Ehrlich-Becker Banking with Financial Intermediation and Systemic Risk

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Abstract

The paper is an extension of Arrow-Debreu theory with costly transfer across states. It extends the Ehrlich and Becker (1972) two contingent-state economy by adding in the financial intermediation that is able to transfer income across uncertain good and bad states of nature. There is a cost in transferring resources across states. This cost is modeled using a financial intermediation production function that is found in the banking literature. Instead of being able to costlessly transfer income between states as in the complete markets of the Arrow-Debreu value theory under uncertainty, now let the cost be given by the financial intermediation of the banking industry and let the states take place at the current time as in Ehrlich and Becker (1972). It reduces down to perfect consumption smoothing as a special case but has consumption tilting towards the good state when there is a positive cost of transferring income. Markets are incomplete by degree as determined by the transfer cost. Aggregate risk can be discussed in this environment from a drop in the productivity of the financial intermediary. Risk-based deposit insurance for financial intermediaries is discussed as the main application.

- Keywords: Arrow-Debreu, good and bad state, insurance, financial intermediation productivity
- JEL: G01, G2, E44

1 Introduction

Many partial equilibrium approaches to banking have been used, typically specifying a convex cost function in order to get an upward sloping marginal cost of banking (eg. Berk and Green, 2004, Wang et al.,2009). A way to get a general equilibrium solution is to follow the banking microeconomic literature (see Inklaar and Wang, 2013). Sealey and Lindley (1977) specified a production function for financial intermediation services, followed by a particular constant-returns-to-scale function by Clark (1984). Clark made labor and capital inputs but also crucially a third factor, the amount of deposited funds. These deposits can be thought of as financial capital rather than physical capital. Hancock (1985) estimated the Clark function and found that data supported its particular CRS specification in labor, capital and deposits. Since then, this function has been at the basis of standard banking estimated functions, and it is known as the "financial intermediation approach" to modeling banking.

The financial intermediation approach of Clark has been used to model exchange credit and in the process solve the King and Plosser (1984) problem of finding a unique equilibrium between the money and exchange credit. It does this because an endogenous upward sloping marginal cost of credit results, from the duality of specifying the production function. Specification of an exogenous convex cost function is avoided as this is done instead through specification of the CRS production function. Having deposits as a factor is the key to the success of this approach.

To get the intuition for the scope of this approach, of which the provision of exchange credit in monetary economies is only a particular case, consider when the financial intermediation is for the transference of income across uncertain good and bad states of nature. Instead of being able to costlessly transfer income between states as in the complete markets of the Arrow-Debreu value theory under uncertainty, now let the cost be given by the financial intermediation of the banking industry and let the transfer across states take place in the same time period as in Ehrlich and Becker (1972).

Typically consumption "tilting" results by including incentive compatibility constraints to help bridge markets that are no longer "complete". The degree of this incompleteness can be marginally measured in the particular economy for example when there is a degree of collateral collection that can approach 100% in the sovereign debt literature. Here a more general abstraction is that these contract enforcement costs, or any other such costs, are summarized in the technology for transferring the income across state. This will also give a degree of consumption tilting that depends on the parameters of the production technology. The transference of deposits across uncertain states is considered presented with a view towards the policy application of risk-based premiums for systemic financial intermediation insurance.

2 Complete Markets

Consider an endowment economy that exists for two periods (labelled 1 and 2). We assume on date 2 a two state analysis with a good and bad state, with probabilities of the states adding to one;

$$p_g + p_b = 1. \tag{1}$$

The representative agent has known first-period income y_1 and uncertain output on date 2 with the endowments $y_2(s)$, $s = g, b$, with $y_2(g) > y_2(b)$. Let c_1 denote consumption on date 1, which must be chosen before a desired two-state contingency consumption plan is resolved for date 2, which we denote by $c_2(s)$, $s = g, b$.

Our analysis under complete markets is such that on date 1 the agent is allowed to buy or sell a complete set of Arrow-Debreu securities with the following payoff structure: the owner (seller) of the security receives (pays) 1 unit of output on date 2 if state s occurs then, but receives (pays) nothing if the other state occurs. Let $B_2(s)$ be the representative individual's net purchase of state s Arrow-Debreu securities on date 1. Let $price(s)/(1+r)$ denote the price, quoted in terms of date 1 consumption, of one of these securities, that is, of a claim to one output unit to be delivered on date 2 if, and only if, state s occurs. Thus $price(s)$ is the price of Arrow-Debreu security, quoted in terms of date 2 consumption unit which delivers one unit of output only conditional on state s occurring.

Banking insurance across states of nature takes place on date 2 only. Good state consumption $c_2(g)$ is lessened by the deposit of funds d for transference to the bad state should that state occur. Now let the payout in the bad state be given as $d\left(1 + R^d\right)$, so that R^d notates the "return" on the deposits d. For a simple insurance example, which is what state uncertainty can be viewed as, the amount d is the "premium" paid in during the good state and the amount $d\left(1 + R^d\right)$ is the insurance coverage paid out in the bad state. Alternatively, it could be stated in terms of a insurance payout of "y", so that prices such as R^d are not used. And another alternative is to let the pay in be for example $p_y y$ and the payout y, so that p_y is the price per unit of insurance coverage. In terms of the $1 + R^d$ notation, this latter case would be such that $1 + R^d = \frac{1}{p_y}$, and so these approaches are equivalent.

As usual in an endowment economy, the value of a country's net accumulation of assets on date 1 must equal the difference between its income and consumption:

$$\frac{price(g)}{1+r} B_2(g) + \frac{price(b)}{1+r} B_2(b) = y_1 - c_1. \tag{2}$$

When date 2 arrives, the state of nature s is observed, and the country will be able to consume the sum of its endowment and any payments on its state s contingent assets and payout of the banking insurance. With d invested in the good state in order to receive $d\left(1 + R^d\right)$ in the bad state, the contingent state budget constraints in period 2 are

$$c_2(g) + d = y_2(g) + B_2(g). \tag{3}$$

$$c_2(b) = d\left(1 + R^d\right) + y_2(b) + B_2(b). \tag{4}$$

To highlight the "actuarially-fair" price of transferring income across states for the bank, consider a probabilistic combination of period 2 contingent state budget constraints into a type of social resource constraint, with expected consumption equal to expected income. Then

$$p_g c_g + p_b c_b = p_g\left[y_2(g) + B_2(g)\right] + p_b\left[y_2(b) + B_2(b)\right], \tag{5}$$

and substituting in from the budget constraints,

$$\begin{aligned} & p_g\left[y_2(g) - d + B_2(g)\right] + p_b\left[d\left(1 + R^d\right) + y_2(b) + B_2(b)\right] && (6)\\ = \;& p_g\left[y_2(g) + B_2(g)\right] + p_b\left[y_2(b) + B_2(b)\right]. && (7)\end{aligned}$$

This representation implies that the expected insurance funds equals the expected payout, as would be the case with a competitive insurance agent that provided this service :

$$p_g d = p_b d\left(1 + R^d\right).$$

More simply the gross return is just the ratio of probabilities

$$\left(1 + R^d\right) = \frac{p_g}{p_b},$$

and this is the standard Arrow-Debreu competition result for the "actuarially-fair" price of transferring income across states. The cost of income in the bad state, or $\frac{1}{(1+R^d)}$, is the ratio of the bad state probability to the good state probability. And this likewise means that the return on an investment in the good state for delivery in the bad state is $1 + R^d = \frac{p_g}{p_b}$.

Using the two equations of the contingent state budget constraints in period 2 to eliminate the $B_2(g)$ and $B_2(b)$ in the asset accumulation identity on date 1, we derive the intertemporal budget constraint for this banking economy:

$$\begin{aligned} c_1 + \frac{price(g)c_2(g)}{1+r} + \frac{price(b)c_2(b)}{1+r} &= y_1 + \frac{price(g)\left[y_2(g) - d\right]}{1+r} + && (8)\\ & \quad \frac{price(b)\left[y_2(b) + d\left(1 + R^d\right)\right]}{1+r}. && (9)\end{aligned}$$

Combining the ability to transfer the income across states in this fashion with expected utility optimization gives the standard consumption smoothing results. The individual's lifetime expected utility on date 1 is

$$\begin{aligned} U_1 &= p_g\left\{u(c_1) + \beta u\left[c_2(g)\right]\right\} + p_b\left\{u(c_1) + \beta u\left[c_2(b)\right]\right\} && (10)\\ U_1 &= u(c_1) + \sum_{s=g,b} p_s \beta u\left[c_2(s)\right]. && (11)\end{aligned}$$

Substituting in from the asset accumulation identity and the contingent state budgets, the representative agent problem is

$$\underset{B_2(s),d}{Max}\ U_1 = u\left(y_1 - \frac{price(g)}{1+r}B_2(g) - \frac{price(b)}{1+r}B_2(b)\right) \quad (12)$$

$$+\left\{p_g\beta u\left[y_2(g)+B_2(g)-d\right]+p_b\beta u\left[y_2(b)+d\left(1+R^d\right)+B_2(b)\right]\right\} \quad (13)$$

The necessary first-order conditions are

$$\frac{price(s)}{1+r}\frac{\partial u(c_1)}{\partial c_1} = p_s\beta\frac{\partial u\left[c_2(s)\right]}{\partial c_2(s)},\ \ s=g,b. \quad (14)$$

The above equation is the related intertemporal Euler equation, now pertaining to an Arrow-Debreu security rather than to a riskless bond. The left-hand side is the cost, in terms of date 1 marginal utility, of acquiring the Arrow-Debreu security for date s. And the right-hand side is the expected discounted benefit from having an additional unit of consumption in state s on date 2. The equation can also be re-arranged to obtain the usual price equilibrium to show that the marginal rate of substitution between c_1 and $c_2(s)$ is equal to the two goods' relative prices:

$$\frac{p_s\beta\frac{\partial u[c_2(s)]}{\partial c_2(s)}}{\frac{\partial u(c_1)}{\partial c_1}} = \frac{price(s)}{1+r},\ \ s=g,b. \quad (15)$$

An implication of the above equation is

$$\frac{p_b\frac{\partial u[c_2(b)]}{\partial c_2(b)}}{p_g\frac{\partial u[c_2(g)]}{\partial c_2(g)}} = \frac{price(b)}{price(g)}. \quad (16)$$

That is, the marginal rate of substitution of the good state for the bad state consumption must equal the relative price of the bad state consumption in terms of the good state consumption. Only when

$$\frac{price(b)}{price(g)} = \frac{p_b}{p_g}. \quad (17)$$

does the above equation imply that $c_2(b) = c_2(g)$, so that it is optimal to equate consumption in different states of nature, in this case, the Arrow-Debreu security prices are actuarially fair. At actuarially fair prices, a country trading in complete asset markets will fully insure against all future consumption fluctuations.

The equilibrium for d, the bank deposits across states of nature is

$$p_g\beta\frac{\partial u\left[c_2(g)\right]}{\partial c_2(g)}(-1) + p_b\beta\frac{\partial u\left[c_2(b)\right]}{\partial c_2(b)}\left(1+R^d\right) = 0. \quad (18)$$

The probability-weighted marginal rate of substitution across states to the marginal product of the deposits:

$$\frac{p_b \frac{\partial u[c_2(b)]}{\partial c_2(b)}}{p_g \frac{\partial u[c_2(g)]}{\partial c_2(g)}} = \frac{1}{(1+R^d)}. \tag{19}$$

Now it is necessary to determine $1 + R^d$. And if given by the above zero profit condition, so that it is the ratio of probabilities, then

$$\frac{p_b \frac{\partial u[c_2(b)]}{\partial c_2(b)}}{p_g \frac{\partial u[c_2(g)]}{\partial c_2(g)}} = \frac{1}{(1+R^d)} = \frac{p_b}{p_g}.$$

And then the consumption smoothing result occurs.

$$\frac{\partial u\left[c_2(b)\right]}{\partial c_2(b)} = \frac{\partial u\left[c_2(g)\right]}{\partial c_2(g)}, \tag{20}$$

and

$$c_2(b) = c_2(g).$$

This standard result depends on the nature of the "insurance agent" problem.

2.1 Incomplete Markets and implication of Bank's costly transfer

Expanding the horizon of this framework, the insurer can be a separate firm problem that can involve costs. If there are costs, then perfect consumption smoothing will not result. First consider when consumption "tilting" typically occurs.

Typically consumption "tilting" results by including incentive compatibility constraints to help bridge markets that are no longer "complete". For example the degree of this incompleteness can be marginally measured in a particular economy when there is a degree of collateral collection that can approach 100% in the sovereign debt literature. Here a more general abstraction is that these contract enforcement costs, or any other such costs, are summarized in the technology for transferring the income across state.

What is missing is a technology for transferring income across states. Incentive compatibility constraints are a way to implicitly assume a technology. Incomplete markets from imperfect collateral is a type of technology of the cost of transferring income across whatever space is being spanned by the particular contract.

One more general approach is to assume a technology that encompasses measurable labor and capital costs as in the actual industries that exist in market economies specifically for the purpose of providing such transfer of income across contingent states, time, and space. The financial intermediary industry is the industry that actually does such transfers in the broad sense. Therefore using the production technology that is found in the financial intermediary sector is a straightforward way to include the cost of transferring income across contingent states.

The trick to specifying the production technology for income transfer is that it must contain the deposits, d notationally, as an factor input. To see this, first a linear technology will be used. Rather than it being linear in either labor or physical capital, the key is that it is linear in deposits d, which could be called financial capital. Perhaps surprisingly, the issue of how to form the linear form of this production function has roots deep in controversy and understandable contradiction in the literature.

Under the case that insurance is actuarially not fair, i.e., if there are costs, then usually the consumer consumes more in the good state. Under this scenario, the Arrow-Debreu securities' prices imply that

$$price(b) > p_b. \tag{21}$$

and

$$price(g) < p_g. \tag{22}$$

henceforth,

$$\frac{price(b)}{price(g)} > \frac{p_b}{p_g}. \tag{23}$$

The implication for costly banking thus would imply that

$$\frac{1}{(1+R^d)} > \frac{p_b}{p_g}, \tag{24}$$

meaning that the marginal product of deposits has effectively fallen.

2.2 Example 1. Linear Technology with Full Insurance

Now let $A_Q = 1$, $p_g = 0.8$, $p_b = 0.2$, $y_1 = 1$, $y_2(g) = 1$, $y_2(b) = 0.3$. Assuming that the time preference rate is equal to the riskless real interest rate $\rho = r = 0.1$, then under actuarially fair insurance, consumption across both time and states of nature will equalize if we assume utility to be log-utility for instance:

$$c = c_1 = c_2(g) = c_2(b) \tag{25}$$

Also $price(g) = p_g$, $price(b) = p_b$ and $1 + R^d = \frac{p_g}{p_b} = \frac{0.8}{0.2} = 4$

Using the intertemporal budget constraint above:

$$
\begin{aligned}
c_1 + \frac{price(g) c_2(g)}{1+r} + \frac{price(b) c_2(b)}{1+r} &= y_1 + \frac{price(g)\left[y_2(g) - d\right]}{1+r} && (26)\\
&\quad + \frac{price(b)\left[y_2(b) + d\left(1+R^d\right)\right]}{1+r} && (27)\\
c_1 + \frac{0.8 c_2(g)}{1.1} + \frac{0.2 c_2(b)}{1.1} &= 1 + \frac{0.8\left[1-d\right]}{1.1} + \frac{0.2\left[0.3+4d\right]}{1.1} && (28)\\
[1 + \frac{0.8}{1.1} + \frac{0.2}{1.1}]c &= 1 + \frac{1.4}{1.1} && (29)\\
c &= 1.1905 && (30)
\end{aligned}
$$

The intertemporal budget constraint above shows us that we can draw a graph in $(c_2(b), c_2(g))$ space and the slope of the budget constraint is 4, so that the transfer of resources across states of nature is done at this actuarially fair rate of the ratio of state probabilities.

Then from writing the above intertemporal budget constraint in terms of the set of Arrow-Debreu securities,

$$
\begin{aligned}
\frac{price(g)}{1+r} B_2(g) + \frac{price(b)}{1+r} B_2(b) &= y_1 - c_1 && (31)\\
\frac{0.8 B_2(g) + 0.2 B_2(b)}{1.1} &= -0.1905 && (32)\\
0.8 B_2(g) + 0.2 B_2(b) &= -0.2095 && (33)
\end{aligned}
$$

Then from the two states budget constraints together with the condition that $c_2(b) = A_Q c_2(g)$, where $A_Q = 1$:

$$
\begin{aligned}
c_2(b) &= d\left(1+R^d\right) + y_2(b) + B_2(b) && (34)\\
d\left(1+R^d\right) &= c_2(b) - y_2(b) - B_2(b) && (35)\\
d\left(1+R^d\right) &= A_Q c_2(g) - y_2(b) - B_2(b) && (36)\\
d\left(1+R^d\right) &= A_Q\left[y_2(g) + B_2(g) - d\right] - y_2(b) - B_2(b) && (37)\\
d &= \frac{A_Q\left[y_2(g) + B_2(g)\right] - y_2(b) - B_2(b)}{A_Q\left(1 + \frac{p_g}{p_b}\right)} && (38)
\end{aligned}
$$

Reducing the above equation to one with the three unknown variables $B_2(g)$, $B_2(b)$ and d:

$$
d = \frac{0.7}{5} + \frac{1}{5} B_2(g) - \frac{1}{5} B_2(b) \qquad (39)
$$

Further, if we reduce the two states budget constraints to an equation with only deposits (d) and Arrow-Debreu securities ($B_2(s)$),

$$c_2(b) = d\left(1+R^d\right) + y_2(b) + B_2(b) \tag{40}$$
$$1.1905 = 4d + 0.3 + B_2(b) \tag{41}$$
$$4d = 1.1905 - 0.3 - B_2(b) \tag{42}$$
$$d = 0.2226 - 0.25B_2(b) \tag{43}$$

and

$$c_2(g) = y_2(g) - d + B_2(g) \tag{44}$$
$$1.1905 = 1 - d + B_2(g) \tag{45}$$
$$d = -0.1905 + B_2(b) \tag{46}$$

Then combining any of either $d = 0.2226 - 0.25B_2(b)$ or $d = -0.1905 + B_2(b)$ into the above $d = \frac{0.7}{5} + \frac{1}{5}B_2(g) - \frac{1}{5}B_2(b)$, for instance substituting $d = -0.1905 + B_2(b)$ into $d = \frac{0.7}{5} + \frac{1}{5}B_2(g) - \frac{1}{5}B_2(b)$ yields:

$$0.8B_2(g) + 0.2B_2(b) = 0.3305 \tag{47}$$

To be completed.

3 Production Technology of Contingent State Transfer

The natural inclination in writing down a production function for any industry is to say the output q is a function of labor l and physical capital k, and often in a Cobb-Douglas fashion. For example, with $\gamma \in (0,1)$, let

$$q = Al^{\gamma}k^{1-\gamma}, \tag{48}$$

where the output is the amount of funds that are available for transfer across contingent state, before any costs incurred in the production process are subtracted. And this approach is actually used in some of the banking literature, and is known as the "production approach to banking" (Matthews, 2008).

While seeming innocuous and accurate in a neoclassical economic or "real business cycle" sense, it cannot replicate the perfect consumption smoothing result in the Arrow-Debreu contingent-state equilibrium using a production technology. This assumes only that physical capital is accumulated in the usual way, that output can be divided into physical capital and consumption as usual, and that output itself is produced with a Cobb-Douglas production function of labor and capital. But the result that it cannot produce Arrow-Debreu as a special case takes some work to show, and requires the more general approach to be introduced.

The other approach that has now been around for at least 25 years, depending on how it is counted, is to include the deposits d as the "third factor" of production. Keeping a Cobb-Douglas form, now specify the output q as

$$q = A_Q (l_Q)^{\gamma_1} (k_Q)^{\gamma_2} d^{1-\gamma_1-\gamma_2}. \tag{49}$$

Here $A_Q > 0$, with l_Q denoting the labor time in production of q, with k_Q denoting the capital used in production of q, and with $\gamma_1 \in (0,1)$, $\gamma_2 \in (0,1)$, and $\gamma_1 + \gamma_2 < 1$.

Now consider using this production function to reproduce the Arrow-Debreu perfect consumption smoothing result. There is a way to do this. In the case with $\gamma_1 = \gamma_2 = 0$, so that no labor or physical capital is used, let $A_Q = 1$. Then $q = d$, a special case involving no costs in the production process.

Consider the opposite extreme, when $\gamma_1 + \gamma_2 = 1$. Then the production function can still be expressed per unit of deposits, as

$$\frac{q}{d} = A_Q \left(\frac{l_Q}{d}\right)^{\gamma_1} \left(\frac{k_Q}{d}\right)^{\gamma_2}, \tag{50}$$

but now this is identical to

$$q = A_Q (l_Q)^{\gamma_1} (k_Q)^{\gamma_2}. \tag{51}$$

There is no return to deposits, as all of the profit from selling the financial services gets used up in labor and physical capital costs. This means in effect that no matter what financial service, such as a simple investment of savings in government bonds for the consumer, the whole of the revenue of the service is used up in the cost of labor and capital. For bond sales, then the revenue is the interest on the government bonds and none of this interest will go back to the consumer since it will be used up in the cost of providing the service.

The case with $\gamma_1 + \gamma_2 = 1$ is an extreme way to model financial intermediation. It assumes that no funds are being ultimately intermediated in that the cost of the intermediation uses up all of the funds. Rather, not only is $\gamma_1 + \gamma_2 < 1$ more plausible, so that some funds are being intermediated in the end, but further $\gamma_1 + \gamma_2$ is much closer to zero than to one in general. In calibrating $\gamma_1 + \gamma_2$, we calculated in Benk et al. (2008) for example a $\gamma_1 + \gamma_2$ in the range of 0.10 to 0.20. It could be even smaller, or larger, depending on the type of intermediation service.

The more realistic limiting case is not that $\gamma_1 + \gamma_2 = 1$, but rather that $\gamma_1 + \gamma_2 = 0$, and only the deposits are used without any labor or physical capital cost. This gives a production function linear in deposits, of $q = A_Q d$, which transfers funds one for one if $A_Q = 1$. In general, it requires labor and physical capital and this means there are more cost and the output is not as high when, $\gamma_1 + \gamma_2 \in (0,1)$ as compared to when $\gamma_1 + \gamma_2 = 0$, for a given A_Q.

The unfortunate fact that there are industries in financial intermediation services that require people, training, buildings, and information technology all make $\gamma_1 + \gamma_2 > 0$, but hopefully not too high. And certainly competition and economies of scale will tend to drive $\gamma_1 + \gamma_2$ down.

3.1 Cost Implications

The production function can also be viewed in terms of the amount of services per unit of deposits, or $\frac{q}{d}$. Here the per unit services are found by dividing through equation (49) by d :

$$\frac{q}{d} = A_Q \left(\frac{l_Q}{d}\right)^{\gamma_1} \left(\frac{k_Q}{d}\right)^{\gamma_2}.$$

The services per unit of deposits are then a function with a scale of less than one, equal to $\gamma_1 + \gamma_2$. The importance of this is that this leads to the key feature that costs are convex, per unit of deposits.

If the comparable good that the financial service has a constant marginal cost per unit, then the upward sloping marginal cost of financial services per unit will have an equilibrium with the constant marginal cost per unit of the alternative. In the case of money, the marginal cost of money is the nominal interest rate R. This is a cost per unit of money, and so it is compared in equilibrium by the consumer to the cost of financial services per unit of deposits.

The per unit perspective gives the key reason of why the third factor, d, is so crucial in the production function. Only with this deposit input entering production, and so mathematically with $\gamma_1 + \gamma_2 < 1$, can the per unit cost of deposits exhibit an upward sloping marginal cost per unit of deposits in general equilibrium. And the factors of production then are the labor and physical capital per unit of deposits, $\frac{l_Q}{d}$ and $\frac{k_Q}{d}$.

Another way to view a financial services production function with CRS in labor and physical capital is the same as any such output function. It is well known that goods output using a function that is CRS in labor and physical capital has a horizontal marginal cost curve. Marginal and average cost is not upward sloping, but rather constant at some level.

For financial services, the per unit cost of equation (48) has a constant marginal cost just as does the function of equation (51). There is no convex cost curve for financial services in such a case. Yet a convex cost curve is apparently always required for a unique equilibrium across the array of financial services that have been studied.

For money demand, if the competing private financial service were exchange credit, and the exchange credit had a constant marginal cost per unit, then there would be no unique equilibrium between money and credit. And this is exactly the point made by King and Plosser (1984).

3.2 Net Transfer and Cost

Production is a constant returns to scale function with the share of labor in income given by γ_1, the share of capital in income given by γ_2, and the share of deposits the remainder of $1 - \gamma_1 - \gamma_2$. The payment of the return to deposits is the payment of the residual profit after labor and physical capital costs are subtracted from revenue. There are diminishing returns to each factor.

Consider letting the amount of income transferred net of cost to be $q - C(q)$. More generally this can be called for now "profit", because of its relation to revenue minus cost. Then the income transfered, which above is described as $\left(1+R^d\right)d$, is now more generally expressed as

$$\left(1+R^d\right)d = \frac{p_g\left[q - C(q)\right]}{p_b}. \tag{52}$$

In the case when $A_Q = 1$ and $q = d$, and there are no labor, physical capital or even financial capital cost, so that $C(q) = 0$, then

$$q - C(q) = A_Q d = d.$$

And it this case

$$\left(1+R^d\right)d = \frac{p_g d}{p_b}, \tag{53}$$

and the standard Arrow-Debreu result returns, with perfect consumption smoothing.

3.3 Linear Production of Transfer

Now try a case of linear production with $A_Q < 1$ but with no other cost so that $C(q) = 0$. Then

$$q - C(q) = A_Q d < d,$$

and

$$\left(1+R^d\right)d = \frac{p_g A_Q d}{p_b}. \tag{54}$$

The result for consumption is very different now from Arrow-Debreu. Since

$$\left(1+R^d\right) = \frac{p_g A_Q}{p_b},$$

combining this with the consumer equilibrium condition it results that

$$\frac{p_b \frac{\partial u(c_b)}{\partial c_b}}{p_g \frac{\partial u(c_g)}{\partial c_g}} = \frac{1}{\left(1+R^d\right)} = \frac{p_b}{p_g A_Q}.$$

Let utility be log-utility and the result is that

$$c_2(b) = c_2(g) A_Q. \tag{55}$$

Consumption in the bad state is less than consumption in the good state. This is the definition of consumption tilting, towards the good state and away from the bad state. It gives any degree of consumption tilting that depends only on the particular specification of $A_Q \leq 1$.

A way to think of why A_Q is less than one is to say there are financial costs so that in the process of producing the contingent state transfer, a fraction of the deposits $(1 - A_Q)$ are used up. In this alternative conceptualization, the output would be $q = d$ but the the costs would be positive at $(1 - A_Q)\,d$, so that

$$q - C\,(q) = d - (1 - A_Q)\,d = A_Q d.$$

This is another way of saying that the net transfer is $A_Q d$. And in the special case when $A_Q = 1$, the economy is immediately, almost trivially, back to the perfect consumption smoothing. Yet it is not a trivial result in that this is the way to show a simple linear production of the income transference in which the consumption smoothing is less than perfect to any degree depending upon $A_Q \leq 1$.

3.4 Standard Cobb-Douglas Transfer

No such simple special case of perfect consumption smoothing is available if instead the transfer technology is Cobb-Douglas in labor and capital, with

$$q = A_Q\,(l_Q)^{\gamma_1}\,(k_Q)^{\gamma_2}\,, \tag{56}$$

and $\gamma_1 + \gamma_2 = 1$. There are costs of using the labor and capital, typically given as the wage cost and rental cost, or $wl_Q + rk_Q$. And the net transfer would be the "profit"

$$q - C\,(q) = A_Q\,(l_Q)^{\gamma_1}\,(k_Q)^{\gamma_2} - wl_Q - rk_Q. \tag{57}$$

In this case, the equilibrium would involve

$$\left(1 + R^d\right) d = \frac{p_g\left[A_Q\,(l_Q)^{\gamma_1}\,(k_Q)^{\gamma_2} - wl_Q - rk_Q\right]}{p_b}. \tag{58}$$

This cannot reduce to a special case of perfect consumption smoothing, which would require

$$A_Q\,(l_Q)^{\gamma_1}\,(k_Q)^{\gamma_2} - wl_Q - rk_Q = q - C\,(q) = d,$$

since it is instead true, given a output price of one, that

$$A_Q\,(l_Q)^{\gamma_1}\,(k_Q)^{\gamma_2} - wl_Q - rk_Q = 0.$$

3.5 Financial Intermediation Transfer

What this comparison of a production technology using only deposits d, versus one using only labor and physical capital, is that in combining all three factors into a production technology, the closer to perfect consumption smoothing that results, the less important will be the labor and physical capital in producing the income transfer. In other words, the parameters γ_1 and γ_2 are low values while $1 - \gamma_1 - \gamma_2$ is a higher value. And so the cost of the transfer may indeed have some labor and physical capital cost but not so much.

The linear technology $q = A_Q d$, with $A_Q = 1$ is the exact opposite in a sense from the Cobb-Douglas technology with $q = A_Q\,(l_Q)^{\gamma_1}\,(k_Q)^{\gamma_2}$, because the linear technology with $A_Q = 1$ means all of the deposits are available for transfer into the bad state, while the Cobb-Douglas technology in labor and physical capital would mean that after costs are subtracted there is nothing left to transfer to the bad state. Also there is the awkward problem with in the case with $q = A_Q (l_Q)^{\gamma_1} (k_Q)^{\gamma_2}$ in that deposits d do not actually appear anywhere and yet these are what are being transferred.

In contrast the more general technology, $q = A_Q (l_Q)^{\gamma_1} (k_Q)^{\gamma_2} d^{1-\gamma_1-\gamma_2}$, solves all of these issues by having deposits appear as part of the output, by having the special case of perfect consumption smoothing when $A_Q = 1$ and $\gamma_1 = \gamma_2 = 0$, and by allowing any degree of labor and capital cost that subtract from what is leftover for transfer to the bad state but do not use up all of the deposits being transferred. This makes sense for when funds are transferred some amount will get used up, but not so much. For example, say one uses Western Union's "wire" money transfer when something really bad happens and one suddenly needs money and cannot get it any other way. There are costs of the transfer, much higher than just using a bank machine to withdraw cash, but certainly the cost is still a small percent of each dollar that is transferred. The same applies to the three-factor transfer technology. With A_Q close to one, and with a small value of γ_1 and γ_2, the labor and physical capital cost of the Western Union transfer is small per dollar that is transferred.

The labor and physical capital do unfortunately increase the cost above what it would be if $q = A_Q d$ were instead the technology and $A_Q < 1$, because the labor and physical capital use up additional resources. So with $q = A_Q (l_Q)^{\gamma_1} (k_Q)^{\gamma_2} d^{1-\gamma_1-\gamma_2}$ the fraction of deposits "lost" is more, as the labor and physical capital usage are more (high γ_1 and γ_2). Given the values of γ_1 and γ_2, the use of labor and physical capital is balanced at the margin in an optimal way, against each other and the use of deposits, so as to minimize the ultimate costs of the transfer. This is true of all such service industries!

3.6 Example 2. Linear Technology with Consumption Tilting

For ease of juxtaposition and without any loss of generality, we abstract from Arrow-Debreu securities from the model, so that the analysis becomes a static one-period problem.

Now let $A_Q = 0.75$, with the result of more good state consumption than bad state consumption. Again with $p_g = 0.8$, $p_b = 0.2$, $y_g = 1$, and $y_b = 0.3$. Then

$$
\begin{aligned}
d &= \frac{A_Q y_g - y_b}{A_Q\left(1+\frac{p_g}{p_b}\right)} = \frac{0.75-0.3}{0.75\left(1+\frac{0.8}{0.2}\right)} = 0.12, && (59)\\
\left(1+R^d\right)d &= \frac{p_g}{p_b}\left(\frac{A_Q y_g - y_b}{1+\frac{p_g}{p_b}}\right) = \frac{0.8}{0.2}\frac{0.75-0.3}{\left(1+\frac{0.8}{0.2}\right)} = 0.36, && (60)\\
c_b &= \left(1+R^d\right)d + y_b = 0.36+0.3 = 0.66, && (61)\\
c_g &= 1-d = 1-0.12 = 0.88, && (62)\\
1+R^d &= \frac{p_g A_Q}{p_b} = 4\,(0.75) = 3. && (63)
\end{aligned}
$$

The premium is now $d - 0.12$, and consumption is 0.1 more in the good than the bad state. Less is invested in contingent state transfer because it is more expensive, and so less consumption smoothing results.

The utility level curve is

$$
\begin{aligned}
u &= 0.8\ln(c_g) + 0.2\ln(c_b)\\
-0.18537 &= 0.8\ln(0.88) + 0.2\ln(0.66),\\
e^{-0.18537} &= (c_g)^{0.8}(c_b)^{0.2},\\
c_b &= \left(\frac{e^{-0.18537}}{(c_g)^{0.8}}\right)^{\frac{1}{0.2}}, && (64)
\end{aligned}
$$

and the production line is

$$
\begin{aligned}
c_b &= \frac{A_Q p_g y_g}{p_b} + y_b - \frac{A_Q p_g c_g}{p_b};\\
c_b &= \frac{0.75\,(0.8)\,1}{(0.2)} + 0.3 - \frac{0.75\,(0.8)}{(0.2)}c_g. && (65)
\end{aligned}
$$

Figure 1 graphs both the $A_Q = 1$ and the $A_Q = 0.75$ equilibria, in lighter red and blue and darker red and blue respectively. Notice that there are both income and substitution effects of the relative price change implicit in $1 + R^d$ falling from 4 to $4\,(0.75) = 3.0$. The substitution and income effects are both towards less bad state consumption, with a fall from 0.86 to 0.66, and the substitution effect is towards more goods state consumption but the income effect is towards less good state consumption. The substitution effect somewhat dominates the income effect with the resulting good state consumption rising to 0.88 from 0.86.

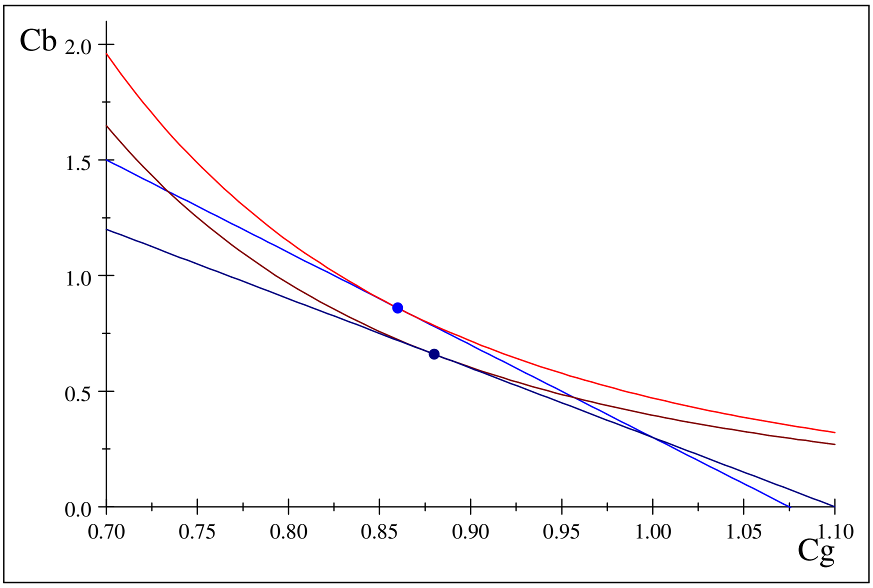

Figure 1. Good State and Bad State Consumption

In $(c_b : d)$ space, utility and production are

$$c_b = \left(\frac{e^{-0.18537}}{(1-d)^{0.8}}\right)^{\frac{1}{0.2}}, \tag{66}$$

$$c_b = \frac{0.75\,(0.8)\,1}{(0.2)} + 0.3 - \frac{0.75\,(0.8)}{(0.2)}\,(1-d)\,, \tag{67}$$

as shown in Figure 2 with both $A_Q = 1$ and $A_Q = 0.75$.

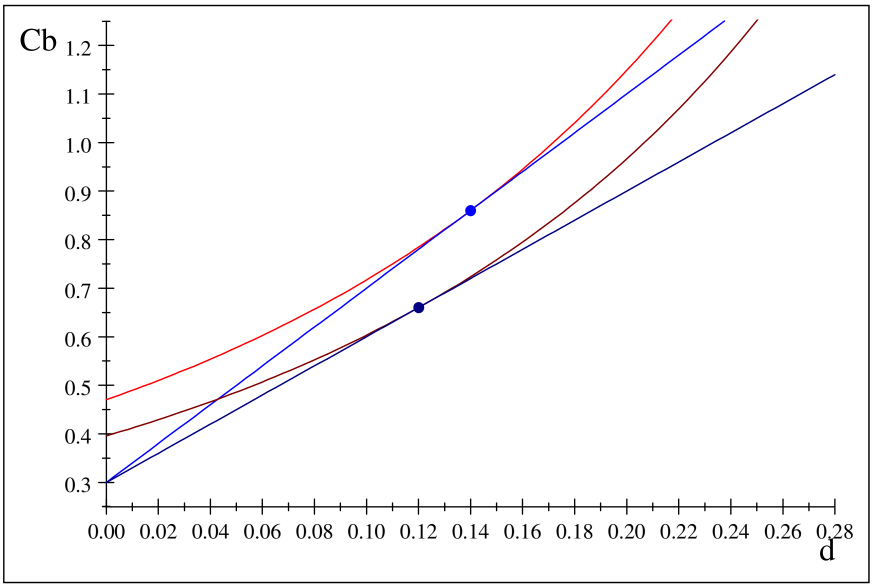

Figure 2. Utility and Production in Example 1

With the substitution effect dominating the income effect of a lower A_Q, the good state consumption rises, and d falls in Figure 2 in terms of $(c_b : d)$ space. This contingent state investment d, or "insurance premium", residually falls somewhat from 0.14 to 0.12. In particular the substitution effect is that the slope of the linear technology line in Figure 2 has become flatter, so that with utility held constant, the result would be a lower value of d. The income effect is that the technology line then shifts down from the substitution point, with the result of increasing d, and decreasing bad state consumption. Since the substitution effect dominates the income effect, the value of d falls.

4 General Financial Intermediation Production

For the production technology as extended by using financial intermediation, a concave production gets drawn into the graphs instead of a line, going all the way back to "self-insurance" between good and bad states for example in Ehrlich and Becker (1972), "Market Insurance, Self-Insurance and Self-Protection". The market insurance comes in once the problem is decentralized. And the self-protection comes in only if there is a focus on decreasing the probability of the bad state through such additional investment.

Now the payoff in the bad state is the production output minus the real resource cost. Take the production function of equation (49), and simplify it by assuming no physical capital so that

$$q = A_Q \left(l_Q\right)^{\gamma} d^{1-\gamma}.$$

The resource cost of the labor is the amount times its value and as before, we abstract from Arrow-Debreu securities from the model, so that the analysis becomes a static one-period problem. Assume in addition a linear production of goods output in this labor only economy, so that

$$y = A_G l.$$

Then the value of any time is A_G factored by the time. This makes the amount available for transfer

$$\left(1+R^d\right) d = \frac{p_g \left[A_Q \left(l_Q\right)^{\gamma} d^{1-\gamma} - A_G l_Q\right]}{p_b}, \tag{68}$$

as a special case of equation (58).

To solve for the input ratio $\frac{l_Q}{d}$ requires extending the consumer problem. It starts still with the good state deposit of d and the bad state receipt of $\left(1+R^d\right) d$, but now this also involves a time allocation choice. If the return is given as

$$\left(1+R^d\right) = \frac{p_g \left[A_Q \left(\frac{l_Q}{d}\right)^{\gamma} - A_G \frac{l_Q}{d}\right]}{p_b},$$

then the consumer problem with log utility is

$$\begin{aligned}
\underset{d,l_Q}{Max}\ Eu &= p_g \ln\left(y_g - d\right) + p_b \ln\left[\left(1+R^d\right) d + y_b\right], \\
\underset{d,l_Q}{Max}\ Eu &= p_g u\left(y_g - d\right) + p_b u\left(\frac{p_g \left[A_Q \left(l_Q\right)^{\gamma} d^{1-\gamma} - A_G l_Q\right]}{p_b} + y_b\right). \qquad (69)
\end{aligned}$$

The equilibrium conditions are

$$\begin{aligned}
d &: -p_g u'\left(c_g\right) + p_b u'\left(c_b\right)\left(\frac{\left(1-\gamma\right) p_g A_Q \left(l_Q\right)^{\gamma} d^{-\gamma}}{p_b}\right) = 0, \\
l_{Qt} &: \gamma A_Q \left(\frac{l_{Qt}}{d_t}\right)^{\gamma-1} = A_G,
\end{aligned}$$

This implies that

$$MRS_{c_g,c_b} \equiv \frac{u'\left(c_g\right)}{u'\left(c_b\right)} = \left(1-\gamma\right) A_Q \left(\frac{l_Q}{d}\right)^{\gamma} \equiv MP_d. \tag{70}$$

This states that the marginal rate of substitution between consumption across states, MRS_{c_g,c_b}, equals the marginal product of investment d in deposits, which is denoted by MP_d

The second equilibrium condition gives the solution for the labor to deposit ratio:

$$\frac{l_{Qt}}{d_t} = \left(\frac{\gamma A_Q}{A_G}\right)^{\frac{1}{1-\gamma}}. \tag{71}$$

Using both equations,

$$\frac{u'(c_g)}{u'(c_b)} = A_Q(1-\gamma)\left(\frac{\gamma A_Q}{A_G}\right)^{\frac{\gamma}{1-\gamma}},$$

and with log utility,

$$\frac{c_b}{c_g} = A_Q(1-\gamma)\left(\gamma\frac{A_Q}{A_G}\right)^{\frac{\gamma}{1-\gamma}}. \tag{72}$$

Solve the complete equilibrium by using the budget constraints

$$\begin{aligned} c_g &= y_g - d, \\ c_b &= \frac{p_g\left[A_Q(l_Q)^{\gamma} d^{1-\gamma} - A_G l_Q\right]}{p_b} + y_b, \end{aligned}$$

and substituting these into the consumption equation (72) that

$$\frac{\frac{p_g\left[A_Q(l_Q)^{\gamma} d^{1-\gamma} - A_G l_Q\right]}{p_b} + y_b}{y_g - d} = A_Q(1-\gamma)\left(\gamma\frac{A_Q}{A_G}\right)^{\frac{\gamma}{1-\gamma}}.$$

This can be rewritten as

$$\frac{\frac{p_g d\left(\frac{l_Q}{d}\right)\left[A_Q\left(\frac{l_Q}{d}\right)^{\gamma-1} - A_G\right]}{p_b} + y_b}{y_g - d} = A_Q(1-\gamma)\left(\gamma\frac{A_Q}{A_G}\right)^{\frac{\gamma}{1-\gamma}}.$$

Substituting in the solution for $\frac{l_{Qt}}{d_t} = \left(\frac{\gamma A_Q}{A_G}\right)^{\frac{1}{1-\gamma}}$ from equation (71),

$$\frac{\frac{p_g}{p_b} d\left(\frac{\gamma A_Q}{A_G}\right)^{\frac{1}{1-\gamma}} \frac{A_G}{\gamma}(1-\gamma) + y_b}{y_g - d} = A_Q\left(\frac{A_Q}{A_G}\right)^{\frac{\gamma}{1-\gamma}}(1-\gamma)\gamma^{\frac{\gamma}{1-\gamma}}.$$

And now solve for d as

$$d = \frac{y_g A_Q(1-\gamma)\left(\gamma\frac{A_Q}{A_G}\right)^{\frac{\gamma}{1-\gamma}} - y_b}{\left(1+\frac{p_g}{p_b}\right) A_Q(1-\gamma)\left(\gamma\frac{A_Q}{A_G}\right)^{\frac{\gamma}{1-\gamma}}}. \tag{73}$$

The gross return $1+R^d$ is

$$\begin{aligned} 1+R^d &= \frac{p_g\left[A_Q\left(\frac{l_Q}{d}\right)^{\gamma} - A_G\frac{l_Q}{d}\right]}{p_b}, \\ &= \frac{p_g A_Q(1-\gamma)\left(\frac{\gamma A_Q}{A_G}\right)^{\frac{\gamma}{1-\gamma}}}{p_b} = \frac{p_g}{p_b}(MP_d). \end{aligned} \tag{74}$$

This return is the probability ratio factored by the marginal product of investment in d. In the Arrow-Debreu costless case, the marginal product in effect equals 1.

And the consumption in each state is

$$c_g = y_g - d = y_g - \frac{y_g A_Q (1-\gamma)\left(\gamma \frac{A_Q}{A_G}\right)^{\frac{\gamma}{1-\gamma}} - y_b}{\left(1+\frac{p_g}{p_b}\right) A_Q (1-\gamma)\left(\gamma \frac{A_Q}{A_G}\right)^{\frac{\gamma}{1-\gamma}}}, \tag{75}$$

$$c_b = \left(1+R^d\right) d + y_b = \frac{p_g}{p_b} \frac{y_g A_Q (1-\gamma)\left(\gamma \frac{A_Q}{A_G}\right)^{\frac{\gamma}{1-\gamma}} - y_b}{\left(1+\frac{p_g}{p_b}\right)} + y_b. \tag{76}$$

And the analytic utility solution follows directly using these functional solutions for consumption.

4.1 Comparison to Linear Production

Compare the equilibrium with the financial intermediation production technology to the linear production case. Consider writing the solution for d as

$$d = \frac{y_g A_Q (1-\gamma)\left(\gamma \frac{A_Q}{A_G}\right)^{\frac{\gamma}{1-\gamma}} - y_b}{\left(1+\frac{p_g}{p_b}\right) A_Q (1-\gamma)\left(\gamma \frac{A_Q}{A_G}\right)^{\frac{\gamma}{1-\gamma}}} = \frac{y_g A_Q z - y_b}{\left(1+\frac{p_g}{p_b}\right) A_Q z}$$

with the factor

$$z \equiv (1-\gamma)\left(\gamma \frac{A_Q}{A_G}\right)^{\frac{\gamma}{1-\gamma}}.$$

In the linear production case, $z = 1$, as $\gamma = 0$. Then d reverts back to $d = \frac{y_g A_Q - y_b}{\left(1+\frac{p_g}{p_b}\right) A_Q}$, as in equation (59). Clearly a decrease in z causes d to fall, since

$$\frac{\partial d}{\partial z} = \frac{y_b \left(1+\frac{p_g}{p_b}\right) A_Q}{\left[\left(1+\frac{p_g}{p_b}\right) A_Q z\right]^2} > 0.$$

So a key issue is how this z factor changes as γ rises from 0. To find this out, take the $\ln z$, and find $\frac{\partial z}{\partial \gamma}$:

$$\ln z = \ln(1-\gamma) + \frac{\gamma}{1-\gamma} \ln\left(\gamma \frac{A_Q}{A_G}\right),$$

$$\frac{\frac{\partial z}{\partial \gamma}}{z} = \ln\left(\gamma \frac{A_Q}{A_G}\right) \frac{(2-\gamma)}{(1-\gamma)^2} = \left(\ln \gamma + \ln \frac{A_Q}{A_G}\right) \frac{(2-\gamma)}{(1-\gamma)^2}.$$

Therefore if $\frac{\gamma A_Q}{A_G} \leq 1$, then $\frac{\partial z}{\partial \gamma} \leq 0$ since $\ln \gamma \leq 0$. This means that as γ goes up, then d will fall if $\frac{\gamma A_Q}{A_G} < 1$. With a low value of γ being typical, then this condition is satisfied for a wide range of A_Q and A_G values. This means the usual case will see less insurance as the labor intensity of transfer production γ rises.

Clearly as γ approach zero, and z approaches 1, the labor to deposit input ratio $\frac{l_{Qt}}{d_t}$ approaches 0 :

$$\lim_{\gamma \to 0} \left(\frac{l_{Qt}}{d_t} \right)^{1-\gamma} = \lim_{\gamma \to 0} \left(\frac{\gamma A_Q}{A_G} \right)^{\frac{1}{1-\gamma}} = 0,$$

meaning that no time is used in the production of the contingent state transfer. Also the equilibrium return $1 + R^d$ approaches the linear production case as $\gamma \to 0$:

$$1 + R^d = \frac{p_g}{p_b} A_Q (1-\gamma) \left(\gamma \frac{A_Q}{A_G} \right)^{\frac{\gamma}{1-\gamma}} = \frac{p_g}{p_b} A_Q z, \tag{77}$$

$$\lim_{\gamma \to 0} \left(1 + R^d\right) = \lim_{\gamma \to 0} \left[\frac{p_g}{p_b} A_Q (1-\gamma) \left(\gamma \frac{A_Q}{A_G} \right)^{\frac{\gamma}{1-\gamma}} \right] = \frac{p_g}{p_b} A_Q.$$

This limiting return is the exact Arrow-Debreu costless case when $A_Q = 1$.

Similarly, the marginal rate of substitution across states is

$$\frac{p_b \frac{\partial u(c_b)}{\partial c_b}}{p_g \frac{\partial u(c_g)}{\partial c_g}} = \frac{1}{(1 + R^d)} = \frac{p_b}{p_g A_Q} \frac{1}{z}, \tag{78}$$

and so for log utility

$$c_b = c_g A_Q z,$$

which approaches the linear case as z approaches 1. In addition with $A_Q = 1$, again, the costless Arrow-Debreu perfect consumption smoothing equilibrium results.

Consumption in the more general financial intermediation production case is tilted away from the bad state towards the good state the higher is γ and lower is z, given the parameter bounds of $\frac{\gamma A_Q}{A_G} < 1$. And since z falls for most calibrations as γ increases, then the higher the share of labor in production the more tilted is consumption.

Consider the graph of $\frac{c_b}{c_g}$. For example, if $A_Q = A_G < 1$, then

$$\begin{aligned} \frac{c_b}{c_g} &= A_Q z = A_Q (1-\gamma) \gamma^{\frac{\gamma}{1-\gamma}}, \\ z &= (1-\gamma) \gamma^{\frac{\gamma}{1-\gamma}}. \end{aligned}$$

Figure 3 shows that the graph of $z(\gamma)$ is strictly decreasing in γ, varying from 0 to 1. And this leads to a simple proposition about when consumption tilting towards the bad state results.

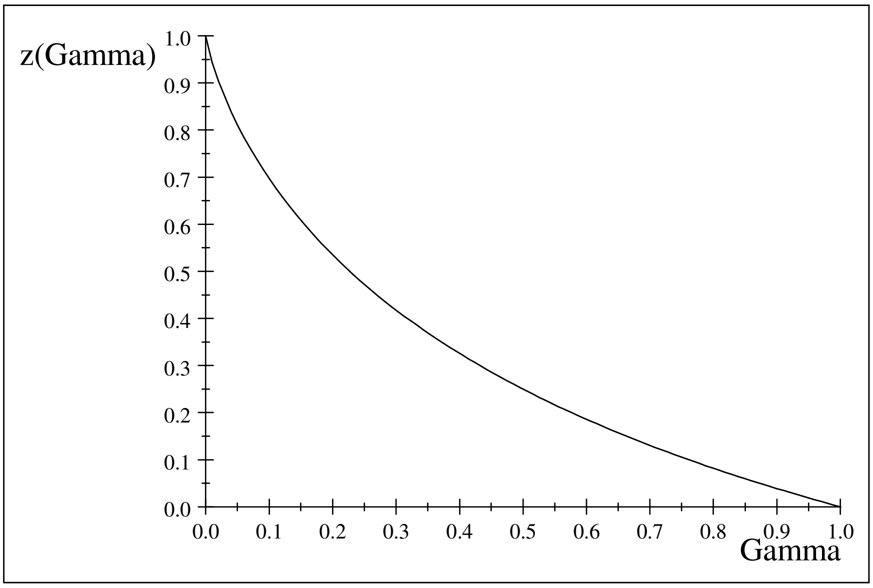

Figure 3.

Proposition 1 *For* $A_G = A_Q < 1,$ *and* $\gamma \in (0,1)$ *, then* $\frac{c_b}{c_g} < 1.$

More generally, the relation between A_G and A_Q is limited in order to establish that $\frac{c_b}{c_g} < 1.$

Proposition 2 *For* $\frac{\gamma A_Q}{A_G} < 1,$ $\gamma \in (0,1)$ *, and* $A_Q \leq 1,$ *there is consumption tilting towards the good state:*

$$c_b < c_g.$$

For example if $\gamma = 0.2$, and $A_G = 0.19$, then $A_Q \leq \frac{0.19}{0.2} = 0.95$ in order for $c_b \leq c_g$. In contrast, a result of $c_b > c_g$ makes little economic sense in that it reverses the basic intuition of contingent state transfers, so this limiting relation between A_G, A_Q and γ is taken to mean that there are constraints on what is a reasonable calibration. Clearly if A_G and A_Q are close in value, then such issues are not relevant in that the limits would not be violated.

Figure 4 illustrates the proposition by letting $A_Q = 0.75$, and $A_G = 1$, and by then graphing the ratio of consumption across states as a function of γ from the equation

$$\frac{c_b}{c_g} = 0.75 \left(\frac{0.75}{1} \right)^{\frac{\gamma}{1-\gamma}} (1-\gamma) \left(\gamma^{\frac{\gamma}{1-\gamma}} \right).$$

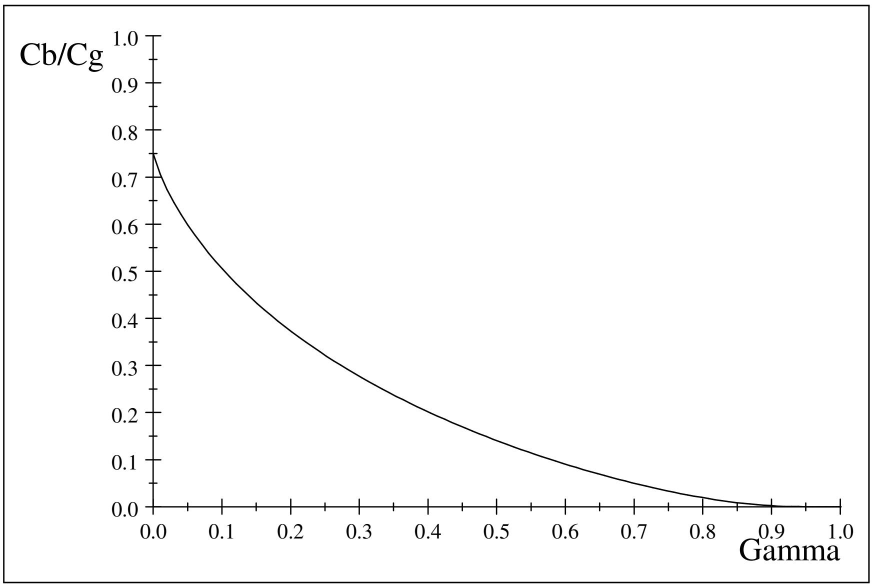

Figure 4.

When $\gamma = 0$ and $A_Q = 0.75$, then the consumption ratio is $\frac{c_b}{c_g} = 0.75$. With γ rising from 0 to 1, then the ratio of consumption in the bad state falls relative to that of the good state and approaches 0.

In terms of the labor per deposit in banking, the consumption ratio from equation (70) can be stated as

$$\frac{c_b}{c_g} = (1 - \gamma) A_Q \left(\frac{l_Q}{d} \right)^{\gamma}, \tag{79}$$

and graphed. For example assuming that $\gamma = 0.1$ and $A_q = 0.75$, Figure 5 graphs equation (79).

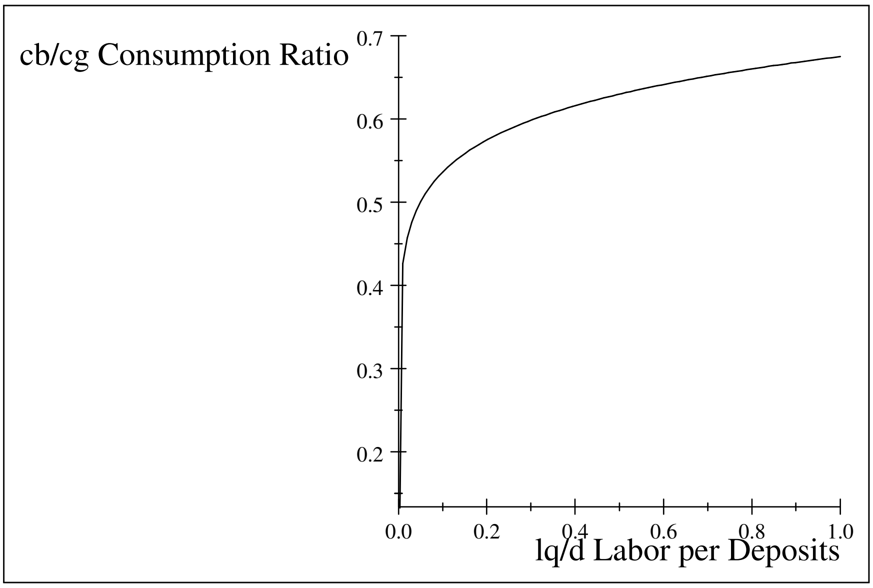

Figure 5.

The Figure graphs the consumption ratio as being equal to the marginal product of deposits for Example 3. Therefore it gives the typical shape of a marginal product that is rising at a diminishing rate as the factor ratio increases. It also shows that given the level of γ, the consumption tilt towards good state consumption is decreased by using the bank sector to produce intermediation of the transfer across states. By using labor, the consumption ratio $\frac{c_b}{c_g}$ rises from 0 and finds an optimal level for $\gamma = 0.1$ near to $\frac{c_b}{c_g} = 0.5$.

4.2 Example 3. Financial Intermediation

For ease of juxtaposition and without any loss of generality, we abstract from Arrow-Debreu securities from the model, so that the analysis becomes a static one-period problem.

Let $p_g = 0.8$, $p_b = 0.2$, $A_Q = 0.75$, $y_g = 1$, $y_b = 0.3$, and now allow for the financial intermediation technology such that $\gamma = 0.1$, and $A_G = 1$.

Then the equilibrium is given by

$$\begin{aligned} c_b &= c_g A_Q \left(\frac{A_Q}{A_G}\right)^{\frac{\gamma}{1-\gamma}} (1-\gamma)\left(\gamma^{\frac{\gamma}{1-\gamma}}\right), \\ c_b &= c_g 0.75 \left(\frac{0.75}{1.0}\right)^{\frac{0.1}{1-0.1}} (1-0.1)\left(0.1^{\frac{0.1}{1-0.1}}\right), \\ c_b &= c_g\,(0.506\,19)\,. \end{aligned} \tag{80}$$

From equation (73), the solution for d is

$$d = \frac{y_g A_Q (1-\gamma)\left(\gamma \frac{A_Q}{A_G}\right)^{\frac{\gamma}{1-\gamma}} - y_b}{\left(1+\frac{p_g}{p_b}\right) A_Q (1-\gamma)\left(\gamma \frac{A_Q}{A_G}\right)^{\frac{\gamma}{1-\gamma}}}, \tag{81}$$

$$= \frac{1\,(0.75)\,(1-0.1)\left((0.1)\,\frac{0.75}{1}\right)^{\frac{0.1}{1-0.1}} - 0.3}{\left(1+\frac{0.8}{0.2}\right)(0.75)\,(1-0.1)\left((0.1)\,\frac{0.75}{1}\right)^{\frac{0.1}{1-0.1}}} = 0.081467. \tag{82}$$

Therefore this a decrease relative to the related linear production of Example 2, in which $\gamma = 0$ and $d = 0.14$. For Example 3 but with a varying γ, Figure 6 graphs the d function that equals 0.0815 with $\gamma = 0.1$.

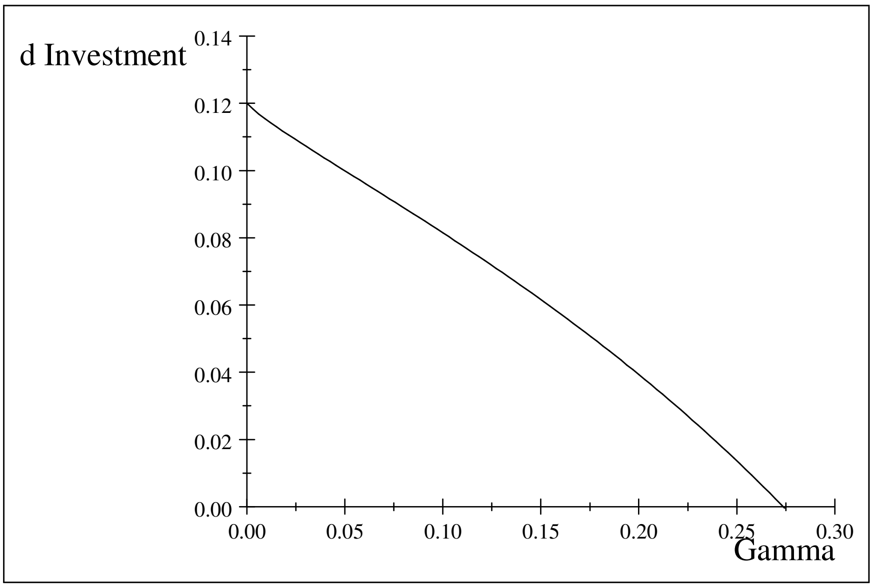

Figure 6.

From the good state, bad state consumption ratio in equation (80), and the consumption budget constraint for the good state of $c_g = y_g - d$, the equilibrium consumption is

$$c_g = y_g - d = 1 - 0.081467 = 0.91853,$$
$$c_b = c_g 0.506\,19 = 0.91853\,(0.506\,19) = 0.464\,95.$$

And in turn the return $1 + R^d$ is residually implied, or alternatively computed from its equilibrium solution:

$$\begin{aligned} c_b &= \left(1+R^d\right)d + y_b, \\ 1+R^d &= \frac{c_b - y_b}{d} = \frac{0.464\,95 - 0.3}{0.081467} = 2.\,0247, \\ 1+R^d &= \frac{p_g}{p_b} A_Q \left(\frac{A_Q}{A_G}\right)^{\frac{\gamma}{1-\gamma}} (1-\gamma)\,\gamma^{\frac{\gamma}{1-\gamma}}, \\ &= 4\,(0.75)\,(1-0.1)\left((0.1)\,\frac{0.75}{1}\right)^{\frac{0.1}{1-0.1}} = 2.\,0247. \end{aligned} \quad (83)$$

The gross return falls from 3 in Example 2 with linear production to 2.025 here. Figure 6 graphs equation (83) except with γ a variable, illustrating how $1+R^d$ in Example 3 varies with γ:

The gross return can also be graphed as in Figure 7 against the labor share parameter γ. From equation (77),

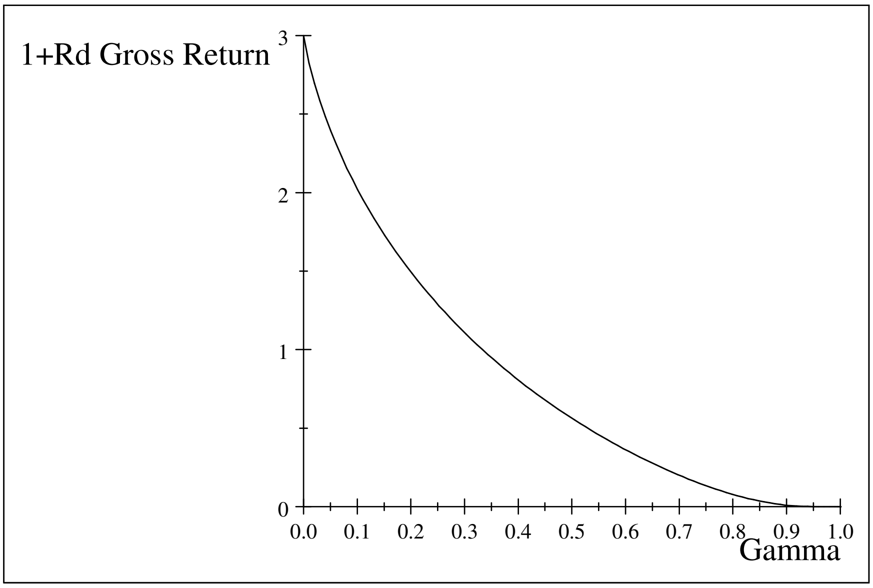

Figure 7.

Finally, the labor usage is given by equation (71), as

$$\begin{aligned} \frac{l_{Qt}}{d_t} &= \left(\frac{\gamma A_Q}{A_G}\right)^{\frac{1}{1-\gamma}} = \left(\frac{0.1\,(0.75)}{1}\right)^{\frac{1}{1-0.1}} = 0.056243, \\ l_{Qt} &= d_t\,(0.056243) = 0.081467\,(0.056243) = 0.0045819. \end{aligned}$$

Thus the labor time is just a small fraction 0.0046 of the total time.

Expected utility is given from equation (69) for Example 3 as

$$EU = p_g \ln c_g + p_b \ln c_b = 0.8 \ln 0.91853 + 0.2 \ln 0.464\,95 = -0.22115.$$

4.3 Graph of Output and Input Space Changes

Bring together the Example equilibrium by showing how it can be graphed in output (c_b, c_g) and input (l_Q, d) spaces. For the output space, the production function specifies determines how c_g is turned into c_b. Consider from equation (68) and that

$$\left(1+R^d\right)d = \frac{p_g\left[A_Q\left(l_Q\right)^\gamma d^{1-\gamma} - A_G l_Q\right]}{p_b}. \tag{84}$$

Now combine this technology statement with the budget constraints in both states,

$$\begin{aligned} c_g &= y_g - d, \\ c_b &= \left(1+R^d\right)d + y_b, \end{aligned}$$

in order to write c_b in terms of c_g for a given equilibrium l_Q :

$$\begin{aligned} c_b - y_b &= \frac{p_g\left[A_Q\left(l_Q\right)^\gamma \left(y_g - c_g\right)^{1-\gamma} - A_G l_Q\right]}{p_b}, \\ c_b &= \frac{p_g\left[A_Q\left(l_Q\right)^\gamma \left(y_g - c_g\right)^{1-\gamma} - A_G l_Q\right]}{p_b} + y_b, \\ c_b &= 4\left[0.75\left(0.0046\right)^{0.1}\left(1-c_g\right)^{1-0.1} - 1\left(0.0046\right)\right] + 0.3. \end{aligned} \tag{85}$$

The expected utility is given by

$$\begin{aligned} Eu &= p_g \ln c_g + p_b \ln c_b = -0.22115, \\ e^{-0.22115} &= c_g^{0.8} c_b^{0.2}, \\ c_b &= \left(\frac{e^{-0.22115}}{c_g^{0.8}}\right)^5 = \frac{e^{-(0.22115)5}}{\left(c_g\right)^4}. \end{aligned} \tag{86}$$

Figure 8 graphs the output equilibrium, with the production function of equation (85) and the utility level curve of equation (86).

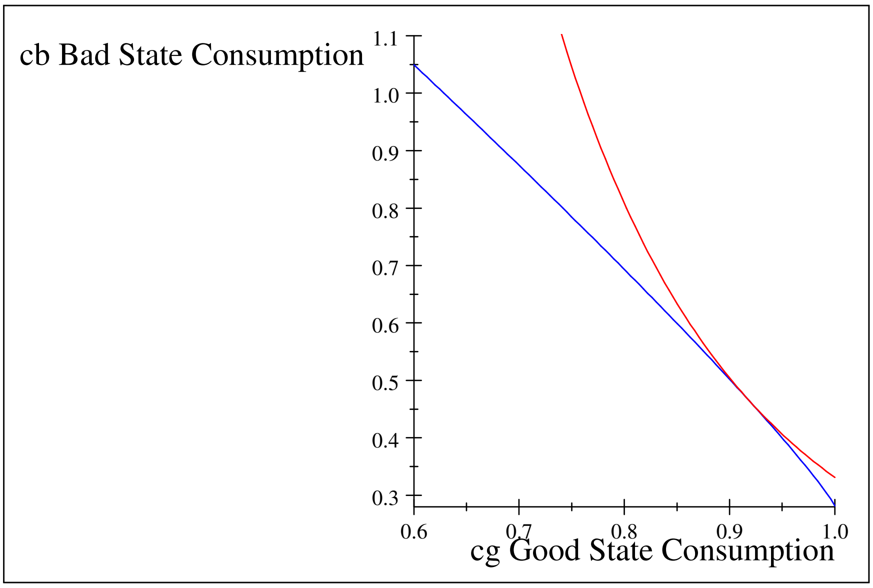

Figure 8

The tangency is where the marginal rate of substitution in production equals the probability weighted marginal product of the transformation of investment deposits, $\frac{p_g}{p_b}(MP_d)$. The production function only has a slight curvature and is close to linear because γ is parameterized at a low value, and little labor time is used for each unit transferred across states. With $\gamma = 0$ it would revert to the linear case. The low γ is probably realistic for calibration but at this level of abstract this example is just an illustration.

Similarly, in input, output, space of (c_b, d), the production function and expected utility level are given functionally by

$$c_b = 4\left[0.75\,(0.0046)^{0.1}\,(d)^{1-0.1} - 1\,(0.0046)\right] + 0.3, \tag{87}$$

$$c_b = \frac{e^{-(0.22115)5}}{(1-d)^4}. \tag{88}$$

Figure 9 represents the Example 3 equilibrium in (c_b, d) space by graphing the production equation (87) in blue and the utility equation (88) in red.

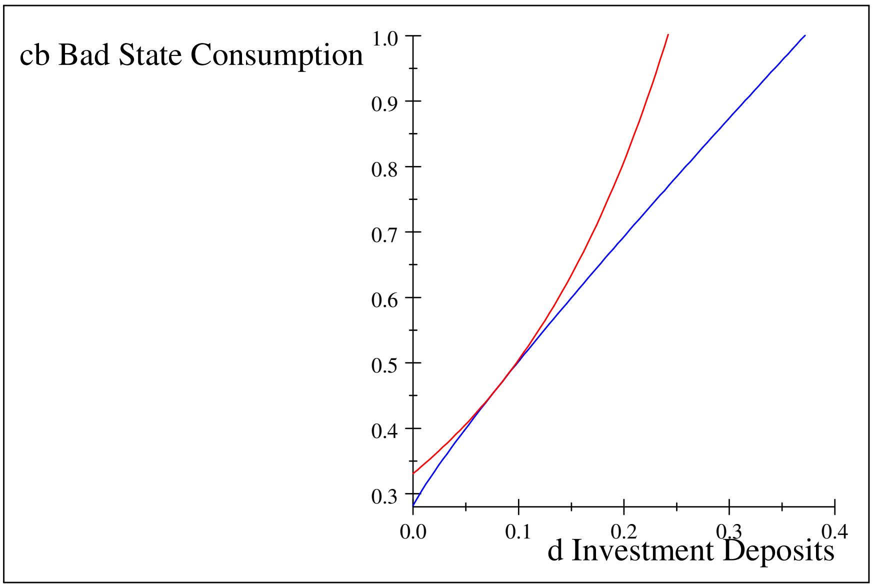

Figure 9.

Increasing deposits d moves the consumer up the production function until the equilibrium tangency with the expected utility level curve is reached.

An isoquant curve can also be drawn with the factor input ratio indicating the equilibrium, whereby $\left(1+R^d\right)d = \frac{p_g\left[A_Q(l_Q)^\gamma d^{1-\gamma} - A_G l_Q\right]}{p_b}$. Then the production function level for Example 3 is

$$\begin{aligned} A_Q\left(l_Q\right)^\gamma d^{1-\gamma} &= \frac{p_b}{p_g}\left(1+R^d\right)d + A_G l_Q \\ &= (0.25)\,(2.0247)\,(0.081467) + (0.0046) = 0.045837. \end{aligned}$$

And the isoquant can then be given in terms of (l_Q, d) as

$$l_Q = \left(\frac{0.045837}{A_Q d^{1-\gamma}}\right)^{\frac{1}{\gamma}} = \frac{(0.045837)^{10}}{(0.75)^{10}\, d^9}. \tag{89}$$

And since $\frac{l_{Qt}}{d_t} = 0.056243$, the input ratio can be represented by the equation

$$l_Q = (0.056243)\, d. \tag{90}$$

Figure 10 graphs the isoquant equation (89) and the input ratio equation (90).

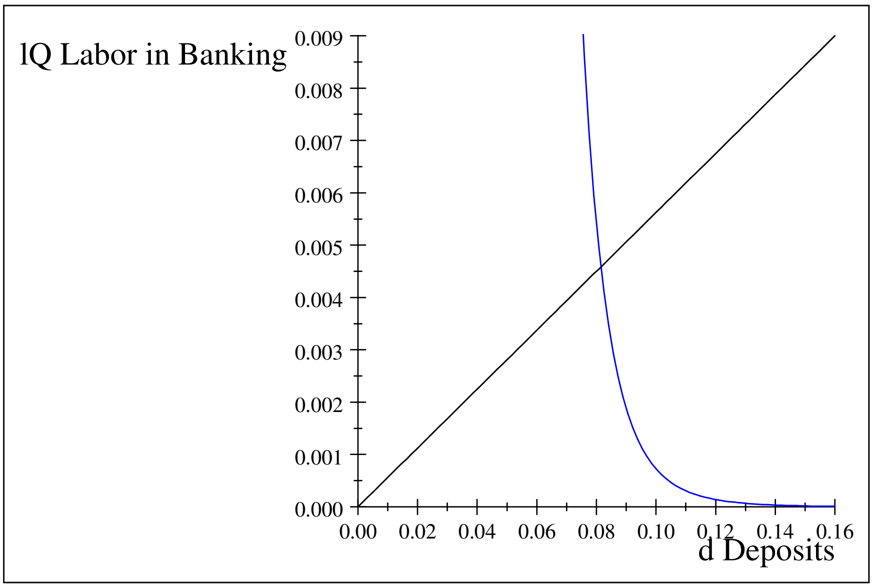

Figure 10. Isoquant and Factor Input Ratio.

5 Decentralized Banking Across Contingent States

Decentralizing the problem between the consumer and the bank allows for an explicit cost function to be seen that is dual to the rising marginal product of the centralized economy. This duality gives a marginal cost that is convex, rising at an increasing rate. It also makes more clear the application of banking in general equilibrium.

Now the consumer receives a market determined return on deposits d, as given in gross terms as $1 + R^d$. While $1 + R^d$ was just a convenient way to present the problem in the centralized economy, with it being the probability ratio factored by the marginal product of deposits, $\frac{p_g}{p_b}(MP_d)$, now $1 + R^d$ is a market price. Of course in equilibrium it is found to equal the same value of $\frac{p_g}{p_b}(MP_d)$.

Consider that when the consumer deposits d in the bank for some financial intermediation service, then in terms of ownership, the consumer can be thought of as buying one share of ownership in the bank for each unit of deposits. The price of the bank is constant at one, and the dividend is the payment of the profits in accordance to the amount of deposits held by the consumer. This is the same as the structure of a "mutual" bank that is owned by virtue of the "equity investment" in accordance to the deposits made in the bank.

Let R^d be the net return per unit of deposits d. Then the total dividend payments is $R^d d$. And this is the profit after the bank pays its costs of labor and physical capital. Should it be that $\gamma_1 + \gamma_2 = 1$, the profit $R^d d$ would be zero.

5.1 Consumer problem

Besides buying and selling Arrow-Debreu securities and investing d in the good state in period 2 by depositing the funds in the bank, for transference when the bad state occurs, the consumer produces goods output with labor time l, with the output equal to $A_g l$. The consumer works l_Q amount of time for the bank in the good state in period 2, receiving the wage value of this time, equal to wl_Q, with w the real wage rate. The allocation of time constraint is

$$1 = l + l_Q. \tag{91}$$

The budget constraint for the good state in period 2 is:

$$\begin{aligned} c_2(g) &= A_g l + wl_Q - d + B_2(g), \\ &= A_g (1 - l_Q) + wl_Q - d + B_2(g). \end{aligned} \tag{92}$$

In the bad state, the consumer receives the dividend return on the deposits of $\left(1 + R^d\right) d$ plus the income from working in the goods sector in the bad state, of A_b together with the payout of the Arrow-Debreu securities, so that consumption is

$$c_2(b) = \left(1 + R^d\right) d + A_b + +B_2(b). \tag{93}$$

The consumer problem is

$$\underset{B_2(s),d,l_Q}{Max} \; U_1 = u\left(y_1 - \frac{price(g)}{1+r}B_2(g) - \frac{price(b)}{1+r}B_2(b)\right) \tag{94}$$

$$+ \left\{p_g \beta u \left[A_g (1 - l_Q) + wl_Q - d + B_2(g)\right] + p_b \beta u \left[\left(1 + R^d\right) d + A_b + +B_2(b)\right]\right\} \tag{95}$$

The necessary first-order conditions are

$$\frac{price(s)}{1+r}\frac{\partial u(c_1)}{\partial c_1} = p_s \beta \frac{\partial u\left[c_2(s)\right]}{\partial c_2(s)}, \; s = g, b. \tag{96}$$

and can also be re-arranged to obtain the usual price equilibrium to show that the marginal rate of substitution between c_1 and $c_2(s)$ is equal to the two goods' relative prices:

$$\frac{p_s \beta \frac{\partial u[c_2(s)]}{\partial c_2(s)}}{\frac{\partial u(c_1)}{\partial c_1}} = \frac{price(s)}{1+r}, \; s = g, b. \tag{97}$$

An implication of the above equation is

$$\frac{p_b \frac{\partial u[c_2(b)]}{\partial c_2(b)}}{p_g \frac{\partial u[c_2(g)]}{\partial c_2(g)}} = \frac{price(b)}{price(g)}. \tag{98}$$

The equilibrium for d, the bank deposits across states of nature is

$$p_g \beta \frac{\partial u\left[c_2(g)\right]}{\partial c_2(g)}(-1) + p_b \beta \frac{\partial u\left[c_2(b)\right]}{\partial c_2(b)}\left(1+R^d\right) = 0. \tag{99}$$

The probability-weighted marginal rate of substitution across states to the marginal product of the deposits:

$$\frac{p_b \frac{\partial u[c_2(b)]}{\partial c_2(b)}}{p_g \frac{\partial u[c_2(g)]}{\partial c_2(g)}} = \frac{1}{(1+R^d)}. \tag{100}$$

The other first-order condition with respect to l_Q simply gives that the real wage equals the marginal product of labor in producing goods in the good state:

$$w = A_g. \tag{101}$$

5.2 Bank Problem with Contingent Payoff

The bank problem is to take in the deposit funds d during the good state and transform them so that they are available in the bad state. The expected profit function of the bank equals expected net revenue in the good state, $p_g\left(A_Q l_Q^{\gamma} d^{1-\gamma} - w l_Q\right)$, minus expect net payouts in the bad state of $p_b\left(1+R^d\right) d$:

$$\underset{d, l_F}{Max}\ E\Pi = p_g\left(A_Q l_Q^{\gamma} d^{1-\gamma} - w l_Q\right) - p_b\left(1+R^d\right) d; \tag{102}$$

The equilibrium condition with respect to the deposited funds d is that

$$1+R^d = \frac{p_g\left(1-\gamma\right) A_Q^{\frac{\gamma}{1-\gamma}}\left(\frac{l_Q}{d}\right)^{\gamma}}{p_b}. \tag{103}$$

The equilibrium condition with respect to the labor l_Q is

$$w = \gamma A_Q\left(\frac{l_Q}{d}\right)^{\gamma-1}, \tag{104}$$

so that

$$\frac{l_Q}{d} = \left(\frac{\gamma A_Q}{w}\right)^{\frac{1}{1-\gamma}}. \tag{105}$$

The solution for $1+R^d$ follows as

$$1+R^d = \frac{p_g A_Q\left(1-\gamma\right)\left(\frac{\gamma A_Q}{w}\right)^{\frac{\gamma}{1-\gamma}}}{p_b}. \tag{106}$$

5.3 Marginal Cost

Considering the cost of producing the transference that is incurred in the good state, this can be formulated as the labor cost of wl_Q. As a marginal cost function is typical in terms of the cost of additional units of output, define the amount of output as

$$q = A_Q l_Q^{\gamma} d^{1-\gamma}.$$

Then l_Q of the cost wl_Q can be written in terms of q so that the cost function can also be in terms of q. From the production function for q, solve l_Q as

$$l_Q = \left(\frac{q}{A_Q} \frac{1}{d^{1-\gamma}} \right)^{\frac{1}{\gamma}}.$$

Then the total cost, call it $C(q)$, is $w \left(\frac{q}{d^{1-\gamma}} \frac{1}{A_Q} \right)^{\frac{1}{\gamma}}$. The cost per unit of d, is found by dividing by d, to get

$$\frac{C(q)}{d} = w \left(\frac{q}{d} \frac{1}{A_Q} \right)^{\frac{1}{\gamma}}.$$

For $\gamma = 1$, this per unit cost becomes linear in the output per unit of d, or in $\frac{q}{d}$. This corresponds to a production function for financial intermediation that does not have deposits as a factor. However, for $\gamma \in (0, 1)$, the per unit cost rises with $\frac{q}{d}$.

For example, using the calibration of Example 3, with $w = y_b = 1$, $A_Q = 0.75$, and $\gamma = 0.1$, then

$$\frac{C(q)}{d} = w \left(\frac{q}{d} \frac{1}{A_Q} \right)^{\frac{1}{\gamma}} = 1 \left(\frac{q}{d} \frac{1}{0.75} \right)^{\frac{1}{0.1}},$$

and Figure 11 graphs in the black curve this convex cost function against the output per unit of deposits:

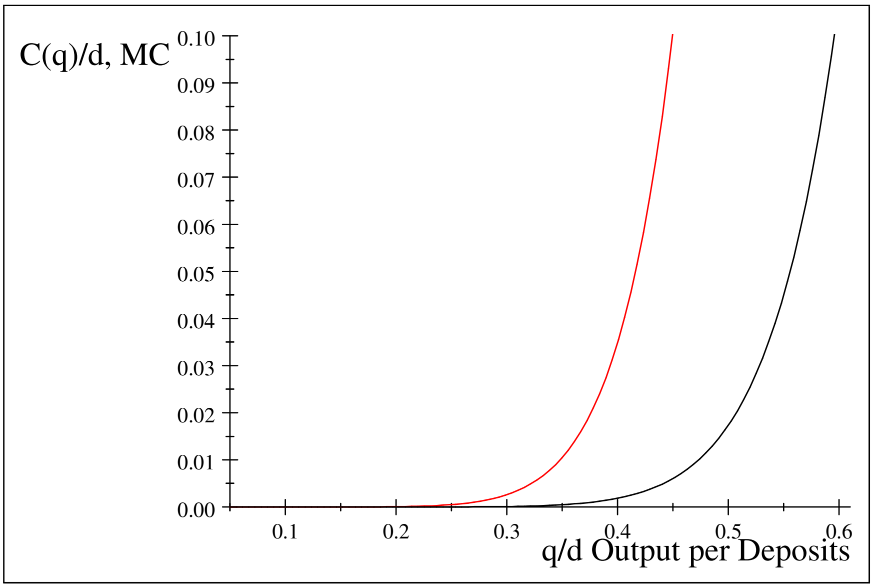

Figure 11.

Further the convex marginal cost per unit of d, denoted by MC, can be computed as

$$MC \equiv \frac{\partial\left(\frac{C(q)}{d}\right)}{\partial\left(\frac{q}{d}\right)} = \frac{1}{\gamma} w \left(\frac{1}{A_Q}\frac{q}{d}\right)^{\frac{1-\gamma}{\gamma}} = \frac{1}{0.1}(1)\left(\frac{1}{0.75}\frac{q}{d}\right)^{\frac{1-0.1}{0.1}},$$

and this is graphed as the red curve in the same Figure.

5.4 Market Clearing

Bringing together the consumer and firm equilibrium conditions, the same equilibrium results as in the representative agent centralized problem.

$$\frac{p_g \frac{\partial u(c_g)}{\partial c_g}}{p_b \frac{\partial u(c_b)}{\partial c_b}} = 1 + R^d = \frac{p_g A_Q (1-\gamma)\left(\frac{\gamma A_Q}{A_g}\right)^{\frac{\gamma}{1-\gamma}}}{p_b}, \tag{107}$$

And with log-utility, $u(c) = \ln c$, and without any loss of generality, we abstract from Arrow-Debreu securities from the model, so that the analysis becomes a static one-period problem. The equilibrium condition simplifies to

$$\frac{c_b}{c_g} = A_Q (1-\gamma)\left(\frac{\gamma A_Q}{A_g}\right)^{\frac{\gamma}{1-\gamma}}. \tag{108}$$

This is the same equilibrium as in the centralized economy.

One difference is that now there is the market budget line in the decentralized economy, with $\left(1+R^d\right)$ being the market determined price. Consider the bad state budget constraint

$$c_b = \left(1+R^d\right) d + A_b; \tag{109}$$

this gives the market line that can be graphed in (c_b, d) space, as in Figure Z. Substituting in for d from the good state budget constraint gives the market line with c_b in terms of c_g :

$$c_b = \left(1 + R^d\right)\left(A_g - d\right) + A_b. \tag{110}$$

5.5 Example 4. Decentralized Economy

For ease of juxtaposition and without any loss of generality, we abstract from Arrow-Debreu securities from the model, so that the analysis becomes a static one-period problem.

Assume the same parameters, $p_g = 0.8$, $p_b = 0.2$, $A_Q = 0.75$, $y_g = 1$, $y_b = 0.3$, $\gamma = 0.1$, and $A_G = 1$, so that the only difference is that now there is a market line with the price $1 + R^d$. Then the equilibrium solution is the same and the only difference is that a green market line now appears in all of the general equilibrium graphs.

The calibrated form of the consumer budget line in terms of (c_b, c_g) space is

$$\begin{aligned} c_b &= \left(1 + R^d\right)\left(1 - c_g\right) + A_b, \\ c_b &= 2.0247\left(1 - c_g\right) + 0.3. \end{aligned} \tag{111}$$

Figure 12 shows Example 3 now with the green budget line of equation (111) also drawn in.

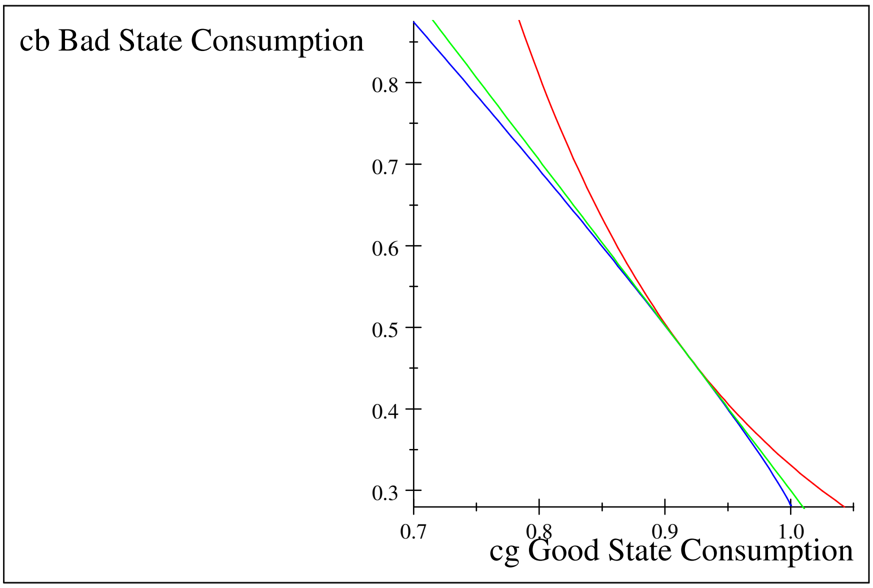

Figure 12

Similarly in the $(c_b : d)$ space,

$$c_b = 2.0247\,(d) + 0.3, \tag{112}$$

and Figure 13 redraws the Figure 12 with the budget line of equation (112) added in:

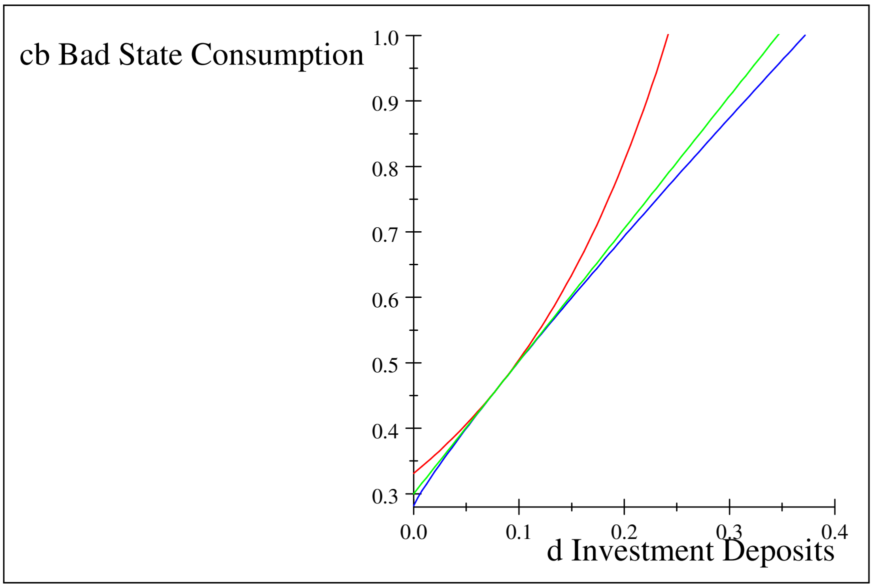

Figure 13

Finally, the isocost line can be added into the isoquant Figure Z of Example 3, in which $p_g = 0.8$ and $p_b = 0.2$. The isocost equation is that the costs equal to the output q, or

$$wl_Q + \frac{p_b}{p_g}\left(1 + R^d\right) d = q = A_Q l_Q^{\gamma} d^{1-\gamma}.$$

Substituting in the equilibrium output level q by putting the Example 3 equilibrium l_Q and d into the production function, then

$$wl_Q + \frac{p_b}{p_g}\left(1 + R^d\right) d = 0.75\,(0.0045819)^{0.1}\,(0.081467)^{1-0.1} = 0.0458.$$

Now adding in the equilibrium factor prices, that $w = y_b = 1$ and $1 + R^d = 2.0247$, the isocost line is

$$\begin{aligned}(1)\, l_Q + \frac{0.2}{0.8}(2.0247)\, d &= 0.0458, \\ l_Q &= 0.0458 - \frac{0.2}{0.8}(2.0247)\, d. \qquad (113)\end{aligned}$$

Figure 14 but now also has the green market, isocost, line included from equation (113).

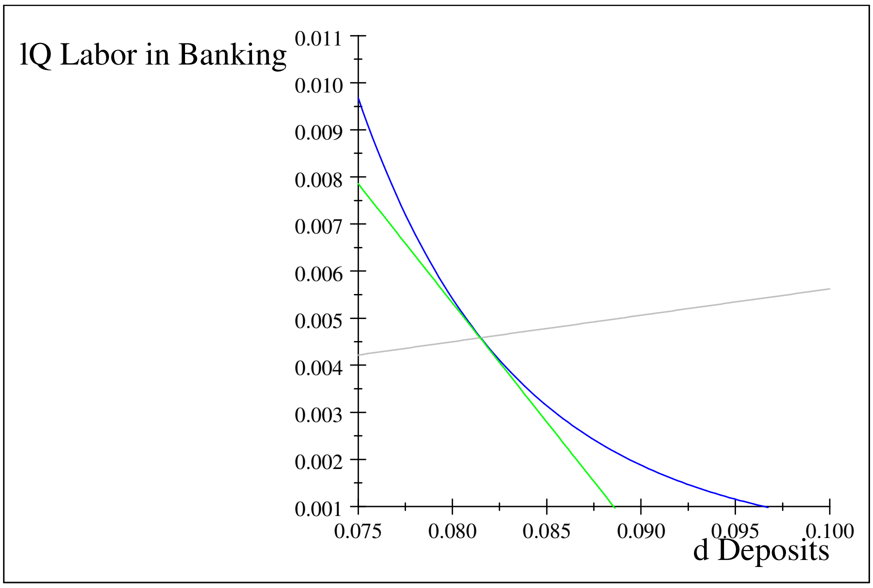

Figure 14.

The isocost line is tangent to the isoquant at the point where the equilibrium input ratio intersects the isoquant. The relative price is the slope of the isocost line and it equals $\frac{p_g}{p_b(1+R^d)}$, which by equation (106) is in turn given by

$$\frac{p_g}{p_b\left(1+R^d\right)} = \frac{1}{A_Q\left(1-\gamma\right)\left(\frac{\gamma A_Q}{w}\right)^{\frac{\gamma}{1-\gamma}}}.$$

As $\gamma \to 0$ and $A_Q \to 1$, the relative price goes to 1.

6 Bank Crises

Bank crises can arise for a variety of reasons, including industry specific risk but more likely from aggregate risk components. For example the bank crisis of 2007-2009 came about after huge debt was undertaken in the US during wartime activity on two fronts in Iraq and Afganistan. Astronomical debt increases pose the threat of future taxes which generally cause lower growth and so lower asset prices. This causes asset prices to fall. An unexpected asset price fall can lead to a bank crisis, if it includes the prices of assets that serve as collateral for loans. In 2007-2009, housing prices fell after rising continuously for a long period of time. When interest rates rose as well, and so loan fees increased, consumers who found the new loan repayments too high could not sell the house and pay back the debt since the house prices fell. This caused bankruptcy and bank failure since many such loans were spread across banks all across the US and Europe.

Both the 1930s and the 2007-2009 crises were characterized by a lack of insurance against so-called aggregate risk arising from the banking sector. The lack of effective insurance in the banking industry in effect can be interpreted as causing a large drop in bank productivity, using the outlines of the baseline model extended to include banking. The difference in the response to the crises was that the governments acted to supply insurance in banking only in 1933 for the US, through the establishment of the Federal Deposit Insurance Corporation, while international governments acted much more quickly in the 2007-2009 crisis, through a variety of measures designed to insure the liabilities of banks, ex post after the crisis occurred. The ad hoc insurance supplied during the modern crisis enabled the recession to be much less pronounced than the 1930s depression.

However the need to use ad hoc bank insurance after the fact results in a very inefficient banking insurance system. Such insurance covered all types of financial intermediation, from commercial banking to investment banking, to insurance (the company AIG was rescued by US government action) to car financing in the form of automobile company aid from governments, especially in the US, and the government financing of new car purchases through a subsidization of old car "scrappage" schemes internationally. But because the shock to the banking system was not insured by the normal insurance elements already in place, the shock ended up acting as aggregate risk that was uninsured.

6.1 Example 5. Increased Bank Productivity

Consider a comparative static increase in bank productivity to illustrate the result on the equilibrium insurance pricing. Let $A_Q = 0.825$, with $p_g = 0.8$, $p_b = 0.2$, $y_g = 1$, $y_b = 0.3$, $\gamma = 0.1$, and $A_G = 1$. The equilibrium quantities are easily calculated by using the general formulation, and then the graphs can be compared to see how the equilibrium changes. From equations (73) to (76) on the general equilibrium, the calibrated solution is

$$\frac{c_b}{c_g} = A_Q (1-\gamma) \left(\gamma \frac{A_Q}{A_G}\right)^{\frac{\gamma}{1-\gamma}}. \tag{114}$$

Substituting in the solution for from equation (71), And now solve for d as

$$d = \frac{y_g A_Q (1-\gamma) \left(\gamma \frac{A_Q}{A_G}\right)^{\frac{\gamma}{1-\gamma}} - y_b}{\left(1+\frac{p_g}{p_b}\right) A_Q (1-\gamma) \left(\gamma \frac{A_Q}{A_G}\right)^{\frac{\gamma}{1-\gamma}}}, \tag{115}$$

$$= \frac{1\,(0.825)\,(1-0.1)\left((0.1)\,\frac{0.825}{1}\right)^{\frac{0.1}{1-0.1}} - 0.3}{\left(1+\frac{0.8}{0.2}\right)(0.825)\,(1-0.1)\left((0.1)\,\frac{0.825}{1}\right)^{\frac{0.1}{1-0.1}}} = 0.093378. \tag{116}$$

$$1+R^{d} = \frac{p_g A_Q (1-\gamma)\left(\frac{\gamma A_Q}{A_G}\right)^{\frac{\gamma}{1-\gamma}}}{p_b}, \tag{117}$$

$$= 4\,(0.825)\,(1-0.1)\left((0.1)\,\frac{0.825}{1}\right)^{\frac{0.1}{1-0.1}} = 2.250\,9 \tag{118}$$

$$l_Q = \left(\frac{\gamma A_Q}{A_G}\right)^{\frac{1}{1-\gamma}} d_t = \left(\frac{0.1\,(0.825)}{1}\right)^{\frac{1}{1-0.1}} d = 0.062526\,(0.093378)\,,$$
$$= 0.0058386.$$

$$c_g = y_g - d = y_g - \frac{y_g A_Q (1-\gamma)\left(\gamma \frac{A_Q}{A_G}\right)^{\frac{\gamma}{1-\gamma}} - y_b}{\left(1+\frac{p_g}{p_b}\right) A_Q (1-\gamma)\left(\gamma \frac{A_Q}{A_G}\right)^{\frac{\gamma}{1-\gamma}}}, \tag{119}$$

$$= 1 - \frac{(0.825)\,(1-0.1)\left((0.1)\,\frac{0.825}{1.0}\right)^{\frac{0.1}{1-0.1}} - 0.3}{(1+4)\,(0.825)\,(1-0.1)\left((0.1)\,\frac{0.825}{1.0}\right)^{\frac{0.1}{1-0.1}}} = 0.906\,62; \tag{120}$$

$$c_b = \left(1+R^{d}\right) d + y_b = \frac{p_g}{p_b}\,\frac{y_g A_Q (1-\gamma)\left(\gamma \frac{A_Q}{A_G}\right)^{\frac{\gamma}{1-\gamma}} - y_b}{\left(1+\frac{p_g}{p_b}\right)} + y_b, \tag{121}$$

$$= 4\frac{(0.825)\,(1-0.1)\left((0.1)\,\frac{0.825}{1}\right)^{\frac{0.1}{1-0.1}} - 0.3}{(1+4)} + 0.3 = 0.51019. \tag{122}$$

$$c_b = c_g A_Q (1-\gamma)\left(\gamma \frac{A_Q}{A_G}\right)^{\frac{\gamma}{1-\gamma}},$$
$$c_b = c_g\,(0.825)\,(1-0.1)\left((0.1)\,\frac{0.825}{1.0}\right)^{\frac{0.1}{1-0.1}},$$
$$= (0.562\,73)\, c_g. \tag{123}$$

The consumption tilting is significantly decreased from $c_b = c_g\,(0.506\,19)$ to $c_b = c_g\,(0.562\,73)$. This represents a fractional increase in bad state consumption by $\frac{0.562\,73-0.506\,19}{0.506\,19} = 0.111\,7$, or 11.2 percent after the 10% increase in bank productivity. The gross return rises to 2.250 9 from 2.0247, also giving the same 11.17% increase. The investment d rises by more from 0.081467 to 0.093378, almost a 15% increase at $\frac{0.093378-0.081467}{0.081467} = 0.146\,21$. Also the labor used in banking increases significantly to 0.0058386 from 0.0045819, a fractional increase of $\frac{0.0058386-0.0045819}{0.0045819} = 0.274\,27$, or 27.4%.

6.2 Output and Input Representations

Graphically, the new equilibrium can compared with substitution and income effects made clear. The production possibility curve in (c_b, c_g) space is

$$\begin{aligned} c_b &= \frac{p_g \left[A_Q (l_Q)^{\gamma} (y_g - c_g)^{1-\gamma} - A_G l_Q\right]}{p_b} + y_b, \\ c_b &= 4\left[0.825\,(0.0058386)^{0.1}\,(1-c_g)^{1-0.1} - 1\,(0.0058386)\right] + 0.3; \quad (124) \end{aligned}$$

expected utility is

$$\begin{aligned} Eu &= p_g \ln c_g + p_b \ln c_b = -0.22115, \\ &= 0.8 \ln 0.906\,62 + 0.2 \ln 0.51019 = -0.213\,02, \quad (125) \\ e^{-0.213\,02} &= c_g^{0.8} c_b^{0.2}, \\ c_b &= \left(\frac{e^{-0.213\,02}}{c_g^{0.8}}\right)^5 = \frac{e^{-(0.213\,02)5}}{(c_g)^4}, \quad (126) \end{aligned}$$

and the budget line is

$$\begin{aligned} c_b &= \left(1 + R^d\right)(1 - c_g) + A_b, \\ c_b &= 2.250\,9\,(1 - c_g) + 0.3. \quad (127) \end{aligned}$$

Figure 15 graph the baseline equilibrium production function, utility level curve and the budget line in darker blue, red and green respectively, along with the new equilibrium.

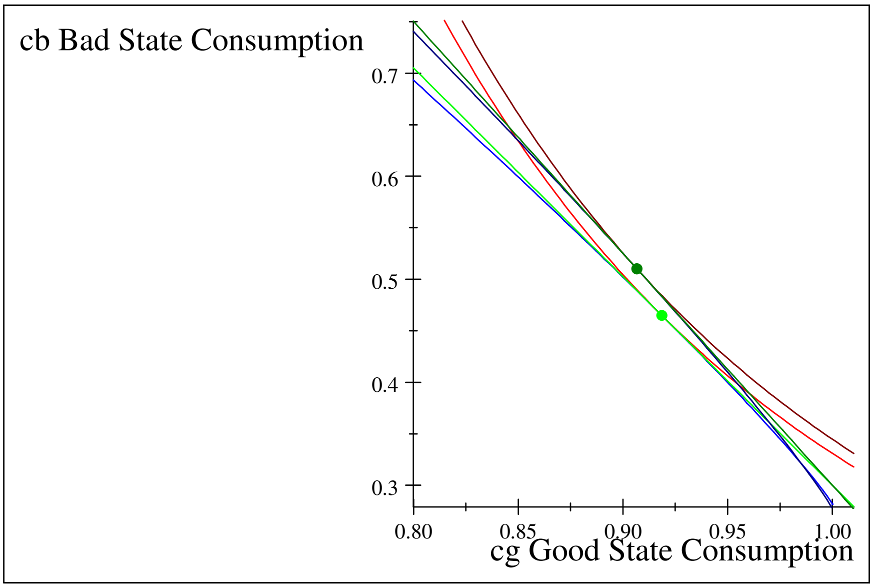

Figure 15.

The graph shows how the production function and budget line both pivot up, and the utility level curve shifts up, to reach an equilibrium tangency between utility and consumption that means the consumer is better off.

The substitution effect of a productivity increase is seen from a steeper budget line, which induces more bad state consumption and less good state consumption. The income effect is to have more consumption in both states. Thus bad state consumption unambiguously rises, and good state consumption depends on which effect dominates. Since good state consumption was 0.91853 in the Example 3 equilibrium, and is 0.906 62 now, the good state consumption decreases as the substitution effect dominates the income effect.

In the $(c_b : d)$ space, the utility level function, budget constraint and production function are

$$c_b = \frac{e^{-(0.213\,02)5}}{(1-d)^4}, \tag{128}$$

$$c_b = 2.250\,9\,(d) + 0.3, \tag{129}$$

$$c_b = 4\left[0.825\,(0.0058386)^{0.1}\,(d)^{1-0.1} - 1\,(0.0058386)\right] + 0.3. \tag{130}$$

Figure 16 graphs the previous equilibrium in light red, green and blue, and the new equilibrium in darker red, green and blue.

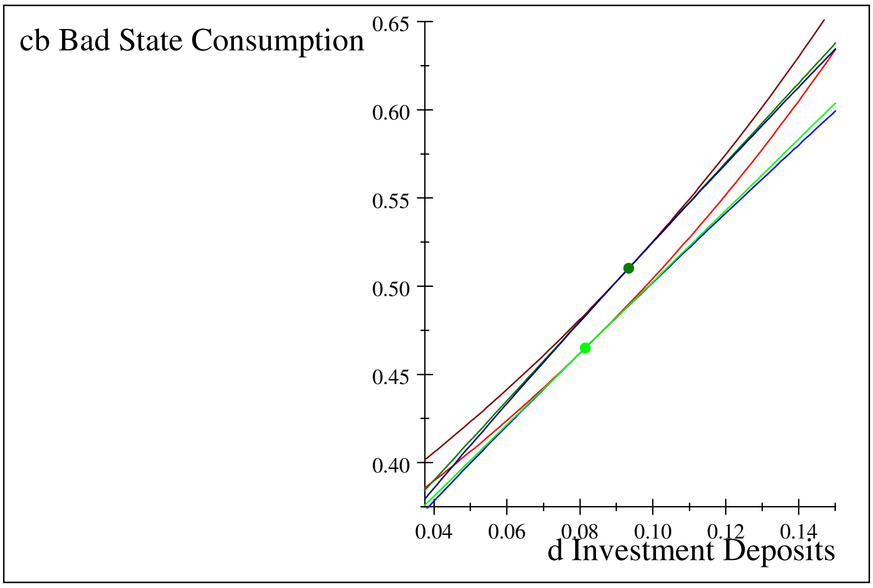

Figure 16.

The graph shows the increase in d as a result of the bank productivity increase, and the increase in bad state consumption. The substitution and income effects both go towards more d and more c_b, causing them both to increase, as utility shifts up as seen by a leftward and upwards shift of the utility level curve tangency at the dark green point.

For the input market, in terms of (l_Q, d) space, the isocost equation is

$$\begin{aligned} wl_Q + \frac{p_b}{p_g}\left(1+R^d\right) d &= q = A_Q l_Q^{\gamma} d^{1-\gamma}, \\ &= 0.825\,(0.0058386)^{0.1}\,(0.093378)^{1-0.1} = 0.058385, \end{aligned}$$

and with equilibrium factor prices of $w = y_b = 1$ and $1 + R^d = 2.2509$, the equilibrium isocost line is

$$\begin{aligned} (1)\,l_Q + \frac{0.2}{0.8}\,(2.250\,9)\,d &= 0.058385, \\ l_Q &= 0.058385 - \frac{0.2}{0.8}\,(2.250\,9)\,d. \end{aligned} \tag{131}$$

The isoquant for Example 5 is

$$\begin{aligned} q &= \frac{p_b}{p_g}\left(1+R^d\right) d + A_G l_Q, \\ l_Q &= \left(\frac{0.045837}{A_Q d^{1-\gamma}}\right)^{\frac{1}{\gamma}} = \frac{0.058385^{10}}{(0.825)^{10}\, d^9}. \end{aligned}$$

The input ratio is

$$\frac{l_Q}{d} = \frac{0.0058386}{0.093378} = 0.062527. \tag{132}$$

$$l_Q = (0.062527)\,d. \tag{133}$$

Figure 17 adds the new equilibrium, in darker blue, green and grey to the previous equilibrium, in lighter blue, green and grey, in the input space.0.081467 to 0.093378

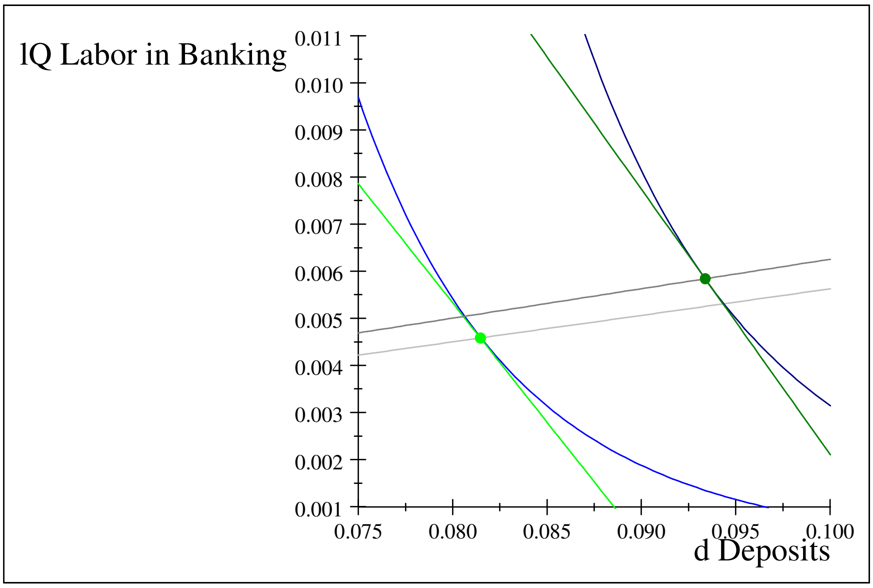

Figure 17.

The isoquant and isocost shift out to the right, to higher levels, and the input price ratio pivots upwards so that it appears in the zoomed-in graph as if it shifts upwards. The equilibrium labor supplied to banking and the deposits both rise.

The dark green budget line has increased the steepness of its slope. This implies that there is a substitution effect along the isoquant towards more labor and less deposits, as the return on deposits rises and makes deposits marginally more expensive to use in production. However the income effect from greater bank productivity is towards more of both labor and deposits. The net effect on deposits usage is that the income effect dominates the substitution effect and increases d, while both substitution and income effects go towards increasing the usage of labor, l_Q.

6.3 Application: Bank Insurance Policy

De facto aggregate risk to the entire financial intermediation sector that is ultimately insured by governments suggests that the risk could have been insured in the first place, in a more efficient, systematic and non-discriminatory fashion. In other words aggregate risk only exists because of incomplete insurance systems. And more complete insurance systems could potentially be devised so that the amount of residual aggregate risk is minimized.

The main policy option for governments to offset the aggregate risk is to ensure the existence of a financial intermediation insurance in a systematic efficient fashion. The cleaning up of bank failures and the minimization of the after-effects of aggregate risk occurring has been a key government policy within countries and at international levels. But with a more complete financial intermediation insurance system, such unexpected decreases in effective bank productivity might be greatly ameliorated and so the need to ex post supply residual insurance could be largely reduced.

6.4 Deposit Insurance with Risk-Based Premiums.

The United States enacted deposit insurance only in 1933, at the height of the depression. Previously, banks had industry-formed "clearinghouses" which acted as a means of insuring depositors at a failing bank. This was designed to minimize the risk that a failure would spread to other banks. Therefore it was a market-based method of providing insurance against aggregate risk.

However with the establishment of the US Federal Reserve Bank in 1913, the government took over the clearinghouse functions and these private insurance mechanisms for the banks were dissolved. Yet when the banks failed during the depression the US failed to insure depositors of failing banks, and the panic then spread to other banks.

Establishment of the Federal Deposit Insurance Corporation (FDIC) in 1933 then created an insurance fund from which to insure depositors, and so avoid the spread of a bank panic. In 2005 the FDIC system was reformed to make the banks pay insurance fees, equal to our d in the analysis, which reflected the risk of the portfolio structure of the bank. These are called "risk-based premiums". And they are designed to insure the system fully by taking into account the different risk factors associated with each bank.

To this day, this system has largely worked, except that financial institutions not covered by the FDIC failed during the 2008 banking crisis. This led to a type of realization of aggregate risk that spread so as to cause a failure of FDIC insured banks. And the US government, using its specially authorized TARP funds, added funding to the FDIC fund so that it could cover the losses of the FDIC-insured bank failures that occurred. So in a sense the FDIC coverage was only partial and did not cover the entire financial intermediation industry. And this can be interpreted as resulting in the manifestation of aggregate risk of bank failure, or a fall in A_F, during the bad state of the concurrent recession of 2007-2009.

For example George Soros lectured at Central European University in 2009 that

> "the Basel Accords made a mistake when they gave securities held by banks substantially lower risk ratings than regular loans: they ignored the systemic risks attached to concentrated positions in securities. This was an important factor aggravating the crisis. It has to be corrected by raising the risk ratings of securities held by banks. That will probably discourage the securitization of loans."

Soros is providing an example of how assets other than commercial banking assets need to be evaluated in terms of the actual risks involved. And this is also necessary in order to provide more efficient insurance to the total set of all financial intermediation liabilities.

6.5 Policy for Global Bank Failure, Aggregate Risk and Moral Hazard

At the international level the widespread failure of banks within interconnected global capital markets has long been a basis for cooperative international government action. One of the main tools has been the International Monetary Fund, or IMF. This agency has been funded by governments internationally, and has acted to intervene to try to contain financial panics within countries or regions when they occur. Latin American intervention during the 1980s, and Asian intervention during the 1997 international bank crisis are key examples.

What the IMF actually does is never pre-determined or clear even after the fact. This has led to criticism of it being a very inefficient way to insure against aggregate risk. And if it allows private banks to be bailed out of their insolvencies, or even just their losses from a regional failure, then criticism has been that the IMF actually increases the probability of such a recurring bank failure. And this is called "moral hazard" when a policy action causes an increase in the probability of the bad state. This increase in the probability of the bad state, as the result of some action or system of actions, is the definition of moral hazard.

6.6 International Finance Systemic Insurance

The success of the FDIC in the US suggests that if such a risk-based premium system were applied to all financial institutions and across all countries, then the aggregate risk of bank failure would be largely eliminated. The first requirement is to consider how to bring all US financial institutions into the FDIC insurance system.

Our analysis is well-suited for this. It suggests that whatever is the form of service that the bank is supplying, its expected outlays can be computed. In our analysis, the expected payout is $\left(1+R^d\right) d$, where R^d includes the cost of the transformation as well as the probabilities of the states occurring. For deposit insurance on commercial, or "retail" banking, such a computation has been fairly straightforward. And so the amount that commercial banks are required to pay into the FDIC is also clear. The problem is the so-called "wholesale" banks, or really the banks that provide services for firms rather than consumers. These investment banks provides loans to firms and help firms sell equity shares to the public markets, in what is called "underwriting". They also help in international risk-pooling through derivative packages and other forms of hedging and risk management.

Investment banks can be brought into a government insurance system in the same way as commercial banks. The only requirement is the assessment of what is R^d on the funds deposited with such investment banks. This assessment is more complicated than for commercial banks because of the variety of risk management vehicles that investment banks offer. And the complication is also that the investment banks themselves invest in a variety of such risk transference vehicles. However, it is clear that such vehicles are subject to evaluation and a risk-premium can be assessed for all deposited funds at investment banks. Allowing these banks to be a part of the FDIC in the US would allow the entire financial sector to take part in the insurance system.

At the international level, a risk-based deposit insurance system that covered both commercial and investment banks could be implemented as a replacement for the ad hoc operation of the IMF in supply such financial insurance. Insurance firms could also be brought into such domestic and global deposit insurance schemes, again on the principle of evaluating the overall average R^d that attaches to each financial intermediary.

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