Speculative Trade Equilibria with Incorrect Price Anticipations

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Abstract

This paper introduces an equilibrium concept for boundedly rational agents who base their demand-supply decisions on incorrect price anticipations. Formally, we differentiate between equilibrium and out-of-equilibrium states. If the agents attach zero prior probability to all out-of-equilibrium states, our equilibrium concept coincides with Radner's (1979) concept of rational expectations equilibria (=REE). In contrast to REE, however, there may exist strict incentives for speculative asset trade whenever boundedly rational agents regard out-of-equilibrium states as possible.

Keywords: Bounded Rationality, Speculative Trade, Rational Expectations, Incorrect Prices

JEL Classification Numbers: D51; D53; G02

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1 Introduction

Equilibrium models of asset markets characterize asset prices through an equilibrium price function which clears markets in every state of the world. The existing literature thereby assumes that the state space is comprehensively described as the space of all possible values of exogenously given economic fundamentals, denoted Ω', such as, e.g., dividend-payments. According to this standard modelling choice, a market participant's uncertainty about future asset prices can be completely reduced to his uncertainty about economic fundamentals driving the adapted price process. By assumption, the agents of these standard models typically understand the market clearing price function and have therefore correct price anticipations in the sense of Radner's (1979) *rational expectations equilibrium* (REE), (cf. Brunnermeier 2001; Yannelis 1991; Glycopantis and Yannelis 2005).

To model the possibility of incorrect price anticipations this paper introduces in Section 2 a new equilibrium concept which is based on the extended state space

$$\Omega \equiv \Omega' \times P. \tag{1}$$

The space of anticipated price vectors $P \subset \mathbb{R}^l_+$ captures the agents' uncertainty about future asset prices on some l-dimensional asset space. For boundedly rational agents, the agents' uncertainty about market-clearing prices cannot be reduced to their uncertainty about economic fundamentals. More specifically, the subjective priors of boundedly rational agents attach strictly positive probabilities to states $(\omega', p) \in \Omega$ such that the anticipated price vector p would not clear the markets whenever the economic fundamentals are determined by $\omega' \in \Omega'$. That is, boundedly rational agents in our sense do not understand the economy's market clearing price mechanism to the effect that they base their demand-supply decisions on incorrect price anticipations.

Formally, we consider expected utility (EU) maximizing agents who decide in an ex ante situation how many units of assets they are going to demand, respectively to supply, in an ex post exchange situation. In this ex post situation markets clear in accordance with the equilibrium price function, denoted $P^X : \Omega \to \mathbb{R}^l_+$. We call $(\omega', p) \in \Omega$ an out-of-equilibrium state if and only if the anticipated price vector p does not clear markets in state (ω', p), i.e., iff $P^X(\omega', p) \neq p$. Conversely, $(\omega', p) \in \Omega$ is an equilibrium state if and only if the anticipated price vector p clears the markets in state (ω', p), i.e., iff $P^X(\omega', p) = p$. According to our interpretation, any possible (ex post) economic reality is comprehensively described by the set of all equilibrium states. Whereas rational agents correctly anticipate market clearing prices by attaching probability mass one to equilibrium states only, boundedly rational agents attach strictly positive probabilities to out-of-equilibrium states.

Technically speaking, our equilibrium concept stands for a specific way of separating the individual ex ante decisions from the measurability of these decisions with respect to the ex post publicly available information revealed through market-clearing prices. Alternative approaches towards such a separation arise around Negishi's (1961, 1972) concept of *subjectively perceived* (rather than objectivly given) demand functions (cf. Dreze and Herings 2008) as well as around the concept of *uncertain delivery* by Correia-da-Silva and Herves-Beloso (2008). In contrast to these approaches, where the agents have no problems to understand the economy's pricing mechanism, our approach emphasizes the agents' bounded rationality with respect to this pricing mechanism.

Tirole (1982) proves the impossibility of speculative trade in a REE with finite time horizon. It is not difficult to demonstrate that our equilibrium concept is observationally equivalent to REE whenever the agents attach zero-probability to out-of-equilibrium states (a formal proof is available from the author upon request). Consequently, speculative trade can only occur in our framework if the agents have incorrect price anticipations. In Section 3 we construct an example that illustrates the possibility of speculative equilibrium trade for agents that attach positive probabilities to out-of-equilibrium states. Because our approach identifies incorrect price anticipations as a possible explanation for speculative trade between EU decision makers, it complements the existing–decision theoretically motivated–literature[1] which establishes the possibility of speculative trade between non-EU decision makers with correct price anticipations.

2 Equilibria with incorrect price anticipations

This section defines an equilibrium concept for an exchange economy under asymmetric information with respect to the extended state space (1). For interpretational convenience we differentiate between an ex ante (decision making) and an ex post (market exchange) situation. In the ex ante situation every agent $i \in \{1, ..., n\}$ decides on a contingent plan of actions which assigns a demand-supply decision to all his anticipated future selves. The anticipated future selves of agent i are formally defined as the members[2] of the following partition on Ω

$$\Pi_i = \{I'_i \times \{p\} \mid I'_i \in \Pi'_i, p \in P\}, \tag{2}$$

[1] The seminal paper is Dow, Madrigal and Werlang (1990). For a more recent overview on the decision-theoretic speculative trade literature see the references in Zimper (2009).

[2] In general, there are more members in Π_i than different future selves that can be actually observed in the ex post situation because $I_i(\omega', p)$ is not observable whenever (ω', p) is an out-of-equilibrium state. That is, the agents of our model are free to anticipate future selves which are actually impossible.

where Π_i' denotes some partition on Ω'. We denote by $I_i(\omega', p)$ the unique member of Π_i that contains state $(\omega', p) \in \Omega$. By construction of (2), each agent expects to learn the market clearing prices in the ex post exchange situation so that $I_i(\omega', p) \neq I_i(\omega', p')$ if $p \neq p'$.

Denote by $\Sigma(\Pi_i)$ the σ-algebra generated by Π_i and define the probability space $(\pi_i, \Omega, \Sigma(\Pi_i))$ where the additive probability measure π_i denotes agent i' subjective belief. We assume that all agents are expected utility maximizers such that $u_i : \Gamma_i \times \Omega \rightarrow \mathbb{R}$ denotes the von vNM utility that agent i obtains in state $\omega \in \Omega$ from the net-trade $\theta_i \in \Gamma_i \subseteq \mathbb{R}^l$ on the l-dimensional asset space. The demand-supply correspondence of agent i is defined as the $\Sigma(\Pi_i)$-measurable set-valued mapping $\varphi_i : \Pi_i \rightarrow 2^{\Gamma_i}$ such that, for all $I_i(\omega', p) \in \Pi_i$,

$$\varphi_i(I_i(\omega', p)) = \arg \max_{\{\theta_i \in \Gamma_i | p\theta_i = 0\}} E[u_i(\theta_i, \omega), \pi_i(\omega \mid I_i(\omega', p))] \tag{3}$$

where $E[u_i(\theta_i, \omega), \pi_i(\omega \mid I_i(\omega', p))]$ denotes the expected utility of agent i's anticipated future self $I_i(\omega', p)$.

An information-belief structure, denoted $\langle \Pi, \pi \rangle$ such that $\Pi = (\Pi_1, ..., \Pi_n)$ and $\pi = (\pi_1, ..., \pi_n)$, collects the agents' information partitions and subjective beliefs, respectively. Given Π, denote by $\bigvee_{i=1}^{n} \Pi_i$ the *joint* (=coarsest common refinement) of all Π_i, $i \in \{1, ..., n\}$. Intuitively speaking, the information cells in $\bigvee_{i=1}^{n} \Pi_i$ would obtain if all agents shared their (anticipated) information. Further, denote by $\Sigma\left(\bigvee_{i=1}^{n} \Pi_i\right)$ the σ-algebra generated by $\bigvee_{i=1}^{n} \Pi_i$.

Definition 1 ***Equilibria with not necessarily correct price anticipations.*** *Fix some information-belief structure $\langle \Pi, \pi \rangle$. An equilibrium with respect to $\langle \Pi, \pi \rangle$, denoted $(P^X, \Theta^X) \langle \Pi, \pi \rangle$, is a mapping*

$$\left(P_1^X, ..., P_l^X; \Theta_1^X, ..., \Theta_n^X\right) : \Omega \rightarrow \mathbb{R}_+^l \times \mathbb{R}^{nl} \tag{4}$$

such that, for all $i \in \{1, ..., n\}$,

1. *P^X and Θ_i^X are $\Sigma\left(\bigvee_{i=1}^{n} \Pi_i\right)$-measurable;*
2. *for all $(\omega', p) \in \Omega$,*
$$\Theta_i^X(\omega', p) \in \varphi_i(I_i(\omega', p)); \tag{5}$$

3. for all $(\omega', p) \in \Omega$ *such that*

$$P^X(\omega', p) = p, \tag{6}$$

$$\sum_{i=1}^{n} \Theta_i^X\left(\omega', P^X(\omega', p)\right) = 0. \tag{7}$$

Measurability Condition 1. ensures that neither equilibrium prices nor allocations can reveal more information about the true state of the world than the commonly shared information of all agents. Observe that the market clearing Condition 3. requires the equilibrium price function P^X to clear markets only in equilibrium states, i.e., in states which satisfy condition (6). We assume that the economic reality is comprehensively described by the set of all equilibrium states so that markets always clear in the ex post situation. Although out-of-equilibrium states are thus never observable ex post, a boundedly rational agent may ex ante attach a positive probability to some out-of-equilibrium state whenever he does not understand the economy's price mechanism.

3 Example: Speculative asset trade

We follow Tirole (1982) and consider a situation of static speculation under asymmetric information. The economy consists of two agents and of a single asset with payoff function $X : \Omega \to \mathbb{R}$. The agents have identical strictly concave vNM utility functions, i.e., $u \equiv u_1 = u_2$, that are strictly increasing with $u(0) = 0$. The agents also have identical priors, i.e., $\pi \equiv \pi_1 = \pi_2$. Agent $i \in \{1, 2\}$ only cares about his vNM utility from the monetary gain $(X(\omega) - p) \cdot \theta_i$, which he receives in the ex post situation in state $\omega \in \Omega$ from either purchasing $(\theta_i = 1)$ or selling $(\theta_i = -1)$ or zero-trading $(\theta_i = 0)$ the asset at the anticipated price p. The demand-supply correspondence (3) of agent $i \in \{1, 2\}$ is thus given as

$$\varphi_i(I_i(\omega', p)) = \arg \max_{\theta_i \in \{-1,0,1\}} E\left[u\left((X(\omega) - p) \cdot \theta_i\right), \pi\left(\omega \mid I_i(\omega', p)\right)\right] \tag{8}$$

for all $I_i(\omega', p) \in \Pi_i$.

We call $\left(P^X, \Theta^X\right)\langle \Pi, \pi \rangle$ a speculative trade equilibrium if and only if, first, the agents share a common prior and, second, there is some equilibrium state ω in which both agents have strict incentives to trade the asset, i.e., for some $\theta_1 \neq 0$,

$$\begin{aligned} (\theta_1, -\theta_1) &= \Theta^X(\omega) \text{ and} & (9)\\ 0 &\notin \varphi_i\left(I_i\left(\omega, P^X(\omega)\right)\right) \text{ for } i \in \{1, 2\}. & (10) \end{aligned}$$

Proposition 1 *A speculative trade equilibrium may exist if the agents attach a strictly positive prior probability to out of-equilibrium-states.*

We prove the proposition by means of a simple but instructive example.

Example. Consider the following space

$$\Omega' = \{\omega'_H, \omega'_L\}, \tag{11}$$

determining all relevant economic fundamentals, as well as the space

$$P = \{p_H, p_L\} \tag{12}$$

capturing the agents' uncertainty with respect to possible asset prices. The asset is characterized by the payoff structure $X : \Omega \equiv \Omega' \times P \to \mathbb{R}$ such that

$$X(\omega', p) = \begin{cases} 2 & \text{if } \omega' = \omega'_H \\ 1 & \text{if } \omega' = \omega'_L \end{cases} \tag{13}$$

Suppose that the common prior of both agents satisfies

$$\pi(\omega) > 0 \text{ for all } \omega \in \Omega. \tag{14}$$

Further suppose that the agents' information partitions are given as

$$\Pi_1 = \{\{(\omega'_H, p_H)\}, \{(\omega'_H, p_L)\}, \{(\omega'_L, p_H)\}, \{(\omega'_L, p_L)\}\}, \tag{15}$$

$$\Pi_2 = \{\{(\omega'_H, p_H), (\omega'_L, p_H)\}, \{(\omega'_H, p_L), (\omega'_L, p_L)\}\}. \tag{16}$$

Agent 1 is thus the "insider" who has perfect knowledge about the economic fundamentals determining the asset's payoff performance, which is "high" in any state $(\omega'_H, \cdot)$ and "low" in any state $(\omega'_L, \cdot)$. In contrast, agent 2 cannot directly observe the economic fundamentals.

Let $p_H = 2 - \varepsilon_1$ for some $\varepsilon_1 > 0$ and observe that the expected utility of agent 2 from selling the asset at price p_H is given as

$$\begin{aligned} & E\left[u\left(\left(X(\omega) - p_H\right) \cdot (-1)\right), \pi\left(\omega \mid \{(\omega'_H, p_H), (\omega'_L, p_H)\}\right)\right] \qquad (17) \\ = \; & u\left((2 - (2 - \varepsilon_1)) \cdot (-1)\right) \cdot \frac{\pi(\omega'_H, p_H)}{\pi(\{(\omega'_H, p_H), (\omega'_L, p_H)\})} \\ & + u\left((1 - (2 - \varepsilon_1)) \cdot (-1)\right) \cdot \frac{\pi(\omega'_L, p_H)}{\pi(\{(\omega'_H, p_H), (\omega'_L, p_H)\})} \end{aligned}$$

whereas agent 2's expected utility from the zero-trade at price p_H is

$$\begin{aligned} & E\left[u\left(\left(X\left(\omega\right)-p_H\right)\cdot\left(0\right)\right),\pi\left(\omega\mid\left\{\left(\omega'_H,p_H\right),\left(\omega'_L,p_H\right)\right\}\right)\right] \\ = \; & u\left(\left(2-\left(2-\varepsilon_1\right)\right)\cdot\left(0\right)\right)\cdot\frac{\pi\left(\omega'_H,p_H\right)}{\pi\left(\left\{\left(\omega'_H,p_H\right),\left(\omega'_L,p_H\right)\right\}\right)} \\ & +u\left(\left(1-\left(2-\varepsilon_1\right)\right)\cdot\left(0\right)\right)\cdot\frac{\pi\left(\omega'_L,p_H\right)}{\pi\left(\left\{\left(\omega'_H,p_H\right),\left(\omega'_L,p_H\right)\right\}\right)}. \end{aligned} \tag{18}$$

Straightforward mathematical transformation shows that

$$\begin{aligned} & E\left[u\left(\left(X\left(\omega\right)-p_H\right)\cdot\left(-1\right)\right),\pi\left(\omega\mid\left\{\left(\omega'_H,p_H\right),\left(\omega'_L,p_H\right)\right\}\right)\right] \\ > \; & E\left[u\left(\left(X\left(\omega\right)-p_H\right)\cdot\left(0\right)\right),\pi\left(\omega\mid\left\{\left(\omega'_H,p_H\right),\left(\omega'_L,p_H\right)\right\}\right)\right] \end{aligned} \tag{19}$$

$$\Leftrightarrow$$

$$u\left(-\varepsilon_1\right)\cdot\pi\left(\omega'_H,p_H\right)+u\left(1-\varepsilon_1\right)\cdot\pi\left(\omega'_L,p_H\right)>u\left(0\right). \tag{20}$$

That is, whenever inequality (20) is satisfied, the uninformed agent 2 strictly prefers selling the asset at price p_H to a zero-trade at price p_H, (which in turn he strictly prefers to buying the asset at price p_H). Since

$$\pi\left(\omega'_L,p_H\right)>0, \tag{21}$$

by assumption, the continuity of the expected utility function implies the existence of some sufficiently small number $\varepsilon_1>0$ such that (20) is satisfied. Fix such $\varepsilon_1>0$ and observe that

$$\varphi_2\left(I_2\left(\omega',p_H\right)\right)=\{-1\}\;\text{ for }\omega'\in\left\{\omega'_H,\omega'_L\right\} \tag{22}$$

whenever $p_H=2-\varepsilon_1$.

Analogously, let $p_L=1+\varepsilon_2$ and observe that the assumption

$$\pi\left(\omega'_H,p_L\right)>0 \tag{23}$$

implies the existence of some sufficiently small $\varepsilon_2>0$ such that

$$u\left(1-\varepsilon_2\right)\cdot\pi\left(\omega'_H,p_L\right)+u\left(-\varepsilon_2\right)\cdot\pi\left(\omega'_L,p_L\right)>u\left(0\right) \tag{24}$$

That is, whenever inequality (24) is satisfied, the uninformed agent 2 strictly prefers buying the asset at price p_L to a zero-trade at price p_L, (which in turn he strictly prefers to selling the asset at price p_L). Consequently,

$$\varphi_2\left(I_1\left(\omega',p_L\right)\right)=\{1\}\;\text{ for }\omega'\in\left\{\omega'_H,\omega'_L\right\} \tag{25}$$

whenever $p_L=1+\varepsilon_2$.

Turn now to the informed agent 1 and observe that $\varepsilon_1, \varepsilon_2 > 0$ implies

$$\varphi_1\left(I_1\left(\omega', p_H\right)\right) = \begin{cases} \{1\} & \text{if } \omega' = \omega'_H \\ \{-1\} & \text{if } \omega' = \omega'_L \end{cases} \tag{26}$$

and

$$\varphi_1\left(I_1\left(\omega', p_L\right)\right) = \begin{cases} \{1\} & \text{if } \omega' = \omega'_H \\ \{-1\} & \text{if } \omega' = \omega'_L \end{cases} \tag{27}$$

In words: The informed agent 1 has a strict incentive to buy, resp. to sell, the asset in states for which the price is below, resp. above, the asset's true value.

Define now the following function $P^X : \Omega \to \mathbb{R}$

$$P^X\left(\omega', p\right) = \begin{cases} p_H & \text{if } \omega' = \omega'_H \\ p_L & \text{if } \omega' = \omega'_L. \end{cases} \tag{28}$$

Observe that P^X is an equilibrium price function because it clears the markets in all equilibrium states given as (ω'_H, p_H) and (ω'_L, p_L). Consequently, there exists a speculative trade equilibrium $\left(P^X, \Theta^X\right)\langle\Pi, \pi\rangle$ such that

$$\Pi = (\Pi_1, \Pi_2) \tag{29}$$

and, for all $(\omega', p) \in \Omega$,

$$\left(P^X; \Theta_1^X, \Theta_2^X\right)\left(\omega', p\right) = \begin{cases} (2 - \varepsilon_1, 1, -1) & \text{if } \omega' = \omega'_H \\ (1 + \varepsilon_2, -1, 1) & \text{if } \omega' = \omega'_L \end{cases} \tag{30}$$

for some sufficiently small numbers $\varepsilon_1, \varepsilon_2$.$\square$

In the above example, the uninformed agent 2 ends up with a bad deal in both economic scenarios. That is, regardless of whether the asset value is high or low, the informed agent 1 uses his informational advantage to rip off agent 2 because agent 2 does not understand the economy's price mechanism. Agent 2's desire to sell the asset at a price $p_H = 2 - \varepsilon_1$, (which is strictly less than the asset's actual value $X\left(\omega'_H\right) = 2$), is based on his incorrect anticipation that the informed agent would also like to buy the asset at this high price if it had a low value. This incorrect price anticipation is formally expressed through the positive probability that agent 2 attaches to the out-of-equilibrium state (ω_L, p_H) in which the asset value is low but the price is high. Similarly, agent 2 attaches a positive probability to the out-of-equilibrium state (ω_H, p_L) thereby expressing his incorrect anticipation that the informed agent 1 would also sell the asset at this low price if it had a high value. Consequently, agent 2 desires to buy the asset at price $p_L = 1 + \varepsilon_1$, which is strictly higher than the asset's actual value $X\left(\omega'_L\right) = 1$.

Speculative trade can only occur because the agents' prior attaches a positive probability to out-of-equilibrium states. To see this consider a prior that attaches zero probability to out-of-equilibrium states so that any equilibrium can be reinterpreted as Radner's (1979) REE for which speculative trade is impossible. The unique equilibrium price function is then given as

$$P^X(\omega', p) = \begin{cases} 2 & \text{if } \omega' = \omega'_H \\ 1 & \text{if } \omega' = \omega'_L \end{cases} \tag{31}$$

which corresponds to the asset's true value in every state of the world so that there is no strict incentive for trading the asset at equilibrium prices.

Finally, note that the speculative trade equilibrium (30) is not *arbitrage free*. To see this observe that both agents would like to trade in both states as many units of the asset as possible. This equilibrium can therefore only exist because, by setting $\theta_i \in \Gamma_i \equiv \{-1, 0, 1\}$ instead of $\theta_i \in \Gamma_i \equiv \mathbb{R}$ for $i = 1, 2$, we violated the (in our opinion: unappealing) assumption of complete markets according to which unlimited amounts of assets can be traded; (cf., e.g., Theorem 1 in Dybvig and Ross (2003) or the Theorem on p. 5 in Duffie (2001) which establish the equivalence between the absence of arbitrage and the existence of a solution to the agents' portfolio choice problem under the assumption that $\Gamma_i = \mathbb{R}$ for all i).

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