Global and National Macroeconometric Modelling: A Long-run Structural Approach Applications

Yongcheol Shin
Leeds University Business School
Seminars at University of Cape Town
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1 A Long-run Structural Model of the UK

- We estimate a VECM of the form:

\[
\Delta y_t = a_y + \alpha_y b_0 - \alpha_y \beta' [z_{t-1} - \gamma (t - 1)] + \sum_{i=1}^{p-1} \Gamma_{yi} \Delta z_{t-i} + \psi_{yo} \Delta p^o_t + (1)
\]

- \( z_t = (p^o_t, y'_t)' \), where \( y_t = (e_t, r^*_t, r_t, \Delta \tilde{p}_t, y_t, p_t - p^*_t, h_t - y_t, y^*_t)' \).

1.1 The Different Stages of Estimation and Testing

(1) A sequence of unrestricted VAR(\( p \)), \( p = 0, 1, 2, \ldots, 6 \) are estimated over 1965q1-1999q4. The order of VAR model is then selected in the light of the Akaike Information criterion (AIC) and the Schwarz Bayesian criterion (SBC).

(2) Cointegration tests are carried out using the trace and the maximum eigenvalue statistics.

(3) We estimate an exactly-identified set of long-run relations, in which \( r^2 \) restrictions are imposed on the cointegrating vectors.

(4) We consider the imposition and testing of over-identifying restrictions on the cointegrating vectors, as predicted by economic theory.
(5) The *fifth* step concerns the interpretation of the results.

### 1.2 Unit Root Properties of the Core Variables

- The results of the Augmented Dickey-Fuller (ADF) and Phillips and Perron (PP) tests are reported in Tables 9.1a and 9.1b.
- Both sets of tests provide relatively strong support for the view that $y_t, y_t^*, r_t, r_t^*, e_t, (h_t - y_t)$ and $p_t^0$ are $I(1)$ series.
- There is, however, some ambiguity regarding the order of integration of the price variables.
  - These raise interesting issues concerning the use of economic theory and statistical evidence in macroeconometric modelling.
    * The Fisher equation requires that inflation and interest rates have the same order of integration. The theoretical literature generally assumes that these series are $I(0)$, but the empirical evidence is mixed. Our approach to this dilemma is a pragmatic one and we treat $r_t, r_t^*, \Delta p_t, \Delta \widetilde{p}_t$ and $\Delta p_t^*$ as $I(1)$ variables.
  - Domestic and foreign prices appear in their *level* in the PPP relationship and this raises the potential difficulty of mixing $I(1)$ and $I(2)$ variables.
    * Following Haldrup (1998) we work with the relative price variables $p_t - p_t^*$, which is unambiguously $I(1)$ according to the ADF statistics.
The decision to include domestic prices in two forms, \((p_t - p^*_t)\) and \(\Delta \tilde{p}_t\), does not create difficulties of inconsistency. Further, there is considerable evidence that the various alternative measures of inflation pairwise cointegrated with a cointegrating vector of \((1, -1)\) and a zero constant. The use of two measures of prices, \(p_t\) and \(\tilde{p}_t\), has no impact on the long-run properties of the model but is likely to capture the short-run dynamics more accurately.

1.3 Testing and Estimating of the Long Run Relations

- Selecting the order of the underlying VAR using AIC and SBC in Table 9.2.
  - A VAR of order two appears to be appropriate.

- Using a VAR(2) model with unrestricted intercepts and restricted trend coefficients, and treating the oil price variable, \(p^o_t\), as a weakly exogenous \(I(1)\) variable, we computed Johansen’s ‘trace’ and ‘maximal eigenvalue’ statistics in Table 9.3.
  - The maximal eigenvalue statistic indicates the presence of just two cointegrating relationships at the 5% significance level. Less reliable.
  - The trace statistics reject the null hypotheses that \(r = 0, 1, 2, 3\) and 4 at the 5 per cent level of significance but cannot reject the null of hypothesis that \(r = 5\). This is in line with our \textit{a priori} expectations based on the long-run theory, which suggests the existence of five possible long-run
relations:

\[ p_t - p^*_t - e_t = b_{10} + b_{11}t + \xi_{1,t+1}, \]  
\[ r_t - r^*_t = b_{20} + \xi_{2,t+1}, \]  
\[ y_t - y^*_t = b_{30} + \xi_{3,t+1}, \]  
\[ h_t - y_t = b_{40} + b_{41}t + \beta_{44}r_t + \beta_{46}y_t + \xi_{4,t+1}, \]  
\[ r_t - \Delta p_t = b_{50} + \xi_{5,t+1}, \]  

- These can be written more compactly as

\[ \xi_t = \beta'_{TH} z_{t-1} - b_0 - b_1(t-1), \]

where

\[ b_0 = (b_{10}, b_{20}, b_{30}, b_{40}, b_{50})', \quad b_1 = (b_{11}, 0, 0, b_{41}, 0)', \]

\[ \xi_t = (\xi_{1t}, \xi_{2t}, \xi_{3t}, \xi_{4t}, \xi_{5t})', \]

and

\[ \beta'_{TH} = \begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & -\beta_{44} & 0 & -\beta_{46} & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \end{pmatrix}. \]

The matrix \( \beta'_{TH} \) imposes all the restrictions necessary to correspond to the long-run relationships and is over-identified.

- We impose 25 exact-identifying restrictions on the cointegrating matrix (in the form of five restrictions on each of the five
cointegrating vectors) so that

\[
\mathbf{\beta}'_{EX} = \begin{pmatrix}
\beta_{11} & \beta_{12} & 0 & 0 & \beta_{15} & 0 & 1 & \beta_{18} & 0 \\
\beta_{21} & 0 & \beta_{23} & 1 & \beta_{25} & 0 & 0 & 0 & \beta_{29} \\
\beta_{31} & 0 & 0 & 0 & 0 & 1 & \beta_{37} & \beta_{38} & \beta_{39} \\
\beta_{41} & 0 & 0 & -\beta_{44} & \beta_{45} & -\beta_{46} & 0 & 1 & 0 \\
\beta_{51} & 0 & 0 & \beta_{54} & -1 & 0 & 0 & \beta_{58} & \beta_{59}
\end{pmatrix} . \tag{9}
\]

- The first vector (the first row of \(\mathbf{\beta}'_{EX}\)) relates to the purchasing power parity (\(PPP\)) relationship defined by (2) and is normalised on \(p_t - p_t^*\);
- the second relates to the interest rate parity (\(IRP\)) relationship defined by (3) and is normalised on \(r_t\);
- the third relates to the “output gap” (\(OG\)) relationship defined by (4) and is normalised on \(y_t\);
- the fourth is the money market equilibrium condition (\(MME\)) defined by (5) and is normalised on \(h_t - y_t\);
- the fifth is the real interest rate relationship (\(FIP\)) defined by (6), normalised on \(\Delta \tilde{p}_t\).

- We then tested the over-identifying restrictions predicted by the long-run theory.
  - There are twenty unrestricted parameters in (9) and there are eighteen theory-based over-identifying restrictions, except for two of the parameters of the money demand equation (\(\beta_{44}\) and \(\beta_{46}\)).
  - There are potentially five further parameters on the trend
terms. There is no economic rationale for including time trends in the IRP, FIP or OG relationships, and the imposition of zeros on the trend coefficients in these relationships provides a further three over-identifying restrictions. The absence of a trend in the PPP relationship is also consistent with the theory, as is the restriction that $\beta_{46} = 0$ is effectively a relationship explaining the velocity of circulation of money.

– Once the long-run theory is fully imposed, there are just two parameters to be freely estimated and there are a total of twenty-three over-identifying restrictions.

1.3.1 Small Sample Properties of the Tests of Restrictions on the Cointegrating Vectors

• We apply the bootstrapping method to accommodate the small sample properties of the log-likelihood ratio (LR) test of over-identifying restrictions, generating a simulated distribution for the test statistic.

• We adopt a bootstrap procedure based on 3000 replications of the LR statistic testing the twenty-three restrictions.

– For each replication, an artificial data set is generated on the assumption that the estimated version of the core model is the true data-generating process, using the observed initial values of each variable, the estimated model, and a set of random innovations.

* These innovations can be obtained by re-sampling with
replacement from the estimated residuals in the light of the evidence of non-normality of residuals

- For each simulated dataset, the cointegrating VAR is estimated subject to the exact-identifying restrictions of (9) and then subject to the over-identifying restrictions of (8).
- The LR test of the over-identifying restrictions is carried out on each of the replicated data sets and the empirical distribution of the test statistic is derived across all replications.

- Figure 9.1 shows the empirical distribution of the test statistic lies substantially to the right of its asymptotic counterpart.
- The LR test for jointly testing the twenty-three over-identifying restrictions takes the value 71.49. The bootstrapped critical values for the joint tests of the twenty-three over-identifying restrictions are 67.51 at the 10 per cent significance level and 73.19 at the 5 per cent level. Using these bootstrapped critical values, the 23 theory restrictions cannot be rejected at the conventional 5 per cent level.

1.4 The Vector Error Correction Model

1.4.1 The Long Run Estimates
Table 9.4. The long-run relations are summarised below:

\[
(p_t - p^*_t) - e_t = 4.588 + \hat{\xi}_{1,t+1} \tag{10}
\]

\[
r_t - r^*_t = 0.0058 + \hat{\xi}_{2,t+1} \tag{11}
\]

\[
y_t - y^*_t = -0.0377 + \hat{\xi}_{3,t+1} \tag{12}
\]

\[
h_t - y_t = 0.5295 - \frac{56.0975}{(22.2844)} r_t - \frac{0.0073}{(0.0012)} t + \hat{\xi}_{4,t+1} \tag{13}
\]

\[
r_t - \Delta \tilde{p}_t = 0.0036 + \hat{\xi}_{5,t+1} \tag{14}
\]

– The first equation, (10), describes the PPP relationship. The finding here that PPP can be readily incorporated into the model is a useful contribution, indicating that the empirical evidence is stronger in a more complete model of the macroeconomy incorporating feedbacks and interactions.

– The second cointegrating relation, defined by (11), is the IRP condition with an intercept interpreted as the deterministic component of the risk premia of approximately 2.3 per cent per annum. The empirical support is compatible with UIP. However, under the UIP hypothesis it is also required that a regression of \( r_t - r^*_t \) on \( \Delta \ln(E_{t+1}) \) has a unit coefficient, but this is not supported by the data.

– The third long-run relationship, given by (12), is the OG relationship with per capita domestic and foreign output (measured by the total OECD output) levels. The co-
trending hypothesis cannot be rejected, suggesting that average long-run growth rate for the UK is the same as that in the rest of the OECD.

- For the $MME$ condition, given by (13), we could not reject the hypothesis that the elasticity of real money balances with respect to real output is equal to unity: an M0 velocity equation. The $MME$ condition, however, contains a deterministic downward trend, representing the steady decline in the money-income ratio experienced in the UK over most of the period 1965-1999. There is also strong statistical evidence of a negative interest rate effect on real money balances.

- Fifth equation, (14), defines the $FIP$ relationship, where the estimated constant implies an annual real rate of return of approximately 1.67 per cent. Our results highlight the important role played by the $FIP$ relationship in a model of the macroeconomy which can incorporate interactions between variables omitted from more partial analyses.

### 1.4.2 Error Correction Specifications

- The short-run dynamics of the model in Table 9.4.
- The estimates of the error correction coefficients show that the long-run relations make an important contribution in most equations and that the error correction terms provide for a statistically significant set of interactions and feedbacks.
• The diagnostic statistics are generally satisfactory as far as the tests of the residual serial correlation, functional form and heteroskedasticity are concerned.

• The assumption of normally distributed errors is rejected in all which is understandable if we consider the three major hikes in oil prices experienced.

• As expected, the exchange rate and domestic interest rate equations appear to have least explanatory power, with $R^2$ of 0.07 and 0.12, and the model struggles to fit the observations of variables during the volatile periods of the seventies.

• The remaining $R^2$ are relatively high [0.25-0.49].

2 Impulse Response and Trend/Cycle Properties of the UK Model

• One important use of macroeconometric models is to conduct counterfactual experiments.
  – Impacts of changes in interest rates or oil prices on output and inflation into the future.
  – ‘trend’ growth or as ‘cyclical’ movements around the trend.

• An analysis of the dynamic impact of shocks is typically carried out using impulse response functions.
  – The estimation of impulse response functions, with respect to shocks applied to observables such as the oil price can be
conducted using the generalised impulse response approach. – In the case of monetary policy shocks or shocks to technology or tastes, the analysis of dynamic impulses is complicated due to the fact that such shocks are rarely observed directly and must be identified indirectly.

• If we wish to identify the effects of monetary policy shocks we require further restrictions to be placed on the contemporaneous relationships amongst the variables. This relates to the “structural” VECM:

\[
A \Delta z_t = \tilde{a} - \tilde{\alpha} \left[ \beta' z_{t-1} - b_1(t - 1) \right] + \sum_{i=1}^{p-1} \tilde{\Gamma}_i \Delta z_{t-i} + \varepsilon_t, \quad (15)
\]

where \( A \) represents the \( 9 \times 9 \) matrix of contemporaneous structural coefficients, \( \varepsilon_t = A u_t \sim (0, \Omega = A \Sigma A') \) are the associated structural shocks.

• Without a priori restrictions on \( A \) and/or \( \Omega \), it is not possible to identify economically-meaningful impulse response functions to shocks. The restrictions on \( A \) require a tight description of the decision-rules followed by the public and private economic agents, incorporating information on agents’ use of information and the exact timing of the information flows.

2.1 Estimates of Impulse Response Functions

• The resultant oil price equation, estimated over the period
$\Delta p_t^o = -0.0039 + 0.04787\Delta p^o_{t-1} + 2.7731\Delta y^*_{t-1} + 0.4199\Delta p^*_{t-1} + 2.4855\Delta r^*_{t-1} + \hat{\epsilon}_{ot}$, \hspace{1cm} (16)


None of the coefficients are statistically significant at the conventional levels.

- These results are in line with the widely held view that oil prices follow a geometric random walk, possibly with a drift. Therefore, we base our computations on:

$\Delta p_t^o = 0.0173 + \hat{\epsilon}_{ot}$, \hspace{1cm} (17)


### 2.2 Effects of an Oil Price Shock

- It is important that special care is taken to separate the output and inflation effects of an oil price shock from those of a monetary shock as they are likely to be positively correlated.

- Achieved by treating oil prices as long-run forcing, and by explicitly modelling the contemporaneous dependence of monetary policy shocks on the oil price shocks, as well as on shocks to exchange rates and foreign interest rates.

- Figure 10.1 provides the persistence profiles of the five long-run relationships.
– All the persistence profiles converge towards zero, thus confirming the cointegrating properties of the long-run relations.

– $PPP$ and output gap relations showing much slower rates of adjustments to shocks.
  * The effect of the oil price shock on the output gap takes some ten years to complete.
  * Deviations from $PPP$ are relatively long lived with the half life of deviations from $PPP$ at about four years.

– Convergence to the $FIP$, $IRP$ and $MME$ relationships is much more rapid, reflecting the standard view that arbitrage in asset markets functions much faster in restoring equilibria.

• Figure 10.2 gives the impulse responses on the levels of all the eight endogenous variables. The oil price shock has a permanent effect, reflecting their unit root properties.
  – Its effect on output has the expected negative sign, reducing domestic output by approximately 0.24% below its base after 2.5 years.
  – Foreign output also declines to the same long-run value but at a much slower speed.
  – On impact, the oil price shock raises the domestic rate of inflation by 0.20%, and by 0.82% after 1 quarter, before gradually falling back close to zero after 3 years.
– Despite the higher domestic prices, the oil price shock generates a small appreciation of the nominal exchange rate. The process starts to reverse after approximately one year. In the long run, the nominal exchange rate fully adjusts to the change in relative prices with \( PPP \) restored.

– The oil price shock is accompanied by increases in both domestic and foreign interest rates, suggesting a possible tightening of the monetary policy.

– The oil price shock affects real money balances both directly and indirectly through its impact on interest rates. The overall outcome is to reduce real money balances by around 1% in the long run. This is indicative of the presence of a strong liquidity effect in our model.

– The oil price shock also causes the real rate of interest to fall, initially by 0.1% and then by 0.7%, before gradually returning to its equilibrium value of zero.

2.3 Effects of a Foreign Output Equation Shock

• Figure 10.3: the persistence profiles.

– The size of the deviations from equilibrium are much smaller than those compared to the oil price shock but the pattern is similar.

– Hence, the \( PPP \) and output gap relations show much slower rates of adjustments to shocks, whilst the convergence to the \( FIP, IRP \) and \( MME \) relationships is much
more rapid.

- Figure 10.4: the impulse responses.
  - In the long run, the effect of a unit shock to the foreign output equation is to increase both domestic and foreign output by 0.2% above their base-line values. However, the gap between domestic and foreign output growths persists even after 20 quarters, with the foreign output level remaining considerably higher.

  - The shock initially reduces domestic prices by 0.23% and appreciates the nominal exchange rate on impact by 0.27%. The fall in inflation is reversed in the following quarter, though, returning to near its base-line value after about 12 quarters. In the long run, the effect of the foreign output shock on the domestic inflation rate is zero, so that the effects are purely temporary.

  - The initial response to the shock is to increase domestic and foreign interest rates by 11 and 6 basis points, respectively. Subsequently the foreign interest rate rises above the domestic interest rate, but eventually this gap disappears.

2.4 Effects of a Foreign Interest Rate Equation Shock

- Figure 10.5 again confirms the varying speeds of adjustments of the long-run relationships as before.

- Figure 10.6 shows the impact effect of the shock to the foreign interest rate equation is to increase the domestic interest rate by
23 basis points whilst domestic output is unchanged. Domestic output falls thereafter, down by 0.37% after 4 quarters and reaching 0.5% below its baseline after 16 quarters.

- The shock to the foreign interest rate equation depreciates the nominal exchange rate on impact by 0.14% and by approximately 0.5% in the long run.

- The fact that the impulse response function for the foreign interest initially slopes upwards reflects the highly persistent nature of the interest rate movements in the short run. The domestic interest rate is much less affected by the shock with the result that, during the first 3 years following the shock, the foreign interest rate tends to rise above the domestic interest rate. This interest rate gap will eventually disappear.

- The initial effect of the interest rate shock on domestic inflation is to increase the rate of inflation by 0.13% followed by 0.57% after one quarter, and 0.88% after four quarters. This effect is reversed from this point onwards, with the inflation rate falling to be approximately 0.1% above its baseline value after about 14 quarters. In the long run, the effect of the interest rate shock on the domestic inflation rate is zero.

- The effects of the shock to the foreign interest rate equation on real money balances are negative.
2.5 Effects of a Monetary Policy Shock

- Turn to the more economically-meaningful monetary policy shocks or, more precisely unanticipated component of the policy.

- Figure 10.7 presents the persistence profiles of the effects of one standard error unexpected increase in the interest rate \(i.e.\) a rise of 91 basis points. The effects of the monetary policy shock on these relations disappear eventually, but the speed with which this occurs varies considerably across the different arbitrage conditions.

  - The interest parity condition is the quickest to adjust followed by the Fisher inflation parity, the monetary equilibrium condition, purchasing power parity and the output gap.

  - On impact, the effect of the monetary policy shock is most pronounced on the money market equilibrium condition, resulting in a 12.7% unexpected excess supply of money.

  - With foreign interest rates unchanged on impact (by construction), the shock raises the domestic interest rate above the foreign interest rate by 91 basis points, but it also raises the real interest rate by 59 basis points while leaving the real exchange rate unchanged.

  - The contractionary impact of the shock on domestic output (relative to the foreign output) begins to be seen after the
second quarter, with domestic output falling below foreign output by 0.29% after two years.

- The impulse response functions in Figure 10.8.
  - After the initial impact, the domestic interest rate declines at a steady rate settling down after approximately four years.
  - In tandem with the fall in the interest rate, the excess supply of money declines to approximately 8.6% after one year, then to 5.0% after two years, reaching its equilibrium after approximately five years. These results clearly show the presence of a sizeable “liquidity effect” in our model following the unexpected tightening of the monetary policy.
  - The monetary policy shock has little immediate effects on the real side of the economy. The contractionary effects of the policy begin to be felt on output and real money balances after one quarter. Each showing a relatively smooth decline to around 0.46% and 0.17% respectively below base after two and half years. The speed of adjustments of the two series differ.
  - Figure 10.8f provides evidence of the well known “price puzzle”, as inflation increases in immediate response to the contractionary monetary shock, falling back to close to zero after three years. But the anomaly are observed in the short run only.
  - The impact effect of the monetary policy shock on the
nominal exchange rate is zero by construction but the shock causes the exchange rate to appreciate by around 0.5% in the following period. The exchange rate remains roughly constant for the subsequent year and then depreciates back to close to its original level after 20 quarters. This pattern is reasonably consistent with the Dornbush (1976) overshooting model or the broader view of overshooting discussed in Eichenbaum and Evans (1995) in which there might be a sequence of periods of appreciation followed by depreciation because of secondary effects of the shock on risk premia, speculative behaviour and information imperfections relating to the permanence of the shock.

- By way of comparison, we provide the time profiles of the effects of shocks to a unit increase in the domestic interest rate equation. Figure 10.9 and Figure 10.10.
  - The time profiles of the impulse responses of Figures 10.9 and 10.10 are very similar, in both size and shape, to those plotted in Figures 10.7 and 10.8.
  - However, there are differences between the two which are important in terms of interpretation of the responses.
    * The response of the exchange rate in Figure 10.10g shows important differences to those in Figure 10.8g, indicating a depreciation of the exchange rate on impact in response to the positive shock to interest rates.
    * The positive differential of domestic over foreign interest
rates observe over the first ten quarters is associated in Figure 10.10g with an *appreciating* exchange rate.

- It is worth emphasising that the GIRs obtained for the shock to the interest rate equation in Figure 10.10 take into account the contemporaneous innovations in the other variables typically observed when the interest rate is shocked.

  * It does not have the interpretation of a monetary policy shock.

- But this comparison illustrates well both the strengths of the GIR analysis of reduced-form shocks and its limitations.

### 2.6 Trend/Cycle Decomposition in Cointegrating VARs

- We consider a decomposition of the variables in the UK model into trends and cycles. The decomposition can be viewed as a multivariate version of the Beveridge-Nelson (BN) permanent/transitory decomposition, but has the advantage that it is characterized fully in terms of the observables (but not unique).

- It is worth noting that the choice of a permanent trend/transitory cycle decomposition relies on *a priori* assumptions on the extent of the correlation between permanent and transitory innovations. Decompositions in the spirit of BN assume that shocks to the transitory component and to the stochastic permanent component have a correlation of one. This is
in contrast to the unobserved components approach which assumes the correlation is zero.

- Suppose we take any arbitrary partitioning of \( z_t = (y'_t, x'_t)' \) into permanent trend, \( z_t^P \), and transitory cycle, \( z_t^C \) components of the form:

\[
z_t = z_t^P + z_t^C, \quad (18)
\]

where

\[
z_t^P = z_{dt}^P + z_{st}^P.
\]

- Define the deterministic and the stochastic trend components of \( z_t \) respectively by

\[
z_{dt}^P = g_0 + gt,
\]

\[
z_{st}^P = \lim_{h \to \infty} E_t (z_{t+h} - z_{dt,t+h}^P) = \lim_{h \to \infty} E_t [z_{t+h} - g_0 - g(t + h)], \quad (19)
\]

where \( g_0 \) is an \( m \times 1 \) vectors of fixed intercepts, \( g \) is an \( m \times 1 \) vector of (restricted) trend growth rates.

- Then we have

\[
z_t^P = \lim_{h \to \infty} E_t (z_{t+h} - gh). \quad (20)
\]

- Even if we are interested in the permanent/transitory decomposition of the endogenous variables, \( y_t \), we would still need to work with the VECM in \( z_t \), under which the permanent/transitory properties of \( x_t \) would have a direct bearing
on those of $y_t$. This point re-affirms the desirability of multivariate approaches over the univariate such as the Hodrick and Prescott (1997).

- It appears that the BN and GRW decompositions are the same in general. But as the applications below illustrate this is not true in the multivariate case where $z_t$ is cointegrated.

2.6.1 An Application to the UK Model

- To compute the decomposition, all the required parameters can be estimated from the ML estimates of the underlying VECM model, except for $g$.
- Under Case IV, the estimation yields an estimate of $\beta'\gamma$, and $\gamma$ can not be separately identified from $\beta$.
- We can estimate $g$ by

$$\Delta z_t = g + \vartheta_t, \quad (21)$$

subject to the restrictions $\beta'g = \beta'\gamma$ with $\beta'\gamma$. A consistent estimate of $g$ can be obtained by application of the SURE procedure to (21) subject to the restrictions, $\beta'g = \beta'\gamma$. 
• In the case of the UK model we have

\[
\hat{\beta}' \mathbf{g} = \begin{pmatrix}
0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\
0 & 0 & 0 & -56.098 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\
\end{pmatrix}
\begin{pmatrix}
g_o \\
g_e \\
g_{r*} \\
g_r \\
g_{\Delta p} \\
g_y \\
g_{p-p^*} \\
g_{h-y} \\
g_{y^*} \\
\end{pmatrix}
\]

\[
= \begin{pmatrix}
ge_e - g_{p-p^*} \\
g_{r*} - g_r \\
g_y - g_{y^*} \\
-56.098g_r \\
g_r - g_{\Delta p} \\
\end{pmatrix}.
\]

Also, the estimated version of the model yields

\[
\hat{\beta}' \gamma = \begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
-0.007300 \\
0 \\
\end{pmatrix}.
\]

This yields the following restrictions

\[
ge_e = g_{p-p^*}, \quad g_{r*} = g_r, \quad g_y = g_{y^*}, \quad g_r = 0.007300/56.098, \text{ and } g_r = g_{\Delta p}
\]

• The presence of a linear trend in the money demand equation
implies a very small but a non-zero value for $g_r (= 0.00013)$. We decided to set $g_r = 0$ as it is unlikely that the trend in the money demand equation could prevail in the very long run.

- Therefore, we estimate $g$ subject to the following restrictions:

$$
ge_e = g_{p-p^*}, \quad g_y = g_{y^*} \quad \text{and} \quad g_{\Delta p} = g_r = g_{r^*} = 0,$$

and obtained the estimate,

$$\hat{g} = (0.01557, 0.002179, 0, 0, 0, 0.005256, 0.002179, -0.007287, 0.005256)$$

- Figure 10.11 plots the actual series and the GRW permanent component of domestic output $y_t$.
  - The GRW permanent component of UK GDP is not as smooth as other trend estimates. However, by construction, the permanent component of $y_t$ is perfectly correlated with the permanent component of $y_t^*$.

- In Figure 10.12, we plot the transitory component of the UK output alongside the transitory component of inflation so that we might look at the cyclical movements in the inflation-output trade off.
  - There is a limited degree of positive co-movement between inflation and output, with a correlation coefficient of just 0.14, which is consistent with a demand-shock view of the business cycle, but the cyclical dependence seems rather weak.

- Figure 10.13 plots GRW and BN transitory components for $y_t$. 
- There is a clear degree of consensus between the two series, reflected in a correlation coefficients of 0.827. The two decompositions yield the same result for the exchange rate due to its random walk property.

- Figure 10.14 plots GRW transitory components of $y_t$ and $y_t^*$. The most noticeable feature is the limited degree of co-movement exhibited by the two series, with a correlation coefficient of 0.28, particularly in the late 1960s, early 1980s and late 1990s.

- Figures 10.15 and 10.16 plot the BN and Hodrick-Prescott (HP) transitory components of the same two series. Both the GRW and BN decompositions show a low degree of co-movement between the transitory components of UK and foreign output at 0.38 and 0.28.

- The degree of co-movement between the HP transitory $y_t$ and $y_t^*$ components is noticeably higher, yielding a correlation coefficient of 0.69.

- The univariate HP filter overstates the degree of co-movement between the transitory components (i.e. induces highly synchronised business cycle for the UK relative to the rest of the world); the multivariate VECM does not support such a high degree of short run synchronisations.

- Figure 10.17 plots the transitory components of $r_t$ and $r_t^*$. Our zero growth rate assumption on $r_t$ and $r_t^*$ implies that
the change in the permanent component of these series is determined purely by the stochastic part.

– The co-movement between domestic and foreign interest rates is positive and reasonably strong, with a correlation coefficient of 0.5.

– The transitory component of domestic interest rates is more volatile but part of this difference reflects that foreign interest rates are measured as the average of interest rates.

- Figure 10.18 plots the actual \( r_t \) series alongside its permanent component.

- A large part of movements in \( r_t \) could be defined as transitory, with the permanent component showing little variations.

3 Probability Event Forecasting with the UK Model

- A number of macroeconomic modelling teams in the UK have recently begun to provide further information on the uncertainties surrounding their forecasts of key macroeconomic variables.

- The Bank of England now routinely publishes a range of outcomes for its inflation and output growth forecasts.

* It remains rare for forecasters to provide the detailed information on the range of potential outcomes. One
explanation of this relates to the difficulty in measuring
the uncertainties associated with forecasts in the large
mainstream macroeconomic models typically employed.

* A second explanation is the perceived difficulty in
conveying the outcomes of complicated macroeconomic
models in a simple and easily understood form.

- Our compact modelling approach provides a practical frame-
work for probability forecasting.

3.1 An Updated Version of the Core Model

- Our VAR(2) model, involving nine variables, represents an
intermediate alternative that is well suited to the generation of
probability forecasts.

- We extend the dataset, so that it covers the period 1965q1-
2001q1 and work with updated versions of the core model.

- Under the assumption that oil prices are “long-run forcing”,
efficient estimation of the parameters can be based on the
following conditional error correction model:

\[
\Delta y_t = a_y - \alpha_y \left[ \beta' z_{t-1} - b_1(t - 1) \right] + \sum_{i=1}^{p-1} \Gamma_{yi} \Delta z_{t-i} + \psi_{yo} \Delta p^t_0 + u_{yt}.
\] (22)

- For forecasting purposes, we specify the process for the change
\[ \Delta p_t^o = \delta_o + \sum_{i=1}^{p-1} \delta_{oi} \Delta z_{t-i} + u_{ot}, \] (23)

- Combining (22) and (23), we have the RF equation:

\[ \Delta z_t = a - \alpha \left[ \beta' z_{t-1} - b_1(t - 1) \right] + \sum_{i=1}^{p-1} \Gamma_i \Delta z_{t-i} + v_t, \] (24)

### 3.1.1 Estimation Results and in-sample Diagnostics

- The results continue to support the existence of 5 cointegrating relations, and are qualitatively very similar as before.

- For the purpose of forecasting, we re-estimated the model over the shorter period of 1985q1-2001q1, taking the long-run relations as given.

(a) The short-run coefficients are more likely to be subject to structural change;

(b) The application of Johansen’s cointegration tests is likely to be unreliable in small samples.

- Table 11.1 gives the estimates of the individual error correcting relations of the benchmark model estimated over the 1985q1-2001q1 period. The estimated error correction equations pass most of the diagnostic tests and compared to standard benchmarks, fit the historical observations relatively well.
3.2 Model Uncertainty

- We adopt the Bayesian Model Averaging (BMA) framework.
- We confined the choice of the models to exactly identified VAR(2) models with \( r = 0, 1, \ldots, 5 \), and two alternative specifications of the oil price equation, namely (23), and its random walk counterpart,

\[
\Delta p_t^o = \delta_o + \mu_{ot}. \tag{25}
\]

Naturally, we also included our benchmark model in the set (for both specifications of the oil price equation), thus yielding a total of 14 models.
- We employed the BMA formulae with the weights set according to Akaike, Schwarz and equal weights (\( w_{iT} = 1/14 \)).
- Only five of the fourteen models appeared as plausible candidates according to the AIC and SBC criteria.
  - Using the AIC, only two candidate models were considered plausible: namely, the exactly-identified 5 cointegrating vector (CV) models with the two alternative oil price specifications. For the estimation period 1985q1-1998q4, the two models had estimated weights of 0.93 and 0.07. These weights gradually changed to 0.60:0.40 for the estimation period 1985q1-2000q4.
  - Using the SBC, the exactly-identified models with 5, 4, 3 and 2 cointegrating vectors, each supplemented
by the random walk model for oil prices, were chosen as plausible candidates. For the estimation period 1985q1-1998q4, the weights of these four models were 0.07:0.86:0.06:0.01 respectively, but these also changed gradually to 0.00:0.01:0.22:0.77 for the estimation period 1985q1-2000q4.

There is considerable variability in the estimated posterior probabilities of these chosen models with relatively minor changes in the sample sizes.

### 3.3 Evaluation and Comparisons of Probability Forecasts

- Each of 14 alternative models was used to generate probability forecasts for a number of simple events over the period 1999q1-2001q1. This was undertaken in a recursive manner.
- The probability forecasts were computed for directional events.
  - In the case of \( p_t - p_t^* \), \( e_t \), \( r_t \), \( r_t^* \) and \( \Delta \tilde{p}_t \), we computed \( \Pr[\Delta(p_t - p_t^*) > 0 \mid \mathcal{I}_{t-1}] \), \( \Pr[\Delta e_t > 0 \mid \mathcal{I}_{t-1}] \), and so on.
  - For the remaining variables, \( (y_t, y_t^*, h_t - y_t \text{ and } p_t^o) \) which are trended, we considered \( \Pr[\Delta^2 y_t > 0 \mid \mathcal{I}_{t-1}] \), \( \Pr[\Delta^2 y_t^* > 0 \mid \mathcal{I}_{t-1}] \), and so on.
- The probability forecasts are computed recursively using the parametric stochastic simulation technique which allow for future uncertainty and the nonparametric bootstrap technique which allow for parameter uncertainty.
- We adopted a statistical approach, using a threshold probability
of 0.5.

- Table 11.2 reports the incidence of the four possible combinations of our directional forecasts based on the benchmark model.
  - For each variable, nine event forecasts are generated over the period 1999q1-2001q1 (nine quarters), thus providing 81 forecasts for evaluation purposes.
  - These event forecasts are compared with their realisations. High values for UU and DD indicate an ability of the model to forecast correctly, while high values of UD and DU suggest poor forecasting ability.
  - For the case of future uncertainty the hit rate is 0.68 versus 0.62 when both parameter uncertainty and future uncertainty are considered.

- The forecasting performance is summarised by the KS, defined by \( H - F \), where \( H \) is the proportion of ups that were correctly forecast to occur, and \( F \) is the proportion of downs that were incorrectly forecast. This statistic provides a measure of the accuracy of directional forecasts of the model, with high positive numbers indicating high predictive accuracy.

- In Table 11.3, we report the KS along with the other forecast evaluation statistics.
  - In the presence of future uncertainty only, the KS suggests that the most accurate forecasts are provided by the
benchmark model.

– Allowing for parameter uncertainty, however, the KS suggests the benchmark model and the AIC average model produce the most accurate forecasts.

• Pesaran and Timmermann (1992) provide a formal directional (market timing) statistical test. The results suggest that the benchmark model performs well under future uncertainty, suggesting the importance of imposing theory-based long-run restrictions for probability forecasting, but that this distinction is removed when both future and parameter uncertainty are considered.

• An alternative approach to probability forecast evaluation would be to use the probability integral transforms

\[ u(z_t) = \int_{-\infty}^{z_t} p_t(x) \, dx, \quad t = T + 1, T + 2, \ldots, T + n, \]

where \( p_t(x) \) is the forecast probability density function, and \( z_t, t = T + 1, T + 2, \ldots, T + n \), the associated realizations. Under the null hypothesis that \( p_t(x) \) coincides with the true density function of the underlying process, the probability integral transforms will be distributed as \( iidU[0, 1] \).

– We first computed a sequence of one step ahead probability forecasts from the over-identified and exactly identified models for the nine simple events over the nine quarters, and hence the associated probability integral transforms,
\( u(z_t) \).

– To test the hypothesis that these probability integral transforms are random draws from \( U[0, 1] \), we calculated the Kolmogorov-Smirnov statistic,

\[
D_n = \sup_x |F_n(x) - U(x)|,
\]

where \( F_n(x) \) is the empirical cumulative distribution function (CDF) of the probability integral transforms, and \( U(x) = x \), is the CDF of \( iidU[0, 1] \).

– For the over-identified benchmark specification, all these statistics are well below the 5% critical alue of Kolmogorov-Smirnov statistic, and the hypothesis that the forecast probability density functions coincide with the true ones cannot be rejected.

– We cannot reject the same hypothesis for the average models but with the noticeable exception of the AIC.

– Overall results do not reject any one model but does provide some evidence, in particular when considering future uncertainty, for supporting the use of the over-identified specification in forecasting.

3.4 Probability Forecasts of Inflation and Output Growth

• Inflation targets have been set explicitly in the UK since October 1992, following the UK’s exit from the European Exchange Rate Mechanism (ERM).
The Chancellor’s stated objective at the time was to achieve an average annual rate of inflation of 2%, while keeping the underlying rate of inflation within the 1%-4% range.

In May 1997, the policy of targeting inflation was formalized further by the setting up of the Monetary Policy Committee (MPC), whose main objective is to meet inflation targets primarily by influencing the market interest rate through fixing the base rate at regular intervals.

Its current remit is to achieve an average annual inflation rate of 2.0%, based on the Harmonised Index of Consumer Prices (HICP), renamed the Consumer Price Index.

We have used the RPI index (as an approximation to the measure previously used by the MPC, the Retail Price Index, excluding mortgage interest payments, RPI-x), where the previous target of 2.5% is argued to be equivalent to the new 2.0% target.

The previous target range of 1.5%-3.5% remains of interest.

Inflation rates outside the target range act as a trigger, requiring the Governor of the Bank of England to write an open letter to the Chancellor explaining why inflation had deviated from the target, the policies being undertaken to correct the deviation, and how long it is expected before inflation is back on target.

The Bank is also expected to conduct monetary policy so as to support the general economic policies of the government, so
far as this does not compromise its commitment to its inflation target.

- Since October 1992, the Bank of England has produced a quarterly *Inflation Report*.
  - The *Report* provides forecasts of inflation over two year horizons, with bands presented around the central forecast to illustrate the range of inflation outcomes that are considered possible (the so-called fan charts). The forecasts are based on the assumption that the base rate is left unchanged.
  - Since November 1997, a similar forecast of output growth has also been provided.

- The fan charts produced by the Bank of England are an important step towards acknowledging the significance of forecast uncertainties in the decision making process.

- However, the approach suffers from two major shortcomings.
  (a) It seems unlikely that the fan charts can be replicated by independent researchers.
  (b) The use of fan charts is limited for the analysis of uncertainty associated with joint events.

### 3.5 Point and Interval Forecasts

- Tables 11.4a and 11.4b provide the point forecasts for domestic inflation rates and output growth over the period 2001q1-2003q1 together with their 95% confidence intervals.
  - The model predicts the average annual rate of inflation
to fall from 2.5% in 2001q1 to 1.8% in 2001q2. This is followed by further falls for the rest of 2001 before returning to approximately 2% to the end of the forecast horizon, 2003q1. These point forecasts are lower than the inflation rates realized during 2000.

- Output growth is predicted to be positive throughout the forecast horizon, falling from an average annual rate of 2.8% in 2000 to 1.3% by the end of 2001, before rising to around 2.0% thereafter. Therefore, we may be tempted to rule out the possibility of a recession occurring in the UK over the 2001-2003 period.

- However, these point forecasts are subject to a high degree of uncertainty.
  - The point forecast of annual inflation in 2003q1 is predicted to be 1.9%, which is well within the announced inflation target range. But the 95% confidence interval covers the range -0.8% to +4.6%. For the quarter on quarter definition, the uncertainty is even larger, with a range of -3.5% to 7.5% around a point forecast of approximately 2.0%.
  - The point forecast of the quarter on quarter annual rate of output growth in 2003q1 is 2.1%, but its 95 per cent confidence interval covers the range -2.6% to +6.7%.
  - It is difficult to evaluate the significance of these forecast intervals for policy analysis.
3.6 Predictive Distribution Functions

- Figures 11.2a and 11.2b: the probability forecasts for the four-quarter moving averages of inflation and output growth for the 1-quarter, 1 and 2-year ahead forecast horizons based on the benchmark model. Computed using the simulation techniques and take account of both future and parameter uncertainties.
  
  - Figure 11.2a presents the estimated predictive distribution function for inflation for the threshold values ranging from 0% to 5% per annum at the three selected forecast horizons.
    * For example, the forecast probability that inflation lies below 3.5% becomes smaller at longer forecast horizons, falling from close to 100% one quarter ahead (2001q2) to 70% eight quarters ahead (2003q1). These forecast probabilities are in line with the recent historical experience.

  - Figure 11.2b plots the estimated predictive distribution functions for output growth.
    * These plots also suggest a weakening of the growth prospects in 2001 before recovering a little at longer horizons. But the estimate is not sufficiently high for it to be much of a policy concern.

3.7 Event Probability Forecasts
• Here we consider three single events of particular interest:

\[ A \]: Achievement of inflation target, the four-quarterly moving average rate of inflation falling within the range 1.5%-3.5%,

\[ B \]: Recession, the occurrence of two consecutive quarters of negative output growth,

\[ C \]: Poor growth prospects, the four-quarterly moving average of output growth being less than 1%,

• The joint events:
  – \[ A \cap \overline{B} \] (Inflation target is met and recession is avoided),
  – \[ A \cap \overline{C} \] (Inflation target is met combined with reasonable growth prospects).

3.7.1 Inflation and the Target Range

• Two sets of estimates of \( \Pr(A_{T+h} | J_T) \) are provided in Table 11.4a and depicted in Figure 11.3.

– The probability that the Bank of England will be able to achieve the government inflation target is estimated to be high in the short run but falls in the longer run.

  * The probability estimate is high in 2001q2, at 0.87 (0.80) for \( \widetilde{\pi} \) (\( \pi \)), but it falls rapidly to nearer 0.45 by the end of 2001/early 2002. This fall in the first quarters of the forecast reflects the increasing likelihood of inflation falling below the 1.5% lower threshold.

  * Ultimately the estimated probability of achieving
inflation within the target range settles to 0.38 (0.35) for \( \tilde{\pi} (\pi) \) in 2003q1. At this longer forecast horizon, the probabilities of inflation falling below and above the target range are 0.32 and 0.30, respectively, using \( \tilde{\pi} \) (or 0.42 and 0.23 using \( \pi \)), so these figures reflect the relatively high degree of uncertainty

* While the likely inflation outcomes are low by historical standards and there is a reasonable probability of hitting the target range, there are also comparable likelihoods of undershooting and overshooting the inflation target range at longer horizons.

### 3.7.2 Recession and Growth Prospects

- Figure 11.4 shows the estimates of the recession probability, \( \Pr(B_{T+h} \mid \mathcal{I}_T) \).
  - \( \pi \) and \( \tilde{\pi} \) are very similar in size across the different forecast horizons and suggest a very low probability of a recession: based on the \( \tilde{\pi} \) estimate, for example, the probability of a recession occurring in 2001q2 is estimated to be around zero, rising to 0.09 in 2002q1.
  - The probability that UK faces poor growth prospects is much higher, in the region of 0.35 at the end of 2001, falling to 0.3 in 2003q1 according to the \( \tilde{\pi} \) estimates.

- For the event \( A_{T+h} \cap \overline{B}_{T+h} \), the joint probability forecasts are similar to those that for \( \Pr(A_{T+h} \mid \mathcal{I}_T) \) alone at every time
horizon. This is not surprising since the probability of avoiding recession is close to one.

- The probability forecasts for $A_{T+h} \cap \overline{C}_{T+h}$ are considerably less than those for $\Pr (A_{T+h} \mid \mathcal{I}_T)$ alone.

- Figure 11.5 plots the values of the joint event probability alongside a plot of the product of the single event probabilities; that is $\Pr (A_{T+h} \mid \mathcal{I}_T) \times \Pr (\overline{B}_{T+h} \mid \mathcal{I}_T)$. This comparison provides an indication of the degree of dependence/independence of the two events.

  - The probabilities are relatively close, indicating little dependence between output growth prospects and inflation outcomes. This result is compatible with the long-term neutrality hypothesis.

- Figure 11.6 also plots the probability estimates of the joint event $A_{T+h} \cap \overline{B}_{T+h}$, but illustrates the effects of taking into account model uncertainty.

  - Three values of the probability of the joint event, each calculated without taking account of parameter uncertainty; the benchmark model, but the other two show the weighted average of the probability estimates obtained from the fourteen alternative models. The plots show that estimated probabilities from the benchmark model are quite close to the ‘equal weights’ estimate, but these are both lower than the AIC-weighted average, by more than 0.1 at some forecast horizons.
3.8 A Postscript

- A real time out of sample forecast evaluation: we compare the point and probability forecasts with the realised values of output growth and inflation for the eight quarters 2001q2-2003q1.

- The difficulty of producing accurate point forecasts are reflected in the size of the forecast errors but where the uncertainty surrounding the point forecasts is so large that in only one case does the realised value exceed the 95% confidence intervals.

- The less volatile four quarterly moving average changes perform reasonably, with a root mean square error (RMSE) of 0.47 and 0.60 percentage points for output growth and inflation.

- The quarter on quarter annual realisations exhibit high volatility, particularly for inflation and as such have larger and more volatile forecast errors. This is reflected in the RMSE’s which take the value of 0.72 and 2.43 for output growth and inflation respectively.
  - On this definition inflation forecasts performs badly. For example, the realised value was 4.86% in 2001q2 as compared to the forecast value of 0.28%.

- The probability event forecasts, which use the same distributions as the point and interval forecasts, perform well
in terms of predicting specific events and as such convey useful information, not always apparent when using the point forecasts.

- If we evaluate the probability event forecasts using the threshold probability of 0.5, then the “hit rate” or percentage of correctly forecasting events, for all the thirty two events regarding inflation defined in Table 11.5a is 84% (27 out of 32) for future uncertainty only and 75% (24 out of 32) for future and parameter uncertainty.

- The hit rate for events associated with output growth exhibit a hit rate of 100% (16 out of 16).

- Joint event probability event predictions also perform well with a hit rate of 69% (11 out of 16).

- Policy makers recognize that their announcements, in addition to providing information on policy objectives, can themselves initiate responses which effect the macroeconomic outcome.

- Central Bank Governors are reluctant to discuss either pessimistic possibilities, as this might induce recession, or more optimistic possibilities, since this might induce inflationary pressures.

- There is therefore an incentive for policy makers to seek ways of making clear statements regarding the range of potential macroeconomic outcomes for a given policy, and the likelihood of the occurrence of these outcomes, in a manner which avoids these difficulties.
• We argue that the use of probability forecasts has an intuitive appeal, enabling the forecaster to specify the relevant “threshold values” which define the event of interest (e.g., a threshold value corresponding to an inflation target range of 1.5% to 3.5%).

• A further advantage of the use of probability forecasts is the flexibility of probability forecasts.

4 Global Modelling Application

• Global modelling is subject to a number of important constraints, including the quantity and quality of data available, the curse of dimensionality, our knowledge of economic theory and institutions and the availability of human and computing resources.

• The Global VAR approach provides a coherent solution by treating the foreign variables as weakly exogenous. This assumption is plausible for small open economies and can be tested empirically.

• The variables in each economy is potentially related to all the variables in other economies by relating the domestic variables of each economy to corresponding foreign variables constructed to match the international trade pattern.

• Once the estimates of the individual country models are obtained, they are combined to generate forecasts or impulse
response functions for all the variables in the world economy simultaneously.

- PSW consider country/region specific quarterly models estimated over the period 1979Q1-1999Q1 for seven countries (namely USA, Germany, France, Italy, UK, Japan, China) along with four broader regions (namely, Western Europe, South East Asia, Middle East, and Latin America).

- Domestic variables of interest: with $i = 0$ (US), $1, \ldots, 10$.
  - real output for area $i$ ($y_{it}$),
  - the rate of price inflation ($\Delta p_{it}$),
  - a real equity price index ($q_{it}$),
  - the real exchange rate ($e_{it} - p_{it}$), where $e_{it}$ is the log of nominal exchange rate in terms of a reference currency (US dollar),
  - an interest rate ($r_{it}$),
  - real money balances ($m_{it}$)

- The foreign variables (indices), denoted by $y_{it}^*$, is a $k_i^* \times 1$ vector are constructed as weighted averages, with region specific weights:

$$
\begin{align*}
  \mathbf{y}_{it}^* &= (y_{it}^*, \Delta p_{it}^*, q_{it}^*, e_{it}^*, r_{it}^*, m_{it}^*)', \\
  y_{it}^* &= \sum_{j=0}^{10} w_{ij} y_{jt}, \quad p_{it}^* = \sum_{j=0}^{10} w_{ij} p_{jt}, \\
  q_{it}^* &= \sum_{j=0}^{10} w_{ij} q_{jt}, \quad e_{it}^* = \sum_{j=1}^{10} w_{ij} e_{jt}, \\
  r_{it}^* &= \sum_{j=0}^{10} w_{ij} r_{jt}, \quad m_{it}^* = \sum_{j=0}^{10} w_{ij} m_{jt},
\end{align*}
$$

(26)
where the weights $w_{ij}$ are based on trade shares. Note that $w_{ii} = 0$, for all $i$.

- Region-specific cointegrating VAR models are estimated treating the relevant foreign variables, along with the price of oil, as exogenous.
- The only foreign variable included by in the US was $e_{it}^*$. 
- A VAR of order 1 is assumed.
- Careful analysis of the cointegrating properties of the data is employed to choose the cointegrating rank of each regional model.
- The underlying exogeneity assumptions are confirmed to be acceptable.
- The regional models are brought together to form a global model. Each of the individual region-specific cointegrating VAR models is written in terms of the variables from all other regions, using the definitions in (26), and these are then stacked in a large global system accommodating all the contemporaneous and lagged interactions across the 60 plus variables of the system.
- The corresponding reduced form representation provides the vehicle for forecasting and impulse response analysis.

4.1 Dynamic Properties of the Global Model

- Using generalized impulse response functions, it is possible to
estimate the effects of shocks to one variable in one country on the other variables in the same country and/or in the rest of the world.

- The effects of a one standard error (unit) negative shock to US equity prices, oil prices and interest rates.

- Figure 12.1 produces the time profiles of the effects of shocks to US equity market on equity prices worldwide.
  - On impact, a fall in the US equity prices causes prices in all equity markets to fall but by smaller amounts: 3.5% in the UK, 4.5% in Germany, 2.4% in Japan, 2.6% in South East Asia, and 4.8% in Latin America, as compared to a fall of 6.4% in the US.
  - The falls in equity prices across the regions generally start to catch up with the US over time, and even get amplified in the case of Italy and Latin America.
  - The relative position and pattern of the impulse response functions confirm the pivotal role played by the US stock market in the global economy and suggest that in the longer run scope for geographic diversifications across equity market might be somewhat limited.

- The time profiles of the effects of the shock to the US equity market on real output are shown in Figure 12.2.
  - The impact effects of the fall in the US equity market on real output are negative for most regions, but rather small.
After one year, real output shows a fall of around 0.31% in the US, 0.25% in Germany, 0.29% in the UK, 0.26% in Latin America, and 0.12% in South East Asia, respectively.

Japanese output only begins to be negatively affected by the adverse US stock market shock much later.

The GVAR model has also been used as a global macroeconomic engine driving credit risk models.

For example, it might be of interest to determine the effects of a contemporaneous 10% drop in the Japanese equity prices on other macroeconomic variables, and the effects that these have on the credit risk.

5 Future Research Direction

There has been considerable interest and activity in the application of the Structural Cointegrating VAR approach to macroeconometric modelling within academia, and from central bankers, government and industry in recent years.

In particular, this approach has been illustrated in models of the US economy, in Anderson et al. (2002), the Canadian economy, in Crowder and Wohar (2004), and for the euro area in Brand and Cussola (2004).

In the US model, a six-variable cointegrating VAR is obtained (including the CPI, the GDP deflator, real money balances, the federal funds rate, the yield on long-term
bonds, and output), and anchored by four long-run relationships suggested by economic theory: namely, a money demand relationship, the Fisher Inflation parity relationship, a term-structure relationship, and a relationship linking the two measures of prices.

– In the Canadian case, the six variables under consideration include disposable income, consumption, wealth, the interest rate, real money balances and the GDP deflator, while the long-run relationships include consumption-income and consumption-wealth relationships, the money demand relationship and the \( FIP \) relationship.

– For the Euro area, Brand and Cussola’s model describes real money balances, inflation, short-term and long-term interest rates and GDP, taking into account a demand for money relationship, a term-structure relationship and the \( FIP \), again motivated with reference to the economic theory of the long run.

• In each case, the models perform well by various statistical criteria and uncover important policy-relevant features.

– The US study concludes that the model provides forecasts that are very similar to those published by government agencies and so could provide a useful tool on which to base policy recommendations, accommodating the steady-state growth model of the economy implicitly shared by many government agencies and private forecasters.
– The Canadian and Euro area studies both focus on the impact of monetary policy change: the Bank of Canada’s shift to a stable price level target in the early eighties was associated with a once-and-for-all shift in the long-run relationships, while no major distortions were found with the advent of Stage Three of Economic and Monetary Union in the Euro area model.

• In all cases, practitioners appreciate the transparency and pragmatism of this modelling approach.

• In the case of the emerging market economies, Akusuwan (2005) develops a small quarterly macroeconometric model for the Thai economy over the period 1980q1-2002q4, and establishes the existence of three long-run relations, namely the Fisher interest parity, the uncovered interest parity, and the long-run money demand. By allowing for a possible break in the domestic variables following the 1997 Asian crisis, she finds that the crisis has significant effects on the short-run structure of the model, but not on its long-run relationships.

• The merits of the application of the approach to macroeconometric modelling are investigated recently by Jacobs and Wallis (2005). The core model of the previous chapters is compared and contrasted with COMPACT, a large-scale simultaneous equation model of the UK. The paper highlights the strengths of these modelling approaches and illustrates the emerging consensus between SEMs and our own approach.
• In particular, we believe there to be considerable scope for the further development and use of the techniques in the sphere of global macroeconometric modelling.

6 Concluding Remarks

• To summarize the book serves to
  – explain and promote the long-run structural modelling approach as a way of undertaking macroeconometric research;
  – provide a comprehensive illustration of the approach through a description of each stage of the development of the core model of the UK economy;
  – demonstrate how a model obtained following this approach can be used in real world decision-making.

• On the issue of promoting the long-run structural modelling approach, our aim has been to present a fully transparent approach.
  – Our explicit description of a macroeconomic theory of the long run aims to highlight the level of abstraction at which an analyst might work in building a model that can be confronted with the data.
  – Our description of the theory of the short run also exposes the extent to which economic theory might realistically inform our attempts to interpret macroeconomic dynamics.
Our comprehensive description of the econometric methods showed how the insights obtained from an explicit statement on the long-run theory can be embedded, and tested, within a statistical model that can both possess economically-reasonable properties and reflect the characteristics of the data.

- In brief, the intention is that the economic theory and the econometric methods complement and enhance each other.
- We hope that our work will contribute to what is an on-going debate and, through our discussion of models employing higher frequency data, hope to focus modelers’ attention on more reasonable identification schemes that can be defended with reference to information flows and the precise timing of decisions.
- The second aim of the book is to provide an illustration of the entire process of building a model, from the description of the underlying economic theory and its empirical counterpart, through the collection of data and its initial characterisation, through the estimation and testing of the model, to its final use in interpreting the macroeconomy and its use in decision-making.
- The third objective is that the work will be useful to practitioners. Practitioners appreciate the transparency and pragmatism of the modelling approach. Moreover, a model obtained following our approach can be readily extended either by its
inclusion within a broader framework (as in the GVAR model), or by linking it to a more detailed ‘satellite’ model of the market of interest.

- In each case, we believe the methods described in the book will contribute to a theoretically-informed and evidence-based analysis of important macroeconomic phenomena.