A Small-Scale Estimated DSGE Model for Forecasting the South African Economy

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Motivation

Forecasting

- Always being “?”
- But demanded!
Motivation

DSGE Forecasting

DSGE models developed in the early 80’s

- In response to critique of Keynesian macro theory
  - Lack of micro foundations
  - Ad hoc treatment of expectations
  - Lucas critique

- “The essence of this new approach is methodologically and reflects a view of how macroeconomics should be done.”

- “Lucas (1972) paved the way for a new macroeconomics based on dynamic stochastic model economics with fully articulated preferences, technologies and rules of the game – hence the name: DSGE modeling.”
Motivation

DSGE Forecasting

Empirical macro models

- Smets and Wouters (ECB), Nel Negro and Schorfheide (FRB-Atlanta)
- Nel Negro, Schorfheide, Smets, and Wouters (FRB-Atlanta + ECB)
- Serious program move from calibrated models to estimated ones
- Using Bayesian estimation techniques
Objective

What We Do in This Study

- Develop DSGE models for forecasting SA economy
- Evaluate forecasting performance:
  - DSGE vs. VARs
- Practical importance:
  - Answer the controversy about empirical research: “Econometricians” vs. “Business Cycle Theorists”
The Model Economy

The Baseline Model

Hansen’s indivisible labor model

- Individuals either work full-time or not at all
- Other features – same as standard RBC model
The Model Economy

The Environment

- The model economy populated by infinitely-lived households
- No money in the model
- Movements of aggregates explained by productivity shock only
The Model Economy

The Preference

- Preferences of households are identical
- Households MAX the expected utility over life time

\[
U(C_t, H_t) = E_t \sum_{t=0}^{\infty} \beta^t (\ln C_t - \gamma H_t), \quad 0 < \beta < 1 \quad \gamma > 0
\]

(1)
The Model Economy

The Technology

Defined as Cobb-Douglas production function

\[ Y_t = Z_t K_t^\rho (\eta H_t)^{1-\rho}, \quad 0 < \rho < 1 \quad \eta > 1 \quad (2) \]

where

\[ K_{t+1} = (1 - \delta)K_t + l_t, \quad 0 < \delta < 1 \quad (3) \]
\[ \log Z_t = (1 - \psi)\log Z + \psi \log Z_{t-1} + \epsilon_t, \quad \epsilon_t \sim i.i.d.(0, \sigma^2) \quad (4) \]
The Model Economy

In equilibrium, household MAXs his or her utility function (1) subject to the aggregate constraints:

\[ Y_t = C_t + I_t \]
\[ Y_t = Z_t K_{t-1}^\rho (\eta_t^t N_t)^{1-\rho} \]
\[ K_{t+1} = (1 - \delta) K_t + I_t \]
The Hybrid Model: A DSGE-VAR Approach

Why Hybrid Model?

- Calibrated model too stylized to be taken directly to the data
- Overcome the singularity problem of RBC models
  - The one-shock assumption makes RBC models stochastically singular
- Improve the forecasting accuracy
  - DSGE models are perceived to do badly in terms of forecasting
The Hybrid Model: A DSGE-VAR Approach

Approaches of Constructing the Hybrid Model

1. Including as many shocks as the number of endogenous variables in the model

2. Bayesian estimation

3. Combination of (1) and (2)

4. DSGE-VAR: Augmenting the linearized solution of the model with unobservable errors that have a VAR representation
The Hybrid Model: A DSGE-VAR Approach

DSGE-VAR Model

- Augmenting a serially correlated residual

\[
\hat{\pi}_t = A\hat{x}_t + \mu_t 
\]

where \( \hat{\pi}_t = [\hat{y}_t \hat{c}_t \hat{i}_t \hat{h}_t]' \); \( \hat{x}_t = [\hat{k}_t \hat{z}_t]' \)

- and

\[
\hat{x}_t = B\hat{x}_{t-1} + C\epsilon_t 
\]
\[
\mu_t = D\mu_{t-1} + \xi_t \quad \xi_t \sim i.i.d.(0, \sigma_\xi^2) 
\]

- Covariance matrix of the innovation in (7), \( E\xi_t\xi_t' = V \), is uncorrelated with the innovation to technology, \( \epsilon_t \)
Estimation

The Data (1970Q1-2000Q4)

- Output (GNP)
- Consumption
- Investment
- Hours worked

Estimation

The Hybrid model consisting of (5), (6), and (7) is in state-space form and can be estimated via maximum likelihood
A Few Words on Bayesian Prior

- Based on the knowledge that more recent values of a variable more likely contain useful information about its future movements than older ones
- Instead of eliminating longer lags, Bayesian technique imposes restrictions on coefficients
- Restrictions imposed by specifying normal prior distributions with zero means and small standard deviations for all coefficients with standard deviations decreasing as lag increases
  - One exception — the coefficient on the first own lag of a variable has a mean of unity
Estimation

Standard Prior

Standard deviation of prior distribution for lag $m$ of variable $j$ in equation $i$ for all $i$, $j$ and $m$ — $S(i, j, m)$

$$S(i, j, m) = [w \times g(m) \times f(i, j)] \frac{\hat{\sigma}_i}{\hat{\sigma}_j}$$

$$f(i, j) = \begin{cases} 
1 & \text{if } i=j, \\
k_{ij} & \text{otherwise, } 0 \leq k_{ij} \leq 1,
\end{cases}$$

$$g(m) = m^{-d}, \ d > 0$$

▷ where

$\hat{\sigma}_i$: Standard Error of Univariate Autoregression for variable $i$;

$w$: overall tightness (also standard deviation on first own lag);

$g(m)$: tightness of lag $m$ relative to lag 1;

$d$: decay factor with $g(m)$ having a harmonic shape;

$f(i, j)$: tightness of variable $j$ in equation $i$ relative to variable $i$
## Forecast Accuracy (2001Q1-2005Q4)

### DSGE vs. VARs

<table>
<thead>
<tr>
<th>Variable</th>
<th>VAR</th>
<th>DSGE</th>
<th>BVARs (w=0.3,d=0.5)</th>
<th>BVARs (w=0.2,d=1)</th>
<th>BVARs (w=0.2,d=2)</th>
<th>BVARs (w=0.1,d=1)</th>
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</thead>
<tbody>
<tr>
<td>GNP</td>
<td>0.003</td>
<td>7.197</td>
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<td></td>
<td>0.00317</td>
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<tr>
<td>CON</td>
<td>0.012</td>
<td>5.291</td>
<td></td>
<td>0.01153</td>
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<tr>
<td>INV</td>
<td>0.051</td>
<td>29.78</td>
<td></td>
<td>0.05120</td>
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<td>EMP</td>
<td>0.043</td>
<td>35.35</td>
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<td></td>
<td>0.02730</td>
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</tr>
</tbody>
</table>

AMAPE: mean absolute percentage error; Variables are in logs.
### Forecast Accuracy: 2001Q1-2005Q4

#### DSGE-VAR vs. VARs

<table>
<thead>
<tr>
<th>QA</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>DSGE-VAR</td>
<td>1.123</td>
<td>1.495</td>
<td>1.431</td>
<td>1.310</td>
</tr>
<tr>
<td>VAR (1)</td>
<td>1.657</td>
<td>2.858</td>
<td>3.814</td>
<td>4.748</td>
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<tr>
<td>VAR (2)</td>
<td>2.036</td>
<td>3.017</td>
<td>3.697</td>
<td>4.260</td>
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</tr>
</thead>
<tbody>
<tr>
<td>DSGE-VAR</td>
<td>1.138</td>
<td>1.489</td>
<td>1.614</td>
<td>1.607</td>
</tr>
<tr>
<td>VAR (1)</td>
<td>1.388</td>
<td>2.179</td>
<td>2.793</td>
<td>3.330</td>
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<tr>
<td>VAR (2)</td>
<td>1.550</td>
<td>2.275</td>
<td>2.790</td>
<td>3.046</td>
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</table>
## Forecast Accuracy (2001Q1-2005Q4)  
**DSGE-VAR vs. VARs**

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</tr>
</thead>
<tbody>
<tr>
<td>DSGE-VAR</td>
<td>4.489</td>
<td>5.982</td>
<td>6.626</td>
<td>7.212</td>
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<tr>
<td>VAR (1)</td>
<td>3.740</td>
<td>6.066</td>
<td>8.378</td>
<td>10.626</td>
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<tr>
<td>VAR (2)</td>
<td>4.510</td>
<td>6.422</td>
<td>7.739</td>
<td>9.410</td>
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</table>

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<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>DSGE-VAR</td>
<td>2.555</td>
<td>3.401</td>
<td>4.176</td>
<td>4.671</td>
</tr>
<tr>
<td>VAR (1)</td>
<td>2.425</td>
<td>3.022</td>
<td>3.478</td>
<td>3.633</td>
</tr>
<tr>
<td>VAR (2)</td>
<td>2.635</td>
<td>3.274</td>
<td>3.730</td>
<td>3.617</td>
</tr>
</tbody>
</table>
Future Research

- NKM
- Bayesian estimation
- SOE
THANK YOU