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Concentration measures as an element in testing the structure-conduct-performance paradigm

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Abstract

The original structure-conduct-performance (SCP) paradigm, according to which market structure determines market conduct and market conduct determines market performance, underlies numerous competition policies. Since its development almost a century ago, the paradigm has been heavily criticised and numerous efforts have been made to test it by correlating measures of seller concentration with measures of market performance. The reliability of seller concentration measures that are frequently used, particularly in South Africa, was tested against the Hannah and Kay criteria, using hypothetical numbers of sellers and market shares. The premise is that a concentration measure must be reliable in the sense that it should lead to a correct conclusion when the relevant concentration curves do NOT cross. The following absolute concentration measures were found to meet the criteria: the Herfindahl-Hirschman index (HHI). the other Hannah and Kay indices $[HKI(\alpha)]$, the Rosenbluth index (RI), the numbers equivalent of the Hannah and Kay indices $[HKIne(\alpha)]$ and the entropy coefficient (EC). The discrete measures, concentration ratios (CRX) and the occupancy count (CRX%), do not always meet the criteria, nor do the relative concentration measures or measures of inequality, namely the Gini coefficient (GC), the variance of logarithms of market shares (VL) and the relative entropy coefficient (REC). The Horvath index (HI), an absolute concentration measure, does not always meet the criteria. Studies that employed the unreliable measures should be disregarded or reworked and students should be forewarned against the use of such measures.

Most competition policies are to some extent based on the original structure-conduct-performance (SCP) paradigm, according to which market structure determines market conduct and market conduct determines market performance. Since the development of the paradigm almost a century ago, numerous efforts have been made to test it by correlating measures of seller concentration with measures of market performance. This paper shows that some of the seller concentration measures are unreliable and that research based on them should be disregarded or reworked.

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1 THE ORIGINAL SCP PARADIGM

The original structure-conduct-performance (SCP) paradigm follows from neoclassical microeconomic theory and, more specifically, the comparison between monopoly and perfect competition found in almost every introductory and intermediate textbook in use in mainstream microeconomics courses. The basic postulate of the original SCP approach is that market structure determines market conduct, which in turn determines market performance.

Market structure consists of the relatively permanent (slow-changing) conditions within which the sellers operate; most importantly seller concentration (the number and size distribution of the sellers), barriers to entry to the market and the homogeneity of the sellers' products.

Market conduct can be described as the efforts of suppliers to market their products and to limit competition between them – that is, marketing conduct and competition-limiting conduct. Marketing conduct is the continuous formulation, implementation and reformulation of policies on pricing, product characteristics, marketing communication (mostly advertising) and distribution. Competition-limiting conduct mainly consists of merging or collusion with competitors, coercion of competitors or would-be competitors, and the use or misuse of political participation to limit competition through government action.

Market performance is the degree to which the suppliers to a market contribute to the economic goals of society, such as efficiency in the use of resources, full employment of resources, equity in the distribution of income and wealth, stability in the general price level and progressiveness, including innovation (the introduction of new products and processes).

According to the original SCP paradigm, a market structure characterised by low seller concentration, a homogeneous product and free entry to and exit from the market (approaching the neoclassical model of perfect competition) leave firms little choice of market conduct. They are price takers, determine their output quantities by setting marginal cost equal to price, must produce efficiently and make only normal profits in the long run. By contrast, a market structure typified by high seller concentration, products that differ appreciably from those of competitors and limited entry to the market (approaching the monopoly model) leave the firm or firms in the market a greater choice of market conduct. Such a firm is regarded as a price maker who chooses the quantity supplied by equating marginal cost with marginal revenue and sets the corresponding market price. The price exceeds marginal cost and the firm may make abnormal profits in the long run, need not maximise profit and can choose not to produce efficiently (within limits). It is deduced that market performance in concentrated markets will be worse than in unconcentrated markets. The price will be higher, fewer units of the product will be supplied, allocative and possibly internal efficiency will be lower, buyers will have a limited choice of sources of the product, service is likely to be worse, and innovation will probably be low

There are many criticisms of the original SCP paradigm, which have led to some adjustments. For example, some authors have added feedback effects to the unidirectional flow of cause and effect from market structure to conduct to performance (Ferguson & Ferguson 1994:18-19) or even argue that the direction of causation runs from market performance to conduct to structure (Reekie 2000:36,67).

2 THE VALIDITY OF THE ORIGINAL SCP PARADIGM

The validity of the original SCP paradigm has been questioned, particularly since the 1970s (Shepherd 1997:6-7). These criticisms seem to be gaining momentum.

There are two broad issues that affect the relevance or validity of the SCP paradigm, namely conceptual problems and the difficulty of empirically testing the paradigm.

2.1 Conceptual issues

As indicated in section 1, the SCP paradigm is based on neoclassical microeconomic theory, in particular the comparison of perfect competition and monopoly. However, neoclassical theories of oligopoly often fail to produce clear-cut conclusions about the relationships between market structure, conduct and performance (Lipczynski et al 2009:16). For example, it is not clear whether a more concentrated oligopoly would result in worse market performance than a less concentrated oligopoly.

It is often difficult to decide if a variable belongs to market structure, conduct or performance (Lipczynski et al 2009:16). For example, "product differentiation" may refer to a condition existing in a market and then it would be an element of structure. However, it may also refer to the act of differentiating one's product from those of competitors and then it would be a form of market conduct. When such a term is encountered, care must be taken to determine its meaning in the relevant context.

The original SCP approach has been criticised for being static in nature, instead of explaining the evolution of structure, conduct and performance over time (Lipczynski et al 2009:17). For example, the existing market structure is assumed to be given and market conduct and performance are deduced from that, while the market structure might be changing and the changes might lead to unexpected forms of market conduct and performance.

The original SCP approach in effect postulated what has become known as the "collusion hypothesis", according to which a positive relationship between seller concentration and the profitability of the firms in the market is interpreted as evidence that the firms are colluding or abusing market power in some other way to increase profits. Later, the Chicago school postulated the so-called "efficiency hypothesis", which states that a positive correlation between seller concentration and profitability reflects that efficient firms are more profitable than their rivals and eventually dominate their markets, meaning that both seller concentration and profitability would be high (Lipczynski et al 2009:17).

In addition, it is quite plausible that intelligent business people could decide for themselves that price-cutting is likely to lead to retaliation and consequently no appreciable increase in turnover – without discussing the matter with competitors. In other words, businesses could keep prices high, turnover low and profits high without collusion. The mere fact that prices and profits are high is no proof of collusion.

Furthermore, even if price fixing does occur in an industry, it might bring benefits to consumers. If price competition is excluded, competition may take other forms and result in high quality of goods, good service and innovation, which might not have occurred otherwise. In such cases, it could be argued that the collusion would not lead to a decline in societal welfare, i.e. to bad market performance.

The conclusion is that competition authorities should not rely on generalisations, but should judge each market on its own merits, based on the facts pertaining to that market.

2.2 Empirical tests

Empirical tests of the SCP paradigm consist of measuring market structure, market conduct and market performance and trying to find statistically significant relationships between them. The original SCP paradigm has not been proven or disproven, owing to problems with the definition of markets, the limitations of the statistical measures employed, data deficiencies, the weakness of statistical relationships found, and biased interpretation of the results.

Definition of markets It is often difficult to define the relevant market. If one considers a particular firm, its "direct" competitors have to be identified. This group of firms may then be considered to form a market. According to neoclassical theory two firms are in the same market when the cross-price elasticity of demand for their products is "high". However, measuring the cross-price elasticity of demand is often difficult in practice, while choosing a threshold between "high" and "low" cross-price elasticity is an arbitrary act.

Measuring market structure Although market structure has a number of dimensions, in empirical work it has often been measured only by seller concentration. Suppose concentration is measured in two different markets using the same measure and the same numerical answer is found. If entry to one of the markets is difficult, but easy in the other case, it is reasonable to expect that prices and profits would be higher (relative to cost) in the market that is difficult to enter. Seller concentration and ease of entry should therefore be measured simultaneously to achieve meaningful results. By the same token, the degree of product differentiation should also be measured. The problem is that these other variables, apart from seller concentration, are more difficult to measure numerically.

Secondly, there are many measures of seller concentration and they yield different results. This is the main focus of this paper, considered in section 3.

Measuring market conduct It is clear from the definition of market conduct that this concept is very difficult to measure quantitatively.

Measuring market performance Market performance is the extent to which the suppliers to a market contribute to social welfare and can theoretically be measured by profits or price (Lipczynski et al 2009:283-289), among other things. Most people would intuitively associate high prices with bad market performance, but may not immediately realise that the SCP paradigm associates good profitability with bad market performance. This association results from the neoclassical view that monopolists may earn economic or abnormal profits in the long term, while perfect competitors cannot. The business manager would associate good profitability with good performance by the FIRM, but firm performance is not the same as MARKET performance, which focuses on the welfare of the society, not that of the firm. The Lerner index (Lipczynski et al 2009:62) may be used to measure market performance – the higher the index, the worse the market performance – but it is difficult to apply in practice. The price-cost margin (Lipczynski et al 2009:286) may be used instead.

Data deficiencies Apart from the problematic characteristics of some measures, researchers have encountered data problems when trying to apply such measures, including those that are not inherently deficient.

Official organizations, such as competition commissions and courts, have the authority to demand information from firms in a market being investigated and can apply any measure of seller concentration. Other researchers often have to rely on statistics published by official compilers of statistics, for example, Statistics South Africa (Stats SA). Such statistics are subject to limitations, including:

- Secrecy provisions in legislation preclude official organizations from publishing information on individual firms; therefore they publish the information for groups of firms. Concentration measures that require the calculation of the market shares of individual firms cannot be calculated from such information. The organizations themselves may calculate and publish such concentration measures, but this is seldom done and the measures are subject to the limitations mentioned below.
- Official statistics are published in accordance with schemes for industry classification, such as the UK's successive Standard Industrial Classifications (SICs). Stats SA and its predecessors devised a similar series of SICs based on the United Nations' International Standard Industrial Classification (ISIC). These classifications rely on criteria that are seldom appropriate for identifying relevant markets where all sellers are in direct competition with one another. Another problem is that the information

is published for "establishments", not firms. In South Africa, this term could indicate a branch plant (e.g. a factory or shop) of a multiplant firm. In other words, a firm with five factories or shops would be counted as five entities, whereas the firm, and not the establishment, is the decision maker of economic theory. The five units are managed by a single decision maker and therefore constitute a single competitor. Basing concentration measures on establishment data understates the degree of concentration. Because of this problem, Stats SA has published some data for firms instead of establishments. These data are still subject to the problem that every registered company is regarded as a separate firm, whereas many are subsidiaries of other companies and are in effect branches of the holding or controlling company and are not decision makers in their own right. Consequently, even measures of concentration based on the Stats SA data for firms would understate the degree of concentration.

- Establishments and firms often produce several types of products that are sold in different markets. Such entities are classified in SICs on the basis of their most important product and their total turnover is shown under that heading. A firm that produces products A and B might be classified in industry A and its total sales of A and B would then be published as if it pertained only to A. Sales of A would be overstated and sales of B understated.
- Turnover figures in official reports on, for example, manufacturing usually only cover sales by domestic producers, whereas imports may constitute an important source of supply to the relevant market and thus influence the nature of competition in that market. Similarly, a portion of a domestic producer's production might be exported and this should be taken into account.

Relating market structure, conduct and performance to one another Numerous studies test simple correlations between some measure of market structure and some measure of market performance. Partly because of the difficulty of measuring market conduct, no effort is made to test the intervening relationships between market structure and market conduct, or between market conduct and market performance. Furthermore, no effort is made to exclude the possible effects of variables other than market structure on the employed measure of market performance, say profits. Many variables influence profits, for example, GDP, employment, inflation, interest rates, exchange rates and tax. Particularly when studying trends in seller concentration and performance, these variables should, for example, be included in multiple regressions, so that their influence may be excluded when determining the relationship between market structure and market performance. Some studies merely measure trends in seller concentration, suggesting that increases in seller concentration have negative consequences for society (without trying to correlate seller concentration with some measure of market performance).

Weak statistical relationships Numerous studies, which aim at determining the relationship between market structure and profit, have been done in the US and the UK (Ferguson & Ferguson 1994:95-96). Where positive correlations were found, such correlations have hardly been statistically significant.

Interpreting results If a study finds no correlation between market structure and market performance, opponents of the SCP paradigm may claim that the finding disproves the paradigm. However, protagonists of the paradigm might claim that this is not the case, since there are problems with the market definition, the measures, the data or the statistical techniques employed.

Suppose a study does find a statistically significant relationship between a measure of market structure and one of market performance. Protagonists of the paradigm would probably claim that the study proves the paradigm. However, even if possible disputes about the definition of the market, the measures, the data and the statistical techniques were ignored, the study would not necessarily prove the paradigm, because the direction of causality might run from performance to structure (Lipczynski et al 2009:287). The possibility of feedback effects inhibits the empirical testing of the original paradigm, since the statistical tests do not indicate the direction of causation.

3 MEASURES OF SELLER CONCENTRATION

The plethora of problems involved in empirical testing of the SCP paradigm may lead to the conclusion that all efforts at such testing should be abandoned. However, many researchers persist in doing such empirical work and it is important to evaluate the measures they use. In this paper, the spotlight falls on measures of seller concentration. Some measures increase when concentration rises – or are supposed to do so – while others decline – or are expected to decline – when concentration increases. The second category of concentration measures is sometimes called "inverse" measures. For want of a better term, the first category is referred to as "positive" measures in this paper.

For the purposes of this paper, concentration measures discussed in textbooks (Lipczynski et al 2009:195-206, Ferguson & Ferguson 1994:39-43) plus additional ones used in South African literature (Leach 1992, Fourie 1996, Fedderke & Szalontai 2009) are tested by means of hypothetical data. These tests are reported on in section 4. The measures are defined in the rest of this section.

3.1 Positive measures of concentration

3.1.1 X-firm concentration ratio

Possibly the best known measures of seller concentration are the x-firm concentration ratio and the Herfindahl-Hirschman index. The x-firm concentration ratio measures the market share of the largest x sellers in a market and is abbreviated CRX, where X can be 3, 4, 5 or another number:

$$CRX = \sum_{i=1}^{n} S_i^2 \tag{1}$$

where S_i is the market share of the ith seller.

A variant of CRX is CRX%, the cumulative market share of the top X% of the sellers in a market (Fourie 1996:100). When there are small numbers of sellers, say 3, it might be difficult to determine, for example, CR5%. The top firm then represents 33% of the sellers. One encounters a similar problem in the case of CRX, when there are fewer than X sellers in a market.

3.1.2 Herfindahl-Hirschman index

The Herfindahl-Hirschman index, abbreviated HHI, is the sum of the squared market shares of all the sellers in a market, where N is the total number of sellers:

$$HHI = \sum_{i=1}^{N} S_i^2 \tag{2}$$

CRX is a so-called "discrete" measure, while HHI is classified as a "summary" measure. The difference can be explained by means of a concentration curve.

To construct a concentration curve, the firms are ranked from largest to smallest, in terms of their market shares. The rank of each firm is plotted on the horizontal axis of a box diagram, up to N. The cumulative market shares are plotted on the vertical axis, the maximum value being 100% or 1 respectively, if the market shares are expressed as percentages or as ratios to 1. An example is given in Diagram 1, using ratios to 1. Hypothetical market shares are used, with N = 5.

CRX is known as a discrete measure because it represents only one point on the concentration curve (Marfels 1975:486). For example, if X=3, the combined market share of the largest three sellers is 0.85 and CR3 represents only point (3, 0.85) on the concentration curve in Diagram 1. To calculate CR3 at this point, one needs only the cumulative sales of the three largest sellers and the total sales of the N sellers involved. By contrast, HHI utilizes all the information represented by the concentration curve and is for this reason called a summary measure (Marfels 1975:488). The market share of each and every firm in the market is squared and added up to calculate the value of the index.

Since market shares are expressed as ratios to 1 in the formulas for most of the seller concentration measures, it is advantageous to use such ratios (instead of percentages) in graphs of concentration curves. Another advantage is that the surface area of the rectangle between (0, 0) and (N, 1) always equals N (base \times height = N \times 1). The larger N, the longer the concentration curve and the larger the area of the rectangle would be. Since a larger number of sellers is associated with more competition, an increase in the length of the concentration

curve indicates lower seller concentration. Conversely, a shorter concentration curve denotes fewer sellers and thus higher seller concentration.

The concentration curve also provides a visual depiction of the other element of seller concentration, namely the size distribution of the sellers. If the market shares of the sellers were equal, the concentration curve would coincide with the diagonal line from (0, 0) to (N, 1). The more unequal the distribution, the higher the curve would rise above the diagonal line before ending at (N, 1). Consequently, the inequality can be measured by the area between the diagonal line and the concentration curve. To paraphrase Marfels (1975:486), a higher and shorter concentration curve depicts a higher level of seller concentration than a lower and longer curve. However, when two concentration curves cross, it is harder to state categorically which one depicts a higher level of seller concentration. In such cases, one would have to rely on some concentration measure to make a judgement.

The premise of this paper is that such a concentration measure must be reliable in the sense that it should lead to a correct conclusion when the concentration curves do NOT cross. The reliability of various concentration measures is tested by using hypothetical numbers of sellers and market shares (see 4).

3.1.3 Hanna and Kay indices

While there is no need to attach weights to the market shares used in the CRX, it is necessary to use such weights in the case of summary measures, since the market shares of all the firms always add up to one. Marfels (1975:488) pointed out that the HHI weighs the market share of each seller by itself. Therefore, the largest seller's market share is multiplied by the highest weight and the smallest market share by the lowest weight. A shift among the market shares of large sellers has a greater impact on the index than a similar shift among the shares of small sellers in a given market.

Hanna and Kay (1977:56) indicate that the HHI is just one of a numerous array of indices that are the sums of the market shares weighed by the shares themselves, raised to some power. In the HHI each weight is the market share raised to the power of one. One could also use weights equal to the market shares raised to the power of 0.5, 1.5, or literally any other power. (For the moment, powers of zero or less are disregarded.) Such an index is denoted $\text{HKI}(\alpha)$, where HKI = "Hanna and Kay Index" and $\alpha =$ the power to which the market share is raised.

Table 1 shows an example of such indices, using the market shares of the five hypothetical firms in Diagram 1. If weights of $S_i^{0.5}$ are used, the weighted values of the market shares will be $S_i^1 \times S_i^{0.5} = S_i^{1.5}$, and so forth. The higher the power to which S_i is raised (α), the lower the weights and the corresponding indices would be.

To aid in interpreting the size of a concentration index, the maximum and minimum values that it can take must be known. The maximum value of all the HKIs, including the HHI, is 1 and occurs in the case of monopoly, as shown in Table 1. The minimum values occur when all the sellers in a market have equal market shares and their precise values depend on the number of sellers, N. Each seller's market share would equal 1/N. The Herfindahl-Hirschman index, for example, would equal $(1/N)^2 \times N = 1/N$. HKI(2.5) would equal $(1/N)^{2.5} \times N = (1/N)^{1.5}$. This is shown in Table 1, along with some examples, with N equal to 5, 10 and 100 respectively. The minimum values of HKI(1.5) equal 0.4472, 0.3162 and 0.1000 respectively, for N equal to 5, 10 and 100. This proves that the index declines as N increases; correctly indicating that seller concentration declines.

Following Adelman (1969), Hannah and Kay (1977:54) themselves prefer the so-called "numbers equivalent" of the indices. This is revisited in section 3.2.2.

3.1.4 The Horvath index

The Horvath index (HI) is a hybrid index in which the market share of the largest seller is not weighed (or weighed by 1), but those of the other sellers are squared, as in the Herfindahl-Hirschman index (HHI). In addition, the square of each market share (except S_1) is "reinforced" by the multiplier (2 - S_i) (Horvath 1970:446; Marfels 1975:490,500):

$$HI = S_1 + \sum_{i=2}^{N} S_i^2 (2 - S_i)$$
(3)

Horvath (1970: 446) originally wrote the multiplier " $(1 + [1 - S_i])$ " (though with a different symbol) and stated that the multiplier reflects "the proportional size of the rest of the industry". For all sellers except the largest one "the square of the fraction of the industry it does have times one plus the fraction of the industry which it does not have" is calculated.

The weight of the largest firm's market share is 1 and those of the other firms are $S_i(2 - S_i)$, as shown in Table 2. S_i obviously declines as firms get smaller, but $(2 - S_i)$ increases. In fact, since $S_i < 1(i = 2, ..., N), (2 - S_i) > S_i$. The second element of the weight does not "reinforce" the first, but more than counterbalances it. Since $(2 - S_i)$ includes the market share of the largest firm, the contribution of the largest firm to HI is reduced further than would be the case if it had merely not been weighed (or weighed by 1).

The maximum value of HI is one and occurs in the case of monopoly: $S_1 = 1$ and the rest of the formula falls away. The minimum value occurs when the sellers have equal market shares and is "a decimal fraction which is higher than the dominant firm's absolute percentage share" (Horvath 1970:448). The decimal fraction equals $[(3N^2 - 3N + 1)/N^3]$ (Marfels 1975:500). The minima are shown in Table 2 for N equal to 5, 10 and 100 respectively. They decline as N increases, correctly indicating the direction of change in seller concentration.

3.1.5 Rosenbluth index

It is not always easy to determine the weights used in other measures of concentration, for example, the Rosenbluth index (RI). This index is defined as follows (Marfels 1975: 500):

$$RI = 1 / \left[\left(2\sum_{i=1}^{N} iS_i \right) - 1 \right]$$
(4)

The sellers are ranked by market share from highest to lowest, as before. The term iS_i indicates that the market share of each seller is multiplied by the rank of the seller. Ignoring for the moment the rest of the formula, it means that the rank of the seller is the weight of its market share. The relevant calculations appear in Table 3, using the same numbers as in the top part of Tables 1 and 2. The market share of the largest seller is weighed by 1, that of the second seller by 2, etc, meaning that the weights increase as the market shares decline. This procedure differs from that utilised in the HHI and other HKIs, where the weights decline as the market shares decrease. Whereas the HHI and other HKIs emphasize the role of the larger sellers, the RI emphasizes that of the smaller sellers. But this is not where the matter ends.

Table 3 shows that the sum of the market shares weighed by rank is 2.2. To calculate the RI, this number is multiplied by 2, yielding 4.4. The answer is reduced by 1, yielding 3.4. Finally, the inverse of the last number is taken, meaning that the RI equals 0.2941 (rounded). It is no longer obvious what the final weights are. In particular, inverting the expression in square brackets means that the weights might now decline when the market shares increase, instead of increasing too.

The RI can be related to the graph of the concentration curve. The expression iS_i represents an area on the graph. If $i = 1, 1S_1$ equals the area OABC (= 0.35) in Diagram 2. Likewise, $2S_2$ equals the area CDEF (= 0.30), etc. The sum of these areas forms the area OABDEGHJKMNP. If the triangles OAB, BDE, EGH, HJK and KMN are removed from this area, the area above the concentration curve (OBEHKNP) remains. The area of each of these triangles equals $(1/2) \times \text{base} \times \text{height} = (1/2)(1)S_i$. Consequently, the area above the concentration curve can be written as:

$$\sum_{i=1}^{N} iS_i - \sum_{i=1}^{N} \left(\frac{1}{2}\right) S_i$$
 (5)

And simplified to:

$$\sum_{i=1}^{N} \left(iS_i - \frac{S_i}{2} \right) = \sum_{i=1}^{N} S_i \left(i - \frac{1}{2} \right)$$
(6)

The market shares are no longer weighed by the ranks of the sellers, but by the rank minus 1/2. If the weights are multiplied by 2, they become 2(i - 1/2) = (2i - 1). The relevant figures in the case of Diagram 2 are shown in Table 3, where

$$\sum_{i=1}^{N} S_i(2i-1) = 3.4 \tag{7}$$

The reciprocal of the expression on the left side of the equation above provides an alternative formula for the Rosenbluth index:

$$RI = 1/\sum_{i=1}^{N} S_i(2i-1)$$
(8)

In this case, RI equals 0.2941, as calculated before. While the weights of the market shares are (2i - 1) in 1/RI, it is not clear what they are once RI is calculated. If the inverse of the individual components of 1/RI are calculated and added up, their sum is not equal to 0.2941. In general, it is difficult to determine the actual weights of individual market shares when the sum of the weighted shares are manipulated after it has been calculated, for example when it is inverted or expressed as a ratio to something else.

The maximum value of RI occurs when N = 1:

$$RI = 1/1(2 \times 1 - 1) = 1/1 = 1 \tag{9}$$

The minimum value of RI occurs when the sellers are of equal size and equals 1/N, which is equal to the market share of anyone of the sellers. RI declines as N increases, correctly indicating that seller concentration declines.

3.1.6 Gini coefficient

The Gini coefficient (GC) is a relative concentration measure, as opposed to the HKIs, HI and RI, which are absolute concentration measures. Relative concentration measures indicate the inequality of firm sizes in a particular market and effectively ignore the number of firms present. Absolute concentration measures combine the number of firms and their size distribution in a single measure. Marfels (1971:754) points out that an absolute concentration measure is the weighted *sum* of the market shares, while a relative concentration measure is their weighted *average*.

GC was originally devised to measure the degree of income inequality in a human population and is associated with the Lorenz curve. To construct a Lorenz curve, members of the population are ranked according to their income from lowest to highest. The cumulative percentages of the population are plotted on the horizontal axis of a box diagram, while the corresponding percentages of cumulative income are plotted on the vertical axis. If the income were equally distributed among the population, the Lorenz curve would be the straight line that connects the points (0, 0) and (100, 100). The area between this diagonal line and the actual Lorenz curve provides a visual impression of the inequality of income among the population. GC is the ratio between this area and the triangular area under the diagonal line.

If the Lorenz curve were used to indicate the size distribution of sellers in a market, the sellers would be ranked by their market shares from smallest to largest and the cumulative percentage of sellers would be plotted on the horizontal axis. The corresponding cumulative percentage shares of the market would be plotted on the vertical axis. Instead of percentages, one could use ratios, as in the case of the concentration curves in Diagrams 1 and 2. Furthermore, the sellers may be ranked from largest to smallest, yielding a graph similar to a concentration curve instead of a Lorenz curve. The only difference would be that the cumulative proportions of the sellers would be plotted on the horizontal axis, instead of their ranks (i/N instead of i), as in Diagram 3. To emphasize the difference, such a curve may be called a "relative concentration curve". The total area of the box diagram equals 1, (instead of N as in Diagrams 1 and 2).

GC would now equal the area between the proportional concentration curve and the diagonal line, divided by the area of the triangle above the diagonal line. Since GC is the ratio of one area to another, the same answer would be derived from the graph of a concentration curve with the firms' ranks on the horizontal axis.

There is a relationship between GC and RI, since both make use of surface areas in box diagrams of concentration curves. While RI uses the area above the concentration curve, GC uses the area between the concentration curve and the diagonal line. The latter is the complement of the area above the concentration curve (relative to the area of the triangle above the diagonal line). As shown in section 3.1.5, when the ranks of the sellers are plotted on the horizontal axis, the area above the concentration curve equals

$$\sum_{i=1}^{N} S_i \left(i - \frac{1}{2} \right) \tag{10}$$

When ratios instead of ranks are used on the horizontal axis (i/N instead of i), the area above the concentration curve becomes:

$$\sum_{i=1}^{N} S_i \left(\frac{i}{N} - \frac{1}{2N} \right) = \sum_{i=1}^{N} \frac{S_i(2i-1)}{2N}$$
(11)

Furthermore, when ratios are used on both axes, the area of the box diagram equals one, and those of the triangles above and below the diagonal line equal 1/2 each. Therefore, the area between the concentration curve and the diagonal line can be written as

$$\frac{1}{2} - \sum_{i=1}^{N} \frac{S_i(2i-1)}{2N} \tag{12}$$

The formula for GC would then be:

$$GC = \left[\frac{1}{2} - \sum_{i=1}^{N} \frac{S_i(2i-1)}{2N}\right] / \frac{1}{2}$$
(13)

which can be simplified to:

$$GC = 1 - \sum_{i=1}^{N} S_i \frac{(2i-1)}{N}$$
(14)

While the area above the concentration curve can be viewed as the sum of weighted market shares with the weights equal to (2i - 1)/N, the area between the concentration curve and the diagonal line cannot be viewed as such. The diagonal line is a different concentration curve that is calculated by means of different market shares (namely equal ones). Furthermore, once the sum has been calculated, it is subjected to further manipulation, obscuring the actual weights.

As Marfels (1971:756,759) points out, GC can be transformed into RI and vice versa, if the number of sellers, N, is known: RI = 1/[N(1 - GC)] or $GC = 1 - 1/(N \times RI)$.

The minimum value of GC is zero and occurs when the sellers have equal market shares. The concentration curve would then coincide with the diagonal line and the area between the concentration curve and the diagonal line would be zero. A numerical example is shown in Table 4.

Some sources state that the maximum value of GC occurs when there is only one seller (e.g. Ferguson & Ferguson 1994:43, Fedderke & Szalontai 2009:242). However, this is not the case. When there is only one seller, the Gini coefficient equals zero. When there is only one firm, the relative concentration curve is the straight line from point (0, 0) to (1, 1) and coincides with the diagonal line. Therefore, the area between the curve and the diagonal line is zero. The formula of GC also equals zero: $GC = 1 - 1(2 \times 1 - 1)/1 = 1 - 1 = 0$. Furthermore, the formula for transforming RI into GC yields GC = 0 when N = 1 and RI = 1: $GC = 1 - 1/(N \times RI) = 1 - 1/(1 \times 1) = 1 - 1 = 0$.

This is reminiscent of an old joke – one person asks another: "What is the difference between a duck?" When the other person looks dumbfounded and says the question is incomplete, the questioner says something like: "Its beak is equally long!" When there is no one to share with, there is no inequality. The issue would not easily arise in the case of a human population because a human population would typically consist of numerous people. However, a single seller is possible in the case of a market.

Lipczynski et al (2009: 205) correctly state that the maximum possible value of GC "corresponds to the case of one dominant firm with a market share approaching one, and (N - 1) very small firms each with a negligible market share." In practice, a small firm would need some minimum market share to stay in business, say 0.05 (i.e. 5%). Table 4 shows three interesting possibilities. If there were two firms, one with a market share of 0.95 and the other with 0.05, GC would be 0.45. If there were five firms, one with a market share of 0.8 and four with market shares of 0.05 each, most economists would agree that there is lower concentration and more competition in this market than in the first. However, GC would be higher in the second market (0.60 instead of 0.45). signifying higher concentration and less competition. GC would be 0.45 in a market consisting of 10 firms, one with a market share of 0.55 and nine with market shares of 0.05 each, correctly indicating lower concentration than in the second market. However, concentration as measured by GC is the same as in the first market. It is clear that the Gini coefficient sends unsatisfactory signals about seller concentration and competitive conditions in various markets.

3.1.7Variance of natural logarithms of market shares

Another relative concentration measure is the variance of the natural logarithms of the market shares (VL). Lipczynski et al (2009:203) put forward the following formula for this measure:

$$VL = 1/N \sum_{i=1}^{N} [\ln S_i - 1/N \sum_{i=1}^{N} \ln S_i]^2$$
(15)

The formula can also be written as (Ferguson & Ferguson 1994: 42):

$$VL = 1/N \sum_{i=1}^{N} (\ln S_i)^2 - 1/N^2 (\sum_{i=1}^{N} \ln S_i)^2$$
(16)

where ln denotes the natural logarithm (log to the base e) and the other

symbols have the same meaning as before. The expression $1/N \sum_{i=1}^{N} \ln S_i$ is the arithmetic average of the natural logarithms of the market shares. It is subtracted from each market share and this difference is squared. The squared differences are added up and the sum is divided by N to calculate the arithmetic average of the squared differences.

Since the market shares are expressed as ratios to one, the natural logarithms of the market shares are negative (except for the case of monopoly where $S_1 = 1$ and $lnS_1 = 0$ and are therefore squared to yield positive numbers. Lipczynski et al (2009:203) point out that in statistics, a variance provides a standard measure of dispersion or inequality within any data set.

Once more, it is difficult to identify the precise weights of the market shares. The minimum VL = 0 and occurs when all the sellers have equal market shares. As in the case of GC, VL also equals 0 in the case of monopoly, while the maximum possible value of VL corresponds to the case of one dominant firm with a market share approaching one, and (N-1) very small firms each with a negligible market share. The same figures as in Table 4 are used to calculate various maxima of VL in Table 5. In this case, the maximum can exceed 1, which makes the measure difficult to interpret. However, the maximum of VL declines consistently as more small firms are added to the market and the market share of the dominant seller is reduced accordingly.

3.2Inverse measures of concentration

The so-called inverse measures of concentration decline as concentration increases and vice versa. Whereas the positive measures of concentration have easily identifiable maxima (with the exception of GC and VL), the inverse measures do not.

The inverse measures include the so-called "numbers equivalent" of some of the measures discussed above, namely the HKIs, including HHI. Other inverse measures include the occupancy count, the entropy coefficient and the relative entropy coefficient.

3.2.1 Occupancy count

The X% occupancy count (OCX%) is the smallest number of sellers that have a combined market share of X%. Leach (1992:390), for example, uses the 80% occupancy count. The lower OCX%, the higher the seller concentration would be. This inverse measure of concentration is a discrete measure, since it represents a specific point on the concentration curve. As in the case of CRX and CRX%, it may be difficult to determine the number of sellers that have a combined market share of exactly X%. For example, the largest three sellers might have a combined market share of 70%, while that of the largest four sellers might be 90%. OC80% would then be 4, but OC90% would also be 4. Meaningful comparisons of different industries by means of OCX% might not be possible.

3.2.2 Numbers equivalent of the Hannah and Kay indices

Suppose the value of an index has been determined for a particular market. Then its numbers equivalent is the number of equal-sized sellers for which the index would have the same value. The numbers equivalent of the HKIs can be written as follows (Hannah & Kay 1977: 55, Lipczynski et al 2009: 200):

$$n(\alpha) = \left[\sum_{i=1}^{N} S_i^{\alpha}\right]^{1/(1-\alpha)}$$
(17)

The top part of Table 1 is reproduced in Table 6. The totals of the weighted values are the relevant HKIs for the hypothetical five-seller market. When $\alpha = 2$, namely when HKI = HHI, the numbers equivalent is:

$$n(2) = \left[\sum_{i=1}^{N} S_i^2\right]^{1/(1-2)} = \left[\sum_{i=1}^{N} S_i^2\right]^{-1} = 1/\left[\sum_{i=1}^{N} S_i^2\right] = 1/HHI$$
(18)

This is the reciprocal of HHI. In the table HHI is 0.2650 and its reciprocal is 3.7736, meaning that a market with 3.7736 equal-sized sellers would also have an HHI of 0.2650. Sellers are counted in whole numbers, so it is rather awkward to refer to 3.7736 sellers. One might interpret it such that an industry consisting of four sellers with equal market shares would have roughly the same HHI as the hypothetical five-seller market. Each of the four sellers has a market share of 0.025, which has a squared value of 0.0625. The sum of the squared market shares = $0.0625 \times 4 = 0.2500$, the HHI of the four-seller market, and is fairly close to the HHI of the five-seller market (0.2650).

When $\alpha = 3$, the numbers equivalent is:

$$n(3) = \left[\sum_{i=1}^{N} S_i^3\right]^{1/(1-3)} = \left[\sum_{i=1}^{N} S_i^3\right]^{-1/2} = 1/\left[\sum_{i=1}^{N} S_i^3\right]^{1/2}$$
(19)

For the five-seller market in Table 6 this is 3.5578. The table also indicates n(1.5) and n(2.5).

The minimum value that any of the numbers-equivalent HKIs can take occurs in the case of a monopoly and equals 1 for all values of α , except $\alpha=1$, when $n(\alpha)$ is not defined. There is no upper bound, but given N sellers with equal market shares, the maximum value of any of the numbers-equivalent HKIs would simply be N, except when $\alpha=1$. A few examples are shown in the bottom part of Table 6.

3.2.3 Entropy coefficient and relative entropy coefficient

The entropy coefficient (EC) is a weighted-sum concentration measure. The weights are the natural logarithms of the reciprocals of the sellers' market shares and are inversely related to the market shares (Lipczynski et al 2009: 202):

$$EC = \sum_{i=1}^{N} S_i \ln\left(\frac{1}{S_i}\right) \tag{20}$$

The top part of Table 7 indicates EC for the hypothetical market shares used in Tables 1 to 6. The weights increase from 1.0498 to 2.9957 as the market shares decline from 0.35 to 0.05. EC equals 1.4306. To aid in interpreting this number, one needs to know the range of values EC can take. The minimum is zero and occurs in the case of monopoly. The maximum values occur when the market shares of the sellers are equal and these maximum values are equal to the respective weights, namely $\ln(1/S_i)$.

The relative entropy coefficient (REC) is defined as EC/lnN:

$$REC = \left(\frac{1}{\ln N}\right) \sum_{i=1}^{N} S_i \ln\left(\frac{1}{S_i}\right)$$
(21)

This can be rewritten as:

$$REC = \sum_{i=1}^{N} S_i \left[\ln \left(\frac{1}{S_i} \right) \right] \left(\frac{1}{\ln N} \right)$$
(22)

The latter expression indicates that REC is a weighted-sum concentration measure like EC, with weights of $[\ln(1/S_i)](1/\ln N)$. The top part of Table 8 indicates the calculation of REC for the hypothetical market shares used in Tables 1 to 7. The weights increase from 0.6523 to 1.8614 as the market shares decline from 0.35 to 0.05. REC equals 0.8889. REC is not defined in the case of monopoly. The maximum values occur when the market shares of the sellers are equal and these maximum values are equal to one, irrespective of the number of sellers. Since REC varies from zero to one, its value is easier to interpret than that of EC, the range of which does not have an absolute upper boundary. However, the fact that REC does not vary with the number of sellers when the sizes of the sellers are equal, is a major drawback. It renders REC a measure of inequality, as in the case of GC and VL.

4 EVALUATION OF MEASURES OF SELLER CONCENTRATION

Hannah and Kay (1977:48-50) put forward a number of reasonable "axioms" or criteria which a concentration measure should always meet, namely (slightly rephrased):

- the concentration curve ranking criterion: if firms are ranked from largest to smallest and plotted (on the horizontal axis) against their cumulative output (on the vertical axis) and the concentration curve of one market lies above that of another market at all points [except (0, 0) and, when N is equal in the two markets, (N, 1)], a positive concentration measure of the first market must be higher than that of the second market, while an inverse concentration measure must be lower
- the sales transfer criterion: measured concentration should increase if customers switch from smaller to larger firms and vice versa
- the entry criterion: if a new firm, smaller than the average size of existing firms, enters the market, measured concentration should decline (assuming that the relative market shares of the existing firms decline proportionately to accommodate the new firm)
- the merger criterion: measured concentration must increase if existing firms merge

The literature covering empirical tests of the SCP paradigm often make use of measures that do not meet the Hannah and Kay criteria, casting doubt on the validity of the tests.

In this section, 11 hypothetical markets are compared pairwise to weigh up the concentration measures defined in section 4 against the Hannah and Kay criteria. The market shares are chosen such that the concentration curves being compared do not cross. Thus the concentration curve ranking criterion is used in each case, along with one of the other criteria.

Two markets are compared in Diagram 4. Market 1 is a monopoly established by the producer of a new product. As patents run out, another seller enters and in time captures half the market. The new situation is called Market 2 and is compared with the original monopoly market. The cumulative market shares are shown below the diagram, as well as the values of the concentration measures. The first 11 measures (CR3 to VL) are positive measures and are expected to decline when new entry occurs. The last seven (OC80% to REC) are inverse measures, which are supposed to increase. The numbers that do not change in the expected direction are shaded. CR3, CR4 and CR5 do not decline when the new entrant arrives, because the number of sellers remains less than 3. GC and VL remain equal to zero, since the market shares of the sellers are equal in Market 2 and there is no firm to share with in Market 1, as explained before. REC is not defined for the case of monopoly (Market 1) and equals 1 in Market 2. The other concentration measures change in the expected direction.

Diagram 5 depicts a market with 4 equally sized sellers (Market 3) into which a fifth seller enters and in time gains an equal portion of the market from each incumbent (Market 4). CR5, GC, VL, OC80% and REC remain unchanged, while the other measures change as expected.

Diagram 6 depicts a market with 5 equally sized sellers (Market 4). The distribution of market shares gradually changes to one where the largest firm captures 35% of the market, while the least successful seller is left with 5% (Market 5). These are the figures used in the top parts of tables 1 to 8. In the jargon of Hannah and Kay, sales transfers have taken place. Most economists would agree that concentration has increased and all but one of the concentration measures move in the expected direction. The exception is CR5.

If the two smallest sellers in Market 5 merge, leading to the formation of Market 6 (Diagram 7), six concentration measures misbehave, namely CR3, CR5, GC, VL, OC80% and REC.

If the two largest sellers merge, resulting in Market 7, only one concentration measure misbehaves, namely CR5 (Diagram 8).

Diagram 9 depicts sales transfers from the third and fourth seller to the first and second sellers, while the market share of the smallest seller remains unaltered (Market 5 is transformed into Market 10). In this case, CR5 does not change in the expected direction, because there are only 5 sellers in the market. The rest of the measures change as expected.

Diagram 10 starts off with Market 2, a duopoly with equal-sized sellers. One of the sellers gradually captures 70% of the market, a situation depicted in Market 11. Since there are only two sellers, CR3, CR4 and CR5 are useless. The other discrete measure, OC80%, also remains constant. Perhaps surprisingly, one of the absolute concentration measures, HI, declines, instead of increasing. This is related to the inconsistent assigning of weights and the strange "reinforcement" of the weight of each market share (except S_1) by the multiplier $(2 - S_i)$. The weight of S_1 remains the same, while that of S_2 is reduced substantially.

Diagram 11 indicates the concentration curve of Market 5, with 5 sellers of unequal size, and that of Market 8, which results from the entry of 6 small firms, each gaining a market share of 1% at the expense of the incumbents. In this case all the concentration measures, except REC, correctly indicate that concentration has decreased.

Finally, Diagram 12 depicts the entry of a large seller into Market 5, which could for instance happen if an existing conglomerate decides to enter in a big way, with a sizeable factory, a well designed and packaged product and an extensive advertising campaign. It is assumed that the entrant quickly gains a market share of 32%, slightly above the share of 30% that the second-largest seller had. Whereas the largest seller loses 2% of its market share, the others shifts downwards in rank and collectively lose 30%. The entry of an additional firm lengthens the concentration curve, which signifies lower concentration. Since the sellers' ranking changes, their size-distribution does not change much. The

identity of the sellers does not play a role in the ranking by market share. The bottom part of the new concentration curve (Market 9) largely coincides with the old one, but the top part lies slightly below the old one. The concentration measures should therefore indicate a decline in concentration. Four of them do not, namely GC, VL, OC80% and REC.

The above analysis leads to the conclusion that the discrete concentration measures, as well as the relative ones, do not yield reliable results and should not be used when measuring seller concentration. In addition, HI is less reliable than the other absolute measures and should best be set aside by researchers in this field.

These findings are nothing new. Based on a slightly different analysis, Hannah and Kay (1977: 50-52) reached similar conclusions more than three decades ago. They found that the concentration ratios do "not necessarily react positively to a merger or sales transfer", but still expressed the opinion they stand up "reasonably well" to the criteria, because they "will never be perverse in the direction of change". However, they concluded that a number of measures of *inequality*, including GC and VL, violate some of the axioms and should not be used to measure seller *concentration*. They emphasised "that inequality and concentration are not the same thing" and "that trends in one do not necessarily shed light on trends in the other", adding the following interesting remark:

"It is tedious to labour what we hope the reader will find an obvious point. Our only justification for doing so is that the arguments above were laid out with great clarity and lucidity by Adelman (1951) all of twenty-five years ago, and that nevertheless economists have regularly continued to make unwarranted inferences about changes in concentration on the basis of measurements of inequality."

It is alarming that these measures are still being used in studies that find their way into peer-reviewed journals and presented in textbooks as measures of seller concentration.

This issue was discussed in the South African Journal of Economics (SAJE) in the early 1990s, starting with Leach's (1992) criticism of the work of Fourie and Smit (1989). Du Plessis (1978) had been the first to publish seller concentration measures in the SAJE, using data from the 1972 manufacturing census. His article was based on work done for the Mouton Commission (1977) and his DComm thesis at the University of Stellenbosch. Thereafter Fourie published a series of seller concentration measures in the SAJE, either on his own or with coauthors, aiming to establish whether or not seller concentration levels in South Africa were high and whether or not they increased from one manufacturing census to the next.

Leach (1992: 386-387) criticized Fourie and Smit (1989) for using the Gini coefficient (GC) to measure seller concentration and reworked the figures for the years 1972 to 1985, using the Rosenbluth index (RI) and the 80% occupancy count (OC80%). Whereas Fourie and Smit found that seller concentration in manufacturing tended to increase from 1972 to 1982, Leach found no positive trend when using OC80% and a negative trend when using RI.

In his 1996 article Fourie did not accept the obvious deficiencies of GC and other indices of inequality and criticised the use of RI on the grounds that it is not used widely in industrial economics and "gives inordinate weight to an increase in the size of the fringe of an industry" and argues that the dominance of the n-factor in RI is extraordinary, since the fringe may increase without affecting the dominance of two or three large firms (Fourie 1996: 101-103).

Hannah and Kay (1977: 50) also list the Hall-Tideman index, which is exactly the same as RI, as one of the "unsatisfactory concentration indices". According to them, this index does not meet either the entry or the merger criterion (the third and fourth axioms). The tests in this paper do not confirm this in respect of the direction of change in RI. It may be that Hannah and Kay thought it not sensitive enough to changes in the number of sellers (the opposite of Fourie's argument).

RI is nevertheless a vast improvement on GC, since it is a measure of concentration, not inequality. Probably the most important reason why many researchers have used GC is that it can be calculated from grouped data. The strength of RI is that it can be calculated from GC if the total number of sellers is known.

Fedderke and Szalontai (2009) use both GC and RI to indicate trends in concentration in South African three-digit manufacturing industries and to find correlations between concentration and certain measures of market performance, mainly because previous South African studies made use of GC and RI (Fedderke & Szalontai 2009: 242).

5 CONCLUSION

The discrete measures, namely concentration ratios (CRX) and the occupancy count (CRX%), do not always meet the Hannah and Kay criteria, nor do the relative concentration measures or measures of inequality, namely the Gini coefficient (GC), the variance of logarithms of market shares (VL) and the relative entropy coefficient (REC). An absolute concentration measure that does not always meet the criteria is the Horvath index. The other absolute concentration measures that have been tested, namely the Herfindahl-Hirschman index (HHI), the other Hannah and Kay indices [HKI(α)], the Rosenbluth index (RI), the numbers equivalent of the Hannah and Kay indices [HKIne(α)] and the entropy coefficient (EC) do meet the criteria.

Seeing that similar findings were made decades ago, it is alarming that measures such as GC, VL and REC are still being used in studies that find their way into peer-reviewed journals. It is even more alarming that measures of inequality are still being put forward in textbooks on industrial organization as legitimate measures of seller concentration. Instead of learning from the mistakes of past generations, new generations of scholars are being set up for failure. Given the use of measures of inequality in the past, they need to be mentioned in textbooks, but today's students should be forewarned against their use as measures of seller concentration. Another implication is that studies on seller concentration that were done in the past (e.g. those listed in Ferguson & Ferguson 1994:95-98) should be revisited and those that make use of GC, VL and REC should be disregarded, unless data is available to calculate RI from GC to reach a new conclusion in a particular case.

References

- Adelman, MA. 1951. The measurement of industrial concentration. The Review of Economics and Statistics 33(4), November: 269-296.
- [2] Adelman, MA. 1969. Comment on the "H" concentration measure as a numbers-equivalent. The Review of Economics and Statistics 51(1), February: 99-101.
- [3] Du Plessis, PG. 1978. Concentration of economic power in the South African manufacturing industry. South African Journal of Economics 46(3), September: 257–270.
- [4] Fedderke, J & Szalontai, G. 2009. Industry concentration in South African manufacturing industry: Trends and consequences, 1972-96. *Economic Modelling 26 (1): 241-250.*
- [5] Ferguson, PR & Ferguson, GJ. 1994. Industrial economics: issues and perspectives. London: Macmillan.
- [6] Fourie, FC van N & Smit, MR. 1989. Trends in economic concentration in South Africa. South African Journal of Economics 57(3), September: 241–256.
- Fourie, FC van N. 1996. Industrial concentration levels and trends in South Africa: completing the picture. South African Journal of Economics 64(1), March: 97–121.
- [8] Hannah, L & Kay, JA. 1977. Concentration in modern industry. London: Macmillan.
- [9] Horvath, J. 1970. Suggestion for a comprehensive measure of concentration. Southern Economic Journal 36(4), April: 446-452.
- [10] Lipczynski, J, Wilson, J & Goddard, J. 2009. Industrial organization: competition, strategy, policy. 3rd edition. Harlow, England: Prentice Hall.
- [11] Leach, DF. 1992. Absolute vs. relative concentration in manufacturing industry: 1972-1985. South African Journal of Economics 60(4), December: 386–399.
- [12] Marfels, C. 1971. Absolute and relative measures of concentration reconsidered. *Kyklos* 24(4): 753-766.

- [13] Marfels, C. 1975. A bird's eye view to measures of concentration. The Antitrust Bulletin 2(3), Fall: 485-503.
- [14] Mouton Commission. 1977. Report of the Commission of Inquiry into the Regulation of Monopolistic Conditions Act, 1955. RP 64/1977. Pretoria: Government Printer.
- [15] Reekie, WD. 2000. Monopoly and competition policy. 2nd edition. FMF Monograph No 24. Sandton: The Free Market Foundation.
- [16] Shepherd, WG. 1997. *The economics of industrial organization*. 4th edition. Upper Saddle River, NJ: Prentice-Hall.

Table 1: Herfi	ndahl-Hirsc	hman an	d Hannal	h and Kay	indices	of conce	ntration		
Seller	Market		Wei	ghts			Weighte	d values	
	share								
i	S _i	S _i ^{0.5}	S _i ¹	S _i ^{1.5}	S _i ²	S _i ^{1.5}	S _i ²	S _i ^{2.5}	S _i ³
1	0.3500	0.5916	0.3500	0.2071	0.1225	0.2071	0.1225	0.0725	0.0429
2	0.3000	0.5477	0.3000	0.1643	0.0900	0.1643	0.0900	0.0493	0.0270
3	0.2000	0.4472	0.2000	0.0894	0.0400	0.0894	0.0400	0.0179	0.0080
4	0.1000	0.3162	0.1000	0.0316	0.0100	0.0316	0.0100	0.0032	0.0010
5	0.0500	0.2236	0.0500	0.0112	0.0025	0.0112	0.0025	0.0006	0.0001
Total	1.0000	2.1264	1.0000	0.5036	0.2650	0.5036	0.2650	0.1434	0.0790
α						1.5	2	2.5	3
HKI(1.5)						0.5036			
HHI = HKI(2)							0.2650		
HKI(2.5)								0.1434	
HKI(3)									0.0790
Maximum							Ind	ices	
N=1	1.0000	1.0000	0.1000	1.0000	1.0000	1.0000	0.1000	1.0000	1.0000
Minimum						1/(N ^{0.5})	1/N	1/(N ^{1.5})	$1/(N^2)$
Equal sizes	Per seller								
N=5	0.2000	0.4472	0.2000	0.0894	0.0400	0.4472	0.2000	0.0894	0.0400
N=10	0.1000	0.3162	0.1000	0.0316	0.0100	0.3162	0.1000	0.0316	0.0100
N=100	0.0100	0.1000	0.0100	0.0010	0.0001	0.1000	0.0100	0.0010	0.0001

Table 2: Horvath			
Seller	Market	Weights	Weighted values
	share		
i	S _i	S _i (2-S _i), i=2 to 5	S _i ² (2-S _i), i=2 to 5
1	0.3500	1.0000	0.3500
2	0.3000	0.5100	0.1530
3	0.2000	0.3600	0.0720
4	0.1000	0.1900	0.0190
5	0.0500	0.0975	0.0049
Total	1.0000	2.1575	0.5989
Maximum			Index
N=1	1.0000	1.0000	1.0000
Minimum			[(3N ² -3N+1)/N ³]
Equal sizes	Per seller	i=2,, N	
N=5	0.2000	0.3600	0.4880
N=10	0.1000	0.1900	0.2710
N=100	0.0100	0.0199	0.0297

Table 3: Ros	ion				
Seller	Market	Wei	ghts	Weighted values	
	share				
i	S _i	i	2i -1	S _i i	S _i (2i-1)
1	0.3500	1	1.0000	0.3500	0.3500
2	0.3000	2	3.0000	0.6000	0.9000
3	0.2000	3	5.0000	0.6000	1.0000
4	0.1000	4	7.0000	0.4000	0.7000
5	0.0500	5	9.0000	0.2500	0.4500
Total	1.0000	15.0000	25.0000	2.2000	3.4000
RI					0.2941
Maximum					Index
N=1	1.0000				1.0000
Minimum					1/N
Equal sizes	Per seller				
N=5	0.2000				0.2000
N=10	0.1000				0.1000
N=100	0.0100				0.0100

Table 4: Gini coefficient			
Seller	Market	Weights	Weighted
	share		values
i	S _i	(2i-1)/N	S _i (2i-1)/N
1	0.3500	0.2000	0.0700
2	0.3000	0.6000	0.1800
3	0.2000	1.0000	0.2000
4	0.1000	1.4000	0.1400
5	0.0500	1.8000	0.0900
Total	1.0000	5.0000	0.6800
GC			0.3200
Maximum			GC
S ₁ =0.95, S ₂ =0.05			0.4500
S ₁ =0.8, S ₂ to S ₅ =0.05			0.6000
S ₁ =0.55, S ₂ to S ₁₀ =0.05			0.4500
Minimum			
Equal sizes	Per seller		
N=1	1.0000		0.0000
N=5	0.2000		0.0000
N=10	0.1000		0.0000
N=100	0.0100		0.0000

Table 5: Variance of natu				
Seller	Market	InS _i	1/N∑lnS _i	(InS _i - 1/N∑InS _i)²
	share			
i	S _i			
1	0.3500	-1.0498	-1.8323	0.6123
2	0.3000	-1.2040	-1.8323	0.3948
3	0.2000	-1.6094	-1.8323	0.0497
4	0.1000	-2.3026	-1.8323	0.2212
5	0.0500	-2.9957	-1.8323	1.3536
Total	1.0000	-9.1616	-9.1616	2.6315
VL				0.5263
Maximum				
S ₁ =0.95, S ₂ =0.05				2.1674
S ₁ =0.8, S ₂ to S ₅ =0.05				1.2300
S ₁ =0.55, S ₂ to S ₁₀ =0.05				0.5175
Minimum				
Equal sizes	Per seller			
N=1	1.0000	0.0000	0.0000	0.0000
N=5	0.2000	-1.6094	-1.6094	0.0000
N=10	0.1000	-2.3026	-2.3026	0.0000
N=100	0.0100	-4.6052	-4.6052	0.0000

Table 6: Number	Table 6: Numbers equivalent of Herfindahl-Hirschman and Hannah and Kay indices of concentration								
Seller	Market		Wei	ights			Weighte	d values	
	share								
i	S _i	S _i ^{0.5}	S _i ¹	S _i ^{1.5}	S _i ²	S _i ^{1.5}	S _i ²	S _i ^{2.5}	S _i ³
1	0.3500	0.5916	0.3500	0.2071	0.1225	0.2071	0.1225	0.0725	0.0429
2	0.3000	0.5477	0.3000	0.1643	0.0900	0.1643	0.0900	0.0493	0.0270
3	0.2000	0.4472	0.2000	0.0894	0.0400	0.0894	0.0400	0.0179	0.0080
4	0.1000	0.3162	0.1000	0.0316	0.0100	0.0316	0.0100	0.0032	0.0010
5	0.0500	0.2236	0.0500	0.0112	0.0025	0.0112	0.0025	0.0006	0.0001
Total	1.0000	2.1264	1.0000	0.5036	0.2650	0.5036	0.2650	0.1434	0.0790
α						1.5	2	2.5	3
							Numbers	equivalen	t
$n(\alpha) = [\Sigma s_i^{\alpha}]^{1/(1-\alpha)}$						3.9426	3.7736	3.6505	3.5578
Minimum									
N=1	1.0000	1.0000	1.0000	1.0000	1.0000	1.00	1.00	1.00	1.00
Maximum	Per seller					Ν	Ν	Ν	Ν
N=4	0.2500	0.5000	0.2500	0.1250	0.0625	4.00	4.00	4.00	4.00
N=5	0.2000	0.4472	0.2000	0.0894	0.0400	5.00	5.00	5.00	5.00
N=10	0.1000	0.3162	0.1000	0.0316	0.0100	10.00	10.00	10.00	10.00
N=100	0.0100	0.1000	0.0100	0.0010	0.0001	100.00	100.00	100.00	100.00

Table 7: Entropy			
Seller	Market	Weights	Weighted
	share		values
i	S _i	ln(1/S _i)	S _i ln(1/S _i)
1	0.3500	1.0498	0.3674
2	0.3000	1.2040	0.3612
3	0.2000	1.6094	0.3219
4	0.1000	2.3026	0.2303
5	0.0500	2.9957	0.1498
Total	1.0000	9.1616	1.4306
			EC
Minimum			
N=1	1.0000	0.0000	0.0000
Maximum	Per seller		ln(1/S _i)
N=5	0.2000	1.6094	1.6094
N=10	0.1000	2.3026	2.3026
N=100	0.0100	4.6052	4.6052

Table 8: Relativ	e entropy co	oefficient	
Seller	Market share	Weights	Weighted values
i	S _i	[ln(1/S _i)]/lnN	S _i [ln(1/S _i)]/lnN
1	0.3500	0.6523	0.2283
2	0.3000	0.7481	0.2244
3	0.2000	1.0000	0.2000
4	0.1000	1.4307	0.1431
5	0.0500	1.8614	0.0931
Total	1.0000	5.6924	0.8889
			REC
Minimum			
N=1	1.0000	Not def	Not def
Maximum	Per seller		[ln(1/S _i)]/lnN
N=5	0.2000	1.0000	1.0000
N=10	0.1000	1.0000	1.0000
N=100	0.0100	1.0000	1.0000









Rank of	Market shares			
seller	Mkt 1	Mkt 2		
1	1.00	0.50		
2		0.50		

Rank of	Cumulative market				
seller	shares				
	Mkt 1	Mkt 2			
0	0.00	0.00			
1	1.00	0.50			
2		1.00			

Measure	Mkt 1	Mkt 2	% change
CR3	1.000	1.000	0.0
CR4	1.000	1.000	0.0
CR5	1.000	1.000	0.0
HKI(1.5)	1.000	0.707	-29.3
HKI(2)=HHI	1.000	0.500	-50.0
HKI(2.5)	1.000	0.354	-64.6
НКІ(З)	1.000	0.250	-75.0
н	1.000	0.875	-12.5
RI	1.000	0.500	-50.0
GC	0.000	0.000	#DIV/0!
VL	0.000	0.000	#DIV/0!
OC80%	1.000	2.000	100.0
HKIne(1.5)	1.000	2.000	100.0
HKIne(2)=1/HHI	1.000	2.000	100.0
HKIne(2.5)	1.000	2.000	100.0
HKIne(3)	1.000	2.000	100.0
EC	0.000	0.693	#DIV/0!
REC	#DIV/0!	1.000	#DIV/0!



Rank of	Market shares		
seller	Mkt 3	Mkt 4	
1	0.25	0.20	
2	0.25	0.20	
3	0.25	0.20	
4	0.25	0.20	
5		0.20	

Rank of	Cumulative market		
seller	shares		
	Mkt 3 Mkt 4		
0	0.00	0.00	
1	0.25	0.20	
2	0.50	0.40	
3	0.75	0.60	
4	1.00	0.80	
5		1.00	

Measures	Mkt 3	Mkt 4	% change
CR3	0.750	0.600	-20.0
CR4	1.000	0.800	-20.0
CR5	1.000	1.000	0.0
HKI(1.5)	0.500	0.447	-10.6
HKI(2)=HHI	0.250	0.200	-20.0
HKI(2.5)	0.125	0.089	-28.4
НКІ(3)	0.063	0.040	-36.0
HI	0.578	0.488	-15.6
RI	0.250	0.200	-20.0
GC	0.000	0.000	#DIV/0!
VL	0.000	0.000	#DIV/0!
OC80%	4.000	4.000	0.0
HKIne(1.5)	4.000	5.000	25.0
HKIne(2)=1/HHI	4.000	5.000	25.0
HKIne(2.5)	4.000	5.000	25.0
HKIne(3)	4.000	5.000	25.0
EC	1.386	1.609	16.1
REC	1.000	1.000	0.0



Rank of	Market shares		
seller	Mkt 4	Mkt 5	
1	0.20	0.35	
2	0.20	0.30	
3	0.20	0.20	
4	0.20	0.10	
5	0.20	0.05	

Rank of	Cumulative market		
seller	shares		
	Mkt 4 Mkt 5		
0	0.00	0.00	
1	0.20	0.35	
2	0.40	0.65	
3	0.60	0.85	
4	0.80	0.95	
5	1.00	1.00	

Measures	Mkt 4	Mkt 5	% change
CR3	0.600	0.850	41.7
CR4	0.800	0.950	18.8
CR5	1.000	1.000	0.0
HKI(1.5)	0.447	0.504	12.6
HKI(2)=HHI	0.200	0.265	32.5
HKI(2.5)	0.089	0.143	60.3
HKI(3)	0.040	0.079	97.5
HI	0.488	0.599	22.7
RI	0.200	0.294	47.1
GC	0.000	0.320	#DIV/0!
VL	0.000	0.526	#DIV/0!
OC80%	4.000	3.000	-25.0
HKIne(1.5)	5.000	3.943	-21.1
HKIne(2)=1/HHI	5.000	3.774	-24.5
HKIne(2.5)	5.000	3.650	-27.0
HKIne(3)	5.000	3.558	-28.8
EC	1.609	1.431	-11.1
REC	1.000	0.889	-11.1



Rank of	Market shares		
seller	Mkt 5	Mkt 6	
1	0.35	0.35	
2	0.30	0.30	
3	0.20	0.20	
4	0.10	0.15	
5	0.05		

Rank of seller	Cumulative market shares		
	Mkt 5 Mkt 6		
0	0.00	0.00	
1	0.35	0.35	
2	0.65	0.65	
3	0.85	0.85	
4	0.95	1.00	
5	1.00		

Measures	Mkt 5	Mkt 6	% change
CR3	0.850	0.850	0.0
CR4	0.950	1.000	5.3
CR5	1.000	1.000	0.0
HKI(1.5)	0.504	0.519	3.0
HKI(2)=HHI	0.265	0.275	3.8
HKI(2.5)	0.143	0.148	3.5
HKI(3)	0.079	0.081	2.8
н	0.599	0.617	3.0
RI	0.294	0.303	3.0
GC	0.320	0.175	-45.3
VL	0.526	0.111	-78.8
OC80%	3.000	3.000	0.0
HKIne(1.5)	3.943	3.714	-5.8
HKIne(2)=1/HHI	3.774	3.636	-3.6
HKIne(2.5)	3.650	3.568	-2.3
HKIne(3)	3.558	3.508	-1.4
EC	1.431	1.335	-6.7
REC	0.889	0.963	8.3



Rank of	Market shares		
seller	Mkt 5	Mkt 7	
1	0.35	0.65	
2	0.30	0.20	
3	0.20	0.10	
4	0.10	0.05	
5	0.05		

Rank of seller	Cumulative market shares		
	Mkt 5 Mkt 7		
0	0.00	0.00	
1	0.35	0.65	
2	0.65	0.85	
3	0.85	0.95	
4	0.95	1.00	
5	1.00		

Measures	Mkt 5	Mkt 7	% change
CR3	0.850	0.950	11.8
CR4	0.950	1.000	5.3
CR5	1.000	1.000	0.0
HKI(1.5)	0.504	0.656	30.3
HKI(2)=HHI	0.265	0.475	79.2
HKI(2.5)	0.143	0.362	152.6
HKI(3)	0.079	0.284	259.2
н	0.599	0.746	24.5
RI	0.294	0.476	61.9
GC	0.320	0.475	48.4
VL	0.526	0.897	70.5
OC80%	3.000	2.000	-33.3
HKIne(1.5)	3.943	2.322	-41.1
HKIne(2)=1/HHI	3.774	2.105	-44.2
HKIne(2.5)	3.650	1.968	-46.1
HKIne(3)	3.558	1.877	-47.2
EC	1.431	0.982	-31.4
REC	0.889	0.708	-20.3



Rank of	Market shares		
seller	Mkt 5	Mkt 10	
1	0.35	0.40	
2	0.30	0.35	
3	0.20	0.12	
4	0.10	0.08	
5	0.05	0.05	

Rank of	Cumulative market		
seller	shares		
	Mkt 5 Mkt 10		
0	0.00	0.00	
1	0.35	0.40	
2	0.65	0.75	
3	0.85	0.87	
4	0.95	0.95	
5	1.00	1.00	

Measures	Mkt 5	Mkt 10	% change
CR3	0.850	0.870	2.4
CR4	0.950	0.950	0.0
CR5	1.000	1.000	0.0
HKI(1.5)	0.504	0.535	6.3
HKI(2)=HHI	0.265	0.306	15.4
HKI(2.5)	0.143	0.181	26.3
HKI(3)	0.079	0.109	38.3
н	0.599	0.646	7.9
RI	0.294	0.327	11.1
GC	0.320	0.388	21.3
VL	0.526	0.666	26.5
OC80%	3.000	3.000	0.0
HKIne(1.5)	3.943	3.488	-11.5
HKIne(2)=1/HHI	3.774	3.270	-13.3
HKIne(2.5)	3.650	3.125	-14.4
HKIne(3)	3.558	3.026	-15.0
EC	1.431	1.340	-6.3
REC	0.889	0.833	-6.3



Rank of	Market shares		
seller	Mkt 2 Mkt 11		
1	0.50	0.70	
2	0.50	0.30	

Rank of	Cumulative market		
seller	shares		
	Mkt 2	Mkt 11	
0	0.00	0.00	
1	0.50	0.70	
2	1.00	1.00	

Measures	Mkt 2	Mkt 11	% change
CR3	1.000	1.000	0.0
CR4	1.000	1.000	0.0
CR5	1.000	1.000	0.0
HKI(1.5)	0.707	0.750	6.1
HKI(2)=HHI	0.500	0.580	16.0
HKI(2.5)	0.354	0.459	29.9
HKI(3)	0.250	0.370	48.0
HI	0.875	0.853	-2.5
RI	0.500	0.625	25.0
GC	0.000	0.200	#DIV/0!
VL	0.000	0.179	#DIV/0!
OC80%	2.000	2.000	0.0
HKIne(1.5)	2.000	1.778	-11.1
HKIne(2)=1/HHI	2.000	1.724	-13.8
HKIne(2.5)	2.000	1.680	-16.0
HKIne(3)	2.000	1.644	-17.8
EC	0.693	0.611	-11.9
REC	1.000	0.881	-11.9



Rank of	Market shares		
seller	Mkt 5	Mkt 8	
1	0.35	0.34	
2	0.30	0.28	
3	0.20	0.17	
4	0.10	0.11	
5	0.05	0.04	
6		0.01	
7		0.01	
8		0.01	
9		0.01	
10		0.01	
11		0.01	

Rank of	Cumulative market		
seller	shares		
	Mkt 5	Mkt 8	
0	0.00	0.00	
1	0.35	0.34	
2	0.65	0.62	
3	0.85	0.79	
4	0.95	0.90	
5	1.00	0.94	
6		0.95	
7		0.96	
8		0.97	
9		0.98	
10		0.99	
11		1.00	

Measures	Mkt 5	Mkt 8	% change
CR3	0.850	0.790	-7.1
CR4	0.950	0.900	-5.3
CR5	1.000	0.940	-6.0
HKI(1.5)	0.504	0.467	-7.3
HKI(2)=HHI	0.265	0.237	-10.5
HKI(2.5)	0.143	0.125	-12.7
HKI(3)	0.079	0.068	-14.5
HI	0.599	0.555	-7.3
RI	0.294	0.243	-17.5
GC	0.320	0.625	95.5
VL	0.526	2.066	292.6
OC80%	3.000	4.000	33.3
HKIne(1.5)	3.943	4.585	16.3
HKIne(2)=1/HHI	3.774	4.216	11.7
HKIne(2.5)	3.650	3.996	9.5
HKIne(3)	3.558	3.847	8.1
EC	1.431	1.672	16.9
REC	0.889	0.697	-21.5



Rank of	Market shares		
seller	Mkt 5	Mkt 9	
1	0.35	0.33	
2	0.30	0.32	
3	0.20	0.16	
4	0.10	0.11	
5	0.05	0.05	
6		0.03	

Rank of	Cumulative market		
seller	shares		
	Mkt 5 Mkt 9		
0	0.00	0.00	
1	0.35	0.33	
2	0.65	0.65	
3	0.85	0.81	
4	0.95	0.92	
5	1.00	0.97	
6		1.00	

Measures	Mkt 5	Mkt 9	% change
CR3	0.850	0.810	-4.7
CR4	0.950	0.920	-3.2
CR5	1.000	0.970	-3.0
HKI(1.5)	0.504	0.487	-3.2
HKI(2)=HHI	0.265	0.252	-4.8
HKI(2.5)	0.143	0.135	-5.5
НКІ(3)	0.079	0.074	-6.0
ні	0.599	0.579	-3.4
RI	0.294	0.275	-6.6
GC	0.320	0.393	22.9
VL	0.526	0.794	50.8
OC80%	3.000	3.000	0.0
HKIne(1.5)	3.943	4.209	6.7
HKIne(2)=1/HHI	3.774	3.962	5.0
HKIne(2.5)	3.650	3.791	3.9
HKIne(3)	3.558	3.669	3.1
EC	1.431	1.521	6.4
REC	0.889	0.849	-4.5