Firms in International Trade

Lecture 2: The Melitz Model

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Essential Reading


What Does This Paper Do?

• Dynamic Industry Model with heterogeneous firms where opening to trade leads to reallocations of resources within an industry

• Opening to trade leads to
  – Reallocations of resources across firms
  – Low productivity firms exit
  – High productivity firms expand so there is a change in industry composition
  – High productivity firms enter export markets
  – Improvements in aggregate industry productivity
  – No change in firm productivity

• Consistent with empirical evidence from trade liberalizations?
Theoretical Model and Evidence

- The theoretical model is consistent with a variety of other stylized facts about industries
  - Heterogeneous firm productivity
  - Ongoing entry and exit
    * Co-movement in (gross) entry and exit due to sunk entry costs
    * Exiting firms are low productivity (selection effect)
  - Explains why some firms export within industries and others do not
    * Contrast with traditional theories of comparative advantage
    * Exporting firms are high productivity (selection effect)
    * No feedback from exporting to productivity
Where Does the Paper Fit in the Literature?

• Theoretical
  – Dynamic industry models of heterogeneous firms under perfect competition
    * Jovanovic (1982) and Hopenhayn (1992)
  – Models of trade under imperfect competition
    * Krugman (1980)
  – Other frameworks for modeling firm heterogeneity
    * Yeaple (2003)

• Empirical
  – Empirical literature on heterogeneous productivity, entry and exit
    * Davis and Haltiwanger (1991)
    * Dunne, Roberts and Samuelson (1989)
    * Bartelsman and Doms (2000)
  – Empirical literature on exports and productivity
    * Bernard and Jensen (1995, 1999)
    * Roberts and Tybout (1996, 1997)
    * Clerides et al. (1998)
  – Empirical literature on trade liberalization
    * Levinsohn (1999)
Road Map

- Overview of Model Structure
- Equilibrium in a Closed Economy
- Equilibrium in an Open Economy
- The impact of the opening of trade
- What did we learn?
Overview of Model Structure

- Single factor: labor (numeraire, $w = 1$)
- Firms enter market by paying sunk entry cost ($f_e$)
- Firms observe their productivity ($\varphi$) from distribution $g(\varphi)$
- Productivity is fixed thereafter
- Once productivity is observed, firms decide whether to produce or exit
- Firms produce horizontally-differentiated varieties, with a fixed production cost ($f_d$) and a constant variable cost that depends on their productivity
- Firms face an exogenous probability of death ($\delta$) due to force majeure events
• CES “love of variety” preferences:

\[ C = \left[ \int_{\omega \in \Omega} q(\omega)^\rho \, d\omega \right]^\frac{1}{\rho}, \quad 0 < \rho < 1, \]

• Dual price index:

\[ P = \left[ \int_{\omega \in \Omega} p(\omega)^{1-\sigma} \, d\omega \right]^\frac{1}{1-\sigma}, \quad \sigma = \frac{1}{1-\rho} > 1, \]

• Equilibrium firm revenue:

\[ r(\omega) = R \left( \frac{p(\omega)}{P} \right)^{1-\sigma}, \]
Production

- Production technology:
  \[ l = f + \frac{q}{\varphi}, \]

- Firms of all productivities behave symmetrically and therefore we can index firms by productivity alone

- Profit maximization problem:
  \[
  \max_{p(\varphi)} \left\{ p(\varphi)q(\varphi) - w \left( f + \frac{q(\varphi)}{\varphi} \right) \right\},
  \]

- The first-order condition yields the equilibrium pricing rule:
  \[
  p(\varphi) = \left( \frac{\sigma}{\sigma - 1} \right) \frac{w}{\varphi} = \frac{1}{\rho \varphi},
  \]

- where we choose the wage for the numeraire, \( w = 1 \)
Firm Revenue

- Substituting the pricing rule into equilibrium revenue:

\[ r(\phi) = (\rho \phi)^{\sigma-1} \, R P^{\sigma-1}, \quad \pi(\phi) = \frac{r(\phi)}{\sigma} - f. \]

- Therefore the relative revenue of any two firms within the same market depends solely on their relative productivities:

\[ r(\phi'') = \left( \frac{\phi''}{\phi'} \right)^{\sigma-1} \, r(\phi'), \quad (1) \]

- The presence of a fixed production cost implies a zero-profit cutoff productivity below which firms exit:

\[ \pi(\phi^*) = 0, \quad \iff \quad r(\phi^*) = \sigma f, \quad (2) \]

- The revenue of any firm can therefore be written as:

\[ r(\phi) = \left( \frac{\phi}{\phi^*} \right)^{\sigma-1} \, \sigma f, \]
Profits and Productivity

\[ \pi(\varphi) \text{ (Autarky)} \]

\[ \pi \]

\[ -f \]

\[ \varphi^* \]

\[ \varphi \]
Firm Entry and Exit

• The \textit{ex post} productivity distribution conditional on successful firm entry is therefore:

\[
\mu(\varphi) = \begin{cases} 
\frac{g(\varphi)}{1 - G(\varphi^*)} & \text{for } \varphi \geq \varphi^* \\
0 & \text{otherwise}
\end{cases},
\]

• The value of a firm with productivity \( \varphi \) is:

\[
v(\varphi) = \max \left\{ 0, \frac{\pi(\varphi)}{\delta} \right\},
\]

• In equilibrium, the free entry condition requires the expected value of entry to equal the sunk entry cost

\[
v_e = \frac{1 - G(\varphi^*)}{\delta} \bar{\pi} = f_e, \tag{3}
\]

– where \([1 - G(\varphi^*)]\) is the probability of successful entry
– where \(\bar{\pi}\) is expected profits conditional on successful entry
Free Entry

- Expected profits conditional on successful entry are:

\[
\bar{\pi} = \int_{\varphi^*}^{\infty} \pi(\varphi) \frac{g(\varphi)}{1 - G(\varphi^*)} \, d\varphi,
\]

- which using the relationship between variety revenues and the zero-profit cutoff condition (2) can be written as:

\[
\bar{\pi} = f \int_{\varphi^*}^{\infty} \left[ \left( \frac{\varphi}{\varphi^*} \right)^{\sigma-1} - 1 \right] \frac{g(\varphi)}{1 - G(\varphi^*)} \, d\varphi,
\]

- Therefore the free entry condition becomes:

\[
\nu_e = \frac{f}{\delta} \int_{\varphi^*}^{\infty} \left[ \left( \frac{\varphi}{\varphi^*} \right)^{\sigma-1} - 1 \right] g(\varphi) \, d\varphi = f_e,
\]

- which is monotonically decreasing in \( \varphi^* \)

- Therefore the model has a recursive structure where \( \varphi^* \) can be determined from the free entry condition alone
Aggregate Variables

- Define a weighted average of firm productivity:

\[
\tilde{\varphi} = \left[ \int_{\varphi^*}^{\infty} \varphi^{\sigma-1} \frac{g(\varphi)}{1 - G(\varphi^*)} d\varphi \right]^{\frac{1}{\sigma-1}}. (5)
\]

- Aggregate variables, such as dual price index \( P \), can be written as functions of mass of firms \( M \) and weighted average productivity:

\[
P = \left[ \int_{\varphi^*}^{\infty} p(\varphi)^{1-\sigma} M \frac{g(\varphi)}{1 - G(\varphi^*)} d\varphi \right]^{\frac{1}{1-\sigma}},
\]

\[
P = \left[ \int_{\varphi^*}^{\infty} (\rho \varphi)^{\sigma-1} M \frac{g(\varphi)}{1 - G(\varphi^*)} d\varphi \right]^{\frac{1}{1-\sigma}},
\]

- where there is a mass of firms with each productivity 

\[
Mg(\varphi) / [1 - G(\varphi^*)]
\]

\[
P = M^{\frac{1}{1-\sigma}} p(\tilde{\varphi}) = M^{\frac{1}{1-\sigma}} \frac{1}{\rho \tilde{\varphi}}. (6)
\]
The closed economy general equilibrium is referenced by the triple \( \{ \phi^*, P, R \} \)

All other endogenous variables can be written in terms of this triple

The steady-state equilibrium is characterized by a constant mass of firms entering each period, \( M_e \), a constant mass of firms producing, \( M \), and a stationary ex post distribution of firm productivity, \( g(\varphi) / [1 - G(\varphi^*)] \)

To determine general equilibrium, we use the recursive structure of the model

Equilibrium \( \phi^* \) follows from the free entry condition (4) alone
Closed Economy General Equilibrium, $R$

- To determine $R$, we use the steady-state stability condition that the mass of successful entrants equals the mass of exiting firms

$$[1 - G(\phi^*)] M_e = \delta M$$

- Using this steady-state stability condition to substitute for $1 - G(\phi^*)$ in the free entry condition (3), competitive entry implies that total payments to labor used in entry equal total firm profits:

$$L_e = M_e f_e = M \bar{\pi} = \Pi,$$

- Total payments to labor used in production equal total revenue minus total firm profits:

$$L_p = R - M \bar{\pi} = R - \Pi.$$

- Therefore total revenue equals total labor payments and the labor market clears:

$$R = L = L_p + L_e.$$
Closed Economy General Equilibrium, $P$

- To determine $P$ in (6), we need to solve for $\tilde{\phi}$ and $M$
- Having determined $\varphi^*$, $\tilde{\phi}$ follows immediately from (5)
- The mass of firms can be determined from:

$$M = \frac{R}{\bar{r}} = \frac{L}{\sigma(\bar{\pi} + f)},$$

- where $\bar{r}$ and $\bar{\pi}$ can be written as a function of $\varphi^*$ and $\tilde{\phi}$, which have both been determined:

$$\bar{r} = \int_{\varphi^*}^{\infty} r(\varphi) \frac{g(\varphi)}{1 - G(\varphi^*)} d\varphi = r(\tilde{\phi}) = \left( \frac{\tilde{\phi}}{\varphi^*} \right)^{\sigma-1} \sigma f,$$

$$\bar{\pi} = \int_{\varphi^*}^{\infty} \pi(\varphi) \frac{g(\varphi)}{1 - G(\varphi^*)} d\varphi = \pi(\tilde{\phi}) = \left[ \left( \frac{\tilde{\phi}}{\varphi^*} \right)^{\sigma-1} - 1 \right] f,$$
Open Economy Model

- Consider a world of symmetric countries
- Suppose that each country can trade with $n \geq 1$ other countries
- Choose the wage in one country as the numeraire, which with country symmetry implies $w = w^* = 1$
- To export a firm must incur a fixed export cost of $f_x$ units of labor
- In addition, exporters face iceberg variable costs of trade such that $\tau > 1$ units of each variety must be exported for 1 unit to arrive in the foreign country
- As firms face the same elasticity demand in both markets, export prices are a constant multiple of domestic prices due to the variable costs of trade:
  \[
  p_x(\varphi) = \tau p_d(\varphi) = \frac{\tau}{\rho \varphi},
  \]
- Consumer optimization implies that export market revenue is a constant fraction of domestic market revenue:
  \[
  r_x(\varphi) = \tau^{1-\sigma} r_d(\varphi) = \tau^{1-\sigma} R(P \rho \varphi)^{\sigma-1},
  \]
Firm Exporting Decision

- Total firm revenue depends on whether or not a firm exports:

\[ r(\varphi) = \begin{cases} 
  r_d(\varphi) & \text{not export} \\
  r_d(\varphi) + nr_x(\varphi) = (1 + n\tau^{1-\sigma})r_d(\varphi) & \text{export} 
\end{cases} \]

- Consumer love of variety and fixed production costs ⇒ no firm will ever export without also serving the domestic market

- Therefore we can apportion the fixed production cost to domestic market and the fixed exporting cost to export market
  - When deciding whether to export, firms compare export market profits to the fixed exporting costs
  - Equivalently, we could compare the sum of domestic and export market profits to the sum of the fixed production and exporting costs

- Given fixed exporting costs, there is an exporting cutoff productivity \( \varphi_x^* \) such that only firms with \( \varphi \geq \varphi_x^* \) export:

\[ r_x(\varphi_x^*) = \sigma f_x. \quad (7) \]
Selection into Export Markets

- A large empirical literature finds evidence of selection into export markets (e.g. Bernard and Jensen 1995, Roberts and Tybout 1997)
  - Only some firms export
  - Exporters are more productive than non-exporters
- From the relative revenues of firms with different productivities in the same market (1), and from relative revenue in the domestic and export markets (18), we have:

  \[ r_d(\varphi_x^*) = \left( \frac{\varphi_x^*}{\varphi^*} \right)^{\sigma-1} r_d(\varphi_d^*), \quad r_x(\varphi_x^*) = \tau^{1-\sigma} r_d(\varphi_x^*). \]

- Therefore using the zero-profit and exporting cutoff conditions, (2) and (7), we obtain the following relationship between the productivity cutoffs:

  \[ \varphi_x^* = \tau \left( \frac{f_x}{f} \right)^{\frac{1}{\sigma-1}} \varphi^*, \quad (8) \]

- where selection into export markets, \( \varphi_x^* > \varphi^* \), requires \( \tau^{\sigma-1} f_x > f \)
Free Entry

• The free entry condition in the open economy becomes:

\[ v_e = [1 - G(\varphi^*)] \left[ \frac{\bar{\pi}_d + \chi \bar{\pi}_x}{\delta} \right] = f_e, \]

where \([1 - G(\varphi^*)]\) is the probability of successful entry, \(\bar{\pi}_d\) is expected domestic profits conditional on successful entry, 
\(\chi = \left[ 1 - G(\varphi^*_x) \right] / \left[ 1 - G(\varphi^*_d) \right]\) is the probability of exporting conditional on successful entry, and \(\bar{\pi}_x\) is expected export profits conditional on exporting.

• Using the relationship between variety revenues and the zero-profit and exporting cutoff conditions, we obtain:

\[ v_e = \frac{f}{\delta} \int_{\varphi^*}^{\infty} \left[ \left( \frac{\varphi}{\varphi^*} \right)^{\sigma - 1} - 1 \right] g(\varphi) d\varphi \]

\[ + \frac{f_x}{\delta} \int_{\varphi^*_x}^{\infty} \left[ \left( \frac{\varphi}{\varphi^*_x} \right)^{\sigma - 1} - 1 \right] g(\varphi) d\varphi = f_e, \]
Average Firm Revenue and Profits

- Average firm revenue and profits are now:

\[
\bar{r} = r_d(\bar{\phi}) + \chi n r_x(\bar{\phi}_x), \quad \bar{\pi} = \pi_d(\bar{\phi}) + \chi n \pi_x(\bar{\phi}_x),
\]

- where average revenue in each market is:

\[
\bar{r}_d = r_d(\bar{\phi}) = \left(\frac{\bar{\phi}}{\phi^*}\right)^{\sigma-1} \sigma f, \quad \bar{r}_x = r_x(\bar{\phi}_x) = \left(\frac{\bar{\phi}_x}{\phi^*_x}\right)^{\sigma-1} \sigma f_x,
\]

- and average profits in each market are:

\[
\bar{\pi}_d = \pi_d(\bar{\phi}) = \left[\left(\frac{\bar{\phi}}{\phi^*}\right)^{\sigma-1} - 1\right] f, \\
\bar{\pi}_x = \pi_x(\bar{\phi}_x) = \left[\left(\frac{\bar{\phi}_x}{\phi^*_x}\right)^{\sigma-1} - 1\right] f_x,
\]
Aggregate Variables

• Define weighted average productivity for the export market:

\[ \tilde{\phi}_x = \left[ \int_{\phi_x^*}^{\infty} \frac{\phi^{\sigma-1} g(\phi)}{1 - G(\phi^*_x)} d\phi \right] \frac{1}{\sigma-1}. \quad (10) \]

• The dual price index \( P \) can be written as a function of the mass of firms supply each market \( M_t \) and overall weighted average productivity \( \tilde{\phi}_t \):

\[ P = M_t^{1-\sigma} p(\tilde{\phi}_t) = M_t^{1-\sigma} \frac{1}{\rho \tilde{\phi}_t}, \]

\[ \tilde{\phi}_t = \left\{ \frac{1}{M_t} \left[ M \tilde{\phi}_x^{\sigma-1} + nM_x \left( \tau^{-1} \tilde{\phi}_x \right)^{\sigma-1} \right] \right\}^{\frac{1}{\sigma-1}}, \]

\[ M_t = M + nM_x, \quad M_x = \chi M, \]
Open Economy General Equilibrium

• The open economy general equilibrium is referenced by the quadruple \( \{ \phi^*, \phi_x^*, P, R \} \)

• All other endogenous variables can be written in terms of this quadruple

• The steady-state equilibrium is characterized by a constant mass of firms entering each period, \( M_e \), constant masses of firms producing and exporting, \( M \) and \( M_x \), and stationary \textit{ex post} distributions of firm productivity in the domestic and export markets, \( g(\varphi) / [1 - G(\varphi^*)] \) and \( g(\varphi) / [1 - G(\varphi_x^*)] \)

• To determine general equilibrium, we use the recursive structure of the model

• Equilibrium \( \phi^* \) can be determined from the free entry condition (10), substituting for \( \phi_x^* \) using the relationship between the cutoffs (8)

• Having determined \( \phi^*, \phi_x^* \) follows immediately from the relationship between the cutoffs (8)
Open Economy General Equilibrium, $R$

- To determine $R$, we use the steady-state stability condition that the mass of successful entrants equals the mass of exiting firms

\[ [1 - G(\varphi^*)] M_e = \delta M \]

- Using this steady-state stability condition to substitute for $1 - G(\varphi^*)$ in the free entry condition (3), competitive entry implies that total payments to labor used in entry equal total firm profits:

\[ L_e = M_e f_e = M [\bar{\pi}_d + \chi \bar{\pi}_x] = \Pi, \]

- Total payments to labor used in production are:

\[ L_p = R - M [\bar{\pi}_d + \chi \bar{\pi}_x] = R - \Pi. \]

- Therefore total revenue equals total labor payments and the labor market clears:

\[ R = L = L_p + L_e. \]

- Labor used in production includes fixed production, fixed exporting and variable production costs
Open Economy General Equilibrium, $P$

- To determine $P$, we can use the expressions for $\tilde{\phi}_t$ and $M_t$ above.
- Having pinned down $\phi^*$ and $\phi_x^*$, we can determine $\chi = \left[1 - G(\phi_x^*)\right] / \left[1 - G(\phi^*)\right]$, $\tilde{\phi}$ and $\tilde{\phi}_x$.
- Having pinned down the probability of exporting and weighted average productivities, we can determine $\bar{r}$ and $\bar{\pi}$.
- We can therefore also determine the mass of firms serving the domestic market and exporting:

$$M = \frac{R}{\bar{r}} = \frac{L}{\sigma(\bar{\pi} + f + \chi nf_x)}, \quad M_x = \chi M,$$

- Having pinned down $M$ and $M_x$, we have determined $M_t$.
- Having pinned down $M_t$, $M$, $M_x$ and weighted average productivities, we have determined $\tilde{\phi}_t$.
- We have therefore determined the price index $P$. 


• The open economy free entry condition provides a downward-sloping relationship between the productivity cutoffs \( \phi^* \) and \( \phi_x^* \)

\[
v_e = \frac{f}{\delta} \int_{\phi^*}^{\infty} \left[ \left( \frac{\phi}{\phi^*} \right)^{\sigma-1} - 1 \right] g(\phi) d\phi \\
+ \frac{f_x}{\delta} \int_{\phi_x^*}^{\infty} \left[ \left( \frac{\phi}{\phi_x^*} \right)^{\sigma-1} - 1 \right] g(\phi) d\phi = f_e,
\]

• The closed economy free entry condition can be obtained by considering the case where trade costs become prohibitive and \( \phi_x^* \rightarrow \infty \)

• **Proposition 1:** The opening of trade raises the zero-profit productivity cutoff below which firms exit, \( \phi^* \)
Profits, Entry and Exit

\[ \pi(\varphi) \] (Trade)

\[ \pi(\varphi) \] (Autarky)
The Effects of Trade

- The opening of trade leads to:
  - Rise in the zero profit cutoff productivity
  - Rise in average firm revenue and profit

- Low productivity firms between $\phi_A^*$ and $\phi_I^*$ exit
  - Increased exit by low productivity firms

- Intermediate productivity firms between $\phi_I^*$ and $\phi_{xI}^*$
  - Contraction in revenue at domestic firms

- Only firms with productivities greater than $\phi_{xI}^*$ enter export markets
  - Selection into export markets
  - Expansion in revenue at exporting firms

- All of the above lead to a change in industry composition that raises aggregate industry productivity

- As the zero profit cutoff productivity and average revenue rise:
  - Mass of domestically produced varieties falls: $M_I < M_A$
  - Total mass of varieties available for consumption typically rises: $(1 + n\chi)M_I > M_A$
  - Welfare necessarily rises due to aggregate productivity gains
What Did we Learn?

- The opening of trade leads to reallocations of resources across firms within industries
  - Low productivity firms exit
  - Intermediate productivity surviving firms contract
  - High productivity surviving firms enter export markets and expand
  - Change in industry composition
- Improvements in aggregate industry productivity
- No change in firm productivity
- Selection into export markets but no feedback from exporting to firm productivity
Subsequent Research

  - Introduces both exports and FDI as alternative means of serving a foreign market
  - Introduces an outside sector to tractably characterize equilibrium with many asymmetric sectors

  - Combines the Melitz model with the Antras (2003) model of incomplete contracts and trade

  - Incorporates the Melitz model into the framework of integrated equilibrium of Helpman and Krugman (1985)
Subsequent Research

  - Provides a simplified static version of the Melitz model without ongoing firm entry and with an outside sector
  - Examines the model's implications for the extensive and intensive margins of international trade

  - Solves the Chaney version of the model without an outside sector
  - Derives a sufficient statistic for welfare of the same form as that in Eaton and Kortum (2002)

- Motivated by the empirical importance of multi-product firms, uses the Melitz (2003) framework to develop a general equilibrium model of multi-product firms
- The model accounts for key observed features of the distribution of exports across firms, products and countries
- Trade liberalization gives rise to measured within-firm productivity growth by inducing firms to focus on their core competencies