



# **On the impossibility of insider trade in rational expectations equilibria**

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# On the impossibility of insider trade in rational expectations equilibria

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## Abstract

Existing no trade results are based on the common prior assumption (CPA). This paper identifies a strictly weaker condition than the CPA under which speculative trade is impossible in a rational expectations equilibrium (REE). As our main finding, we demonstrate the impossibility of speculative asset trade in an REE whenever an insider is involved who knows the asset's true value. To model insider trade as an equilibrium phenomenon an alternative equilibrium concept than the REE is thus required.

*Keywords:* Levin-Coburn Report; Goldman Sach; Insider Trade; Rational Expectations

*JEL Classification Numbers:* D51; D53; G02

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# 1 Introduction

The Levin-Coburn report (2011) describes in some detail how Goldman Sachs sold off in 2007 collateral debt obligations (CDO) to gullible investors. At this point Goldman Sachs already knew that these CDOs were worthless so that they made gains from trade by exploiting their informational advantage. Existing no trade results (Tirole 1982; Milgrom and Stokey 1982; Sebenius and Geanakoplos 1983) tell us, however, that gains from trade based on informational advantage are impossible in a rational expectations equilibrium (REE) (Radner 1979) whenever the economic agents share a common prior. According to these no trade results, Goldman Sachs could thus only sell the CDOs because the investors either violated the rationality requirements of an REE or they violated the common priors assumption (CPA).

This paper presents a no trade result which establishes the impossibility of speculative trade whenever the agents' beliefs are *ex post homogenous*. Ex post homogeneity of beliefs is a strictly weaker condition than the CPA and, in contrast to the CPA, it might be trivially satisfied in relevant situations. In particular, we demonstrate that speculative asset trade is impossible in an REE whenever it involves an insider with perfect knowledge about the asset's true value. That is, for any specification of the priors of Goldman Sachs and of the investors, respectively, Goldman Sachs should not have been able to sell the CDOs to investors who had been rational in the sense of an REE.

It is a common perception in the literature that the CPA is crucial to no trade results. For example, in an influential article Morris (1995) writes:

“Aumann’s work stimulated work on *no trade* results which establish that, in the absence of *ex ante* gains from trade, asymmetric information cannot generate trade. In particular, Sebenius and Geanakoplos (1983)—extending Aumann’s argument—showed that (under the common prior assumption) it cannot be common knowledge that risk neutral individuals are prepared to bet against each other, that is, that one individual’s posterior beliefs exceed another’s. Milgrom and Stokey (1982) showed an analogous result in a more general setting of risk averse traders. Since no trade results can be shown to underlie many important results in microeconomic theory, it had by now become clear that the common prior assumption was critical.”  
(p. 230)

Because of this perception, the controversy about whether no trade theorems have much practical relevance or not has become entangled with the controversy about the appeal

of the CPA (Gul 1998; Aumann 1998). For example, I have met more than one colleague who would argue that no trade results are practically irrelevant because the CPA has not much realistic appeal.

In contrast, this paper shows that the question about the relevance of no trade results can be (at least to some degree) disentangled from the question about the appeal of the CPA. Moreover, our analysis suggests that violations of the rational expectations paradigm rather than different priors may be the reason for the occurrence of speculative trade such as the selling of CDOs by Goldman Sachs. Motivated by the analysis in this paper, Zimper (2013) constructs a non REE competitive equilibrium framework such that boundedly rational agents may have strict incentives for engaging in speculative trade.

We proceed by introducing in Section 2 the economy and the relevant equilibrium concept. In Section 3 a no trade result (Lemma 1) is presented which establishes the impossibility of speculative trade whenever the agents' beliefs satisfy ex post homogeneity. Because ex post homogeneity always holds if there is an insider agent who knows the asset's true value (Lemma 2), Section 4's main result (Proposition)—stating the impossibility of insider trade in an REE—immediately follows. A stylized Goldman Sachs example illustrates the impossibility of insider trade by showing that—regardless of the agents' priors—there does not exist any REE such that Goldman Sachs could have been able to sell the CDOs to the investors. The discussion in Section 5 shows that neither impersonalized markets nor non-expected utility decision making can explain insider trade as an equilibrium phenomenon. Section 6 gives an outlook on a general equilibrium concept developed in Zimper (2013) in which—unlike as in an REE—boundedly rational agents may not fully understand the market clearing price mechanism to the effect that insider trade may occur in an equilibrium.

## 2 Economy

We consider an economy given as a situation of static speculation under asymmetric information. The economy consists of  $n$  agents and a single risky asset with payoff function  $X : \Omega \rightarrow \mathbb{R}$  for some finite state space  $\Omega$ . Agent's  $i$  private information is described by some partition  $\Pi_i$  on  $\Omega$ .

Denote by  $\bigvee_{i=1}^n \Pi_i$  the *join* (coarsest common refinement) of all  $\Pi_i$ ,  $i \in \{1, \dots, n\}$ , with generic element  $I(\omega)$ . Intuitively speaking, the information represented by the partition  $\bigvee_{i=1}^n \Pi_i$  obtains if all agents shared their private information, i.e., the information partition

$\bigvee_{i=1}^n \Pi_i$  stands for the full communication information available in this economy. Further, denote by  $\Sigma \left( \bigvee_{i=1}^n \Pi_i \right)$  the  $\sigma$ -algebra generated by  $\bigvee_{i=1}^n \Pi_i$ . Agent  $i$ 's prior belief is given by the probability measure  $\pi_i$  defined on the events in  $\Sigma \left( \bigvee_{i=1}^n \Pi_i \right)$ .

An information-belief structure, denoted  $\langle \Pi, \pi \rangle$  with  $\Pi = (\Pi_1, \dots, \Pi_n)$  and  $\pi = (\pi_1, \dots, \pi_n)$ , collects the agents' private information partitions and their subjective beliefs, respectively. We further assume that  $\pi_i(I) > 0$  for all  $I \in \bigvee_{i=1}^n \Pi_i$  with  $i \in \{1, \dots, n\}$  whereby the information  $I$  can be observed with positive objective probability in this economy. On the one hand, this assumption is technically convenient because it implies that the conditional probability measures  $\pi_i(\cdot | I)$  are well-defined. On the other hand, this assumption is in line with the (weak) rationality requirement that all agents regard information as possible that is also objectively possibly in this economy.

Each agent  $i$  is an expected utility maximizer with concave and strictly increasing vNM utility function  $u_i$  defined over the gains from trade  $(X - p) \cdot \theta_i$  which result from buying (i.e.,  $\theta_i \geq 0$ ), respectively selling (i.e.,  $\theta_i \leq 0$ ),  $\theta_i \in \Gamma \subseteq \mathbb{R}$  units of the asset at price  $p \in \mathbb{R}_+$ . We assume that  $0 \in \Gamma$ , implying that the zero-trade (i.e.,  $\theta_i = 0$ ) is always a possible option to any agent  $i$ . We define the demand-supply correspondence of agent  $i$  as the set-valued mapping  $\varphi_i : \bigvee_{i=1}^n \Pi_i \times \mathbb{R}_+ \rightarrow 2^\Gamma$  such that, for all  $(I(\omega), p) \in \bigvee_{i=1}^n \Pi_i \times \mathbb{R}_+$ ,

$$\varphi_i(I(\omega), p) = \arg \max_{\theta_i \in \Gamma} E[u((X(\omega') - p) \cdot \theta_i), \pi_i(\omega' | I(\omega))]. \quad (1)$$

**Definition. Full Communication Equilibrium (FCE).** Fix some information-belief structure  $\langle \Pi, \pi \rangle$ . An FCE with respect to  $\langle \Pi, \pi \rangle$ , denoted  $(P, \Theta) \langle \Pi, \pi \rangle$ , is a mapping

$$(P; \Theta_1, \dots, \Theta_n) : \Omega \rightarrow \mathbb{R}_+ \times \mathbb{R}^n \quad (2)$$

such that, for all  $i \in \{1, \dots, n\}$ ,

1.  $P$  and  $\Theta_i$  are  $\Sigma \left( \bigvee_{i=1}^n \Pi_i \right)$ -measurable;

2. for all  $\omega \in \Omega$  and all  $I(\omega) \in \bigvee_{i=1}^n \Pi_i$ ,

$$\Theta_i(\omega) \in \varphi_i(I(\omega), P(\omega)); \quad (3)$$

3. for all  $\omega \in \Omega$ ,

$$\sum_{i=1}^n \Theta_i(\omega) = 0. \quad (4)$$

By the measurability condition 1, neither equilibrium prices nor allocations can reveal more information about the true state of the world than the full communication information.

If there is an FCE  $(P, \Theta) \langle \Pi, \pi \rangle$  such that

$$P(I) \neq P(I') \text{ for all } I, I' \in \bigvee_{i=1}^n \Pi_i \text{ with } I \neq I', \quad (5)$$

we call  $(P, \Theta) \langle \Pi, \pi \rangle$  *revealing*. Intuitively speaking, in a revealing FCE every agent learns the information of all other agents because the equilibrium price function is in an one-to-one correspondence with the full communication information available in the economy.

Recall that a rational expectations equilibrium (REE)  $(P^{REE}, \Theta^{REE}) \langle \Pi, \pi \rangle$  in the sense of Radner (1979) is characterized by the consistency condition that every agent  $i$  bases his equilibrium demand-supply decision  $\Theta_i^{REE}$  on his private information  $I_i(\omega) \in \Pi_i$  augmented with the common information revealed through equilibrium prices

$$I(P^{REE}(\omega)) = P^{REE}(\omega)^{-1}. \quad (6)$$

Given some revealing  $(P, \Theta) \langle \Pi, \pi \rangle$ , let

$$P^{REE} = P \quad (7)$$

so that agent  $i$ 's private information augmented with the information revealed through equilibrium prices becomes

$$I_i(\omega) \cap I(P^{REE}(\omega)) = I(P^{REE}(\omega)) \quad (8)$$

with  $I(P^{REE}(\omega)) \in \bigvee_{i=1}^n \Pi_i$ . As a consequence, the demand-supply correspondence of the REE  $(P^{REE}, \Theta^{REE}) \langle \Pi, \pi \rangle$  is identical to the demand-supply correspondence of the revealing FCE  $(P, \Theta) \langle \Pi, \pi \rangle$ , i.e.,

$$\varphi_i^{REE}(I(P^{REE}(\omega)), P^{REE}(\omega)) = \varphi_i(I(\omega), P(\omega)) \quad (9)$$

with  $I(\omega) \in \bigvee_{i=1}^n \Pi_i$ . That is, any revealing FCE can be equivalently interpreted as a revealing REE in the sense of Radner (1979). In the remainder of this paper, we restrict attention to equilibria given as revealing FCE (i.e., revealing REE).

### 3 No trade under ex post homogenous beliefs

We speak of a speculative trade equilibrium if there is some state  $\omega \in \Omega$  in which agents strictly prefer to sell or to buy the asset to the zero-trade. More specifically,  $(P, \Theta) \langle \Pi, \pi \rangle$  is a speculative trade equilibrium iff there is some  $i \in \{1, \dots, n\}$  and some  $\omega \in \Omega$  such that agent  $i$ 's demand-supply correspondence evaluated at information cell  $I(\omega)$  and at equilibrium price  $P(\omega)$  only contains non-zero trade positions, i.e., iff

$$\varphi_i(I(\omega), P(\omega)) \neq \emptyset \text{ and } 0 \notin \varphi_i(I(\omega), P(\omega)). \quad (10)$$

**Definition. Ex post homogeneous beliefs.** *We say that the agents' beliefs are ex post homogenous iff, for all  $I \in \bigvee_{i=1}^n \Pi_i$ ,  $\pi_i(\omega | I) = \pi_j(\omega | I)$  for all  $\omega \in \Omega$  and all  $i, j \in \{1, \dots, n\}$ .*

Tirole (1982, Proposition 1) proves the impossibility of speculative trade if the CPA holds, i.e., if  $\pi_i = \pi_j$  for all  $i, j \in \{1, \dots, n\}$ . Note that ex post homogeneity of beliefs is implied by the CPA whereas the converse statement is not true. Consequently, the following Lemma 1 (proved in the Appendix) provides a generalization of Tirole's no-trade result from common priors to ex post homogenous beliefs.

**Lemma 1.** *Speculative trade is impossible, if the agents' beliefs are ex post homogenous.*

The question arises how relevant this generalization of Tirole's no-trade result is. That is, how relevant are situations in which the CPA is violated whereas ex post homogeneity is satisfied? The following example helps to illustrate this point.

**Example.** Consider the state space

$$\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}. \quad (11)$$

Further suppose that there are two agents such that we have for agent 1 that

$$\Pi_1 = \{\{\omega_1, \omega_2, \omega_3\}, \{\omega_4\}\}, \quad (12)$$

$$\pi_1(\omega_1) = x, \pi_1(\omega_2) = \delta x, \pi_1(\omega_3) = a, \pi_1(\omega_4) = b \quad (13)$$

with

$$x, \delta x, a, b > 0, \quad (14)$$

$$(1 + \delta)x + a + b = 1, \quad (15)$$

whereas we have for agent 2 that

$$\Pi_2 = \{\{\omega_1, \omega_2\}, \{\omega_3, \omega_4\}\} \quad (16)$$

$$\pi_2(\omega_1) = y, \pi_2(\omega_2) = \delta y, \pi_2(\omega_3) = c, \pi_2(\omega_4) = d \quad (17)$$

with

$$y, \delta y, c, d > 0, \quad (18)$$

$$(1 + \delta)y + c + d = 1. \quad (19)$$

Note that

$$\bigvee_{i=1}^2 \Pi_i = \{\{\omega_1, \omega_2\}, \{\omega_3\}, \{\omega_4\}\}, \quad (20)$$

so that we obtain the following posterior beliefs

$$\pi_1(\omega_1 | \{\omega_1, \omega_2\}) = \pi_2(\omega_1 | \{\omega_1, \omega_2\}) = \frac{1}{1 + \delta}, \quad (21)$$

$$\pi_1(\omega_2 | \{\omega_1, \omega_2\}) = \pi_2(\omega_2 | \{\omega_1, \omega_2\}) = \frac{\delta}{1 + \delta}, \quad (22)$$

$$\pi_1(\omega_3 | \{\omega_3\}) = \pi_2(\omega_3 | \{\omega_3\}) = 1, \quad (23)$$

$$\pi_1(\omega_4 | \{\omega_4\}) = \pi_2(\omega_4 | \{\omega_4\}) = 1. \quad (24)$$

Consequently, the above beliefs are ex post homogenous for arbitrary—subject to the constraints (14)-(19)—values of  $x, \delta, a, b, y, c, d$  whereas the CPA only holds if

$$x = y, a = c, b = d. \quad (25)$$

□

Two important observations emerge from the above example. First, whenever the agents do not fully learn the true state of the world, i.e., when they receive information  $\{\omega_1, \omega_2\}$ , ex post homogenous beliefs require some sort of alignment across the agents' beliefs, here in form of the common factor  $\delta$ . Although beliefs can thus be ex post homogenous for much more general parameter values than implied by the CPA, an argument can be made that ex post homogeneity, like the CPA, only holds for a set of



beliefs which is ‘non-generic’ with respect to all possible beliefs whenever the information is not fully revealing.<sup>1</sup>

Second, the situation is different whenever the agents learn the true state of the world as in the states  $\omega_3$  and  $\omega_4$ . In this case, ex post homogeneity holds regardless of the agent’s prior beliefs as can be seen from the following lemma.

**Lemma 2.** *Suppose that the full communication information available in the economy reveals the true state of the world, i.e.,*

$$\bigvee_{i=1}^n \Pi_i = \{\{\omega\} \mid \omega \in \Omega\}. \quad (26)$$

*Then speculative trade is impossible for arbitrary priors  $\pi_1, \dots, \pi_n$ .*

**Proof.** If (26) holds, then, for all  $\omega \in \Omega$  and all  $i \in \{1, \dots, n\}$ ,

$$\pi_i(\omega \mid I(\omega)) = \pi_i(\omega \mid \omega) = 1. \quad (27)$$

That is, the agents’ beliefs are ex post homogenous for arbitrary priors  $\pi_1, \dots, \pi_n$  so that the impossibility of speculative trade follows from Lemma 1.  $\square$

## 4 Insider trade and Goldman Sachs revisited

We speak of an insider agent, if this agent knows the asset’s true value. By Lemma 2, any speculative trade that includes at least one insider is impossible in a fully revealing REE regardless of whether the CPA is satisfied or not. We formally state this main insight of our paper in the following proposition.

**Proposition.** *Regardless of the specification of priors  $\pi_1, \dots, \pi_n$ , insider trade is impossible in an REE.*

We conclude this section with an example that illustrates the above proposition by revisiting the stylized Goldman Sachs story from the introduction.

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<sup>1</sup>It is beyond the scope of this paper to make this argument mathematically precise. For our purpose it suffices to observe that  $\pi_1(\omega_1) = x_1$ ,  $\pi_1(\omega_2) = x_2$  and  $\pi_2(\omega_1) = y_1$ ,  $\pi_2(\omega_2) = y_2$  would only satisfy  $x_1 = x$ ,  $x_2 = \delta x$  and  $y_1 = y$ ,  $y_2 = \delta y$  on a subset of all possible beliefs in  $\Delta^3 \times \Delta^3$  that has Lebesgue measure zero (where  $\Delta^3$  denotes the open 3-dimensional simplex for fixed  $a, b, c, d$ ).

**Illustrative example.** According to the Levin-Coburn report, there was a point in time when non-US investors did not know the true value of the CDOs whereas US investors, in particular Goldman Sachs, already knew that the CDOs were worthless. Among many pieces of according evidence, the Levin-Coburn report states:

“The Goldman sales manager for Europe and the Middle East suggested that Mr. Sparks focus the CDO sales efforts abroad, because the clients there were not involved in the U.S. housing market and therefore were “not feeling pain” [...]” (p. 493)

To formalize the uncertainty of the representative non-US investor, consider the state space

$$\Omega = \{\omega_H, \omega_L\} \quad (28)$$

and suppose that an CDO is an asset characterized by the following payoff structure

$$X(\omega) = \begin{cases} 1 & \text{if } \omega = \omega_H \\ 0 & \text{if } \omega = \omega_L \end{cases} \quad (29)$$

That is,  $\omega_H$  stands for the state of the world in which the CDO has a high value whereas  $\omega_L$  stands for the state in which the CDO is worthless.

To capture the asymmetric private information between the insider, i.e., Goldman Sachs, on the one hand, and the representative non-US investor, on the other hand, define the following private information partitions

$$\Pi_{GS} = \{\{\omega_H\}, \{\omega_L\}\}, \quad (30)$$

$$\Pi_{Inv} = \{\{\omega_H, \omega_L\}\}. \quad (31)$$

That is, whereas Goldman Sachs knows the true state of the world the investor does not. Furthermore, observe that  $\bigvee_{i \in \{GS, Inv\}} \Pi_i = \{\{\omega_H\}, \{\omega_L\}\}$  so that condition (26) of Lemma 2 implies that both agents' beliefs are ex post homogenous for arbitrary priors  $\pi = (\pi_{GS}, \pi_{Inv})$ .

Let the set of possible trade positions be given by  $\Gamma = \{-1, 0, 1\}$  so that any agent  $i \in \{GS, Inv\}$  can either sell (i.e.,  $\theta_i = -1$ ), zero-trade (i.e.,  $\theta_i = 0$ ), or buy (i.e.,  $\theta_i = 1$ ) the CDO. It is easy to see that for arbitrary priors  $\pi = (\pi_{GS}, \pi_{Inv})$  any revealing FCE  $(P, \Theta) \langle \Pi, \pi \rangle$  must satisfy

$$P(\omega) = \begin{cases} 1 & \text{if } \omega = \omega_H \\ 0 & \text{if } \omega = \omega_L \end{cases} \quad (32)$$

Observe that the equilibrium price function correctly prices the asset in the sense that the price of the CDO coincides in every state of the world with its true value. Because there are no gains from trade in any equilibrium, the zero-trade gives every agent the same utility as any other asset position. Consequently, there does not exist any insider trade equilibrium in this example regardless of whether the CPA is satisfied or not.  $\square$

## 5 Discussion

Radner's (1979) REE concept is the standard theoretical justification of the *information efficient market hypothesis* (EMH) according to which asset prices fully reflect all available information in the economy. Under the EMH prices are always 'correct' so that *mispricing* cannot occur. Although the empirical occurrence of mispricing is generally not easy to prove (cf. Fama 1970), Barberis and Thaler (2003) have collected empirical evidence of "financial market phenomena that are almost certainly mispricings, and persistent ones at that" (p. 1061). The occurrence of insider-trade by Goldman Sachs, as described in the Levin-Coburn report, would be a striking example for mispricing because a positive price for completely worthless CDOs cannot be the 'correct' price in any asset-pricing model.

No-trade results establish that mispricing cannot happen in an REE for situations of static speculation. There are no strict incentives for trading the asset in an REE because the equilibrium price coincides with the asset's fundamental value (i.e., is 'correct') when all information is revealed to the market participants. The question now arises what conceptual extensions of Radner's REE, if any, would enable general equilibrium theory under asymmetric information to accommodate mispricing or, more specifically, insider trade in a situation of static speculation. In the remainder of this section, we briefly look at two possible candidates for such concepts, namely, impersonalized markets, on the one hand, and decision theoretic alternatives to EU theory, on the other hand.

Unlike as in a fully revealing REE, where private information does not lead to any personal advantage, Hirshleifer (1971) derives private benefits from information acquisition in an impersonalized market in which "one individual's choices would negligibly affect the ruling prices" (p. 564). The notion that a single individual's demand-supply decision is insignificant is easy to model within a *large* economy where each individual corresponds to a point of measure zero. It is not obvious, however, how this notion can be translated into a consistent general equilibrium concept for a finite number of market participants. For economies with finitely many participants, an individual's demand-supply decision is not insignificant because it will (typically) affect the market clearing

price. Although the assumption of large markets might be a good approximation for some real-world situations with many market participants, there are other market exchange situations with only a few participants that clearly violate the assumption of negligible demand-supply decisions as, e.g., in our Goldman Sachs example.

Consider now an economy with finitely many participants that is impersonalized in such a way that some market participant does not know whether he is offered, e.g., CDOs from an insider or from some non-insider. We could then model the uncertainty about the traders' identity by two different, mutually exclusive 'submarkets'—one market with the insider, one market with the non-insider—such that each submarket is cleared through an REE. If these REEs were different (which is, by Radner's formal argument, generically the case), it would be revealed through the submarket REEs whether the insider or the non-insider is offering the shares. But this would bring us back to fully revealed information in an equilibrium. At this point, it is far from obvious how the assumption of an impersonalized market might help to establish the existence of insider trade in an REE (or an according 'sub-market perfect' version of an REE). I regard it an interesting avenue for future research to look into the details of this argument but I also suspect that one would have to leave the realm of general equilibrium theory and include game theoretic concepts. For example, some market participants might have a strategic incentive to not fully reveal their private information through their demand-supply decisions. Such strategic considerations under asymmetric information cannot be modelled within general equilibrium theory but would have to be addressed by either adverse selection or signaling models (cf., e.g., Batabyal 2012).

In their survey on behavioral finance, Barberis and Thaler (2003) convincingly argue that *cumulative prospect theory* (CPT) can contribute towards understanding several mispricing puzzles (e.g., the volatility and the equity premium puzzles). CPT (Tversky and Kahneman 1992; Wakker and Tversky 1993) allows for the expression of ambiguity attitudes and/or likelihood insensitivity (Wakker 2010; Abdellaoui et al. 2011) and it explicitly models the psychological phenomenon of loss aversion. Although cumulative prospect theory has thus a greater realistic appeal than EU theory, it is of no help for explaining the existence of insider trade. More specifically, CPT decision makers have no strict incentives for speculative trade in our Goldman Sachs example because with only two deterministic outcomes (one versus zero in (29)) CPT reduces to EU theory (also see Dow et al. 1990; Zimper 2009; Dominiak and Lefort 2013).

## 6 Outlook

Motivated by the above discussion, I develop in Zimper (2013) an equilibrium concept under asymmetric information which generalizes Radner's (1979) REE concept by admitting for boundedly rational agents who base their demand-supply decisions on *incorrect price anticipations*. I further demonstrate in Zimper (2013) that insider trade can become an equilibrium phenomenon if and only if the agents are boundedly rational in the above sense. The intuition for this result is straightforward: If a boundedly rational agent 'irrationally' assumes that the insider might sell (resp. buy) the asset below (resp. above) its true value, he might have a strict incentive to buy (resp. sell) the asset at incorrect prices so that he ends up being exploited by the insider in an equilibrium.

More precisely, I consider EU maximizing agents who decide in an ex ante situation how many units of assets they are going to demand, respectively to supply, in an ex post exchange situation on an  $l$ -dimensional asset space. In this ex post situation, markets clear in accordance with some equilibrium price function, denoted  $P^X : \Omega \rightarrow \mathbb{R}_+^l$ . The crucial generalization of Radner's (1979) REE concept comes from the extended state space

$$\Omega \equiv \Omega' \times P \tag{33}$$

where  $P \subset \mathbb{R}_+^l$  denotes the space of possible anticipated price vectors and  $\Omega'$  is the space of economic fundamentals. The extended state space (33) allows to distinguish between *equilibrium states*, i.e., all  $(\omega', p) \in \Omega$  such that  $P^X(\omega', p) = p$ , versus *out-of-equilibrium states*, i.e., all  $(\omega', p) \in \Omega$  such that  $P^X(\omega', p) \neq p$ . Because of this distinction it is possible to define an equilibrium concept which requires market clearing in equilibrium but not necessarily in out-of-equilibrium states.

If an agent's prior attaches positive probabilities only to equilibrium states, he is rational in the sense that he fully understands the economy's price mechanism and we are back to Radner's (1979) REE concept. However, if an agent attaches a strictly positive probability to some out-of-equilibrium state  $(\omega', p)$ , he thereby expresses his irrational belief that markets may possibly clear at the anticipated price vector  $p$  whenever the economic fundamentals are pinned down by  $\omega' \in \Omega'$  whereas markets are actually cleared at the different price vector  $P^X \neq p$ . By assumption, the economic reality is comprehensively described by markets that clear; that is, we will ex post only observe equilibrium states. Although out-of-equilibrium states are thus impossible to observe in reality, any positive probabilities attached to these impossible states by boundedly rational agents may result in observable equilibrium allocations that are inconsistent with an REE. As a consequence, it becomes possible to construct equilibria that give rise to insider trade whenever the agents are boundedly rational. For further details of this (non-trivial)

formal argument, I would like to refer the reader to Zimper (2013).

## Appendix: Proof of Lemma 1

The proof proceeds in three steps.

**Step 1.** Consider an arbitrary equilibrium  $(P, \Theta) \langle \Pi, \pi \rangle$ . Observe that the zero trade guarantees at all  $I \in \bigvee_{i=1}^n \Pi_i$

$$E[u_i((X(\omega) - P(\omega)) \cdot 0), \pi_i(\omega | I)] = u_i(0). \quad (34)$$

By concavity of the vNM utility function,

$$\begin{aligned} & u_i(E[(X(\omega) - P(\omega)) \cdot \Theta_i(\omega), \pi_i(\omega | I)]) \\ & \geq E[u_i((X(\omega) - P(\omega)) \cdot \Theta_i(\omega)), \pi_i(\omega | I)] \end{aligned} \quad (35)$$

so that, for all  $i$ ,

$$E[(X(\omega) - P(\omega)) \cdot \Theta_i(\omega), \pi_i(\omega | I)] \geq 0 \quad (36)$$

because  $u_i$  is strictly increasing.

**Step 2.** Suppose now that  $(P, \Theta) \langle \Pi, \pi \rangle$  is a speculative trade equilibrium. Then there is some  $i \in \{1, \dots, n\}$  and some  $I \in \bigvee_{i=1}^n \Pi_i$  such that

$$E[u_i((X(\omega) - P(\omega)) \cdot \Theta_i(\omega)), \pi_i(\omega | I)] > u_i(0) \quad (37)$$

for some  $\Theta_i(\omega) \in \Gamma$  with  $\Theta_i(\omega) \neq 0$ . By strictly increasing and concave vNM utility functions,

$$E[(X(\omega) - P(\omega)) \cdot \Theta_i(\omega), \pi_i(\omega | I)] > 0. \quad (38)$$

Combining (38) with (36) implies for any speculative trade equilibrium  $(P, \Theta) \langle \Pi, \pi \rangle$  that

$$\sum_{i=1}^n E[(X(\omega) - P(\omega)) \cdot \Theta_i(\omega), \pi_i(\omega | I)] > 0. \quad (39)$$

**Step 3.** By the market clearing condition (4), we have for every equilibrium  $(P, \Theta) \langle \Pi, \pi \rangle$  that, for all  $\omega \in \Omega$ ,

$$0 = \sum_{i=1}^n \Theta_i(\omega) \quad (40)$$

$$= (X(\omega) - P(\omega)) \cdot \sum_{i=1}^n \Theta_i(\omega) \quad (41)$$

$$= \sum_{i=1}^n (X(\omega) - P(\omega)) \cdot \Theta_i(\omega). \quad (42)$$

This implies for arbitrary beliefs  $\pi_i$  that satisfy the ex post homogeneity condition

$$\sum_{\omega \in \Omega} \pi_i(\omega | I) \cdot \sum_{i=1}^n (X(\omega) - P(\omega)) \cdot \Theta_i(\omega) \quad (43)$$

$$= \sum_{i=1}^n E[(X(\omega) - P(\omega)) \cdot \Theta_i(\omega), \pi_i(\omega | I)] \quad (44)$$

$$= 0. \quad (45)$$

But this is a contradiction to (39).  $\square$



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