0.187143	0.087639	-0.387790	-1.355489^{***}
RGOLDP	1.693462^{**}	-0.078400	0.666949	7.983027^{***}	5.110445^{***}
DUM95	0.018627	0.059399^{***}	0.040779^{***}	0.191110	0.108753^{***}
· + ++++++++++++++++++++++++++++++++++	- - - -			د ب ب	
Notes: ***, **, * indicat	es significant at 15	%,5% and 10% res	pectively. eq stan	ds tor equation. ¹	he variables are
defined in section 4.					

Source: Output using Eviews 8.

Table 9: EGARCH(1.1) results

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				1	
			GARCH(1	,1)	
	RERCPI	RERWPI	RUSNOM	REER	NEER
Q-test: v_t					
O(2)	2.0625(0.357)	2.0334(0.362)	1.2700(0.530)	0.3042(0.859)	1.3215(0.516)
	A A150(0 353)	6 0016(0 109)	1 7851(0 775)	9 3609(0 670)	1 4550(0 834)
	(000.0) 0011-F	(761.0)0160.0		(010.0)2000.2	T. ±000(0.00±.1
Q(6)	4.4917(0.610)	7.0069(0.320)	1.8693(0.931)	5.9833(0.425)	5.9960(0.424)
Q-test: v_t^2					
Q(2)	0.5023(0.778)	1.2471(0.536)	0.0247(0.988)	3.5327(0.171)	0.6145(0.735)
O(4)	5.1905(0.268)	1.5755(0.813)	0.5093(0.973)	3.6964(0.449)	0.6742(0.954)
$\mathbf{O}(6)$	6 3750(0 382)	5 5020(0 470)	$4\ 9339(0\ 645)$	4.6683(0.587)	3 7986(0 704)
	0.0100(0.0002)			0 L107(0 1100)	
ARCH-LM	0.2295(0.0317)	0.0003(0.9801)	0.0004(0.9832)	2.5127(0.1129)	0.0008(0.9343)
Log likelihood	754.1531	774.8246	774.8326	920.2729	822.4597
AIC	-4.4347	-4.5592	-4.5592	-5.4458	-4.8634
SIC	-4.2284	-4.3529	-4.3529	-5.2275	-4.6331
			EGARCH(1	(1)	
$\overline{\text{O-test: } v_t}$					
$\mathbf{Q}(2)$	0.9681(0.616)	1.0351(0.596)	0.8492(0.654)	0.3002(0.861)	1.9718(0.373)
O(4)	3.2902(0.510)	1.3807(0.848)	2.8105(0.590)	0.9172(0.922)	4.8371(0.304)
(-)(e)	9 7041(0 70E)	(630.0)(620.1)	9 1076(0 705)	9 9 5 7 6 (0 7 6 9)	
$\mathcal{Q}(0)$	3.1941(U.1U3)	(.01/9(0.203)	3.1U/0(U./95)	3.3370(U.703)	(
Q-test: v_t^2					
$\mathrm{Q}(2)$	2.0938(0.351)	1.0351(0.596)	0.0900(0.956)	3.9200(0.141)	1.9697(0.373)
$\mathrm{Q}(4)$	5.0341(0.284)	1.3807(0.848)	0.8412(0.933)	5.9449(0.203)	2.7384(0.603)
Q(6)	9.4561(0.150)	7.6779(0.263)	4.6388(0.591)	8.9929(0.174)	6.3968(0.380)
ARCH-LM	0.2136(0.6440)	0.1870(0.6654)	0.0006(0.9798)	2.0755(0.1497)	1.2537(0.2628)
Log likehood	777.5297	777.5982	786.9701	944.3293	847.5239
AIC	-4.5695	-4.6012	-4.6263	-5.6179	-4.9911
SIC	-4.3517	-4.3352	-4.4086	-5.3519	-4.7733
Notes: Q-test: v_t	and Q-test: v_t^2 are	tests for the presenc	e of serial correlatic	n and remaining Al	3CH/GARCH effects conducted on
Ljung-Box Q-stat:	istic of standardised	l and squared standa	rdised residuals res	pectively. The numb	ber in brackets is the p-value. The
ontimal lac lenoth	(k) of 6 for the $O-t$	est statistic is chosen	n according to the s	morestion by Tsav (3003) that k=ln(T) where T is the
UPULLIAL 145 LULAU				UBBUDUUU UY INUU	ATTA AT T ATATTA (T)TTT-V ADTTA (7007

Table 10: Diagnostic tests

number of observations. AIC is Akaike info criterion and SIC is Schwarz criterion. Source: Output using Eviews 8.

Table 11: Pairwise Granger Causality Tests for ZAR volatility and Gold price volatility

Null Hypothesis	lags	obs	F-statistic	P-value
$NEERvola \Longrightarrow Goldpvola$	1	332	0.60854	0.4359
$Goldpvola \Longrightarrow NEERvola$	1	332	3.15102	0.0768*
$REERvola \Longrightarrow Goldpvola$	1	332	0.37146	0.5426
$Goldpvola \Longrightarrow REERvola$	1	332	4.14174	0.0426^{**}
RERCPIvola⇒Goldpvola	1	332	0.71132	0.3996
$Goldpvola \Longrightarrow RERCPIvola$	1	332	5.41292	0.0206^{**}
RERWPIvola⇒Goldpvola	1	332	0.39736	0.5289
$Goldpvola \Longrightarrow RERWPIvola$	1	332	5.67514	0.0178^{**}
RUSNOMvola⇒Goldpvola	1	332	1.54394	0.2149
Goldpvola⇒RUSNOMvola	1	332	5.97100	0.0151**

Notes: \implies stands for "does not Granger Cause". NEER =nominal effective exchange rate vola=volatility. Goldp=gold price.REER=real effective exchange rate. RERCPI=real bilateral exchange rate using consumer price indices.RERWPI= real bilateral exchange rate using wholesale price index and CPI.RUSNOM=nominal bilateral exchange rate. obs=observations.

Source: Output using Eviews 8.

8.1 The Model

This model considers a small open economy with the nontraded goods sector characterised by monopoly and sticky-price problems, while the traded sector has a single homogeneous output, which is priced competitively in the world markets. Each representative home household j is endowed with a constant quantity of the traded good each period, \bar{y}_T , and has a monopoly power over one of the nontradables $z \in [0, 1]$. The model assumes that all households have identical preferences characterised by an intertemporal utility function that depends positively on consumption and real money balances, but negatively with production. With this, the intertemporal utility function of the household is given by

$$U_t^j = \sum_{S=t}^{\infty} \beta^{S-t} \left[\varphi \ln C_{T,S}^j + (1-\varphi) \ln C_{N,S}^j + \frac{\chi}{1-\varepsilon} \left(\frac{M_S^j}{P_S} \right)^{1-\varepsilon} - \frac{\kappa}{2} y_{N,S}(j)^2 \right]$$
(17)

where C_T represents the consumption of traded goods and C_N is the composite consumption of nontraded goods, defined as:

$$C_N = \left[\int_0^1 c_N\left(z\right)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}}$$
(18)

Consumption based price index, P, is defined as the minimum cost of buying an additional unit of real consumption $C_T^{\varphi} C_N^{1-\varphi}$. This price index is represented as:

$$P = \frac{P_T^{\varphi} P_N^{1-\varphi}}{\varphi^{\varphi} (1-\varphi)^{1-\varphi}} \tag{19}$$

where P_T represents the price of tradables and P_N is the nontraded goods price index, defined as

$$P_N = \left[\int_0^1 p_N(z)^{1-\theta} dz\right]^{\frac{1}{1-\theta}}$$
(20)

where $p_N(z)$ is the price of nontraded good z. Domestic prices P_T are linked to a constant world prices P_T^* via the exchange rate, E. This is represented as follows:

$$P_T = EP_T^* \tag{21}$$

In addition, the model assumes the existence of an international bond market with real bonds denominated in terms of tradables. The constant world net interest rate in tradables is denoted by r and $\beta(1+r) = 1$. The intertemporal budget constraint for the representative household j is denoted by

$$P_{T,t}B_{t+1}^{j} + M_{t}^{j} = P_{T,t}(1+r)B_{t}^{j} + M_{t-1}^{j} + p_{N,t}(j)y_{N,t}(j) + P_{T,t}\bar{y_{T}} - (22)$$
$$P_{N,t}C_{N,t}^{j} - P_{T,t}C_{T,t}^{j} - P_{T,t}\tau_{t}$$

where τ_t denotes taxes per capita in terms of the tradable goods while B_t represents the bond portfolio. Abstracting from government spending, the model assumes that the government balances its budget in units of tradables and its constraint is as follows:

$$\tau_t + \frac{M_t - M_{t-1}}{P_{T,t}} = 0 \tag{23}$$

Finally the preferences take the Constant Elasticity of Substitution (CES) form. This results in producers of non-traded goods facing the following demand curve:

$$y_N^d(j) = \left[\frac{p_N(j)}{P_N}\right]^{-\theta} C_N^A \tag{24}$$

where C_N^A represents the aggregate consumption of nontraded goods.

Solving the household's optimisation problem requires maximising equation 17 subject to equations 22 and 24 with respect to the choice variables $B_{t+1}^{j}, M_{t}, C_{N,t}$ and $y_{N,t}$. This results in the following four first-order conditions(FOC):

$$C_{T,t+1}^{j} = C_{T,t}^{j} \tag{25}$$

$$\frac{\varphi}{C_{T,t}^{j}} = \chi \frac{P_{T,t}}{P_{t}} \left(\frac{M_{t}}{P_{t}}\right)^{-\varepsilon} + \beta \frac{P_{T,t}}{P_{T,t+1}} \left(\frac{\varphi}{C_{T,t+1}^{j}}\right)$$
(26)

$$C_{N,t}^{j} = \frac{1 - \varphi}{\varphi} \left(\frac{P_{T,t}}{P_{N,t}}\right) C_{T,t}^{j}$$
(27)

$$y_{N,t}^{\frac{\theta+1}{\theta}} = \left[\frac{\left(\theta-1\right)\left(1-\varphi\right)}{\theta\kappa}\right] \left(C_{N,t}^{A}\right)^{\frac{1}{\theta}} \frac{1}{C_{N,t}^{j}}$$
(28)

Equation 25 shows the Euler condition for optimal intertemporal consumption smoothing for traded goods. Equation 26 shows the utility maximising trade-off between consumption spending in period t and a combination of one-period money holding and consumption spending in period t + 1. Equation 27 states that the marginal utility of traded and nontraded consumption must be equal at any given time. Equation 28 depicts the condition for optimal monopolistic price setting. Hau (2002) states that equation 28 is used to show the marginal consumption utility of an additional unit of nontraded good as well as the marginal disutility of production of an additional unit. As a result, a mark up of $\theta/\theta - 1$ is added by monopolistically competitive firms.

Substituting equation 25 into equation 26 leads to the following expression for the demand for real balances:

$$\frac{M_t}{P_t} = \left\{ \frac{\frac{\chi}{\varphi} C_{T,t} \frac{P_{T,t}}{P_t}}{\left(1 - \beta \frac{P_{T,t+1}}{P_{T,t+1}}\right)} \right\}^{\frac{1}{\varepsilon}}$$
(29)

implying the demand for real balances depends on consumption of tradables, $C_{T,t}$, changes in the price of tradables, $P_{T,t} / P_{T,t+1}$ and changes in the real price of tradables, $P_{T,t} / P_t$.

Including government spending, the model assumes that government spending, G, is dissipative and does not affect productivity or private utility. It also assumes that government's real consumption index takes the same general form as the private sector's. With government spending, equation 23 becomes:

$$G_t = \tau_t + \frac{M_t - M_{t-1}}{P_t}$$
(30)

Given that preferences take the CES form, the producers of non-traded goods take the form:

$$y_{N,t}^{d} = \left[\frac{p_{N,t}(j)}{P_{N,t}}\right]^{-\theta} \left(C_{N}^{A} + G_{N}^{A}\right)$$
(31)

where C_N^A is home country's private demand for non-traded goods and G_N^A is home country's public demand for non-traded goods. Implying solving the optimisation problem including the government spending leads to equation 28 only, changing to

$$y_{N,t}^{\frac{\theta+1}{\theta}} = \left[\frac{\left(\theta-1\right)\left(1-\varphi\right)}{\theta\kappa}\right] \left(C_{N,t}^{A} + G_{N,t}^{A}\right)^{\frac{1}{\theta}} \frac{1}{C_{N,t}^{j}}$$
(32)