The Equity Premium and Risk Free Rate Puzzles in a Turbulent Economy: Evidence from 105 Years of Data from South Africa

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Preliminaries

Equity premium: definition

:=Aggregate stock market return – risk-free rate

Central input for:

- Asset allocation/Pensions/Social Security
- Cost of capital estimation
- Asset pricing; CAPM; MFMs with F(k)=M; eg:

$$R_i - R_f = \underbrace{\left(\frac{\sigma_{iM}}{\sigma_M}\right)}_{\substack{\text{quant risk}\\ \text{in asset i}}} \underbrace{\left(\frac{\mathrm{E}R_M - R_f}{\sigma_M}\right)}_{\substack{\text{market price}\\ \text{of risk}}} = \underbrace{\left(\frac{\sigma_{iM}}{\sigma_M^2}\right)}_{\beta_i} \underbrace{\left(\mathrm{E}R_M - R_f\right)}_{\text{"Equity Premium"}}$$

“Puzzle” (Mehra & Prescott, 1985)

Equity premium in US, 1889-1978

$$R_f = 1.0080,\ \overline{R}_M = 1.0698,\ \overline{R}_M - R_f = 0.0618\ (6.18\%)$$

Macro-financial data (mean growth & var cons):

$$\overline{x} = 1.0180,\ \sigma_x = 0.0360$$

Canonical consumption-based asset pricing model, with α=10 (>>3), β=0.99, and (equil.) $\rho(R_M,x)$=1 :

$$\ln R_f = -\ln\beta + \alpha\mu_x - 0.5\alpha^2\sigma_x^2 = 0.124, \text{ or } R_f = 1.132$$

$$\ln \mathrm{E}(R_M) = \ln R_f + \alpha\sigma_x^2 = 0.136, \text{ or } \mathrm{E}(R_M) = 1.146$$

$$\Rightarrow \mathrm{E}R_M - R_f = 0.014\ (1.4\%)$$

US over last half-century:

- 1946-2005: Equity premium ↑ to 7.48%
- But: "we had no banking panics, and no depressions; no civil wars, no constitutional crises (…). If **any** of these things had happened, we might well have seen a calamitous decline in stock values, and I would not be writing about the equity premium puzzle."

(Cochrane (2005), Asset Pricing, page 461.)

South Africa over last half-century:

- Four distinct constitutional dispensations
- Series of official States of Emergency
- Organised resistance to apartheid
- Recurrent political instability

(Fedderke, de Kadt, and Luiz (2001))

- Armed regional civil confrontation pre-1994
- Unemployment rate (at t) officially >=23% (SSA, 2009); ≈ rate of unemployment in the US at nadir of Great Depression (Romer (1993,2009))
- Currency crises 1996, 1998, 2001 (Aron and ElBadawi (1999), Bhundia and Ricci (2005))

South Africa: other facts

1. Highly capitalised economy
- Stock market is world's 14th largest (market cap)
- 8th largest in EAME region; 6th largest emerging market stock exchange
- Government debt market among world's ten most liquid
- Market value of stock market close to 100 percent of GDP

(Hence better proxy for aggregate wealth (claim to aggregate consumption) than in some advanced economies (eg. Italy and Germany)

2. Also

One of few countries, and only non-advanced economy, with capital market data for over a century.

Why long data?

Variation in year-to-year market returns.

- Three-quarters of countries for which a century of data are available experienced intervals of negative stock market returns (in inflation-adjusted terms) lasting more than two decades.
- Japan, France, and Germany experienced periods of over half a century during which cumulative real equity returns remained negative.

(Dimson, Marsh and Staunton (2008))

- No evidence on "**puzzle**"from EMs over long (>50, 100 years) data

Data

Stock market returns, JSE:

Equity index, 1900 - 2005

Firer and Staunton (2002)

Dimson, Marsh and Staunton (2002, 2008)

Money market rates, South Africa:

Long:Bond index/JSE Actuaries All Bond Index

Short: NCDs; T-Bill rates

Macro:

CPI inflation; per-capita consumption non-durables & services

Arithmetic vs Geometric Averages

- Expected value of initial R1 investment obtained by compounding the average return
- Average: arithmetic or geometric
- Most common (esp. US): arithmetic
- = reliable mean terminal value if returns are serially uncorrelated; otherwise use geometric
- US: corr. ≈0; SA>>0 (7-15%)
- If correlated, arithmetic av. overstates terminal payoff by approx 1/2 variance
- 1/2 SA variance ≈ 2% points
- eg. 105 years; 5%: 1→168; 7%: 1→1.217

Estimation of long (inflation-adjusted) premium for South Africa, 1900-2005, geometric averages, %

Equity return	Approx risk-free return	Equity premium	
7.38	1.89	5.49	Over Bond
7.38	1.08	**6.30**	Over NCD/Bills

Sub-periods, Year-to-year variation, illustration

→

Sub-periods: post 1960 and post 1975 (equity and bond data reliability)

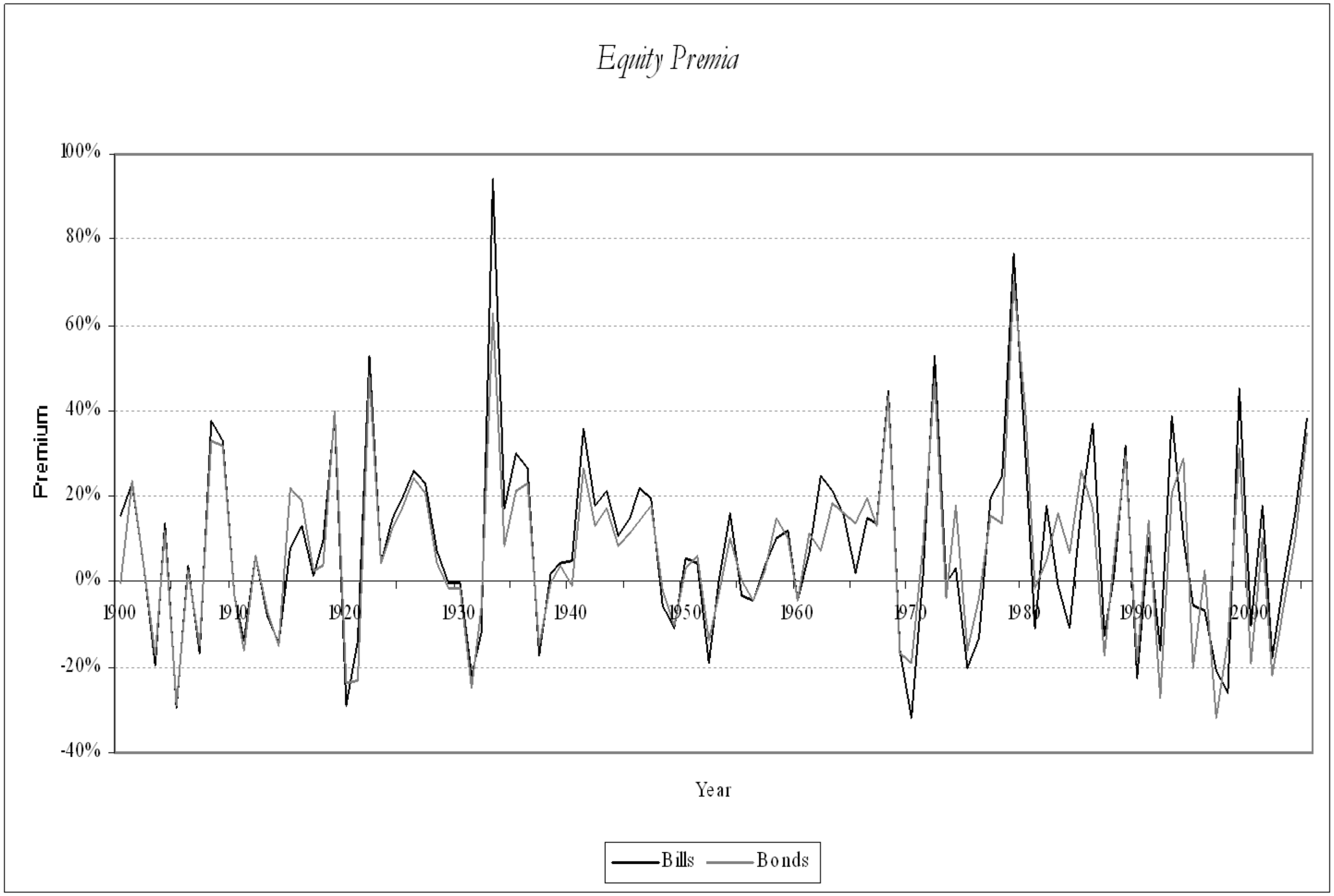
- Sub-period: 1960-2005

Equity	Risk-free	Premium	
8.36	1.78	6.59	Bond
8.36	2.05	**6.32**	Bill

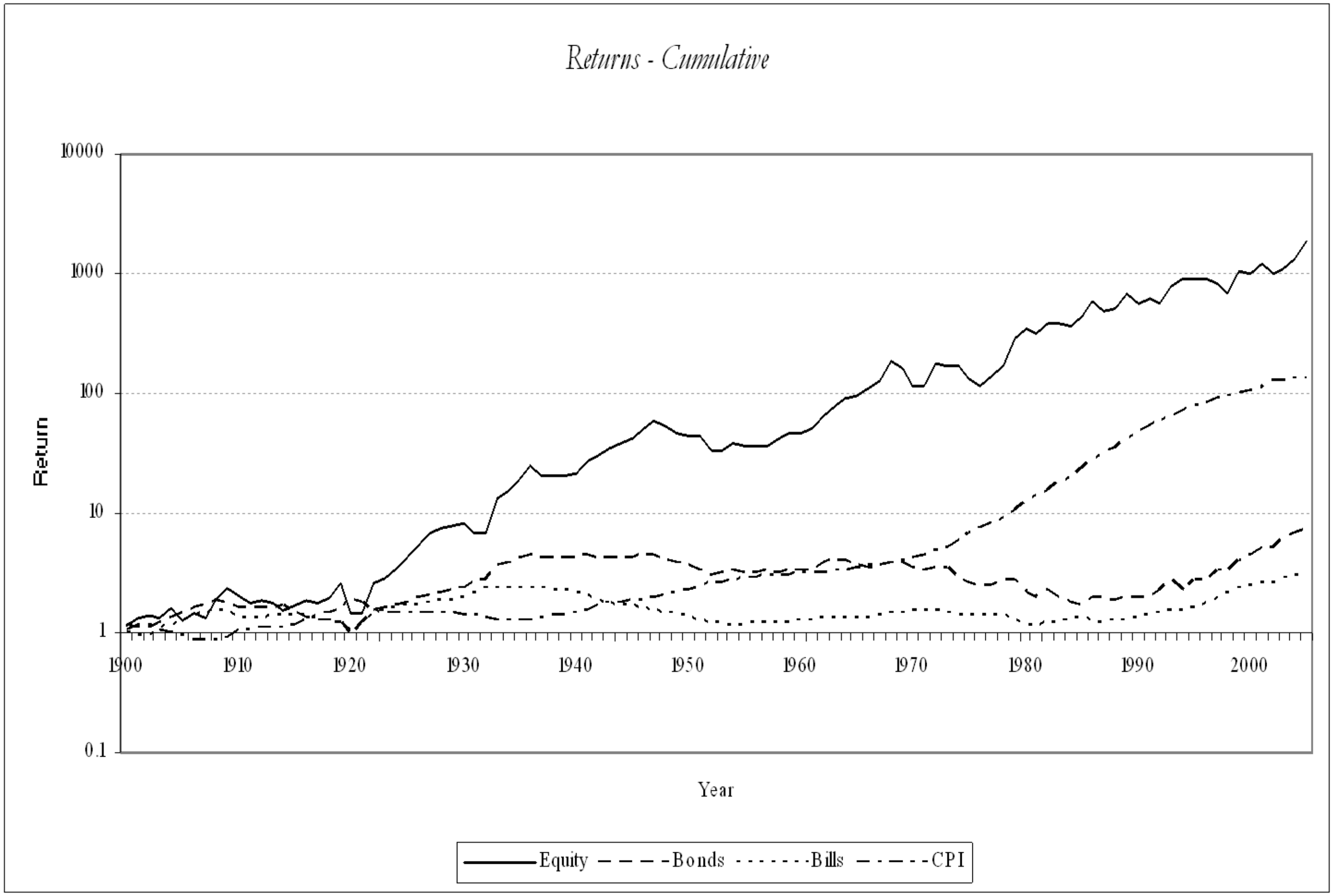
- Sub-period: 1975-2005

Equity	Risk-free	Premium	
8.07	2.31	5.76	Bond
8.07	0.73	**7.34**	Bill

Equity premium: year-to-year variation
Note: short-term risks; long-term?



Wealth-creation implications: evolution of R1 initial investment



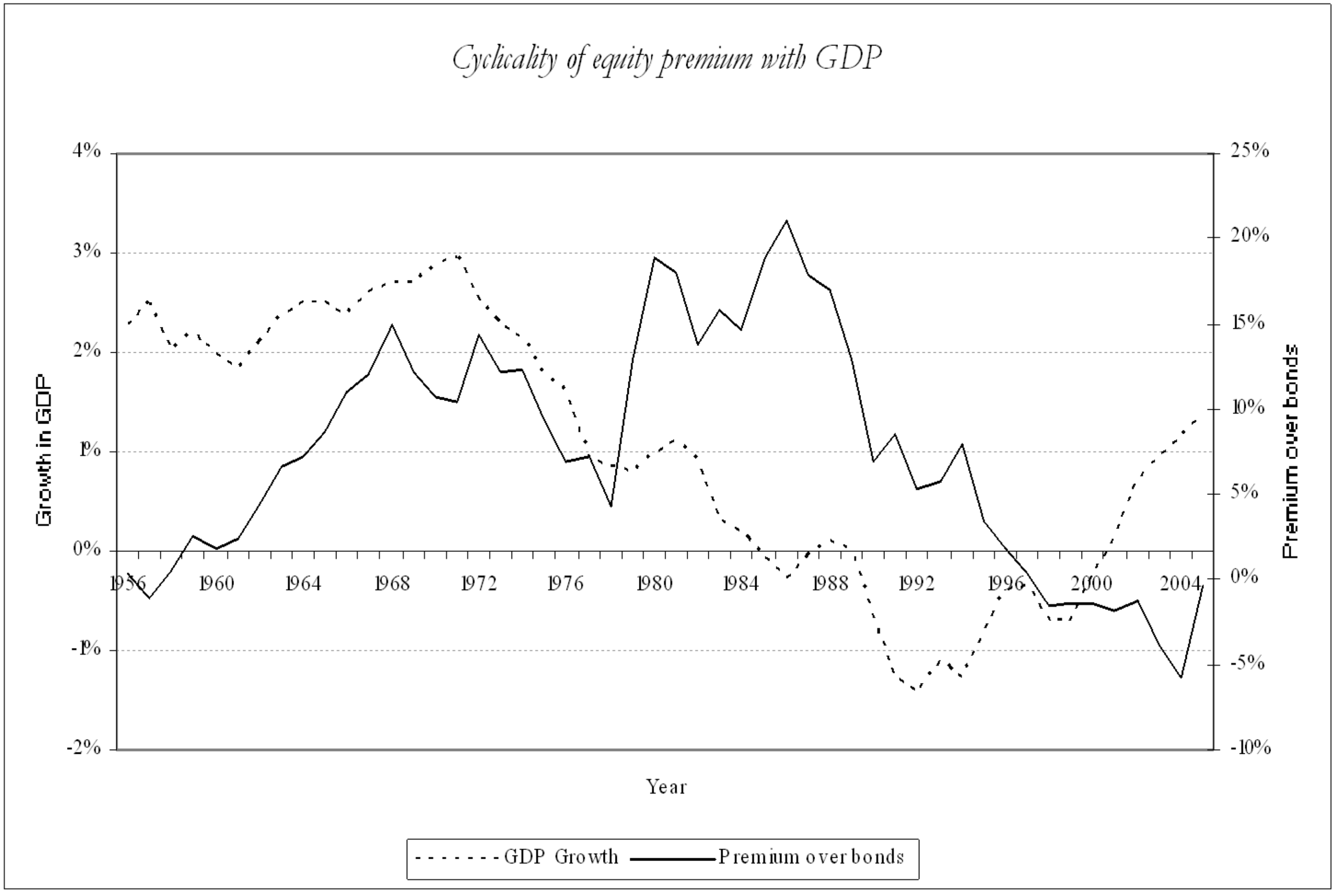
Wealth creation potential: comparison

- Real terminal value of R1 Invested

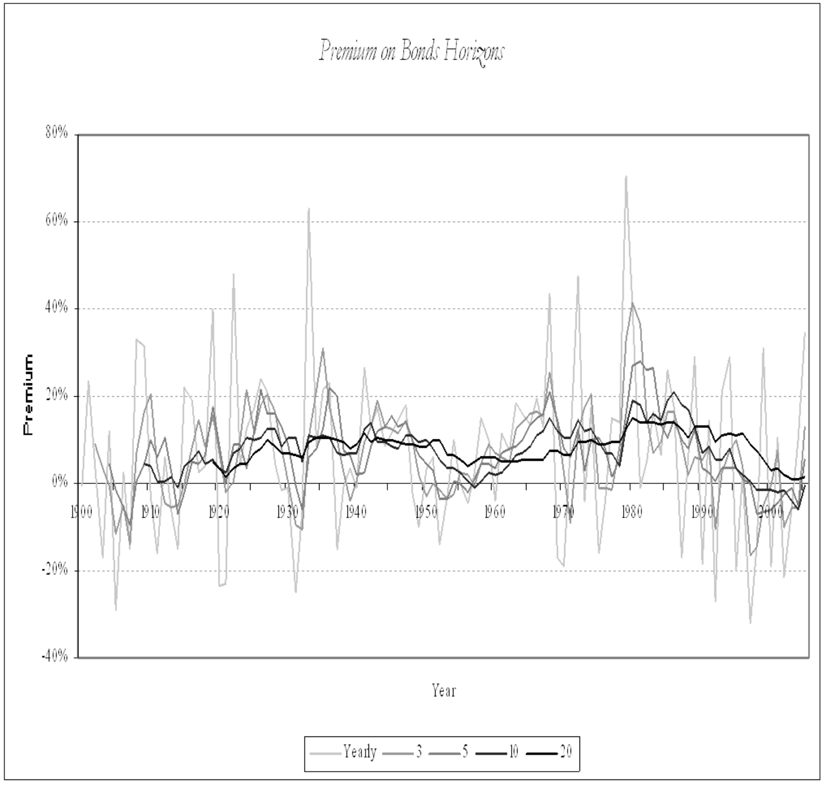
Investment period	Stocks	T-Bills/NCDs	Ratio
1900-2005	1.644,5	3,06	538
1960-2005	34,22	2,44	14
1975-2005	9,49	1,23	8

- No evident cyclicality
- **Not risky** over >=20 year horizon
- →

Cyclicality



Significance of investment time horizon



- Longer holding period = larger yet less volatile realised premium

Eg: Let $Z = ER_M - R_f$

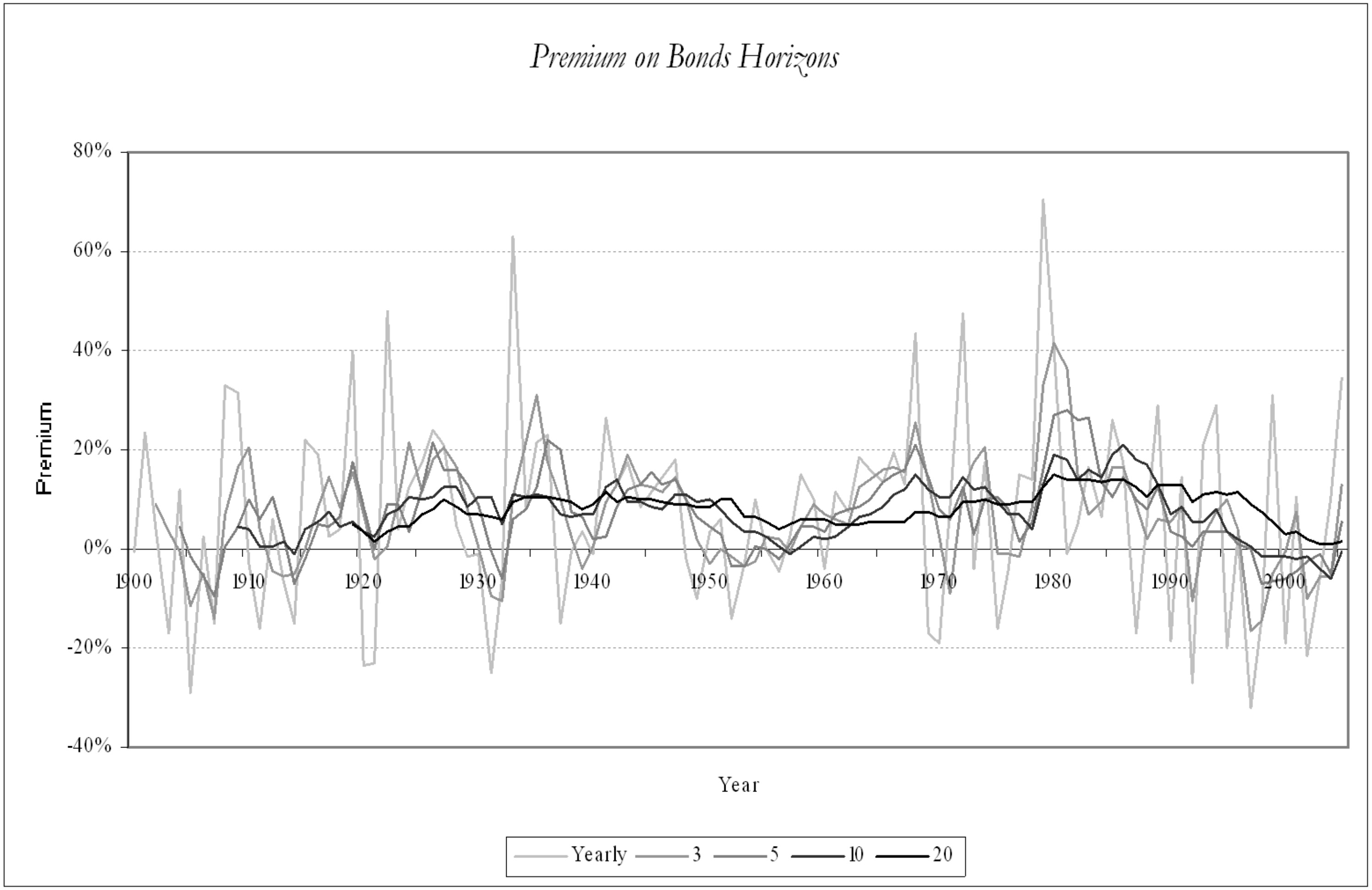
Then:

1-year: E(Z)=6.17%; σ=21.7%

5-year: E(Z)=7.62%; σ=8.6%

20-year: E(Z)=8.94%; σ=3.49%

- At 20-year horizon, investor would not have had a single negative realised (real) equity-premium over the entire 105-year period!



Premium on Bonds Horizons
80%
60%
40%
20%
0%
-20%
-40%
Premium
1900
1910
1920
1930
1940
1950
1960
1970
1980
1990
2000
Year
Yearly
3
5
10
20

Is the South African equity premium compensation for non-diversifiable risk?

Macro-financial data (mean growth & var cons), 1961-2005:

$$\bar{x} = 1.014, \sigma_x^2 = 0.02$$

Canonical consumption-based asset pricing model, with α=10 (>>3), β=0.99, and (equil.) $\rho(R_M,x)$=1:

$$\ln R_f = -\ln\beta + \alpha\mu_x - 0.5\alpha^2\sigma_x^2 = 0.\text{exp, or } R_f = 1.137$$

$$\ln \mathrm{E}(R_M) = \ln R_f + \alpha\sigma_x^2 = 0.\text{exp, or } \mathrm{E}(R_M) = 1.142$$

$$\Rightarrow \mathrm{E}R_M - R_f = 0.005\ (0.5\%)$$

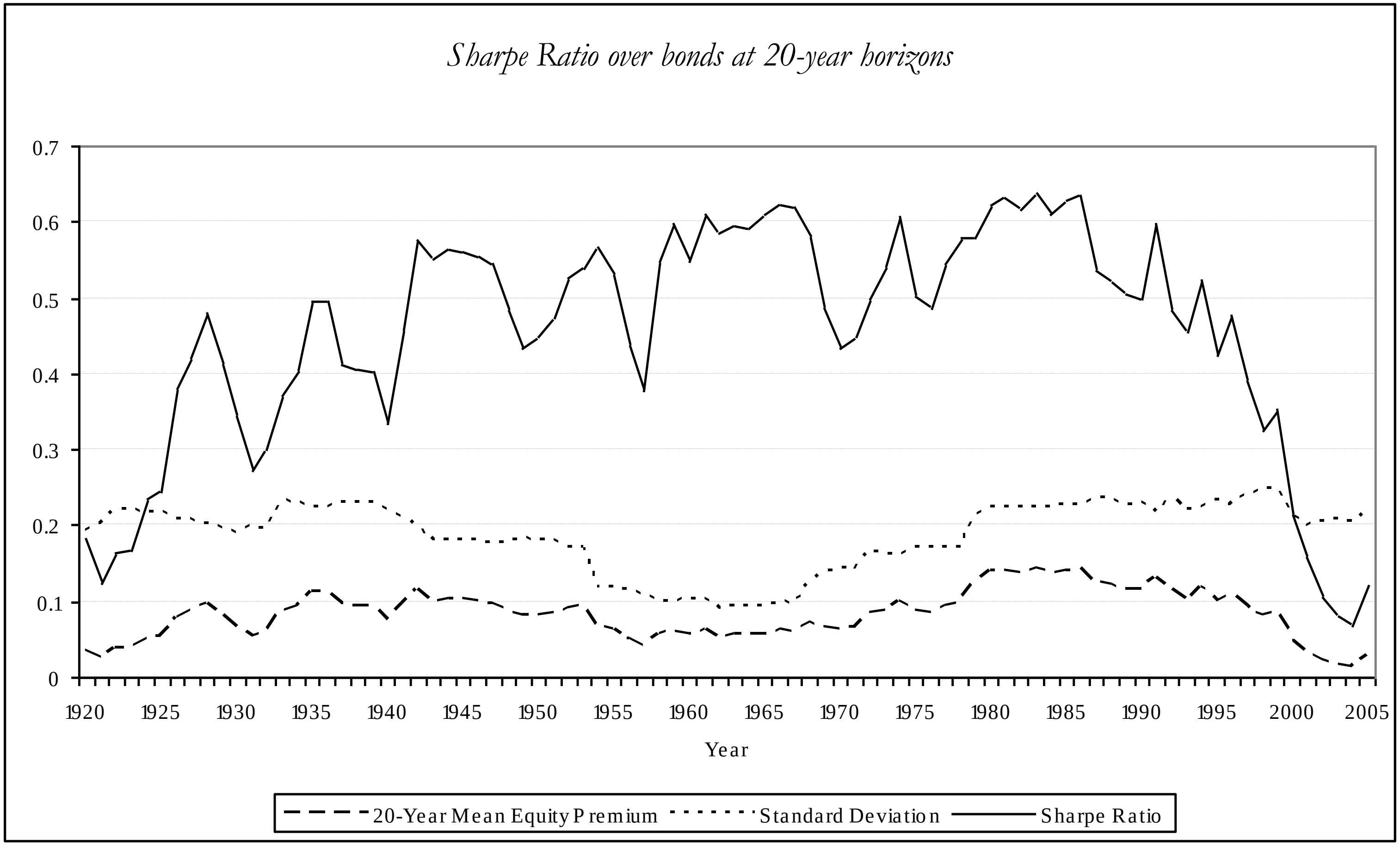
Coefficients of risk aversion to reconcile basic pricing equation with South African data

- With $\rho(R_M,x)=1$ need CRRA $\in\{20,22,17,20\}$

- Without $\rho(R_M,x)=1$ have

$$\ln \mathrm{E}(R_M) = \ln R_f + \alpha\sigma_{x,R_M}$$

- South Africa: $Cov(R_M,x)\approx 0$; need CRRA = 233!
- “Reasonable” CRRA: 3; upper bound: 10
- Sign/direction right; magnitude seriously wrong
- Future? $\rightarrow$



Sharpe Ratio over bonds at 20-year horizons
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1920
1925
1930
1935
1940
1945
1950
1955
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
Year
20-Year Mean Equity Premium
Standard Deviation
Sharpe Ratio

Sharpe Ratio Bounds for the JSE

Based and from Hassan and Wohlmann (in progress)

Aim

Obtain:

- restriction on set of (stochastic) discount factors that can price a given set of returns; and
- restriction on set of returns we will see given a specific (stochastic) discount factor

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21

Sharpe ratio

…From, $E_t[R(t+1)-R_f(t+1)] =$

$-Cov_t[m(t+1),R(t+1)] / E[m(t+1)]$

Use $\rho(m_{t+1}, R_{t+1}) = \text{cov}(m_{t+1}, R_{t+1}) / \sigma(m_{t+1})\sigma(R_{t+1})$

Have, $E_t(R_{t+1} - R_{t+1}^f) = -\dfrac{\rho(m_{t+1}, R_{t+1})\sigma(m_{t+1})\sigma(R_{t+1})}{E(m_{t+1})}$

or:

$$\frac{E_t(R_{t+1} - R_{t+1}^f)}{\sigma(R_{t+1})} = -\frac{\sigma(m_{t+1})}{E(m_{t+1})}\rho(m_{t+1}, R_{t+1})$$

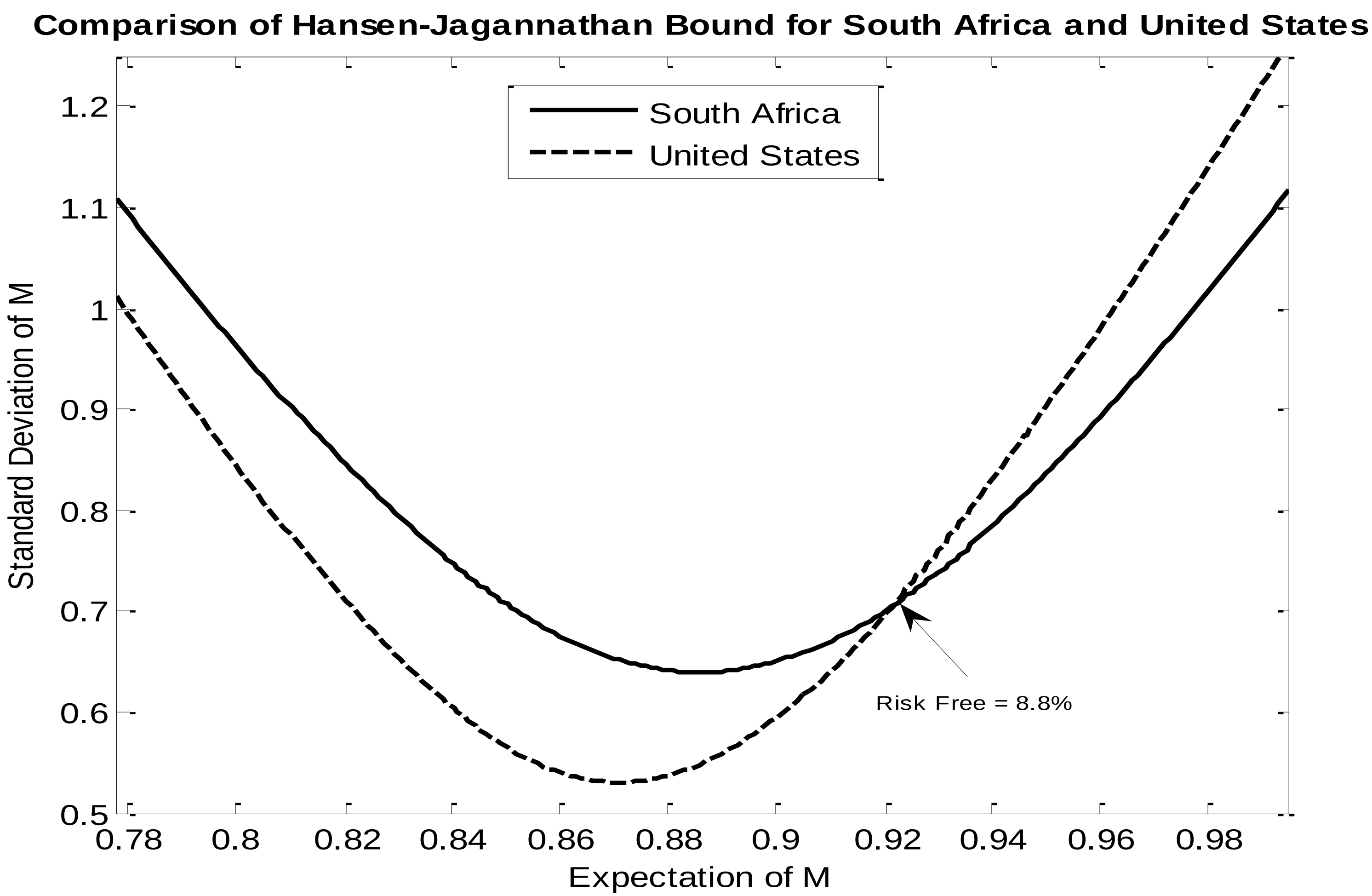
Hansen-Jagannathan bound

$$\rho(m_{t+1}, R_{t+1}) \in [-1,1]$$

Thus

$$\left| \frac{E_t(R_{t+1} - R_{t+1}^f)}{\sigma(R_{t+1})} \right| \leq \frac{\sigma(m_{t+1})}{E(m_{t+1})}$$

Estimated bound for South Africa



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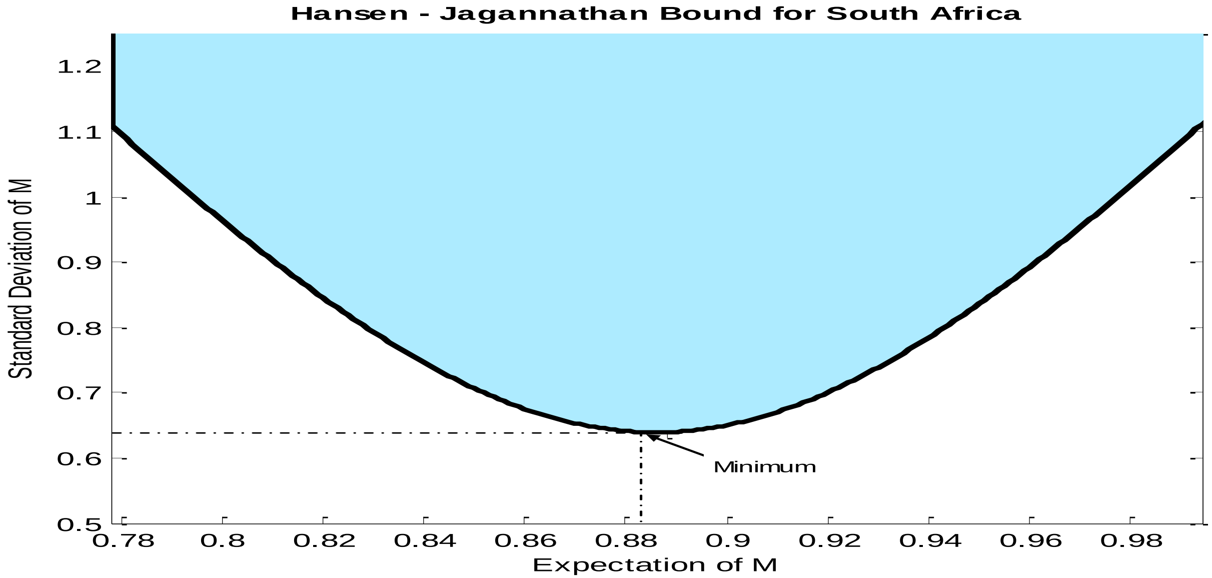
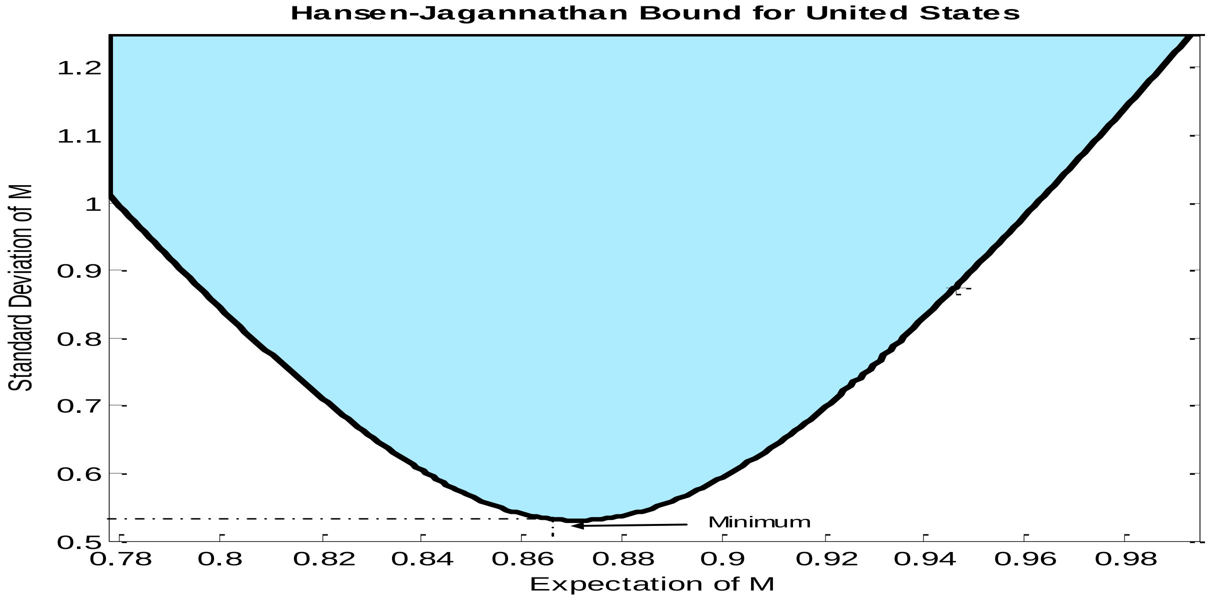

Hansen - Jagannathan Bound for South Africa
Standard Deviation of M
Expectation of M
Minimum
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0.78
0.8
0.82
0.84
0.86
0.88
0.9
0.92
0.94
0.96
0.98


Hansen-Jagannathan Bound for United States
Standard Deviation of M
Expectation of M
Minimum
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0.78
0.8
0.82
0.84
0.86
0.88
0.9
0.92
0.94
0.96
0.98

Savings

- Find valid specification of SDF
- H-J bound gives maximum S.Ratio can expect

$\Rightarrow$

- Input into long-term asset allocation decisions
- Bound on expected terminal value of South African savings