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Aylit Tina Romm

ERSA working paper 323

January 2013

Economic Research Southern Africa (ERSA) is a research programme funded by the National Treasury of South Africa.

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Retirement Date Effects on Saving Behavior: The Case of Non-Separable Preferences*

Aylit Tina Romm[†]

August 2012

Abstract

In this paper we demonstrate that the magnitude of the reaction of saving behavior to a change in the anticipated retirement date is largely determined by the degree to which utility is additively separable in consumption and leisure. We show that the *relative* decrease in saving in response to a later anticipated retirement date is *larger* when preferences are non-separable in consumption and leisure, and the cross-derivative of the utility function is negative, than when preferences are separable. In particular, based on our simulations, the short term decrease in aggregate pre-retirement saving in response to a later anticipated retirement date may be up to 61.5% in the non-separable case as against 31% in the separable case. In the long-term, the decrease in pre-retirement saving would be as much as 28.5% in the non-separable case, as against 16.5% in the separable case.

JEL classification: D91; J26

Keywords: Non-separable preferences; retirement date; saving

*I would like to thank Professor Alexander Zimper for his input in writing this paper. I would also like to thank Professor Martin Wittenburg, as well as seminar participants at the University of the Witwatersrand for their useful comments.

[†]Senior Lecturer, School of Economics and Business Sciences, University of the Witwatersrand, Johannesburg. Email: aylit.romm@wits.ac.za

1 Introduction

The past decade has seen an increase in average retirement ages in many OECD countries. Resulting from a gradual increase in the Normal retirement age (NRA), in combination with other factors, these trends are expected to continue for some time into the future. An increase in the NRA has already been observed in countries such as Hungary, Italy, The United States, The Czech Republic and New Zealand. Some countries, such as Australia, Portugal and Switzerland, have raised only the female NRA, leaving the NRA for males unchanged. These trends are expected to gain momentum, with many countries set to experience even more significant changes in the future. Countries such as the UK, Austria, Denmark, Germany, South Korea, France, Greece and Spain have all started, or announced their intention to start increasing their NRA's starting from some specified date in the near future¹. An important policy issue relates to the implication of these later retirement dates for wealth accumulation over the life-cycle, and hence aggregate saving rates in the economy. Saving rates are important in that they influence the accumulation of capital, and hence growth in the economy. Standard life-cycle models of saving (Modigliani and Brumberg, 1954; Friedman, 1957) presume that individuals receive utility from consumption only. As a consequence, later retirement dates, and hence greater lifetime income, in these models will always lead to greater lifetime consumption of individuals, and for utility maximizers, greater consumption in every time period. The implication would be a reduction in savings at earlier stages of the life-cycle. Under the more realistic assumption, however, that an individual's utility is affected by consumption as well as leisure, the impact of a change in the retirement date is not as straightforward.

In this paper, we study the effects of changes in the retirement date on pre-retirement saving behavior under the assumption that utility is a function of both consumption and leisure. As our main contribution, we demonstrate that the magnitude of the reaction of saving behavior to a change in the retirement date is largely determined by the degree to which utility is additively separable in consumption and leisure.

Starting with Heckman (1974), many authors have suggested that preferences are non-separable in consumption and leisure. The testing of separability between consumption and leisure was first addressed by authors such as Jorgenson and Lau (1975), Ghez and Becker (1975), Abbot and Ashenfelter (1976, 1979), Blackorby et al. (1978), Barnett (1979, 1981), Atkinson et al. (1981), Deaton (1982), Browning et al. (1985), Murphy and Thom (1987), Browning

¹Starting from April 2010, the NRA for women in the UK started increasing from age 60, and is set to reach age 65 by 2018, at which point it will be in line with that of men. At that point both the male and female NRA will start increasing further, reaching age 68 by 2046. Starting from 2024, Austria plans to raise the NRA for women from 60 to 65 by 2033. By 2027 and 2029, Denmark and Germany, respectively, intend on increasing the retirement age for both men and women from the current age of 65 to 67. These changes are set to start in 2024 and 2012, for the two countries respectively. In 2013, South Korea's current NRA of 60 will rise to 61, thereafter increasing by one year every five years until it reaches age 65. The most recent announcements have come from France, Greece and Spain.

and Meghir (1989), Kaiser (1993), and more recently by authors such as Basu and Kimball (2002), Ham and Reilly (2002), French (2005), Laitner and Silverman (2005), Ziliak and Kniesner (2005) and Kiley (2007). All these studies conclude that preferences are non-separable in consumption and leisure, and in particular, that the marginal utility of consumption is negatively related to leisure. Given this empirical evidence, it is thus fitting that we analyze the effect of changing retirement dates on saving behavior in the case where preferences are non-separable in consumption and leisure.

We show that while younger individuals save less in response to a later anticipated retirement date, the *relative* decrease in saving is *larger* when preferences are non-separable in consumption and leisure, and the cross-derivative of the utility function is negative (marginal utility of consumption negatively related to leisure), than when preferences are separable. In particular, based on our simulations, the short term decrease in aggregate pre-retirement saving may be up to 61.5% in the non-separable case as against 31% in the separable case. In the long-term, the decrease in pre-retirement saving would be up to 28.5% in the non-separable case, as against 16.5% in the separable case. Key to our finding is that if preferences are non separable in consumption and leisure, and if the marginal utility of consumption is negatively related to leisure, the positive effect on consumption of an increase in lifetime resources induced by a later anticipated retirement date, is dampened by a negative effect on consumption caused by a decrease in the path of future leisure. More specifically, if preferences are non-separable in consumption and leisure, the effect of a change in the retirement date can no longer be viewed as the same as that caused by a change in future income due to any other reason. For a given date of retirement, an expected, say, capital gain or inheritance is not accompanied by a decrease in leisure. An increase in lifetime resources due to a later retirement date is, however, accompanied by a decrease in expected retirement leisure. We show that this nuance is not significant for preferences that are separable in consumption and leisure, where a change in the expected date of retirement will induce changes in consumption, and hence saving, analogous to the case where utility is considered a function of consumption alone. If, however, preferences are non-separable in consumption and leisure, our model shows that this nuance changes the analysis in a non-trivial manner.

While authors such as Heckman (1974), and more recently French (2005) and Hurd and Rohwedder (2005), have cited non-separable preferences as a possible explanation for the drop in consumption at retirement, our paper explicitly models the optimal consumption path under non-separable preferences, and then shows the effect of variation in the retirement date on *pre-retirement* consumption/saving behavior.

We proceed with the paper as follows. In section 2 we start off with a model in which utility is an additively separable function of consumption and leisure. We show the response of saving to a postponement in the anticipated retirement date. In section 3 we consider a model with non-separable preferences in consumption and leisure, and show how the response of agents' saving decisions differ from the separable case. In Section 4 we provide simulations to show

how saving responses differ under the two different preference structures. We conclude in section 5.

2 Separable Preferences in Consumption and Leisure

We consider a deterministic model in which we have a rational agent whose aim is to maximize lifetime utility. We assume that the agent lives till (and including) period T . Within this period he will spend a certain amount of time working full time and the rest of the time in retirement, during which time he will live off savings accumulated during his working years and social security (and/or private pension) income. We assume that in order to maintain his lifestyle post retirement, savings are necessary to supplement social security/pension income. Assuming that the agent does not face any liquidity constraints in that he is able to borrow against future income, we now proceed to analyze the effect of variation in the anticipated retirement date on pre-retirement consumption/saving decisions.

The agent's instantaneous utility at time t is given by $\hat{u} = [u(c_t) + v(l_t)]$, where $u(c_t)$ is the utility derived from consumption, and $v(l_t)$ is the utility derived from leisure. That is, we assume utility to be a separable function of consumption and leisure. We further assume time separability. We define leisure, l_t , to be 1 before retirement, and equal to $\bar{l} > 1$ every period after retirement, with $v'(l_t) > 0$, so that $v(l_t)$ is greater after retirement than before retirement.

For a given anticipated date of retirement, t_{ret} , (and hence a given $v(l_t)$ in every period), the agent's aim at time t is to maximize utility as follows:

$$\max_{(c_t \dots c_T)} \sum_{k=t}^T \beta^{k-t} (u(c_k)) \quad (1)$$

where β is the discount factor = $\frac{1}{1+\rho}$, where, ρ , is the rate of time preference. The dynamic budget constraint at any time t is given by:

$$\begin{aligned} x_t &= (x_{t-1} - c_{t-1}) \cdot R + y_t \\ &= a_t \cdot R + y_t \end{aligned} \quad (2)$$

where x_t is "cash on hand"; R is the fixed gross return on assets, a_t , and is equal to $(1+r)$, where r is the interest rate common to borrowing and lending; and y_t is non-capital income. We assume, further, that

$$y_t = \begin{cases} I_t & \text{if } t < t_{Ret} \\ i_t & \text{if } t \geq t_{Ret} \end{cases} \quad (3)$$

where I_t is labor income, and i_t is social security/pension income. We assume $I_t > i_t$.²

²This assumption is certainly valid in the context of most developed countries where the old

Human capital wealth, h_t , is the sum of discounted non-capital income and is given by

$$\begin{aligned} h_t &= \sum_{k=t+1}^T y_k \cdot R^{-(k-t)} = \frac{h_{t+1} + y_{t+1}}{R} \\ &= \sum_{k=t+1}^{t_{ret}-1} I \cdot R^{-(k-t)} + \sum_{k=t_{ret}}^T i \cdot R^{-(k-t)} \end{aligned} \quad (4)$$

Finally,

$$w_t = x_t + h_t \quad (5)$$

where w_t is total worth at time t , and evolves according to the following equation:

$$w_t = (w_{t-1} - c_{t-1}) \cdot R \quad (6)$$

We also have

$$\sum_{k=t}^T \frac{c_k}{R^{k-t}} = w_t \quad (7)$$

with terminal condition

$$w_{T+1} = 0 \quad (8)$$

That is, the present value of all future consumption must equal total worth, and further, the binding constraint in equation 7 and terminal condition given by equation 8 imply that an agent's total worth must be consumed by the time he dies³.

Observation 1: w_t is a strictly increasing function of t_{ret} .

In particular, the change in human capital as a result of increasing the retirement date from t_{ret}^1 to t_{ret}^2 is equal to

$$\left[\sum_{k=t_{ret}^1}^{t_{ret}^2-1} (I - i) \right] \cdot R^{-(k-t)} \quad (9)$$

Thus, delaying the date of retirement allows the agent to substitute labor income for social security income between t_{ret}^1 and t_{ret}^2 , increasing human capital wealth and hence total worth⁴.

The utility maximization problem at time period, t , conditioned on the budget constraint, can be written as the following standard dynamic programming problem:

age pension is earnings related, i.e., the old age pension replaces a *percentage* of pre-retirement income.

³For the purpose of this model, we abstract from the bequest motive.

⁴We assume, for simplicity, that i_t is independent of the retirement date.

$$J(a_t, I, i, t_{ret}) = \max_{(c_t \dots c_T)} \sum_{k=t}^T \beta^{k-t} (u(c_k)) \quad (10)$$

where $J(a_t, I, i, t_{ret})$ is the value function, which depends on assets, a_t , pre retirement income, I , post retirement social security/pension income, i , and the date of retirement, t_{ret} .

Proposition 1 *Assuming separable preferences in consumption and leisure, and that $u(c_k)$ is of the standard constant relative risk aversion (CRRA) form,*

$$u(c_k) = \frac{c_k^{1-\theta}}{1-\theta} \quad \text{with } \theta \neq 1 \quad (11)$$

for all time periods k , optimal consumption at time period t is given by

$$c_t = \left(\frac{R^{T-t}}{\sum_{j=0}^{T-t} (\beta R)^{\frac{j}{\theta}} \cdot R^{T-t-j}} \right) \cdot w_t \quad (12)$$

with the marginal propensity to consume out of total worth equal to $\left(\frac{R^{T-t}}{\sum_{j=0}^{T-t} (\beta R)^{\frac{j}{\theta}} \cdot R^{T-t-j}} \right)$

Proof. See appendix ■■

Now, taking the natural log of expression 12, we have

$$\ln c_t = \ln \left(\frac{R^{T-t}}{\sum_{j=0}^{T-t} (\beta R)^{\frac{j}{\theta}} \cdot R^{T-t-j}} \right) + \ln w_t \quad (13)$$

and,

$$\frac{\Delta \ln c_t}{\Delta t_{ret}} = \frac{\Delta \ln \left(\frac{R^{T-t}}{\sum_{j=0}^{T-t} (\beta R)^{\frac{j}{\theta}} \cdot R^{T-t-j}} \right)}{\Delta t_{ret}} + \frac{\Delta \ln w_t}{\Delta t_{ret}} \quad (14)$$

Since $\ln \left(\frac{R^{T-t}}{\sum_{j=0}^{T-t} (\beta R)^{\frac{j}{\theta}} \cdot R^{T-t-j}} \right)$ is constant with respect to t_{ret} , we have

Proposition 2

$$\frac{\Delta \ln c_t}{\Delta t_{ret}} = \frac{\Delta \ln w_t}{\Delta t_{ret}} \quad (15)$$

That is, when preferences are separable in consumption and leisure, the relative change in consumption at time t with respect to a unit change in the anticipated retirement date, is equal to the relative change in total worth at time t for a unit change in the anticipated retirement date.

Now saving at any point in time, t , is given by:

$$s_t = y_t - c_t \quad (16)$$

Taking the natural log of both sides

$$\ln s_t = \ln(y_t - c_t) \quad (17)$$

and using the law for the log of a summation/subtraction,

$$\ln s_t = \ln(y_t) + \ln(1 - e^{(\ln c_t - \ln y_t)}) \quad (18)$$

Proposition 3 As the change in t_{ret} gets very small,

$$\frac{\Delta \ln s_t}{\Delta t_{ret}} \approx \left(-\frac{1}{(1 - e^{(\ln c_t - \ln y_t)})} \cdot e^{(\ln c_t - \ln y_t)} \cdot \frac{\Delta \ln c_t}{\Delta t_{ret}} \right)^5 \quad (19)$$

where, $\frac{\Delta \ln s_t}{\Delta t_{ret}}$ shows the relative change in saving at time t , for a unit change in the retirement date.

Since $\frac{\Delta \ln c_t}{\Delta t_{ret}} > 0$, and $(\ln c_t - \ln y_t) < 0$ (implying $0 < e^{(\ln c_t - \ln y_t)} < 1$), we have corollary 1

Corollary 1 $\frac{\Delta \ln s_t}{\Delta t_{ret}} < 0$

Thus, an increase in the anticipated retirement date will result in the agent saving less in that, and every subsequent period, thereby accumulating less asset wealth over time⁶.

⁵Note that this expression is the derivative of the expression $\ln y + \ln(1 - e^{(\ln c - \ln y)})$ with respect to t_{ret} ($\ln y$ is independent of t_{ret}). $\frac{\Delta \ln s_t}{\Delta t_{ret}}$ approximates this expression as Δt_{ret} gets very small and tends to the continuous time situation, where the derivative expression is appropriate.

⁶It should be clear from equation 2 that $a_t = a_{t-1} \cdot R_{t-1} + s_{t-1}$.

3 Non-separable preferences in consumption and leisure

So far we have restricted our utility function to being additively separable in consumption and leisure. We now relax this assumption, and assume instead that preferences are non-separable in consumption and leisure. Assume the same budget and leisure constraints as in the separable case, except now instantaneous utility at time t is given by the following cobb douglas isoelastic utility function:⁷

$$\begin{aligned}
 u(c, l) &= \frac{(c_t^\eta (l_t)^{1-\eta})^{1-\theta}}{1-\theta} \\
 &= \begin{cases} \frac{(c_t^\eta)^{1-\theta}}{1-\theta} & \text{for } t < t_{ret} \\ \frac{(c_t^\eta (l_t)^{1-\eta})^{1-\theta}}{1-\theta} & \text{for } t \geq t_{ret} \end{cases} \quad (20)
 \end{aligned}$$

where $0 < \eta < 1$ represents the share of consumption in utility (of course $1 - \eta$ represents the share of leisure in utility), and $\theta \neq 1$ ⁸ will influence whether the cross-derivative of the utility function is positive or negative, with $\frac{1}{\theta}$ being the intratemporal elasticity of substitution between consumption and leisure.

Now,

$$\begin{aligned}
 u_c &= \eta (c_t^\eta (l_t)^{1-\eta})^{-\theta} c_t^{\eta-1} (l_t)^{1-\eta} \\
 &= \eta c_t^{\eta(1-\theta)-1} (l_t)^{(1-\eta)(1-\theta)} \quad (21)
 \end{aligned}$$

and

$$u_{cl} = (1-\theta)(1-\eta)c_t^{\eta(1-\theta)-1} l_t^{(1-\eta)(1-\theta)-1} \quad (22)$$

If $\theta > 1$ ($\frac{1}{\theta} < 1$), then $u_{cl} < 0$, i.e. the marginal utility of consumption decreases as leisure increases. Since marginal utility of consumption is lower at times when leisure is high, consumption will also be lower. Conversely, if $\theta < 1$, then $u_{cl} > 0$ ⁹.

Most empirical estimates suggest that $\theta > 1$. Ghez and Becker (1975) report a value of $\frac{1}{\theta} = 0.83$ ($\theta = 1.20$). Auerbach and Kotlikoff (1987) report values of $\frac{1}{\theta}$ between 0.3 and 1.5 (θ between 0.8 and 3.33), but select a value of 0.8 ($\theta = 1.25$) as their base value in simulations. Attanasio and Weber (1995) report an estimate of $\theta = 2.2$, while Barsky et al. (1997) estimate that most people have a value of θ greater than 2, and many have a value greater than

⁷Examples of authors who have used such a utility function are: French (2005), Hurd and Rohwedder (2003), Low (2005), and Laitner and Silverman (2005).

⁸If $\theta = 1$, then the function reduces to a log utility function which is additively separable in consumption and leisure.

⁹Authors such as Hurd and Rohwedder (2003), Low (2005), and Laitner and Silverman (2005) refer to consumption and leisure being *Frisch* substitutes if $\theta > 1$. They refer to Frisch complements if $\theta < 1$.

4. Altig et al. (2001) select a parameter value for $\frac{1}{\theta}$ of 0.8 ($\theta = 1.25$) for their simulations, while Diamond and Zodrow (2007, 2008) use a value of 0.6 ($\theta = 1.67$) in their benchmark simulation. French's (2005) estimates imply an intratemporal elasticity of substitution between 0.18 and 0.45 (θ between 2.2 and 5.6), and Ziliak and Kniesner (2005) report values ranging from 0.09 to 0.23 (θ between 4.34 and 11.11). We thus proceed with our model, concentrating on the case where $\theta > 1$.

For a given retirement date, and hence a given value of l_t in every period, our maximization problem at time t is given by:

$$J(a_t, I, i, t_{ret}) = \max_{(c_t \dots c_T)} U = \sum_{k=t}^T \frac{(c_k^\eta (l_k)^{1-\eta})^{1-\theta}}{1-\theta} \quad (23)$$

Proposition 4 *In the case where preferences are non-separable in consumption and leisure, and the utility function is of the isoelastic form given above, optimal consumption at time t is given by*

$$c_t = \left(\frac{R^{T-t}}{\sum_{j=0}^{T-t} \left[R^{T-t-j} \cdot (\beta R)^{-\frac{j}{\eta(1-\theta)-1}} \cdot \left(\frac{l_t}{l_{t+j}}\right)^{\frac{(1-\eta)(1-\theta)}{\eta(1-\theta)-1}} \right]} \right) \cdot w_t \quad (24)$$

With the marginal propensity to consume out of total worth (mpc) equal to

$$\left(\frac{R^{T-t}}{\sum_{j=0}^{T-t} \left[R^{T-t-j} \cdot (\beta R)^{-\frac{j}{\eta(1-\theta)-1}} \cdot \left(\frac{l_t}{l_{t+j}}\right)^{\frac{(1-\eta)(1-\theta)}{\eta(1-\theta)-1}} \right]} \right)$$

Proof

See Appendix . ■

Observation 2: *The mpc in the non-separable case is a function of the retirement date.*

Observation 3: *For $\theta > 1$, the marginal propensity to consume out of total worth is higher when the agent is working, than when the agent is retired.*

Proof

See Appendix . ■

Observation 4: *An agent working in time period t will experience a higher marginal propensity to consume out of total wealth in the non-separable case where $\theta > 1$, than he would in the separable case.*

Proof

This should be obvious simply by comparing the mpc in the separable and non-separable cases. ■

Observation 5: *An agent who is working in time period t , whose marginal utility of consumption is negatively related to leisure, and who anticipates a*

postponement in his retirement date, will experience a decrease in the marginal propensity to consume out of total worth at time t .

Proof

See appendix . ■

Now, taking the natural log of expression 24 we have

$$\ln c_t = \ln \left(\frac{R^{T-t}}{\sum_{j=0}^{T-t} \left[R^{T-t-j} \cdot (\beta R)^{-\frac{j}{\eta(1-\theta)-1}} \cdot \left(\frac{l_t}{l_{t+j}} \right)^{\frac{(1-\eta)(1-\theta)}{\eta(1-\theta)-1}} \right]} \right) + \ln w_t \quad (25)$$

and hence

Proposition 5

$$\frac{\Delta \ln c_t}{\Delta t_{ret}} = \frac{\Delta \ln w_t}{\Delta t_{ret}} + \frac{\Delta \ln mpc}{\Delta t_{ret}} \quad (26)$$

where $\frac{\Delta \ln w_t}{\Delta t_{ret}} > 0$, and $\frac{\Delta \ln mpc}{\Delta t_{ret}} < 0$

Thus, when preferences are non-separable in consumption and leisure, the effect of later retirement dates on consumption is twofold. The positive effect on consumption caused by an increase in total worth, is dampened by a second negative effect on consumption caused by a decrease in the path of future leisure, and hence the marginal propensity to consume out of total wealth. Thus, the relative increase in consumption is smaller than in the separable case.

Corollary 2 *The effect of later retirement dates on consumption approaches the separable case as $\theta \rightarrow 1$. This is since the magnitude of the second effect diminishes under such conditions.*

Proof

See appendix . ■

Corollary 3 *The effect of later retirement dates on consumption deviates to a larger extent from the separable case as \bar{l} gets larger. This is since the magnitude of the second effect increases under such conditions.*

The proof of this corollary is very simple and simply results from the fact that $\left| \frac{\Delta \ln mpc}{\Delta t_{ret}} \right|$ is larger if \bar{l} is larger. Note that for this effect to dominate the wealth effect, would require an unrealistically high value of \bar{l} , and it is thus unreasonable to expect that an increase in the retirement date would ever lead to a decrease in consumption.

Proposition 6 *The relative decrease in saving of an agent at time t , in response to a later anticipated retirement date, is greater in magnitude in the non-separable case, where the cross-derivative of the utility function is negative ($\theta > 1$), than in the separable case.*

Proof. see Appendix ■

Corollary 4 *The effect of later retirement dates on saving approaches the separable case as $\theta \rightarrow 1$.*

Corollary 5 *The effect of later retirement dates on saving deviates to a larger extent from the separable case as \bar{l} gets larger.*

4 Simulations

In this section we define parameters for the models described above in order to quantitatively simulate the effect of later anticipated retirement dates on the saving behavior of the working population. We look at the effect of a gradual increase in the retirement age from age 65 to 67. We assume that at time t_0 , the government announces that starting at time period t_4 , the normal retirement age will start increasing from age 65, rising in six month intervals each year, from age 65, reaching age 67 at time t_7 . Table 1 shows the retirement age faced by various segments of the working age population at t_0 . We define the working population, according to the OECD definition, as that aged 15 to 64 (i.e. the working population under the assumption of the initial retirement age of 65). We look, first, at the immediate short-term effect, that is, the effect on saving behavior at the time of the announcement, i.e., t_0 . We then look at the long-term effect, that is the effect on saving behavior at t_{49} — the time at which the entire working population has faced a retirement age of 67 from the start of their working lives (thus, taking into account even those 64 year olds that started working at 15).

Table 2 shows the parameters we use in our calculations. We normalize income, with $I = 1000$ and $i = 600$, so that the pension income replacement rate is 0.6, consistent with the OECD average. We assume that assets at age 15, a_{15} , are zero. We let $T = 79.5$, since this is the average life expectancy in the OECD. Note that for the sake of simplicity we abstract from the interest rate and rate of time preference and set $\rho = r = 0 \Rightarrow \beta = R = 1$. This should not be an issue for the purpose of our analysis. η is set equal to 0.6 and \bar{l} equal to 2, implying leisure time doubles after retirement.

At each age between 15 and 64, we calculate the relative change in saving for an individual of such age given the change in retirement date that he faces at t_0 ¹⁰. We then aggregate by weighting the relative change in saving of each

¹⁰For the purposes of this paper, we assume that our representative individual of each age expects to retire at the normal retirement age.

age group by the proportion the particular age group constitutes of the entire working population between 15 and 64 (looking at an OECD average (OECD 2011)). Figure 1 shows the relative changes in saving at t_0 (short term effect) in both the separable and non-separable cases for varying values of θ ¹¹. Figure 2 shows the scenario at t_{49} (long-term effect). It is clear that the immediate, or short term effect, is far more dramatic than the long term effect. At the time of the announcement of a later retirement age, older individuals abruptly decrease their savings, realizing they have saved too much up to this point under the impression they were retiring at age 65. In the long term, however, individuals in their sixties, say, have retirement date expectations, consistent with those they had in their late teens, or early twenties¹². Thus, the decrease in saving is a gradual process spread over the course of the life-cycle.

In addition, we note that in both the short and long term, the relative decrease in saving is greater in the non-separable case than in the separable case. The effect in the non-separable case approaches that of the separable case as $\theta \rightarrow 1$. In particular, in the short-term (at t_0), aggregate pre-retirement saving will decrease by as much as 61.5% in the non-separable case as against 31% in the separable case. In the long term (at t_{49}), aggregate pre-retirement saving will decrease by as much as 28.5% in the non-separable case, as against 16.5% in the separable case.

¹¹We demonstrate the effect for values of θ up till 5, since values beyond this point would be considered rather extreme.

¹²Obviously, within the realms of rational Bayesian learning.

age at t_0	retirement age
62-64	65
61	65.5
60	66
59	66.5
15-58	67

Table 1: Retirement Age faced by the Various Segments of the Working Aged Population at t_0

I	1000
i	600
a_{15}	0
T	79.5
\bar{l}	2
η	0.6
$\rho = r$	0

Table 2: Parameters

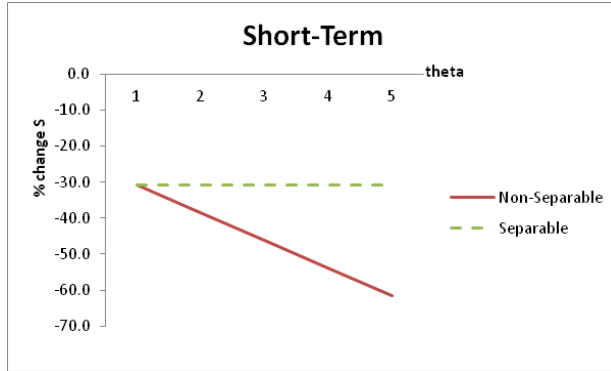


Figure 1: Short-Term Effect

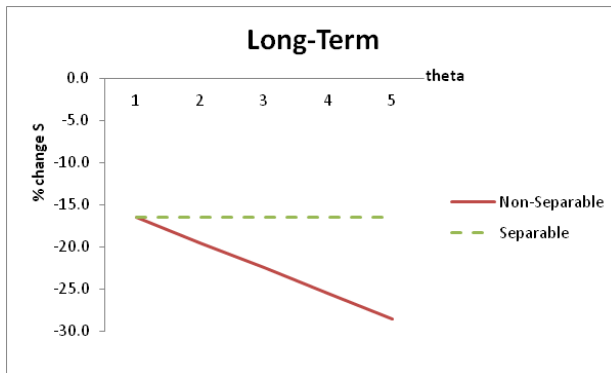


Figure 2: Long Term Effect

5 Conclusion

A postponement in the date of retirement will result in the working population saving less prior to the initial anticipated retirement date. Further, the relative decrease in saving is larger in the case where preferences are non-separable in consumption and leisure—and the marginal utility of consumption is negatively related to leisure, than in the case where preferences are separable.

In light of this theoretical outcome, the upward trend in retirement dates that many OECD countries have experienced of late, and that is expected to persist for some time into the future, is likely to have had, and continue to have an adverse effect on the saving behavior of the young. More so, in light of evidence supporting the view that the marginal utility of consumption is negatively related to leisure, this adverse effect is worse than would be the case if utility is seen as a function of consumption only, or if utility were separable in consumption and leisure. Policies in OECD countries promoting later retirement ages are for good reason. The burden on the social security system of the baby boomers entering retirement, as well as increasing life expectancy, are amongst the most important of these. Cognizance, however, needs to be taken of the unintended adverse effect on saving behavior.

In concluding this paper we take note of the following caveats. Firstly, we need to acknowledge that we are analyzing the saving *behavior* of the *initial* working population (aged 15-64 in our context). Aggregate saving in an economy is determined by the aggregation of the saving of the young and the dissaving of the old. In addition to the behavioral effects of individual saving behavior addressed in this paper, there is a compositional element at the aggregate level which is induced by a change in retirement dates (c.f. Romm and Wolny, 2012). That is, with later retirement dates there is an increase in the percentage of the working population relative to the non-working population. This compositional effect implies that there is also a greater percentage of savers. Thus, while the aim of this paper is to study how later retirement dates affect the saving behavior of younger individuals, we need to be aware that at the aggregate level there is a positive compositional effect in addition to this negative behavioral effect. However, to the extent that the separability of preferences influences the magnitude of the behavioral effect, this is the focus of our paper.

Secondly, it is important to realize that in analyzing the effect of an exogenous change in the retirement date, we are analyzing the partial effect on consumption/saving behavior of a change in the retirement date. In reality, it is likely that some of the factors influencing later retirement dates are endogenous to the consumption/saving decision, and that there are multiple effects at play. We do not, in this paper, attempt to analyze the general equilibrium relationship between retirement dates and savings in the economy. The point of this paper is merely to analyze one effect—the partial effect of later retirement dates on pre-retirement saving behavior—under varying preference structures. While this effect is one of many at play in the complex relationship between retirement dates and savings behavior, it is none the less very relevant to the overall dynamics.

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Appendix

Proof of Proposition 1

The first order conditions pertaining to consumption for the above maximization problem, conditioned on the budget constraint result in the following:

$$u'(c_t) = \beta R u'(c_{t+1}) = \dots \beta^{T-t} R^{T-t} u'(c_T) \quad (27)$$

\Rightarrow

$$\frac{u'(c_t)}{u'(c_{t+1})} = \beta R \quad (28)$$

Let us assume that the utility function is of standard constant relative risk aversion (CRRA) form,

$$u(c_k) = \frac{c_k^{1-\theta}}{1-\theta} \text{ with } \theta \neq 1 \quad (29)$$

for any time period k . θ reflects the curvature/concavity of the utility function with a higher value of θ reflecting a more concave utility function. $\frac{1}{\theta}$ reflects the intertemporal elasticity of substitution.

Now,

$$u'(c_k) = c_k^{-\theta} \quad (30)$$

and from equation 28

\Rightarrow

$$\left(\frac{c_t}{c_{t+1}} \right)^{-\theta} = \beta R \quad (31)$$

We now use recursive methods, as illustrated by Stockey et al. (1989).

From condition 8, we know that all worth should be exhausted by the end of time T . Thus

$$c_T = w_T \quad (32)$$

In general, we can write:

$$c_T = m_T \cdot w_T \quad (33)$$

where $m_T = 1$ is the marginal propensity to consume out of total worth in period T .

Now, by equation 31 we have $c_{T-1} = (\beta R)^{-\frac{1}{\theta}} c_T$, and by equation 6

$$(\beta R)^{\frac{1}{\theta}} \cdot c_{T-1} = (w_{T-1} - c_{T-1}) \cdot R \quad (34)$$

\Rightarrow

$$c_{T-1} = \left(\frac{R}{(\beta R)^{\frac{1}{\theta}} + R} \right) \cdot w_{T-1}$$

i.e.

$$m_{T-1} = \left(\frac{R}{(\beta R)^{\frac{1}{\theta}} + R} \right) \cdot m_T$$

and by continuing recursively, we have in general

$$c_{T-g} = \left(\frac{R^g}{\sum_{j=0}^g (\beta R)^{\frac{j}{\theta}} R^{g-j}} \right) \cdot w_{T-g} \quad (35)$$

and

$$m_{T-g} = \left(\frac{R^g}{\sum_{j=0}^g (\beta R)^{\frac{j}{\theta}} R^{g-j}} \right) \cdot m_T \quad (36)$$

Thus, at time period t

$$c_t = \left(\frac{R^{T-t}}{\sum_{j=0}^{T-t} (\beta R)^{\frac{j}{\theta}} \cdot R^{T-t-j}} \right) \cdot w_t$$

Proof of Proposition 4

Maximization gives rise to the same first order condition as in the separable case —

$$u'(c_t) = \beta R u'(c_{t+1})$$

From equation 21, we have

$$c_t = (\beta R)^{\frac{1}{\eta(1-\theta)-1}} \cdot \left(\frac{l_{t+1}}{l_t} \right)^{\frac{(1-\eta)(1-\theta)}{\eta(1-\theta)-1}} \cdot c_{t+1} \quad (37)$$

which for $t < t_{ret} - 1$, and $t \geq t_{ret} \Rightarrow$

$$c_t = (\beta R)^{\frac{1}{\eta(1-\theta)-1}} \cdot c_{t+1} \quad (38)$$

and between $t_{ret} - 1$ and t_{ret}

$$c_{t_{ret}-1} = (\beta R)^{\frac{1}{\eta(1-\theta)-1}} \cdot (\bar{l})^{\frac{(1-\eta)(1-\theta)}{\eta(1-\theta)-1}} \cdot c_{t_{ret}} \quad (39)$$

Again we have

$$c_T = w_T$$

which now implies by equations 6 and 38

$$c_{T-1} \cdot \left[(\beta R)^{-\frac{1}{\eta(1-\theta)-1}} \left(\frac{l_{T-1}}{l_T} \right)^{\frac{(1-\eta)(1-\theta)}{\eta(1-\theta)-1}} \right] = (w_{T-1} - c_{T-1}) \cdot R \quad (40)$$

i.e.

$$c_{T-1} = \left(\frac{R}{R + (\beta R)^{-\frac{1}{\eta(1-\theta)-1}} \left(\frac{l_{T-1}}{l_T} \right)^{\frac{(1-\eta)(1-\theta)}{\eta(1-\theta)-1}}} \right) \cdot w_{T-1} \quad (41)$$

and in general

$$c_{T-g} = \left(\frac{R^g}{\sum_{j=0}^g \left[R^{g-j} (\beta R)^{-\frac{j}{\eta(1-\theta)-1}} \left(\frac{l_{T-g}}{l_{T-g+j}} \right)^{\frac{(1-\eta)(1-\theta)}{\eta(1-\theta)-1}} \right]} \right) \cdot w_{T-g} \quad (42)$$

so that at time t

$$c_t = \left(\frac{R^{T-t}}{\sum_{j=0}^{T-t} \left[R^{T-t-j} \cdot (\beta R)^{-\frac{j}{\eta(1-\theta)-1}} \cdot \left(\frac{l_t}{l_{t+j}} \right)^{\frac{(1-\eta)(1-\theta)}{\eta(1-\theta)-1}} \right]} \right) \cdot w_t \quad (43)$$

Proof of Observation 3:

Suppose the agent is retired in period $t + j$ for some $j \neq 0$.

Then $\frac{l_t}{l_{t+j}} = \begin{cases} 1 & \text{if } t \geq t_{ret} \text{ (agent is retired in } t) \\ \frac{1}{\bar{l}} < 1 & \text{if } t < t_{ret} \text{ (agent still working in } t) \end{cases}$

Now for $\theta > 1$,

$$\frac{\Delta mpc}{\Delta \frac{l_t}{l_{t+j}}} < 0 \quad (44)$$

Thus, if the agent is retired in period t , the mpc will be smaller than if the agent is working in period t .

Proof of Observation 5:

Suppose that the agent is *not* retired in period t . Then the mpc can be expressed as

$$\left(\frac{R^{T-t}}{\sum_{j=0}^{t_{ret}-1} \left[R^{T-t-j} \cdot (\beta R)^{-\frac{j}{\eta(1-\theta)-1}} \right] + \sum_{j=t_{ret}}^{T-t} \left[R^{T-t-j} \cdot (\beta R)^{-\frac{j}{\eta(1-\theta)-1}} \cdot \left(\frac{1}{\bar{l}}\right)^{\frac{(1-\eta)(1-\theta)}{\eta(1-\theta)-1}} \right]} \right) \quad (45)$$

since

$$\frac{l_t}{l_{t+j}} = \begin{cases} 1 & \text{if } t+j < t_{ret} \text{ (agent still working in } t+j) \\ \frac{1}{\bar{l}} < 1 & \text{if } t+j \geq t_{ret} \text{ (agent retired in } t+j) \end{cases}$$

Now, for $\theta > 1$,

$$\left[R^{T-t-j} \cdot (\beta R)^{-\frac{j}{\eta(1-\theta)-1}} \right] > \left[R^{T-t-j} \cdot (\beta R)^{-\frac{j}{\eta(1-\theta)-1}} \cdot \left(\frac{1}{\bar{l}}\right)^{\frac{(1-\eta)(1-\theta)}{\eta(1-\theta)-1}} \right]$$

Therefore,

$$\frac{\Delta \left(\sum_{j=0}^{t_{ret}-1} \left[R^{T-t-j} \cdot (\beta R)^{-\frac{j}{\eta(1-\theta)-1}} \right] + \sum_{j=t_{ret}}^{T-t} \left[R^{T-t-j} \cdot (\beta R)^{-\frac{j}{\eta(1-\theta)-1}} \cdot \left(\frac{1}{\bar{l}}\right)^{\frac{(1-\eta)(1-\theta)}{\eta(1-\theta)-1}} \right] \right)}{\Delta t_{ret}} > 0$$

and hence

$$\frac{\Delta mpc}{\Delta t_{ret}} < 0 \quad (46)$$

Proof of Corollary 2:

If $\theta \rightarrow 1$, the term $\left(\frac{l_t}{l_{t+j}}\right)^{\frac{(1-\eta)(1-\theta)}{\eta(1-\theta)-1}} \rightarrow 1$.

The mpc then becomes independent of $\left(\frac{l_t}{l_{t+j}}\right)^{\frac{(1-\eta)(1-\theta)}{\eta(1-\theta)-1}}$, and hence the retirement date. Therefore, the effect of a change in the retirement date on consumption tends to the separable case, i.e.,

$$\frac{\Delta \ln c_t}{\Delta t_{ret}} \rightarrow \frac{\Delta \ln w_t}{\Delta t_{ret}} \quad (47)$$

Proof of proposition 6

We know

$$\frac{\Delta \ln s_t}{\Delta t_{ret}} \approx \left(-\frac{1}{(1 - e^{(\ln c_t - \ln y_t)})} \cdot e^{(\ln c_t - \ln y_t)} \cdot \frac{\Delta \ln c_t}{\Delta t_{ret}} \right) \quad (48)$$

We also know that 1) $\frac{\Delta \ln c_t}{\Delta t_{ret}}$ is smaller in the non-separable case than it is in the separable case (from Proposition 5) and 2) $\ln c_t$ is larger in the non-separable case than it is in the separable case (see Observation 4), so that $\frac{1}{(1 - e^{(\ln c_t - \ln y_t)})} \cdot e^{(\ln c_t - \ln y_t)}$ is larger in the non-separable case than it is in the separable case.

Which effect is dominant? It is easy to show that the relative *increase* in $\frac{1}{(1 - e^{(\ln c_t - \ln y_t)})} \cdot e^{(\ln c_t - \ln y_t)}$ when moving from the separable to non-separable case, is greater than the relative *decrease* in $\frac{\Delta \ln c_t}{\Delta t_{ret}}$ when moving from the separable to non-separable cases. Thus, $\frac{\Delta \ln s_t}{\Delta t_{ret}}$ is greater in absolute value in the non-separable case than in the separable case.