Presumptions and prior odds in merger decision rules

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Abstract

Merger decisions can be described as Bayesian decision rules, incorporating prior odds of mergers being anti-competitive. Courts rely on structural presumptions, based on concentration, to reflect prior odds. This involves an approximation of the true prior odds, which is subject to error, both substantively and in terms of measurement. The alternative – to normatively set the prior odds ratio to 1 – does not solve the problem. It introduces case-specific variance in priors, but, more important, it results in larger error in those merger cases likely to go to court.

JEL: *K21, L40*

Keywords: *Bayesian rules, decision theory, mergers, presumptions, prior odds*

1. Merger analysis as a decision rule

Merger analysis can be characterised as a classification problem, in which the analyst must classify a proposed merger as either pro-competitive or anti-competitive in effect. Given that most mergers involve a combination of pro- and anti-competitive effects, the analyst must judge the preponderance of evidence: that is, the analyst judges whether the anti-competitive harm associated with the merger (labelled H) exceeds the efficiency-related benefits of the merger (labelled B). Given that the case evidence (labelled e) is necessarily imperfect, this judgment is a problem of inference: the analyst must infer from the evidence whether it is likely that $H > B$. Following the literature, this burden of proof can be formulated in probabilistic terms. In particular, it involves a weighing of two probabilities, $P(B|e)$ and $P(H|e)$. The preponderance rule specifies that a merger must be judged anti-competitive if:

$$\frac{P(H|e)}{P(B|e)} > 1 \qquad (1)$$

Using the Bayesian Theorem, one can re-express the odds ratio:

$$\frac{P(H|e)}{P(B|e)} = \frac{P(e|H)}{P(e|B)}\frac{P(H)}{P(B)} = LR\frac{P(H)}{P(B)}$$

The relative odds of the merger being judged anti-competitive depend on the prior odds ratio ($\frac{P(H)}{P(B)}$) and the likelihood ratio ($LR = \frac{P(e|H)}{P(e|B)}$). Put differently, the preponderance rule (1) can be expressed as:

$$\frac{P(e|H)}{P(e|B)}\frac{P(H)}{P(B)} > 1$$

$$\therefore LR > \frac{P(B)}{P(H)} \qquad (2)$$

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A merger must therefore be judged as anti-competitive if the LR exceeds the prior odds ratio. A rational judgment of a merger must therefore not be based on likelihood only, but on a comparison of its size relative to prior odds. For example, the evidence from the case must sufficiently adjust the prior expectation that a merger is pro-competitive in the opposite direction, in order to lead to a decision of anti-competitive effect.

The rule in (2) can be further adjusted to reflect that a merger decision relies on a forward-looking assessment. Judgment errors in this forward-looking assessment carry social welfare costs, for which an optimal decision rule must account (Kaplow, 2014). The literature therefore introduces two types of error and error costs by relying on the following statistical formulation. A merger decision can be interpreted as statistical inference based on the following hypotheses[2]:

H_0: The merger is pro-competitive given e

H_a: The merger is anti-competitive given e

Inference is subject to Type I error, rejecting a pro-competitive merger (i.e. incorrectly rejecting H_0), and Type II error, accepting an anti-competitive merger (incorrectly 'accepting' H_0). Each error carries a particular social loss: the loss associated with Type I error is denoted L_1 and that associated with Type II error L_2. Each loss has an associated probability, allowing the calculation of expected losses as follows:

$$E(L_1|e) = L_1 P(B|e)$$

$$E(L_2|e) = L_2 P(H|e)$$

Again using the Bayesian Theorem, the optimal merger decision rule in (2) can be re-expressed as:

$$\frac{P(e|H)}{P(e|B)} > \frac{L_2}{L_1}\frac{P(B)}{P(H)}$$

$$\therefore LR > \frac{L_2}{L_1}\frac{P(B)}{P(H)} \qquad (3)$$

The relative welfare costs therefore alter the benchmark set by the prior odds ratio. For both the optimal rule in (3) and the preponderance rule in (2), the key result is that the prior odds ratio determines a presumption about the competitive effects of any one merger: the weight of evidence in any particular case must convincingly counter the presumption in order to be accepted. The paper assumes that the court adopts an equal loss ratio, in order to focus on the role of prior odds in merger decisions.

The prior odds ratio represents the total sum of knowledge from extant literature and case law on the competitive effects of a specific type of merger. Consequently, both proponents and opponents of such rules have highlighted the so-called practical challenge of evaluating prior probabilities, given their "high information requirements" (Kaplow, 2012). The challenge derives from the need to identify the appropriate reference class and then to summarize extant literature and case law as it pertains to the chosen reference class. Opponents of these rules therefore argue that decision rules incorporating prior odds ratios are neither fair (as the criteria for the selection of the appropriate reference class is not clear) nor practicable (given the large amount of data that the court will have to consult in any

[2] The choice of null hypothesis is not determinative and it is possible to derive the same conclusions by taking the anti-competitive hypothesis as the null.

particular case). The subsequent sections investigate the two main challenges, dealing first with the practical challenge of inferring a prior odds ratio from literature and second with the issue of selecting and relying on a reference class.

2. Inferring prior odds from a larger literature

A merger decision rule without an equal prior odds ratio does not avoid the practical challenge of consulting a large information set. Consider a version of the optimal rule in which the loss ratio and the prior odds ratio are set to one. That is, a merger is accepted if:

$$\frac{P(e|H)}{P(e|B)} > 1 \qquad (4)$$

The decision-maker has to evaluate only whether the case evidence are more likely to be associated with net harm than with net benefit. The court can assess these likelihoods only if it has a sufficient grasp of economic theory, prior research and case law. This required understanding of economic theory and literature can be better understood, if one explores how statisticians would implements the general decision rule derived above if they were to classify a merger. In the statistics literature, a decision rule for a particular dataset is arrived by assuming specific probability density functions $f_H(e)$ and $f_B(e)$ for $P(e|H)$ and $P(e|B)$ respectively. This changes the simple rule in (4) to:

$$\frac{f_H(e)}{f_B(e)} > 1 \qquad (5)$$

Statisticians estimate the parameters of $f_H(e)$ and $f_B(e)$ using data on mergers known to be either harmful or beneficial. This existing data is called the 'training' data. Akin to the statistical classification process, the court consults or 'learns' from other cases when it judges $P(e|H)$ and $P(e|H)$. It is true that legal decision rules do not make particular assumptions about the probability distribution. Put differently, we do not formalise how the court translates the evidence it observes into a judgment of probability. Yet the analogy highlights that the judging of probabilities for a particular merger case relies on an understanding of a larger body of previous cases that provide the 'training' data for the court. Consequently, it would be quite difficult to judge a merger only 'on truth' without reference to the existing body of literature. If merger analysis is a forward-looking exercise aimed at understanding economic effects, reference to the larger literature is unavoidable.

Nevertheless, the greater practical challenge with incorporating prior odds into legal decision rules is probably that of narrowing the information set to a specific reference class. The following section argues that courts have followed structural presumptions as a way of narrowing the set of information used in judging prior odds, while it also establishes a burden of proof that does not vary on a case-by-case basis.

3. Prior odds and merger presumptions

Cheng and Pardo (2015) argue that an important challenge facing the use of prior odds ratios in law is the selection of an appropriate reference class. The selection of the appropriate reference class is difficult and may vary on a case-by-case basis – a feature of decision rules that these authors prefer to avoid. The substantive issue of whether the burden of proof should vary in merger cases fall outside the scope of this paper[3]. Even so,

[3] One can argue that the greater reliance on economic analysis in antitrust supposes/requires a greater role for prior literature in guiding decisions (see Doane et al (2013) in the context of cartels and Cooper et al (2005) in the context of vertical restraints for a discussion of the need for adjusting prior odds assumptions depending on the findings in extant literature).

the following sections argue that the criticism of case-dependent priors is less forceful in the context of merger analysis.

2.1 *Presumptions as prior odds*

Traditionally, US merger guidelines feature structural presumptions, which can be interpreted as an attempt to incorporate prior odds into merger decisions (Salop 2014). The presumptions in merger law, by construction, restrict the amount of information used in arriving at the prior odds ratio to concentration-related information: the assumption is that the true prior odds ratio can be approximated by $\frac{P_C(B)}{P_C(H)}$, a ratio based on concentration (C):

$$\frac{P_C(B)}{P_C(H)} \xrightarrow{D} \frac{P(B)}{P(H)} \qquad (6)$$

This implies that the court relies on an alternative version of (3) to judge whether a merger should be rejected:

$$LR > \frac{L_2}{L_1}\frac{P_C(B)}{P_C(H)} \qquad (7)$$

2.2 *Error of approximating prior odds with structure-based prior odds*

The extent to (6) is satisfied can be explored by considering the following decomposition:

$$\frac{P_C(B)}{P_C(H)} = \frac{P(B)}{P(H)} + \left(\frac{P_C(B)}{P_C(H)} - \frac{P(B)}{P(H)}\right) = \alpha + \beta$$

where α is the 'true' prior odds ($\frac{P(B)}{P(H)}$) and β the error in the approximation of the true odds from prior odds suggested by the concentration literature and case law.

The condition in (6) requires $\beta \to 0$, which is not generally satisfied in literature: there is strong evidence that concentration is an imperfect predictor of competitive effects. One can interpret the widening of merger presumptions – to include structural features other than concentration – as an attempt to improve the approximation. Formally, this involves replacing $\frac{P(B)}{P(H)}$ with $\frac{P_{\boldsymbol{S}}(B)}{P_{\boldsymbol{S}}(H)}$, where $\boldsymbol{S}$ is a vector that includes C. The larger information set suggest an expectation that:

$$\frac{P_C(B)}{P_C(H)} - \frac{P(B)}{P(H)} > \frac{P_{\boldsymbol{S}}(B)}{P_{\boldsymbol{S}}(H)} - \frac{P(B)}{P(H)}$$

Put differently, expanding the presumptions to include a larger set of structural features reflects an assumption that the error of approximation will be reduced:

$$\alpha + \beta > \alpha + \beta_{\boldsymbol{S}}$$

Nevertheless, even $\beta_{\boldsymbol{S}}$ does not approach 0, which Salop (2014) argues explains the adoption of weaker structural presumptions.

2.3 *Measurement error for structure-based prior odds*

The assessment of $\frac{P_C(B)}{P_C(H)}$, and hence the presumption, depends on an *estimate* of concentration ($\hat{C}$):

$$\frac{\hat{P}_C(B)}{\hat{P}_C(H)} = \frac{P_{\hat{C}}(B)}{P_{\hat{C}}(H)}$$

The market definition literature emphasises that the delineation of markets is subject to significant uncertainty (Boshoff, 2015). This implies a potential error in a decision maker's estimate of C. One can therefore decompose the estimated concentration-based prior odds ratio as follows:

$$\frac{\hat{P}_C(B)}{\hat{P}_C(H)} = \frac{P_C(B)}{P_C(H)} + \left(\frac{P_{\hat{C}}(B)}{P_{\hat{C}}(H)} - \frac{P_C(B)}{P_C(H)}\right) = (\alpha + \beta) + \gamma$$

where γ is the measurement error of the prior odds due to measurement error of C.

The decomposition suggests that a correct assessment of the prior odds ratio is not only a function of how well concentration represents prior odds (β), but also a function of the court's ability to estimate concentration correctly (γ). This exposition suggests an additional explanation for weaker merger presumptions. Apart from developments in literature, a reliance on structural presumptions has also been tempered by a more realistic assessment of the court's ability to define markets. That is, the court assumes that $\gamma \neq 0$.

3. Structural presumptions during the investigation stage

Merger assessment can be split into an investigation phase – during which enforcement agencies analyse a proposed merger – and an adjudication phase, during which courts judge the merger. Enforcement agencies may adopt presumptions in order to prioritise resources for more effective antitrust policy. These presumptions enable agencies to construct first screen tests to identify cases deserving closer scrutiny. Typically, such screens – similar to the legal presumptions – relate to structural aspects of the market.

There is no particular requirement for enforcement presumptions to reflect legal presumptions in court. Enforcement presumptions should reflect the aims of antitrust authorities, which may include – but should not be limited to – promoting the administration of justice via simple structural presumptions. Nevertheless, enforcement agencies must consider the likelihood of a merger challenge succeeding in court, which implies that they should consider the extent to which structural presumptions are at play in the court's decision. It is possible to adopt a broader prior odds ratio that more closely reflects policy aims and that is consistent with structural presumptions.

Concentration measures are based on market shares, which are calculated after a relevant market has been defined. Market definition requires an analysis, however cursory, of current and potential substitutes for the merging parties' products. If the enforcement agency is in any event required to conduct an analysis of substitutability in order to assess concentration, there would be no loss if it adopted a prior odds ratio based on substitutability. Research and case law is clearer in their prediction of the effects of a merger in a market characterized by limited substitutability than in their prediction of merger effects in the face of high market concentration. It therefore offers a superior method for deciding which mergers require further scrutiny and which must ultimately be challenged in court. Social welfare is thereby increased by a better allocation of scarce resources at agencies. Furthermore, it is still possible to calculate concentration following the assessment of substitutability.

This view of enforcement presumptions effectively sees the concentration measure as a summary measure aiming to capture the insights gained from an assessment of the prior literature based on limited substitutability. The use of an imperfect summary measure such as concentration is attributable to the need for easily quantifiable and defensible measures.

In this view, weaker structural presumptions indicate an appreciation of the imprecise link between the underlying substitution analysis (which is performed by the enforcement agencies) and the summary variable (concentration). Weaker structural presumptions do not indicate that courts are shifting away, or should shift away, from prior probabilities in their judgment of mergers. Instead, weaker presumptions reflect the search for better summary variables.

4. Equal prior odds do not reduce errors in judgment

Opponents of prior odds ratios would maintain that, even if they could be useful in an enforcement context, legal fairness requires the court to set the prior odds ratio to 1: if structural characteristics do not allow an accurate approximation of the true prior odds ratio, relying on these features introduces bias into the legal process. According to this argument, equal prior odds are more acceptable, as it does not prejudice the decision based on prior evidence. There are two problems with an equal prior odds assumption: firstly, it implies a case-by-case adjustment of the true prior odds ratio to 1 and, secondly, it results in greater error in estimating prior odds in those cases that are most likely to come to courts.

4.1 *Equal prior odds introduce case-by-case variation*

Critics argue that prior odds ratios in merger decisions introduce case-specificity, by implying varying burdens of proof. We noted earlier that we do not necessarily consider varying burdens of proof problematic. Yet, in the case of merger analysis, it is important to note that the use of structural presumptions result in a quite broad reference class for calculating prior odds. Structural presumptions in merger law are formulated in terms of general concentration benchmarks and do not vary by industry or market.

Normatively restricting the prior odds ratio to unity, as is often suggested in the literature criticising prior odds, will remove the generality and result in a case-specific criterion. Setting the prior odds ratio equal to 1 requires scaling the prior odds ratio to 1:

$$\frac{P(B)}{P(H)} + \delta = 1$$

$$\therefore \delta = 1 - \alpha$$

The scaling term δ may involve either an increase or a decrease in the prior odds ratio, depending on the size of α: when $\alpha > 1$, $\delta < 0$ and when $\alpha < 1$, $\delta > 0$. Consequently, the scaling term is case-specific, as α depends on the concentration calculation for the case.

4.2 *Equal prior increase the likelihood of allowing anti-competitive mergers*

A scaling of prior odds ratio to unity relies on the assumption that the true prior odds ratio is 1. If the true prior odds ratio α does not equal 1, the scaling is subject to error. An equal prior odds ratio implies a more accurate prediction of the true prior odds ratio, only if the expected error of scaling is smaller than the expected error $\beta + \gamma$ previously explored:

$$1 - \alpha < E(\beta + \gamma) \qquad (7)$$

It is necessary to consider the expected error $E(\beta + \gamma)$ in (7), rather than the values of $\beta + \gamma$ for a particular case, given that a prior odds ratio of unity is a general restriction applied to all cases.

The market definition literature highlights the significant uncertainty associated with market definition. However, there is no conclusive evidence in the literature that markets are systematically defined as either overly broad or overly narrow. Consequently, there is no basis for arguing that concentration is systematically under-estimated or over-estimated. It is therefore appropriate to assume that $E(\gamma) = 0$, if we are to derive a general condition representative of the larger literature. The condition in (7) can then be reformulated as:

$$\alpha + E(\beta) > 1 \qquad (8)$$

The recent literature suggests that merger decisions that are based on concentration are subject to significant Type-I error. The simulation evidence in Foncel, Ivaldi and Khimich (2013) suggest that the probability of incorrectly rejecting a pro-competitive merger (i.e. when, in truth, $\alpha > 1$) could approximate 0.6, when relying on the HHI change due to the merger. In contrast, the probability of incorrectly accepting an anti-competitive merger (i.e. when, in truth, $\alpha < 1$) could approximate only 0.03. The recent work on so-called 'first-order' approaches to merger analysis suggest that simulation evidence can be subject to significant bias, due to incorrect specification of demand functions. Yet, as a device to explore hypothetical cases, simulation modelling of the type conducted by Foncel, Ivaldi and Khimich (2013) can be useful.

The implications from this literature for an assessment of (8) can be summarised as follows. Consider the case where the true prior odds of the merger being anti-competitive are large ($\alpha < 1$). The literature suggest that $E(\beta) = 0$. In this case it is impossible to satisfy the condition $\alpha + E(\beta) > 1$. In other words, in those cases where an evaluation of the prior odds will lead an enforcement agency to oppose a merger, the court's decision to rely on an equal prior odds ratio will result in the merger decision being subject to a greater average error. It is clear that this is because the scaling term is positive: when the prior odds are set to unity, it is far easier for merging parties to succeed in obtaining approval, than it would be to convince the court that the LR exceeds prior odds pointing to harm.

In the case where the true prior odds of the merger being anti-competitive are small ($\alpha > 1$), the condition in (8) has a probability of being satisfied. A conservative interpretation of the literature mentioned above would suggest that:

$$P(\alpha + \beta < 1) = P(\alpha + \beta > 1) \text{ when } \alpha > 1$$

In other words, an equal prior odds ratio *may* lead to fewer errors in those cases where the prior odds suggest that the merger is pro-competitive. Yet these are also the cases that are less likely to be pursued by the enforcement agencies, which, as we argued earlier, do not rely solely on concentration measures in deciding whether to oppose mergers.

5. **Conclusions**

Bayesian decision rules, featuring prior odds, have been developed for a variety of antitrust areas, including vertical restraints, cartels and merger analysis. It is useful to investigate how antitrust authorities have implemented prior odds in antitrust. In particular, this paper investigates how prior-odds ratios are incorporated into merger decision rules via structural presumptions. Critics and proponents alike appear to hold that prior odds introduce a significant additional analytical burden on competition authorities. The paper questions that position. Even so, it is important to explore the extent to which a concentration-based prior-odds ratio approximates the true prior-odds ratio. In addition, the paper also explores the role of measurement error, given that structural presumptions rely on the definition of the relevant market. Despite the identified errors of approximation, the paper holds that it is not appropriate to set the prior-odds ratio to 1 for all cases. In particular, for those merger cases likely to go to court, it is likely that setting the ratio to unity will involve a greater error than relying on concentration-based measures.

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