An Agent Based model of Banking Regulation

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25th February 2014

Preamble

This research project is aimed at constructing a computational simulation based macroeconomic model for policy evaluation when the stable and efficient functioning of a potentially fragile banking system is crucial to real economic welfare.

The model building is still in an early phase, as I have had to teach myself several new modeling approaches and computational tools. Additionally, given the complexity of the decision problems I describe I have had to make some rather stark simplifications/assumptions in order to render the decisions computable. These are all subject to change and I would greatly appreciate any form of criticism or suggestions of alternatives that could either simplify the model of be more convincing without losing important features of the analysis.

1 Introduction

The global financial crisis that became apparent in 2008 has highlighted the need for an accurate and nuanced understanding of how the institutional and operational features of the banking sector are related to macroeconomic outcomes. Moreover, for this understanding to be systematically built into the type of models used to design and evaluate macroeconomic policy. It would be wrong to suggest that the discipline was not aware of the need before the crisis - some of the most nuanced approaches to the problem reviewed in this proposal were published well before the build-up to the crisis, each the culmination of long standing streams of research. The crisis has, however, greatly emphasized the urgency of succeeding in the goal of effectively integrating this knowledge into the standard arsenal of the economist, so that the global economy has some hope of avoiding the worst of potential future crises and recovering as efficiently from those we do not manage to side-step.

This research project is aimed at constructing a computational macroeconomic model for policy evaluation when the stable and efficient functioning of a potentially fragile banking system is crucial to real economic welfare. I will motivate below that for proper macroeconomic policy making, it may be necessary to simultaneously analyze a whole spectrum of models: on the one end, models that are accurate and informative when the economy is close to “normal times" (e.g. the standard dynamic stochastic general equilibrium models that are studied in approximate solutions around a unique steady state equilibrium); and on the other end models that study the causes of and necessary responses to large and potentially disastrous departures from normal times. In brief, the motivation for having a spectrum rather than a single all encompassing approach is technical: at the present state of the art in the discipline, we can only evaluate the implications of very complicated models in approximate form near locally stable equilibria. This means our understanding is sufficiently accurate only on the presumption that the economy remains near these equilibria. In order to study large departures from equilibria, or large, sudden, endogenous structural shifts between equilibria, models must be more non-linear, and hence simpler on other dimensions. The research plan I outline here aims at add to the more non-linear "crisis origin and recovery" end of the spectrum of macroeconomic models with functional financial sectors.

In order to motivate the scope and form of my model, I will begin by presenting number stylized facts on on the proximate causes of the most recent financial crisis, constrained mostly to those features directly related to the novel modeling features that form the basis of this paper. I condense the following mostly from du Plessis (2012):

First, the period preceding the crisis was notably less volatile for the large economies of the world than preceding decades had been. The breaking of the crisis in 2008 ended almost two decades of relatively stable growth with low volatility in most macroeconomic indicators in the US - this is typically called the "great moderation" since Stock and Watson (2003). This long period of relative stability had the impact that any purely backward looking measure of the riskiness of the global environment would have induced optimism of similar stability into the future (at least for the US). While there had been indications of a potential asset bubble near the end of this period, the general consensus at the time had been that monetary policy should not pay special attention and "mop up after the bubble eventually bursts". This was the policy response to the dot-com bubble between 1997 and 2000, and this policy did not seem to have disturbed the great moderation.

Second, policy interest rates were lower than a "standard benchmark" would have suggested they should have been: over the last decade of the great moderation, there are indications that the policy rate adopted by the Federal Reserve Board (Fed) were unusually low relative to the predictions of a benchmark such as a standard Taylor rule would have suggested. This served to depress the yield curve through which commercial banks engaged in the "classical business" of liquidity conversion of short term liabilities (bank deposits) into longer term assets (investments in loans) in principle make their profits, encouraging the search for more profitable business models, leading them into riskier behaviour in equilibrium (some evidence of this increase in effective riskiness of operations in response to low policy rates can be seen in e.g. Delis and Kouretas, 2011).

Third, regulatory changes encouraged over-exposure to more risky types of real and derivative assets on the balance sheets of commercial banks. This combined with a history of implicit bail-out behaviour by central banks that globally reduced the incentives for individual banks to properly manage and asses their exposure to risks at a systemic level. The regulatory changes included three saliently distinct components that combined to be particularly destabilizing: (1) politically motivated promotion of the attractiveness of the sub-prime housing market, (2) increased competition to commercial banks from investment banks as the regulatory lines between them began to blur and (3) a move toward mark-to-market based accounting schemes that combined with asset price bubble effects to hide significant balance sheet risks (Shin, 2009) across the banking system as a whole.

When interest rates began to rise between 2004 and 2006 (du Plessis, 2012:15), bad debts began to rise and the fragile nature of the interconnected banking system was revealed, accompanied by large real losses and an extended global contraction that has become known as the Great Recession.

A one sentence summary of the above can be stated as follows: the combination of stochastic environment, policy stance and regulatory changes induced a misguided decentralized "coordination" in the incentives that govern the financial sector to induce a large misallocation of real resources into risky, relatively unproductive investment. A number of theoretical theses can be made as to the mechanism that leads to such a large scale misallocation of resources in a sophisticated financial system. These in turn would inform policy as to avoiding and/or recovering efficiently, if they can be modeled in a nuanced enough way.

Based on the above stylized facts on the financial crisis, I argue that for a theoretical model to be able to capture the nature of the interactions that led to the crisis, it requires a three important features in the treatment it gives to banks as a special type of derivative firm:

1. Bank operations must have real consequences Banks in the model should be important savings and gearing instruments for consumers who must make consumption and productive real investment decisions, so that the operational choices made by bank managers affect real production levels and growth in the economy. An important nuance here is that the functioning of banks as operators of financing structure for real investment, is that their actions may affect not only the level of investment in the various productive assets in the economy, but may also affect *which* assets are invested in, and so endogenously influence the nature of the stochastic shock processes that drive the macro economy. This last is usually (and uniformly in the researched reviewed here) treated as exogenously given. My proposed treatment will be a novel feature of my contribution to the literature that I think is crucial for understanding what happened in the crisis: the wrong assets were invested in (e.g. too many US houses).

2. Bank operations must be endogenous The way banks make operational choices must respond endogenously to nature of observed stochastic processes as well as to perceived changes to policies and regulations that affect them. There are two obvious ways to do this: through the budget constraint or through the utility function. The path through the budget constraint has been explored in the literature reviewed below - Kiyotaki and Moore (1997) is a prime example where changes to the value of collateral can induce pro-cyclical variations tightness of a bank's liquidity constraints. The path through the utility function is yet absent, and I believe this is part of why our models fail: it may be that the change in policy, say, changes who tends to own and run banks - e.g. that the low interest regime in the low variability era induced more risk loving individuals to be selected guide the practices of banks which lead to an increase risk appetite of banks, which would be cumulative with what happens to the budget constraint. This interaction may lead to wide enough swings to destabilize a local equilibrium. In order to design policies institutions better able to avoid fragility like this, the models must include a testable set of things to consider. <fix sentence>

3. Interbank exposure to liquidity and solvency risks must be endogenous The highly geared networks of cross-bank exposure to the balance sheet deterioration of multiple entities was induced by a number of factors, including rapid increase in new derivative instruments in the face of a specific policy and regulatory environment. But this happened in the US, not in SA. A model of fragility must allow these inter dependencies to be possible and evolve endogenously in order for it to provide insight on how they might be prevented from inducing systemic risk.

In section 2 I present a literature review of a few salient approaches in the mainstream body of economics as well as some from the less conventional corners. I will use this review to argue that the attempts so far do not capture the three aspects I focus on here, and it is unlikely that this will truly be feasible in the type of General Equilibrium models that currently stand at the heart of policy analysis in modern macroeconomics. I will argue that there is scope for this type of modeling on the more computational end of a spectrum of equilibrium definitions, but that the ideas raised in this document have not yet been modeled as I propose to do.

The central question that this research project aims to investigate is whether the incentives created by regulation and institutions that surround a banking market can induce cyclicality characterized by systemic fragility. The main driver in this model will be the incentives created for risk-taking by institutional features of the banking sector, and the evolutionary implication of these incentives on the aggregate functioning of the economy as a whole.

As the focus is on the functioning of the banking sector, the production side of the model will be simple and there will be no labour. It is thus a consumption - investment model with an exogenously given production technology and variable institutional setup, but one where banks have a productive role to play in aggregating savings and allowing geared investment in potentially productive projects.

To model the idea that institutions and policies can induce more/less patient behaviour endogenously one requires a model where lending/saving decisions are made endogenously, not because of a pre-imposed assumed impatience.

The standard approach to thinking about macro-economic stabilization is policies aimed at reducing the variability of aggregate economic outcomes in response to aggregate shocks with exogenously given stochastic properties.

An “aggregate shock” presupposes an exogenously given process that affects aggregate conditions. There are very few such that one can think of. Fiscal policy relating to taxation certainly only affects those agents that pay tax. Spending directly affects only those who receive or produce the things spent on.

While there will always be things that seem to be “aggregate shocks” they must themselves be the aggregation of the consequences of individual decisions that were based on individual perceptions of all relevant shocks.

This is not yet interesting: it makes no difference to policy making whether that the are aggregate level shocks that are aggregate in origin or individual in origin *so long as the fundamental nature of the aggregate level shock is exogenous to the policy decision.* The model I build and apply to banking sector is meant to apply to a situation where this does not hold. Or to put positively: it is a modeling approach to study situations where aggregate level policy choices endogenously and systematically induce specific type of individually experienced stochastic outcomes in an economy as a consequence of rational decisions by consumers.

This happens because of what alternative investment choices really are: they are choices to be exposed to a certain stochastic process that has certain internal and external consequences.

Individuals are in practice exposed to diverse investment options, and either because of local variation or belief differences, these are to a very important extent individual specific. Which risk-return path is acceptable depends on beliefs about current and future states of the world, which include policy and other decisions and their outcomes.

The value that the financial sector and money in general gives is that agents are able to vicariously invest in a far larger array of productive processes than would otherwise be possible. This means that the low productivity son of a millionaire can to mutual gain invest in the productivity of a poor but brilliant inventor. This is complicated to coordinate, which is why banks exist and make profits in providing the aggregating and gearing services that allows a far more optimal allocation of real investment possibilities than would otherwise.

Almost none of the individual components I use here are novel, but the combination of them in the structure below is quite new, so requires focused discussion.

2 Literature Review

The most prominent paradigm for modern analysis of monetary policy employs a Dynamic Stochastic General Equilibrium (DSGE) model in what has become known as the “New Keynesian” tradition which embeds a range of monetary and real rigidities that allows monetary actions to have real effects in the short run[1]. These models are micro-founded and non-linear; and describe both the production and consumption side of the economy in an infinite horizon framework as being the consequence of the decisions of optimizing agents operating in an uncertain (stochastic) economy. By far the largest fraction of these models are in the representative agent framework - i.e. the agents that populate the model are all identical (hence the existence of a unique aggregate solution). A small but growing subset of these models allows for some non-vanishing agent heterogeneity, but constrained in important ways that i turn to below. I will survey a number of attempts to bring some form of a banking system into the analysis in the following subsections, but first I wish to argue that one can view the "general equilibrium" part of these models as one end of a spectrum of models with different solution concepts.

A general equilibrium for an arbitrary model is *a unique, global fixed point* in the set of functions mapping exogenous parameters and stochastic outcomes to values of endogenous (choice) variables that has the following characteristics:

1. All decision-makers are individually optimizing (i.e. there exists no unilateral change in decision for any agent that would lead to a more preferable outcome for that agent).

2. All markets clear (this can be modified to include concepts like "natural" unemployment as a part of the equilibrium).

3. Aggregate variables are deterministically related to each other and distributions of non-degenerate individual level heterogeneity are invariant. This last feature is only part of the equilibrium in a model where there is non-trivial agent heterogeneity included as part of the

This paradigm is dominant as the solution concept of the model, the general equilibrium, is usually the unique point of attraction of the system of equations that describe individual behaviour. While the fully non-linear global solution of these models can typically not be characterized, they can provide strong predictions for dynamic responses to small perturbations near the equilibrium (which also corresponds to a "steady state" of the model - i.e. a point of rest in which the model is deterministic if no stochastic events occur). This is also the major drawback - these models cannot be solved characterized globally, and hence cannot speak accurately as to the likely dynamics of adjustment far away from equilibrium, which is typically what one would define a "crisis" to be.

Given this lack, one might consider alternative approaches such as Agent Based Computational (ABC) models. These are models that impose simple decision rules on inherently heterogeneous individual agents but does not restrict in any way the nature of "an acceptable point of attraction" - indeed the main focus of this type of modeling is indeed whether or not it tends to predict the same type of equilibria as general equilibrium models. The decision rules are specified so that they evolve and adapt. Typically, the adaptation and evolution

[1] Woodford (2003) is probably the most complete codification of this approach as a tool of monetary policy analysis

of decision rules is made operational by assuming some form of tournament type selection criterion built into the model along with mutation operators that explores in a chaotic way the nature of decision rules that map parameters and observed events (by the agents) into choices. The tournament selection rule then selects the locally "fittest" decision rules in each iteration of the model until the system either collapses or stabilizes around a local fixed point. A prime example is Howitt and Clower (2000) who study the emergence of economic organization from a system of autarky where no two agents that meet necessarily have mutually corresponding wants and offerings. They find that (as is typical in these models) that there are standard points of attraction that emerge in most of the model simulations that correspond to what a general equilibrium would be imposed to look like - trade using a single commodity as numeraire, but also "systemic collapses" in which the model returns to autarky. It is this feature that bodes well for these models as interesting candidates for the study of the origin and consequences of crises.

The benefit of ABC models is that they can explore a wider array of potential fixed points or "regions of attraction" that may exist in a non-linear system of equations than the single general equilibrium that is typically the object of interest in DSGE models. The cost is that the ABC models are purely computational and adaptive with very little "aggregate structure" imposed, and hence the underlying micro-structure that can be allowed for is typically very simple and heuristic in comparison to the complexity and number of different type of decisions problems that can be built into a DSGE model.

In my studies of these two types of models I have come to the conclusion (which I will explore in depth in my thesis work) that one can view these different paradigms as varying primarily according to their assumed solution concepts - DSGE models impose that the equilibrium should be unique and universally optimal without studying how the equilibrium comes into being, while the ABC framework studies how equilibria are coordinated on and what their characteristic are without imposing that they be "optimal" in any external sense. I believe a strong argument can be made that interesting results can be obtained by using some of each approach. In that sense, the modeling I propose will look structurally very similar to that of the standard DSGE models, but use something more like the definition of equilibrium as is found in the ABC framework.

The rest of this review is organized as follows: Section 2.1 reviews a number of salient attempts at building in some features of banking into the standard paradigm, and in Section 2.2 I briefly mention the only extant ABC model with a role for banking that I have found to date.

2.1 Infinite Horizon Dynamic Stochastic General Equilibrium models in the New Keynesian tradition

The most important dimension of distinction among the DSGE models I review is that of agent heterogeneity. The first section reviews the models that remain in the representative agent framework. This is problematic as, since all consumers (or "households") are identical, there is no clear functional role for banks: there are no borrowers or savers that are distinct entities among the consumers. This means that "banks" are at most another level or source of distortions in the economy.

Solving a DSGE model with heterogeneous agents is more complicated - a general equilibrium that has non-degenerate agent heterogeneity requires the specification/derivation of the invariant distribution in equilibrium as well as the equilibrium transition dynamics of the entire distribution of agents, which greatly limits the type of heterogeneity that can be allowed for.

2.1.1 Representative Agent Models

As mentioned above, the role of banks in these models without distinct savers and borrowers is a problematic concept. In all of the papers reviewed banks are imposed as necessary intermediaries that provide a variety of services: in financial intermediation for transaction purposes (Goodfriend and McCallum, 2007), engage in saving aggregation and direct investment capital production (Gertler and Karadi, 2011) or maturity transformation (Angeloni and Faia, 2013).

The models differ notionally in how the banks are modeled in terms of operational decisions: Gertler and Karadi (2011) and Angeloni and Faia (2013) assume an exogenously determined equilibrium flow of a fraction of agents in each household between roles of "worker/saver" and the role of "banker" in a capital owner sense. In Gertler and Karadi (2011) the objective of a "banker agent" is to maximize terminal wealth from the exogenous positive return from converting savings to loans, while in Angeloni and Faia (2013) the banks are assumed to be necessary but subject to a time varying risk of experiencing a bank run. Oddly in the latter model, it is deposits at banks that are considered to be the primarily risky asset rather than capital, and it is the bank that actively decides the size of the deposit it will accept - depositors are passive suppliers. The distinction is irrelevant however, as the model assumes that banks are the only vehicle for investments, and the participation fractions are exogenous.

Angeloni and Faia (2013) label their model as one where banks are "fragile" but this is not really a valid use of the word as it is meant in the empirical sense - that banks are fragile if they can cause themselves to fail. In Angeloni and Faia (2013) banks are simply subject to a time-varying probability of a "bank run" and hence are risky. But even this "bank run" is a risk only in the sense that the rate of return is variable. The fragility is not a function of how the banks are run, nor does it represent any risk that a bank might fail or yield zero return.

Universally in these models, the "loan" side of the balance sheet of banks is the capital investment in the firms assumed to populate the production side of the economy and are assumed to be pure profit maximizers subject to standard nominal rigidity specifications - Banks are therefore the decision-makers that determine capital. Banks are employers in some of the models, or capital earnings vehicles in others, but in all cases these functions are assumed to be quite mechanical.

The results of these models are all presented as a set of impulse response functions of the dynamics of a linearized version of each model around its general equilibrium path, and hence are at most variants of degree of results the type of policy analyze that is standard in DSGE models without banks. Banks might make certain responses more volatile or others change direction, but since it is always analyze in context of a single, exogenously driven equilibrium, banks can have little structural impact on economic welfare in the sense of changing the equilibrium.

2.1.2 Heterogeneous Agent Models

In models where there is heterogeneity among agents, there is more scope for an interesting banking sector: if there are borrowers and savers with non-trivial behavioural aspects, there can be a spread between deposit and loan rates - i.e. a credit spread that is an important element of empirical discussions of the financial sector. I discuss one example in a full infinite horizon DSGE and two examples two-period stochastic general equilibrium models. The two period models manage to be much more interesting along the dimension of fragility - borrowers and banks can actually become illiquid and/or insolvent in the two period models which is not allowed for in the infinite horizon models, but this comes at the significant cost. that the results of the two period models can be at most at the level of comparative statics when it comes to policy analysis.

Infinite horizon models Cúrdia and Woodford (2009) present a model in which agents are heterogeneous in that they can have a high or low risk aversion. In order to give some dynamic character to their model, they assume that households switch type according to an exogenous Markov process. This then gives a possibility for an interesting bank sector with a spread between deposits and loans. This is achieved by simply assuming that households can only intermittently trade directly with one another in an insurance market and at other times must resort to the banking system to make their portfolio decisions match. The more risk averse type of consumer will tend to deposit and the more risk loving ones to borrow.

The bank operations in this model remains rather mechanical: they assume an exogenous cost of intermediation as well as a possibly time varying markups to give some dynamic to the bank's decisions, but the profits made by the bank is distributed to the households as wage earnings for supplying some of their labor to the bank. There is therefore no behavioural link between the bank's objectives and the state of the economy, only through the budget constraint.

The shock that they include in the banking side is a time varying fraction of loans that become non-performing, but this is specified as an entirely exogenous process. There is no way that the bank can avoid or exacerbate the change in risks.

This last feature is reflected in their results: some gains on welfare can be made if the monetary policy decision responds to credit spreads but there is little impact from the level of credit.

Two period models Dubey *et al.* (2005) present a two period model in which there are a whole distribution of different consumer types and a sector of perfectly competitive banks where some agents will save and some borrow in any equilibrium. In the second period they allow for default that may be strategic, which makes it possible for banks to go under too. Since this is a two period model, there must be some mechanism for all agents not to choose strategic default, and the authors assume there is some "exogenous utility cost" to defaulting. Heuristically, they link this to lower access to credit markets in non-modeled future periods, but since this is a non-dynamic thing, it is somewhat unsatisfying.

Goodhart *et al.* (2006) uses a similar "utility cost of default" idea to make the model yield interesting results, but with a fully specified interbank market which allows them to analyze monetary equilibria where the policy variables (interest rates and capital requirements) affect bank outcomes endogenously. In setup, this model is quite closely related to the computational model I propose below. Their model remains highly stylized in application however, as it is only solved for the two period case. This is crucial in their very densely detailed model for the general equilibrium to have any hope of being characterized analytically.

2.2 Agent Based Computational Models

Howitt and Clower (2000) is an example of a paper on the opposite extreme of from general equilibrium models spectrum. They study the emergence of economic organization among autonomous decision making agents faced with economic problems of trade coordination without specifying any "optimizing behaviour" other than a evolutionary search for the best behaviour rules. In one interpretation, one can view their contribution as analyzing why banks exist at all - i.e. how it emerges from a autarkic system that firms come into being that specialize in "being intermediaries". This question is undoubtedly interesting and important, but the nature of primitives that must be used in such an exploration makes it unsuitable for urgent policy related questions relating to the functioning of banks (with observable and exploitable functional structure suitable to modeling) outside of the strictures imposed by the solution concept that governs general equilibrium models.

There are very few papers the banking sector using the ABC framework. The closest to the intent of this paper is Ashraf *et al.* (2011) who augment the self organizing trade model above with fiat money, banks and a central bank and fiscal authority that follow typical rule based policy approaches (the central bank follows a standard Taylor rule.)

They find some evidence that the ABC approach can illuminate events like the financial crises: in most simulated runs, the economy tracks the general equilibrium self correcting type behaviour that is imposed in the previous section, but in a small number of cases, the economy breaks down completely.

Their findings are interesting for policy purposes: Banks can act as either stabilizers or "accelerators" i.a. depending on how far the system is from a "well-functioning" equilibrium. Moreover, micro and macro prudential policies are mutually reinforcing in normal times but can be conflicting in "crash" times. This is exactly the kind of result that a policy relevant model of bank fragility should contain.

The particulars of their model is, unfortunately, too primitive for the model to be useful full policy analysis comparable to the type employed by e.g. Cúrdia and Woodford (2009), as the production sector is where the agent based interaction occurs and is built on an unwieldy production technology assumption. Furthermore, the banking side is rather mechanical and hence cannot capture the type of equity market effects that are in Goodhart *et al.* (2006) or that I build into my model. While I view it as essential to model banks in an ABC framework, it is important that the banks are the focus of the evolutionary aspect of the model, which is not the case in Ashraf *et al.* (2011).

2.3 Summary and Conclusions from the Literature

Focusing exclusively on the aspects pertaining to my proposed research and its placement in the broader macroeconomic literature:

There are multi-period general equilibrium models that do well in capturing the response of the financial market's financing constraints on macroeconomic events and the consequent knock-on effects, but the dimensions along which the financial side of the model responds to macro variation is still underdeveloped. There is no multi-period DSGE model that allows borrowers and banks to default, while those general equilibrium models with realistic equity and lending markets where default is possible can only be characterized (at this stage) in two period models. This means these models are not truly dynamic enough to capture events like those that lead up to and followed on the financial crisis well.

It is difficult to imagine that general equilibrium models complicated enough to have interesting equity and lending markets (i.e. with time varying sets of borrowers and equity holders) can be developed soon given the enormous complexities. This suggests that in order to obtain interesting results, the general equilibrium definition of a solution may have to be replaced by a more computational approach, where there is no comparable analysis as yet.

3 Model

The model is populated by overlapping generations of a large but finite population of economic decision making agents that must make consumption, saving/lending and entrepreneurial decisions (from here on just "agents"). Agents in this model will endogenously choose to be bank equity holders, deposit holders or loan financed entrepreneurs depending on individual circumstance, aggregate conditions and regulatory environment.

A crucial component is the explicit derivation of bank objectives and knowledge: Since some agents will choose to be bankers, the goals of and information sets available to banks should be some aggregation of the preferences and information of the individuals that run it.

The population is partitioned into K regions. Each region has a local bank that sells equity, takes deposits, invests in loans to agents and trades in the interbank market. For a feasible model, some restriction on the relationship between banks, depositors, equity holders and lenders across regions is necessary. These restrictions can take different forms, to model specific cases of interest, but I will present here only the most restricted case: agents can only deal with their own regional bank, but regional banks can trade with each other and the central bank. This can be seen as imposing an extreme form of "relationship banking".

Imposing that regional banks are few in number relative to the number of agents allows for an interesting role for banks and an interbank market with systemic risk. In later versions of this model, I intend to allow self selection of clients to banks, which will most probably have interesting implications along adverse selection lines, but for now I will impose the partitioning exogenously to focus on other elements of analysis.

3.1 Notational Conventions

- Superscripts are used to identify different financial instruments.
- Subscripts are used for bank (since there is one bank per region, bank or region are interchangeable), individual and time indices.
  - Banks/regions will be indexed by subscripts k or h while
  - agents will be indexed by subscripts i or j.
  - $\sum_{i \in k}$ is the sum over all agents in region k.
  - The population (of each generation) in region k is N_k so that the total population in the model is $N = \sum_k N_k$.
- The rate of returns on any instrument will be indicated with the letter R. R_{kt}^z is the gross rate on instrument z in region k in period t.

3.2 Policy Space

The potential policy space in this model is consists of:

- a policy interest rate pair $\left\{ \ R^{\underline{r}}, \quad R^{\overline{r}} \ \right\}$.
- a capitalization requirement on banks, initially specified as a required equity:loanbook capitalization ratio q, and
- a regulation on insolvency settlements: a fraction $\delta(g)$ of a borrower's wealth is attachable when in default where g refers to gearing, and $\delta'(g) > 0$.

I will initially restrict some of these to be strictly exogenously fixed, but will in the end product of the thesis fully explore the most interesting/relevant combination of these three policy instruments. Since there is no direct, standard macro policy related to δ I will probably leave it exogenous and only consider sensitivity analyses of results related to this parameter. Eventually I intend to calibrate this with data on insolvency outcomes.

3.2.1 Modeling insolvency outcomes and policy

The assumption I make on the consequences of insolvency is a compromise between realism and modeling simplicity. It is known that limited liability in absolute terms (i.e. specifying a minimum guaranteed consumption level for all agents) induces bottom truncated utility possibility outcomes which tends to lead to corner solutions or ill-defined maximization problems and model instability.

To keep things smooth in this model, I make a harsh assumption on the nature of insolvency laws: I assume that when an agent is insolvent the bank can at most recover a fraction δ of the wealth of the borrower in default (below I make δ a function of gearing). This means if the the contracted repayment is L and the agent has random wealth realization W, both the bank and the borrower correctly anticipate in their optimization problems that the true repayment B will be non-linear in the wealth outcome:

$$B = \min\{\delta W, L\}$$

This has the double benefit of allowing for partial default while remaining easily modeled from both sides of the loan agreement. The repayment is piecewise defined but continuous (the first derivative of the repayment will of course be discontinuous).

3.3 Financial Instrument Space

The central modeling feature of this paper is that there is an exogenously given set of real (productive) assets, but only an endogenous subset of the available assets is invested in (i.e. the asset market is in a sense "endogenously incomplete"). All other assets are "pure financial instruments". That is: loans, deposits, bank equity or any other instrument in the asset market are derivatives of the underlying set of assets invested in. The return on derivative assets will thus be a function of which underlying assets are actually invested in.

To be precise: In my model there is a fixed set of *individual specific* productive assets available (each an exogenously given stochastic process with strictly positive returns). Moreover, these assets are the *only* production technology available. All current consumption and investment must be sourced out of the returns from past investments by agents in their individual technologies. I think of "real investment" here as all investment whether physical or human capital that has some random positive outcome independently of the financial or monetary system.

A key novel feature of my model (as far as my reading has allowed me to judge) is that only some agents will invest in their individual specific asset; others will invest only in financial instruments. This is chosen to eventually allow the model to capture events like the over-investment in the US housing market.

The model is set up so that agents only choose to invest in their own unique technology if they believe it a productive and safe enough investment to have in their portfolio given the alternatives: the set of financial assets available from banks. Agents thus face a standard optimal portfolio problem where the risk-return structure of the equity, deposits or any other liability of a specific bank is directly dependent on the portfolio of loans and other assets that the bank chooses to hold. The performance of the loanbook depends on the performance of investments in the real productive assets that the loans finance. In sum: what is novel in this model is that economic performance in aggregate (the cyclicality and growth in the income distribution) will be an endogenous consequence of the lending and financing decisions made by individuals and banks. These decisions are in their turn shaped by the incentives created by financial regulation and monetary policy.

I use "individual specific assets" as the foundation of this model based on the simple accounting identity that must hold in a closed system: the real GDP (in a perfectly measurable world) is the sum of (some real valuation of) all production in that takes place, which must be the sum over individuals of production attributable to the efforts and resources invested by each individual in some production process or in derivative assets that ultimately finance productive investment.

I will explicitly derive the relationships between the real and derivative assets below, but for now I just list all assets to fix notation. All quantities are in real terms (units of consumption good) at this stage - once the model judged to be sufficiently well functioning to be of interest, the nominal dimension will be added.

3.3.1 Financial Instruments available to agents

Agent i in region k can:

- invest a_i in her own agent specific **asset** with stochastic gross return $R_i^a(a)$;
- invest e_i to buy a share of the **equity** of bank k, with stochastic gross return R_k^e;
- invest d_i in a **deposit** at bank k, with gross return[2] R_k^d

On the liability side, agent i can, subject to her loan being approved,

- borrow l_i from bank k, where the **loan** must be repaid at gross rate R_k^l, conditional on the borrower being solvent.
- Given that a loan will not be repaid at its promised rate R_k^l with certainty, additional notation is needed to characterize bank decisions and outcomes. Since the outcomes for any specific loan is conditional on the performance of the other assets of the borrower, the actual gross return on any loan *is* a borrower specific random variable:
  - The actual random gross return per dollar loaned to agent i is denoted $\widetilde{R}_{ki}^l$ or $\widetilde{R}_i^l$ when considering a fixed region.

3.3.2 Agent Specific Asset

Each agent is endowed with an individual specific "entrepreneurial" stochastic production technology with return process $R_i^a(a)$ that *only* she can invest in. She will do so only to the extent that she believes is optimal, conditional on the financing options and alternative portfolios available.

If agent i invests a real quantity a in her technology it yields the following stochastic real income out of which the subsequent periods consumption and any loan repayments must be financed:

$$R_i^a(a) = \theta_i a^\alpha \varepsilon$$

Where θ_i is an agent specific mean productivity level and ε an agent specific shock possibly correlated across individuals, such that $\mathbb{E}\left[R_i^a(a)\right] = \theta_i a^\alpha$. Initially, for computational reasons, I will keep the distribution of ε very simple and discrete.

[2] This will possibly be stochastic, possibly be safe - I will eventually consider different underlying institutions that will result in one or the other - In the treatment in the rest of the paper, however, I still assume that depositors believe the asset to be safe - as there is some probability that a bank might become insolvent, this treatment is still incorrect. Once the base model is working satisfactorily I intend to study the impact of deposit insurance on outcomes in this model.

3.3.3 Distribution of effective loan repayment induced by default regulation

Given that investment in the agent specific asset is risky, there is positive probability that the loan an agent obtained to finance the investment cannot be repaid at the promised rate. Combined with my assumed default regulation, the amount an agent who has borrowed has to pay back must leave her with at least fraction $1 - \delta(g)$ of whatever income the asset produce.

Thus, for an agent with productivity θ who experiences shock ε, the distribution of total repayment ($\widetilde{R}^l l$) on a loan of size l with promised rate R^l that was used to finance investment of a (implying gearing $g = \frac{R^l l}{a - R^l l}$) in her private asset is:

$$\widetilde{R}^l l = \begin{cases} R^l l & \text{if } R^l l \leq \delta(g)\,\theta a^\alpha \varepsilon \\ \delta(g)\,\theta a^\alpha \varepsilon & \text{otherwise} \end{cases}$$

or in terms of probabilities:

$$\widetilde{R}^l l = \begin{cases} R^l l & \text{with probability } \Pr\left(\varepsilon \geq \frac{R^l l}{\delta(g)\theta a^\alpha}\right) \\ \delta(g)\,\theta a^\alpha \varepsilon & \text{with probability } \Pr\left(\varepsilon < \frac{R^l l}{\delta(g)\theta a^\alpha}\right) \end{cases}$$

<<Graphs of Probability dependence on each parameter>>

3.3.4 Financial Instruments available to banks

Each regional bank will, of course, be the counterpart in each of the equity, deposit and loan instruments above that are taken up by the agents in that region.

Additionally, banks have access to the interbank (money) market as well as the central bank as fundamental provider of fiat currency. As it is computationally simple, the first Central Bank operating system I consider is a corridor system. In order for the interbank market to have an interesting function, it is necessary for the Central Bank to also maintain a interest differential: It must pay a lower return on reserves kept by commercial banks at the Central Bank than it charges on reserves lent out. When there is possibility of confusion I refer to “commercial banks” as opposed to the Central Bank.

Summarizing the financial instruments available to banks:

- a bank may borrow cash **reserves** from the central bank at the policy lending rate $R^{\overline{r}}$.
- a bank may keep cash reserves at the central bank, at gross return $R^{\underline{r}} < R^{\overline{r}}$.
  - Since in any period some banks will hold reserves at the bank and others will lend from the bank, I occasionally need to refer to the rate at the central bank without specifying which one is relevant to an arbitrary bank. For this purpose I will use R^r_k to mean:

$$R^r_k = \begin{cases} R^{\underline{r}} & \text{if bank } k \text{keeps reserves at Central Bank} \\ R^{\overline{r}} & \text{if bank } k \text{borrows from Central Bank} \end{cases}$$

- bank k borrows from or lends to bank h at **money**[3] market rate $R^m_{h,k}$

Note that the last three points together imply restrictions on possible outcomes in the interbank market: The central bank is assumed never to default since it can autonomously expand its budget constraint. This means the lowest guaranteed return on any financial asset is $R^{\underline{r}}$. Its policy rate is the rate at which a bank with a short term cash deficit can borrow freely, hence no bank will pay more than $R^{\overline{r}}$ for funds on the interbank market.

In order to solve for outcomes in the interbank market, I will assume that some matching process combined with Nash bargaining governs interactions (see section 3.6), but it is immediate that at any period the following must hold: $R^{\underline{r}} \leq R^m_{h,k} \leq R^{\overline{r}}$ for any pair of banks that have access to the Central Bank's facilities.

3.4 Nature of Banks (Preliminary)

For the current version of this model, banks are not yet dynamic entities - they form in the beginning of one period and dissolve at the beginning of the next period and new banks form. This is to facilitate the development of the rest of the model while keeping the bank equity market comparatively simple and static.

For the model to be truly interesting, this is obviously undesirable - one would require a bank that is a dynamic vehicle with equity that can be transferred across generations at some sort of equilibrium price to capture collateral valuation effects.

As I am still at the phase of the project where the sub-structure needs to be sorted out, I leave the bank structure in this simple form.

[3] I call the interbank market the "money market" purely to be able to use m as an identifying superscript

3.4.1 Bank Decisions

I assume a bank passively accepts all equity investments and deposits from the agents in the region, but makes makes two distinct optimizing decisions as a cooperative entity:

1. Given the set of loan applications induced by the lending and deposit rates (which were fixed in the previous period), banks must choose the optimal subset of loans to approve (derived in section 3.7.2); and,

2. Before dissolving, banks must set the interest rates on loans and deposits that will apply in the next period (discussed in section 3.7.2).

Banks make decisions based on a value function that represents some aggregation of the preferences of their equity holders.

The static nature of banks makes the interest rate decision rather awkward to motivate as a bank that is being dissolved sets the interest rate for an entirely new bank that will make decisions the following period. This is done since the final model will not be complete before there is a sensible equity market allowing banks to be dynamically valued entities, in which case the interest decision will be a well defined value maximization decision: i.e. to choose the set of interest rates that will maximize the value of equity that will be sold in the following period.

3.5 Timing of Events and Outcomes within a period

The model, being of a finite population Agent Based kind, does not allow (as far as I can see) a feasibly charaterizable analytical or computable general equilibrium. As such I am currently focusing on a simulation based computed sequential market equilibrium. For this purpose I have assumed a sequence of events that occur each period in order to render the different problems feasible to solve.

Parameters drawn for each young generation at birth

- wealth distribution $\mathbf{w} = [w_1 ... w_N]$
- risk aversion parameter distribution $\boldsymbol{\rho} = [\rho_1 ... \rho_N]$
- productivity distribution $\boldsymbol{\theta} = [\theta_i ... \theta_N]$

State variables that enter at the start of the period (determined by decisions in the previous period)

- Policy interest rates $R^{\underline{r}}$ and$R^{\overline{r}}$ and required capitalization ratio q.
- Bank rates $\left\{R_k^l\right\}_{k=1}^M$ and $\left\{R_k^d\right\}_{k=1}^M$ (where k is the index of the bank/region)

Events:

1. Individual Agent Problems solved: Agents from the young generation solve their individual optimization problems conditional on parameters and state variables and choose to be either entrepreneurs that invest in their private asset or bankers that invest in bank equity.
   This determines:

   - the vectors of individual: consumption, loan application, deposit, equity and individual specific asset investment choices,
   - the total bank equity for each bank: $E_k = \sum_{i \in k} e_i$, and
   - the maximum size of total loanbook allowed by regulation: $L_k^{max} = \frac{E_k}{q}$.

2. Individual Bank Problems solved: Banks review loan applications (from all agents that chose not to become equity holders), approve/deny loans.
   This determines:

   - the vectors of approved loans of accepted deposits,
   - the total loan- and depositbooks for each bank: $L_k = \sum_{i \in k} l_i$ and $D_k = \sum_{i \in k} d_i$,
   - the required money market position for each bank: $M_k = E_k + D_k - L_k$.

3. Interbank Market Clearing: The interbank market opens, surplus and deficit banks agree on interbank positions and rates. Residual positions not cleared in the interbank market are taken up with the Central Bank which operates a corridor system (i.e. "unlimited" facilities at fixed rates).
   This determines:

   - the matrices of interbank positions $\{M_{hk}\}$ and agreed returns $\{R_{hk}^m\}$, and
   - the vectors of commercial banks' positions and returns at the Central Bank: $\{M_{0k}\}$ and $\{R_k^r\}$.

4. All beginning of period flows occur

   (a) equity investments and deposits flows into banks,
   (b) approved loans are paid over to borrowers,
   (c) beginning of period interbank flows occur,
   (d) first period consumption occurs for the young generation,
   (e) investments in private assets are made.

5. Shocks and returns realize and end of period flows occur

   (a) real investments in private assets yield their returns and loans are repaid (fully or partially as determined by the default regulations)

(b) bank positions are settled (only partially if a borrowing bank is insolvent)

(c) banks profits (if any) are paid out to equity holders (but the capital amounts are not - that happens at point 9 below).

6. Central Bank sets new policies (rates and capital requirement) for following period

7. Commercial banks set lending and deposit rates for the following period (once banks are made dynamic entities this step will make more sense economically: current period owners of equity will set rates to maximize value of the bank since they will sell this to the young generation in the beginning of the next period).

8. Period ends, next period begins

9. Old generation consumption occurs (which means banks disband and all capital is paid out to the equity holders. Once banks become dynamic entities, sale of equity from the old to the young occurs here: since this is an OLG model, steps 1 and 9 occur simultaneously - this is the point of overlap between the generations)

Since optimal investment decisions of individual agents (Event 1) depend on the nature of optimal decisions by individual banks (Event 2), and similarly the optimal decisions of individual banks (Event 2) depend on the operation of and outcomes in the Interbank Market (Event 3), it is necessary to characterize the above sequence of decision problems in an "intra-period backward induction" approach, starting with the Interbank Market

3.6 The Interbank Market

3.6.1 Functioning of the interbank market

In this section I set up the matching and bargaining mechanism that I assume clears the interbank markets. The central bank operations will constitute the residual of the interbank market: any bank that still has funds to invest or a cash deficit to cover after the closing of the interbank market has to resort to the Central Bank standing facilities.

Recall that the interbank market opens only after all loan approval decisions have been carried out. At this stage bank k either has a known cash surplus $M_k > 0$ that it wishes to invest in the money market, or a cash deficit $M_k < 0$ it has to borrow on the money market[4]. I refer to these jointly as "desired money market positions".

[4] It is possible that some banks may have $\widetilde{M}^k = 0$, but these do not enter the money market and can be ignored.

Define the following index sets of surplus and deficit banks respectively:

$$\mathcal{M}^{+} := \{k \,|M_k > 0\}$$

and

$$\mathcal{M}^{-} := \{h \,|M_h < 0\}$$

First note that nothing ensures that the aggregate cash surplus matches the aggregate cash deficit. In fact, the central bank is always (in this model) the residual position holder in the money market.

Put differently, if the central bank holds no instruments other than bank reserves or loans, the total quantity of central bank issued money M_0 in the economy is given by:

$$M_0 = \sum_{k=1}^{K} M_{0k} = \sum_{k \in \mathcal{M}^{+}} M_k + \sum_{h \in \mathcal{M}^{-}} M_h$$

On the left hand side is nominal money supply and on the right hand side, nominal money demand.

I discuss first the surplus available from an arbitrary match and how it is shared between the two parties of the match, then how the open positions on the two sides of the the interbank market are matched.

Surplus and surplus sharing: For any pair $\{k \in \mathcal{M}^{+}, h \in \mathcal{M}^{-}\}$ of banks, the desired market positions, $\{M_k, M_h\}$,are of opposite sign but will, in general, not be of equal magnitude. I assume that the smaller of the two determines the transaction size that takes place:

$$M_{kh} = \min\{|M_k|\,, |M_h|\}$$

Since all cash deficit banks can lend indefinitely at $R^{\overline{r}}$ from the central bank and all cash surplus banks can invest at $R^{\underline{r}}$ at the central bank, the maximum surplus available in this typical pairwise transaction is:

$$\left(R^{\overline{r}} - R^{\underline{r}}\right) M_{kh}$$

It is obvious that the lending bank prefers that the rate of return on this transaction be the highest (par with central bank's lending rate), while the borrowing bank prefers that it be the lowest.

I have yet to explore what would be a "market consistent" rule here, but for purposes of the first rounds of computation I assume without further motivation that a Nash bargain occurs where the agreed promised rate of return on this transaction is a weighted geometric average of the two extreme rates of return, with the weights equal to relative size of each bank's equity. I.e. I assume that the outcome of the bargaining is a rate

$$R^{m}_{kh} = \left(R^{\overline{r}}\right)^{\left[\frac{E_k}{E_k + E_h}\right]} \left(R^{\underline{r}}\right)^{\left[\frac{E_h}{E_k + E_h}\right]}$$

I assume the remainder of any match is invested at or borrowed from the central bank:

$$M_k + M_h = \begin{cases} M_{0k} & \text{if } M_n + M_h \geq 0 \\ M_{0h} & \text{otherwise} \end{cases}$$

Matching: This part of the interbank market can be made to resemble the observed reality at some point, but for the first computational rounds of the model I keep it simple: the largest position in each of the sets is set off against the corresponding one in the other.

For computational speed I prefer an simple once-off match algorithm that can be thought of to "imbed" a type of temporal friction in the market.

To be precise, I assume that each bank gets only one money market match every period according to the following algorithm:

Algorithm 1 Interbank Market Matching and Surplus Sharing

Step1: In general, the number of elements in and $\mathcal{M}^+$ and $\mathcal{M}^-$ will not be the same. If they differ, add zeros to the smaller set until they have equal number of elements. This corresponds to selecting the central bank as the counterpart for those banks in the larger set that would not otherwise have had a match. The central bank absorbs any residual.

Step2: Let $\ddot{\mathcal{M}}^+$ and $\ddot{\mathcal{M}}^-$ be vectors of equal length containing the indices of the banks in the corresponding sets in descending order of absolute position size (since the Central Bank has no desired position, it always occupies the bottom position in whichever vector the zeros were added). The list of index pairs is the matches for the period in interbank market is the rows of the two column matrix $\ddot{\mathcal{M}} = \left[\ddot{\mathcal{M}}^+ \ddot{\mathcal{M}}^-\right]$.

Step3: for each entry $[k,h] \in \ddot{\mathcal{M}}$, the following is defined as above:
$M_{kh} = \min\{|M_k|, |M_h|\} = -M_{hk}$
$R^m_{hk} = \left(R^{\overline{r}}\right)^{\left[\frac{E_k}{E_k+E_h}\right]} \left(R^{\underline{r}}\right)^{\left[\frac{E_h}{E_k+E_h}\right]}$
$M_{0k} = \max\{0, M_k + M_h\}$
$M_{0h} = \min\{M_k + M_h, 0\}$
$R^r_{0k} = R^{\underline{r}}$
$R^r_{0h} = R^{\overline{r}}$

In sum - commercial banks are matched - largest positions first - with either another commercial bank, in which case the interbank surplus is shared and the bank with a residual position takes it up at the central bank. If the commercial bank is matched with the central bank, it must take its full position at the central bank at the corresponding rate of return.

3.7 Banks

The minimum components that must be allowed for in any model that wishes to study the institutional, regulatory and monetary policy related causes of the recent banking crises are the following:

- banks must have a productive function in the economy so that decisions made by those who run banks affect the aggregate real outcomes in the economy; and
- the institutional, regulatory and monetary policy features of the economy must *endogenously* affect the incentives of agents that manage banks. For this purpose, banks in the model make autonomous, discreet approval decisions on loans that may default. Put differently: banks must be able to make "mistakes" or "be unlucky" in which loans are approved and which disapproved. This is the only way that there can be an endogenously positive probability that the bank will default unless there are back stop measures like deposit insurance.
- A functional interbank market that allows
  - wholesale funding on the interbank market as a main source of working cash flow.
  - systemic risk to develop due to endogenous inter-dependencies between banks
  - a modern run on a bank when it cannot borrow on the interbank market. (this part cannot not yet occur - the mechanism that allows this to happen is still to be thought up)

To achieve the first, I make the stark assumption that it is prohibitively expensive for an agent with relatively low own productivity but relatively high available resources to find and trust a more productive agent enough to invest with him. Banks provides "counterpart insurance" and economies of scale by aggregating and centralizing the process. In other works banks serve as cost effective deposit aggregators and providers of gearing to agents in control of productive technologies. I am thus implicitly assuming that banks can (costlessly) enforce repayments of loans subject to the insolvency laws given above.

Recall that banks are indexed by k or $h \in \{1, ..., K\}$ (I use the convention when describing the decision problem facing a typical bank that the "decision making bank" is k and "possible counterpart" banks are indexed by h).

This makes it convenient to label the Central Bank as bank "0" so that M_{0k} is the debt/asset position of bank k at the central bank

I make some timing assumptions that serve to turn the bank problem into a static one whose funding is wholesale: i.e. loanbook is funded (at the margin) out of the interbank market and the reserve market run by the Central Bank. The static nature derives from the fact that I only allow single period maturity instruments. In future versions, I will build in a functional maturity structure.

3.7.1 Bank Preferences and Information Sets

A near universal assumption in economic theory is that large firms in uncertain environments act as expected profit maximizers. This assumption is equivalent to assuming risk neutrality, as the implied objective function is a linear function of possible profit realizations. A common motivation (beyond that of analytical ease) is that large firms can more readily access fair insurance and hence will in equilibrium be fully insured and act like a risk neutral firm.

In this model, this assumption is undesirable for two reasons: first, all banks will be subject to aggregate risk that cannot be insured or diversified away; second, banks are *owned and controlled* by risk averse expected utility maximizers.

Assuming that banks are "passive" expected profit maximizers does not allow one to build in behaviour such as "over optimism" in any satisfactory way: profit maximizers have linear utilities that are entirely determined by the expected value of any random gamble. At the cost of significantly more complicated modeling, I choose a different route, largely motivated by my personal belief that no banker will keep his job if he claims or acts as if he did not care about variance (at least). What we need is a theory of where the risk preferences of banks originate, ideally one where the risk appetite endogenously evolves from the conditions and regulations in the market.

I start off with a very simple and still unrealistic assumption that is at least a step away from simple profit maximization:

I assume that the preferences that drive a bank's decisions are some aggregation of the preferences of the owners of that bank, using some function of their proportional shareholdings. For the purposes of my longer term research agenda, I will begin by investigating several exogenously imposed aggregation rules; I will not consider endogenous, strategic or collusive behaviour among subsets of shareholders, nor will I consider principal-agent problems in the management of banks. These are undoubtedly important in analyzing incentive effects and consequent financial market fragility, but are too complicated in an already busy model.

For the purposes of this proposal I stick to the most "optimistic neoclassical" of assumptions about the internal functioning of banks:

Bank Assumption 1: The coefficient of relative risk aversion that represents the preferences of a bank is an equity-share weighted average of coefficients of relative risk aversion that represent the preferences of its equity holders[5], and the objective of the bank is to maximize the valuation of gross return on equity.

[5]Even excluding strategic, collusive and agency problems, there are a number of reasonable alternatives that one can use, such as a majoritarian assumption that only the preferences of the largest shareholders who collectively own greater than 50% of equity are aggregated in the preferences of the bank. These simple alternatives will be easy to add as alternatives after the computational algorithm of the model is complete.

The share of equity that is held by agent i is:

$$s_i = \frac{e_i}{\sum_{j \in k} e_j}$$

This means the preferences of bank k are represented by the value function

$$\begin{aligned} v_k(z) &= \frac{z^{(1-\rho_k)} - 1}{(1-\rho_k)} \\ \text{with} & \\ \rho_k &= \sum_{i \in k} s_i \rho_i \end{aligned}$$

The information available to the bank (or equivalently, the beliefs of the bank) can also have several equally convincing forms. For now, I make a parallel assumption to assumption on the preferences of the bank.

Bank Assumption 2: The beliefs of a bank are an equity-share weighted average of the beliefs of its equity holders. (this assumption will only be necessary and have bite when information is no longer perfect)

3.7.2 Bank decision 1: Optimal Loan Approval Algorithm

This section derives the selection algorithm of the set of loans to approve for arbitrary bank k (so I suppress bank subscripts for most of this section). First I discuss how the bank views the potential cash flows from the individual loan application, then how this is internalized in the algorithm that selects the optimal sub-set of loans to approve.

Given the financial structure of this economy, some portfolio restrictions arise which I impose to facilitate computation (I argue later that these should arise endogenously even if not imposed). This is the following: Agents either invest in a portfolio with a loan (gearing their investment in their private asset) or a portfolio with bank equity, but no portfolio choice contain both. As loans are not approved with certainty, agents must plan a "back-up" portfolio of investment in own asset and bank deposit (see section 3.8.4).

This means, from the point of view of the bank, since the loan and deposit rates are fixed, each loan application is a 4-tuple: $(\text{loan application})_i = \left[\begin{array}{cccc} \theta_i & l_i & g_i & d_i \end{array}\right]$ where: θ_i is the productivity of the underlying asset (which determines the likelihood of default), l_i the size of the loan, g_i the level of gearing (determined by the size of the loan relative to the size of the investment it is financing), and d_i the deposit that will be made if the loan is not approved. This last element will require some motivation which I postpone to section 3.8.4. For now, take it as given that for every loan denied there is a corresponding deposit to accept instead. This means that the final loanbook L and the final depositbook D and hence the final desired money market position M are each a function of the set of loans that are approved $\mathcal{L}$.

The following construction is useful for the rest of this section: Suppose the set of N loan applications can be ordered by individual desirability to the bank[6] and consider the sequence of subsets of approved loans $\{\mathcal{L}_n\}$ defined as follows:

$$
\begin{aligned}
\mathcal{L}_0 &= \{\varnothing\} \text{ (no loan application is approved)} \\
\mathcal{L}_1 &= \{i \,|i \text{ is the most desirable loan application}\} \\
&\vdots \\
\mathcal{L}_n &= \{i \,|i \in [n \text{ most desirable loan applications}]\} \\
&\vdots \\
\mathcal{L}_N &= \{i \,|i \in [\text{all } N \text{loan applications}]\}
\end{aligned}
$$

I start from the position for the bank if no loan is approved: If no loan is approved, the total initial inflow of cash into the bank (sum of all deposits and equity choices of the agents $E + D(\mathcal{L}_0)$) must be offered on the interbank market. This will receive variable return $\widetilde{R}_k^m$ which depends on the relative position of the regional bank to all other regional banks which is influenced by the set of loans approved. Thus total terminal cash flow from operations out of which returns to equity is to be financed if all loans are denied is given by:

$$CF(\mathcal{L}_0) = \widetilde{R}^m (E + D(\mathcal{L}_0)) - R^d D(\mathcal{L}_0)$$

I use the following preliminary[7] approach to modeling the anticipated variability in $\widetilde{R}^m$ from the point of view of the individual bank:

First, note that if the bank if the bank approves no loan, it will necessarily be a surplus bank on the interbank market. If it approves all loan applications (or the maximum allowed by regulation) it may be a deficit bank on the interbank market (if the total value of the final loanbook is large enough to exhaust available funds from equity and deposits). With perfect information and the mechanism assumed for the interbank market above, it is technically conceivable that a bank should be able to project who its counterpart will be on the interbank market and how much of a residual will have to be taken up with the Central Bank. This projection would be very complex to build in the current model and would not fit in with the eventual form of the model in which regional populations (and hence banks) have far more restricted information about any other regions. With this eventual final form of the nature information in this model in mind, I make the assumption (technically incongruent with the perfect information assumed elsewhere) that a bank, when approving loans, does not

[6] In the the current simple version of the model, loans have independent stochastic returns and equal variances, so the set of loan applications can be sorted in desirability by sorting along expected individual returns.

[7] There are several more "realistic" alternatives that I am considering - the one chosen here is mostly for computational convenience - any criticism or suggestion on this part is especially welcome

know with certainty which bank will be its counterpart in the interbank market, because it cannot project which banks will be surplus banks or which deficit banks, but knows the equity of all banks, so can form an idea of its bargaining power relative to that of the "average counterpart bank" it may meet.

Now, for any set of approved loans $\mathcal{L}_n$ either $M_k(\mathcal{L}_n) = E_k + D_k(\mathcal{L}_n) - L_k(\mathcal{L}_n) > 0$ or $M_k(\mathcal{L}_n) < 0$. Defining $\overline{E}_{-k}$ as the average equity position of all banks other than bank k, I assume bank k believes:

$$\widetilde{R}^m = \begin{cases} \left(R^{\overline{r}}\right)^{\left[\frac{E_k}{E_k+\overline{E}_{-k}}\right]} \left(R^{\underline{r}}\right)^{\left[\frac{\overline{E}_{-k}}{E_k+\overline{E}_{-k}}\right]} & \text{if } M_k(\mathcal{L}_n) > 0 \\ \left(R^{\overline{r}}\right)^{\left[\frac{\overline{E}_{-k}}{E_k+\overline{E}_{-k}}\right]} \left(R^{\underline{r}}\right)^{\left[\frac{E_k}{E_k+\overline{E}_{-k}}\right]} & \text{if } M_k(\mathcal{L}_n) < 0 \end{cases}$$

I show in section 3.3.3 that the effective rate of repayment on loans ($\widetilde{R}_i^l$) is a function of the stochastic productivity shock (ε_i) experienced by the borrowing investor. Denote this dependence by $\widetilde{R}_i^l(\varepsilon_i)$. This means the final cash flow implication of approving an arbitrary loan i when agent i experiences shock ε_i is given by:

$$CF_i(\varepsilon_i) = \widetilde{R}_i^l(\varepsilon_i)\, l_i + R^d d_i - \widetilde{R}^m(l_i + d_i)$$

Since approving the loan

- exposes the bank to the return from the loan $\widetilde{R}_i^l(\varepsilon_i)\, l_i$ (which is random if borrowing agent has positive probability of becoming insolvent),
- removes an outflow of the deposit plus interest $R^d d_i$ that would have existed if the loan had not been approved.
- Reduces the cash surplus (or increases the cash deficit) by $(l_i + d_i)$ which can be invested with (or must be borrowed from) the interbank market or the Central Bank at rate $\widetilde{R}^m$.

The final cash flow from an arbitrary set of approved loans $\mathcal{L}_n$ is

$$CF(\mathcal{L}_n) = CF(\mathcal{L}_0) + \sum_{i \in \mathcal{L}_n} CF_i(\varepsilon_i)$$

I use the following algorithm to find the optimal set of approved loans:

Algorithm 2 Optimal Set of Loans to approve

Step1: For each $\mathcal{L}_n$, draw a Monte Carlo sample path $\{\varepsilon_{it}^{mc}\}$ of T^{mc} possible stochastic realizations for each $i \in \mathcal{L}_n$.

Step2: For each $\mathcal{L}_n$ and each Monte Carlo draw ε_{it}^{mc} $i \in \mathcal{L}_n, t \in \{1, 2, ..., T^{mc}\}$ construct a Monte Carlo draw of the potential cash flow implications of approving the loans in set $\mathcal{L}_n$:

$$CF_t^{mc}(\mathcal{L}_n) = CF(\mathcal{L}_0) + \sum_{i \in \mathcal{L}} CF_i(\varepsilon_{it}^{mc})$$

Step3: For each $\mathcal{L}_n$, construct the Monte Carlo Expected Valuation of the corresponding cash-flow possibilities as:

$$\mathbb{E}^{mc}[v(CF(\mathcal{L}_n))] = \frac{1}{T^{mc}} \sum_t v(CF_t^{mc}(\mathcal{L}_n))$$

Step4: The optimal set of loans to approve is

$$\begin{aligned} \mathcal{L}^* &= \arg\max_{\mathcal{L}_n} \mathbb{E}^{MC}[v(CF(\mathcal{L}_n))] \\ &\quad \text{subject to} \\ &\quad \sum_{i \in \mathcal{L}_n} l_i \leq L^{max} \end{aligned}$$

3.7.3 Bank Decision 2: The optimal choice of future deposit and loan rates

The last function of the current owners of any bank is to set the rates at which the next periods deposits and loans will be priced.

I assume that owners act as if they will hold exactly the same equity in the same bank next period (nothing prevents this, although it would be more appropriate to assume that owners act to maximize their anticipated reward conditional on simultaneously planning a possible different optimal level of equity holdings, but this is too complicated for now)

That is, R_{t+1}^d and R_{t+1}^l are chosen to solve:

$$\max_{R_{t+1}^d, R_{t+1}^l} \mathbb{E}^{mc}\left[v\left(R_{t+1}^e\left(\mathcal{L}_{t+1}\left(R_{t+1}^d, R_{t+1}^l\right)\right) E_t\right)\right]$$

I have yet to make satisfactory decisions on how to approach this in the current model.

As a short run solution, I search over a grid of interest rates simulating the events above for a fixed population.

3.8 Overlapping Generations of Agents

Agents live for two periods and are standard risk averse expected utility maximizing individuals with a somewhat non-standard set of assets to invest.

3.8.1 Agent heterogeneity

For interesting results, a model such as this requires agent heterogeneity.

In each period, a fixed number N of agents are born, each with an individual specific level of wealth w_i, risk aversion ρ_i and inherent productivity θ_i. Eventually, I will allow beliefs to be the consequence of decisions in the past, but I do not do so yet in this version of the model.

The variation in risk preference may be necessary to my modeling questions as it allows the following hypothetical event to arise: in a situation of yield curve squeeze, banks need to be riskier to obtain an adequate return on equity (as their normal activities are less profitable) hence relatively more risk loving individuals may choose to become bankers. At the same time, relatively risk averse agents may seek the safest asset which, in this economy, is deposits. If this occurs, it means that the risk averse are subject to the luck of the risk loving. This could therefore imply that interest rates can be sub-optimally low.

Formally, agent i born in period t in region k attempts to maximize the present discounted value of life-time utility derived from consumption, where the utility function is a standard Constant Relative Risk Aversion (CRRA) function, subject to a budget constraint determined by the available assets in region k and the agent's initial wealth w_i.

This implies the agent solves the following problem:

$$\max_{\{c^{young}, c^{old}, \text{asset portfolio}\}} \mathbb{E}\left[u_i\left(c^{young}\right) + \beta u_i\left(c^{old}\right) | w_i\right]$$

Where

$$\begin{aligned} u_i(c) &= \frac{c^{(1-\rho_i)} - 1}{(1-\rho_i)} \\ \text{with} & \\ \rho_i &\in (1, \rho^{max}) \end{aligned}$$

3.8.2 Agent Specific Assets and Financing Technologies

Each agent is endowed with an individual specific "entrepreneurial" stochastic production technology with return process $R^a_{i,t}$ that *only* she can invest in. She will do so only to the extent that she believes is optimal, conditional on the financing options and alternative portfolios available.

If agent i invests a real quantity a in her technology it yields the following stochastic real income out of which the subsequent periods consumption and any loan repayments must be financed:

$$R^a_i(a) = \theta_i a^\alpha \varepsilon$$

Where θ_i is an agent specific mean and ε an agent specific shock possibly correlated across individuals, such that $\mathbb{E}\left[R_i^a(a)\right] = \theta_i a^\alpha$. Initially, for computational reasons, I will keep the distribution of ε very simple and discrete.

The other financial instruments available from the local bank are:

- equity with random return R^e;
- bank deposits with promised return R^d (this return is considered safe conditional on bank solvency); and
- bank loans with a known interest rate charged, subject to the borrower remaining solvent: R^l.

In order to avoid certain types of corner solutions that give unstable model results I have found it necessary to make the fraction of attachable income depend on the level of gearing which I define[8] as the ratio of promised personal loan repayment to "expected value of personal equity" (=Expected value of (assets - liabilities)):

$$g = \frac{R^l l}{a - R^l l}$$

I require this complicated structure in order to study the features of the banking market related to the recent financial crisis, but this means I must make strong simplifications elsewhere in order to have a manageable model. I label these as a succession of assumptions on the structure and functioning of the asset markets in this economy.

Asset Market Assumption 1: An agent must in each period choose to invest *either* in a portfolio that includes bank equity but no loan *or* in a portfolio that includes a loan but no bank equity. Moreover this decision must be made before the loan application is approved and cannot be reversed.

I need this assumption to keep the equilibrium in asset markets and bank's optimal decision problem manageable: the equity market will open only once in each period, and those who invest in a bank gain (some) control over the operating procedures of that bank. Thus the "goals of the bank" will be a function of the goals of the *owners* of the bank.

While this "one-or-the-other asset" is a potentially restrictive assumption, it greatly simplifies the problem that the agents and banks have to solve (as well as the model solution as a whole), as mid-period changes in equity will make the "goals of the bank" too difficult to handle.

[8]The appropriate definition of gearing has been problematic - initially I used pure ex ante gearing: $\frac{l}{a-l}$ but this induced loan demand functions that were upward sloping in interest rates and downward sloping in individual productivity. As such I have found it necessary to make gearing also be increasing in the interest rate on loans, giving the current form.

Other alternatives are: $\frac{R^l l}{\theta_i a^\alpha - R^l l}$ or $\frac{\mathbb{E}\left(R_i^l l\right)}{\theta_i a^\alpha - \mathbb{E}\left(R_i^l l\right)}$. I will explore different options and their impacts and defensibility as the project develops.

Assumption 1 implies that no agent borrows to finance equity. While this might seem a restrictive assumption to impose, I conjecture that it will necessarily hold in any equilibrium in this model, even if it is not imposed, for the following simple reason: bank equity returns are financed out of interest charged on loans minus interest paid on deposits and other sources of capital, i.e. equity at any bank must, on average, have a lower return than the loan rate at that bank, so it will not be optimal to borrow to buy equity. Similarly, since there is no liquidity need for deposits, no borrower will choose to hold both a deposit *and* a loan. Imposing this assumption outright reduces the complexity of the solution algorithm (without additional loss of generality if my conjecture is true). What is certainly restrictive is that no arbitrage across regions is allowed, but that is a consequence of the restrictive regional partitioning, not of assumption 1.

I do not wish to restrict the options further: deposit holders (who do not borrow) may invest out of their own wealth in their private asset. Since deposit rates at any bank will be strictly below lending rates (at any bank) in this model, there is a discontinuity between the lending and savings decisions of entrepreneurs that must be treated with discrete optimization methods.

An important operational feature of banks in this proposed model in the long run is that they will choose whether or not to approve a loan based on incomplete information. That means some loans will be rejected that ought to have been accepted and some accepted that ought to have been rejected.

For now, assume that agent i who wishes to make a loan believes it will be approved with probability[9] $0 < \pi^i < 1$ but that applying for a loan is costless, so that any entrepreneur that wants to borrow submits a loan request irrespective of how small she believes the probability of success is.

In order to reduce the complexity of the loan approval process, I make another strong assumption:

Asset Market Assumption 2: Loan applications are only approved or denied in full. No partial loans are made.

Initially I allowed equity investors to also invest in deposits (this would serve as a kind of preference share that reduces the risk of investing in a bank - it confers no voting rights but is a higher tier investment if the bank fails). It turns out in all initial simulations that the optimal choice of deposits were zero, at the cost of instability, so I pre-empt this option by assumption for now, although future modifications my suggest the reintroduction of this possibility.

Asset Market Assumption 3: Equity investors do not hold deposits.

With these simplifications in hand, I can characterize the structure of the individual optimization problem that must solved by any agent conditional on her beliefs, which I turn to next.

[9]This probability-belief will eventually be endogenously different from the actual approval probability, but for now I keep it exogenously fixed

3.8.3 Agent Beliefs

For the model to truly be classified as an agent based model, some evolutionary features must be built in. Eventually I intend that this will occur entirely on the beliefs regarding the productivity of the economy (more appropriately, each agent's view of her own productivity relative to that of all other agents). That is: agents will use observed outcomes in the economy to learn about their relative productivity and hence optimal investment portfolio.

The model is not yet at a stage where I can sensibly allow for this, so for this version I assume all agents have perfect information, although this makes part of the solution algorithm seem unusual, as it is designed for imperfect information of a specific nature (discussed below).

3.8.4 Two Period Agent Optimization Problem

A young agent born in period t is constrained by exogenous endowment of wealth w_{it} and makes decisions about uncertain outcomes based on his position in the productivity distribution $\boldsymbol{\theta}_t$ of his generation.

Given the assumptions on the asset market, the agent must choose whether to invest in a portfolio that includes bank equity but no loan or one that includes a loan but no bank equity. The additional risk to choosing a portfolio that includes a loan is that the loan may not be approved[10].

[10]Here the current (initial) version of the model with full information is a little awkward - with perfect information, each agent should be able to predict perfectly which other agents will become equity holders, hence be able to predict the loan approval decision of the bank with certainty. This makes the type of decision problem that I characterize below technically incorrect (I assume a much more naive/adaptive process than is warranted by the the perfect information assumption).

Since the end form of the model will only be interesting if it involves imperfect information, the algorithm I develop here is intended to be "appropriate" to that situation - i.e. a situation where each agent almost surely has a unique information set and has no credible information about the information set of another. Or put differently, all agents have some limited means to learn about the information relevant to their investment decision and does so the optimal degree (which remains unmodeled for now). The perfect information version of my model then is the zero measure event that all agents happen to have the correct belief about the productivity vector that characterizes the economy (but they do not know this).

In the beginning of the period, she must choose whether to invest in the private asset or in the bank equity market. She makes this choice by constructing the following contingent plans and selecting the plan that yields the highest expected welfare:

- Plan A: Invest in the private asset
  - Plan A1: Optimal consumption, loan size and investment assuming that the loan request is approved
  - Plan A2: Optimal consumption, deposit size and investment assuming the loan is not approved
    - Note: if Plan A2 gives higher value than Plan A1, then the agent will not apply for a loan at all. Since all three plans have to be solved for before we can make this judgment, this possibility does not complicate the characterization of the problem further
- Plan E: Invest in the equity market with a contingent plan of optimal consumption and equity holdings.

Assume that the agent believes her loan will be approved with probability π. This object requires some development as it is necessarily dependent on the equity composition of the bank. For now I assume a naive backward looking belief on the part of the agents - the probability of loan approval this period is believed to equal the loan approval rate of the previous period. That renders the variable exogenous to this period and allows me to treat the Asset plan and the Equity plan independently. Future versions may explore alternatives on this assumption.

Plan A1 - Invest in Private Asset financed by a loan Combining all the assumptions above, an agent whose loan application is approved knows that she must pay back an amount that leaves her with at least fraction $1-\delta(g)$ of whatever income the asset produces. Her anticipated gross rate of repayment (R_i^l) of a loan of size l_i to finance investment of a_i (implying gearing $g_i = \frac{R^l l}{a-R^l l}$) in her private asset is :

$$\widetilde{R}_i^l l = \min\left\{\delta(g)\,\theta_i a^\alpha \varepsilon, R^l l\right\}$$

$V_i^{A1}(w_i)$is the value of the optimal private investment plan if the loan application is approved:

$$\begin{aligned} V^{A1}(w_i,\theta_i,\rho_i) &= \max_{a,l}\left\{u_i\left(c^{young}\right)+\beta\mathbb{E}\left[u_i\left(c^{old}\right)\right]\right\} \\ \text{subject to:} & \\ c^{young}+a &= w+l \\ c^{old} &= \theta_i a^\alpha \varepsilon - \widetilde{R}_i^l l \end{aligned}$$

Plan A2 - Invest in Private Asset and Bank deposit out of own wealth $V_{i,t}^{A2}(w_{i,t})$is the value of the optimal private investment plan if the loan application is denied (or if no loan application is made) and is the solution to the following problem:

$$\begin{aligned} V^{A2}(w_i, \theta_i, \rho_i) &= \max_{a,d} \left\{ u_i\left(c^{young}\right) + \beta \mathbb{E}\left[u_i\left(c^{old}\right)\right]\right\} \\ \text{subject to:} & \\ c^{young} + a &= w_i - d \\ c^{old} &= \theta_i a^{\alpha} \varepsilon + R^d d \end{aligned}$$

Plan A - Invest in Private Asset rather than Bank Equity The problems above (plans A1 and A2) are each simple to solve numerically for any given parameter set by a simple search over a constrained 2 dimensional space.

Given solutions to the plans above, the expected lifetime utility value of investing in the private asset is then given by:

$$V^A(w_i, \theta_i, \rho_i) = \begin{cases} \pi V_{A1}(w_t) + (1-\pi) V_{A2}(w_t) & \text{if } V_{A1}(w_t) > V_{A2}(w_t) \\ V_{A2}(w_t) & \text{otherwise} \end{cases}$$

I assume that an agent chooses to invest in equity whenever $V_E(w_i) \geq V_A(w_i)$ (i.e. if there is a tie, the agent chooses equity).

Plan E - Invest in Bank Equity rather than Private Asset (Infeasible Individual version) The ideal for this project would be to model the equity decision as the exact analogue of the individual decision to invest in the private asset. That would be given by $V_i^E(w_i, \theta_i, \rho_i)$, the solution to the following problem:

$$\begin{aligned} V^{E}(w_i, \theta_i, \rho_i) &= \max_{e} \left\{ u_i\left(c^{young}\right) + \beta \mathbb{E}\left[u_i\left(c^{old}\right)\right]\right\} \\ \text{subject to:} & \\ c^{young} &= w - e \\ c^{old} &= \widetilde{R}^e e \end{aligned}$$

or more simply:

$$V^E(w_i, \theta_i, \rho_i) = \max_e \left\{ u_i(w-e) + \beta \mathbb{E}\left[u_i\left(\widetilde{R}^e e\right)\right]\right\}$$

where $\widetilde{R}^e$ is the random return to bank equity. Characterizing the solution to this problem is severely complicated by the fact that the return on equity $\widetilde{R}^e$ is itself dependent on the total equity composition of the bank: more equity implies a greater ability to spread investments across many uncorrelated projects which decreases the likelihood of bad returns. Thus the optimal level of equity for agent i will depend on the levels of equity chosen by all others *as well as* on the characteristics of the set of non-equity holders (the loan applicants). This means the appropriate solution concept here is a Nash equilibrium among all N_k agents in the region. I do not yet attempt to solve this problem as it involves the simultaneous maximization of a vector of complicated, interrelated value functions with many discontinuities in terms of a vector of choice variables subject to constraints that also contain discontinuities. As such I choose a different solution concept that employs a restricted type of local "benevolent social planner" that acts only on behalf of the equity holders in the bank. I call this hypothetical construct the "bank specific social planner".

Plan E - Invest in Bank Equity rather than Private Asset (Feasible Social Planner version)

For an arbitrary vector of equity holdings $\mathbf{e} = \begin{bmatrix} e_1 & \cdots & e_{N_k} \end{bmatrix}$ with $e_i \geq 0$, let the return on equity induced be denoted $\widetilde{R}^e(\mathbf{e})$, and recall that the share of equity in the bank is given by $s_i = \frac{e_i}{\sum_{j \in k} e_j}$.

The expected current value of lifetime utility for agent i given $\mathbf{e}$ is

$$U_i(e_i | \mathbf{e}) = \left\{ u_i(w_i - e_i) + \beta \mathbb{E}\left[u_i\left(\widetilde{R}^e(\mathbf{e}) e_i \right) \right] \right\}$$

The hypothetical social planner tasked to choose the optimal equity investment vector does so by solving the problem:

$$V^{SP} = \max_{\mathbf{e}} \left[\sum_{i=1}^{N_k} s_i U_i(e_i | \mathbf{e}) \right]$$

Note that this problem is general to any set of equity holders, as s_i is zero when e_i is (or put differently, those that choose to become entrepreneurs fall out of the objective of the planner).

The solution algorithm I propose for this problem is based on the following observations:

1. The interest rates charged/paid by each bank in a specific period are set in the previous period
2. The solutions to Plans A1 and A2 for all agents can be done independently[11] of Plan E, by assumption 1.
3. For agent i, the Value of Plan A is strictly increasing in θ_i (as consumption in both the default state and the non-default state are increasing in θ_i and the probability mass of the default state is decreasing in θ_i).
4. For agent i, given a fixed set of productivity parameters for all other agents, the value of Plan E is independent of her own productivity parameter θ_i, as by assumption 1, the bank returns will not depend on investment in her private asset as she cannot borrow to invest in it if she invests in equity.

[11] By assumption on the nature of agent beliefs about the probability of loan approval

These observations above imply the following

1. If it is optimal for an agent with productivity θ to choose Plan E, it will also be optimal for any agent with productivity $\theta' < \theta$ to invest in equity as well.

2. For a fixed pair of interest rates $\{R^l, R^d\}$ there is a distribution of productivity $\overbrace{\boldsymbol{\theta}}$ such that no agent prefers Plan E (this will not be a permissible distribution as it will imply that the bank does not come into existence, so that none of the plans that were conditional on an extant bank would have been valid) and a distribution of productivity $\underbrace{\boldsymbol{\theta}}$ such that no agent prefers Plan A (this is permissible, as this bank will necessarily be a surplus bank that will invest in the interbank market).

3. For any pair of interest rates $\{R^l, R^d\}$ for which there is some agent for whom it is optimal to choose Plan A over Plan E and some agent for whom it is optimal to choose Plan E over Plan A, there is a threshold value of productivity $\overleftrightarrow{\theta}$ such that all agents with $\theta > \overleftrightarrow{\theta}$ choose Plan A and all agents with $\theta \leq \overleftrightarrow{\theta}$ choose Plan E.

A problem I have not yet overcome satisfactorily is the endogenous nature of the interbank market rates $\widetilde{R}^m$ to equity outcomes in all regions. To treat this "correctly" would entail solving for the Nash Equilibrium between the K bank specific social planners. Since this would again be highly time-consuming I make an alternative assumption that all equity investors/bank social planners believe that they will face the equally weighted geometric average of the central bank's corridor rate for all interbank transactions. I.e. that $\widetilde{R}^m = \left(R^{\overline{r}}\right)^{\left[\frac{1}{2}\right]} \left(R^{\underline{r}}\right)^{\left[\frac{1}{2}\right]}$.

State variables and parameters that enter the equity decision:

- Interest Rates: Lending and Deposit rates for the regional bank, Lending and deposit policy rates at the Central Bank

- Potential pool of investment opportunities: the list of loan applications from all agents in the region $\left\{\left[\begin{array}{cccc}\theta_i & l_i & g_i & d_i\end{array}\right]\right\}_{i=1}^{N_k}$, in descending order of expected return (θ_i) so that $\theta_1 > \theta_2 > ... > \theta_{N_k}$

- The individual valuations of each agent for plan A (the outside option to the equity decision).

The algorithm I develop below starts from the extreme case where no agent chooses to become an entrepreneur - i.e. all agents in the region choose to become bank equity holders - there are no deposits and no loans, so the entire bank equity is invested in the interbank market - then sequentially solves the problems with one less equity holder (hence one more loan applicant) in each round.

Algorithm 3 Optimal Individual Equity investment

State Variables: $\left\{R^l, R^d\right\}$

Initialisation: Solve Plan A for all agents in the region.
This yields the set of valuations of Plan A $\left\{V_i^A\right\}_{i=1}^{N_k}$ and the corresponding set loan applications $\left\{\left[\begin{array}{cccc}\theta_i & l_i & g_i & d_i\end{array}\right]\right\}_{i=1}^{N_k}$
Sort each of these sets by θ_i in descending order so that $\theta_1 > \theta_2 > ... > \theta_{N_k}$

Iteration 1: Start from the extreme case where no agent chooses to become an entrepreneur - i.e. all agents in the region choose to become bank equity holders (i.e. $e_i > 0, s_i > 0 \forall i$): there are no deposits and no loans, so the entire bank equity is invested in the interbank market earning $\widetilde{R}^m$.
- Find the vector

$$\mathbf{e}^{1*} = \arg\max_{\mathbf{e}} \left[\sum_{i=1}^{N_k} s_i U_i\left(e_i \left|\mathbf{e}\right.\right)\right]$$

- If $U_1\left(e_1^{1*}\left|\mathbf{e}^{1*}\right.\right) < V_1^A$:
conclude that agent with productivity θ_1 prefers to be a loan applicant/entrepreneur and move to the next iteration
- Otherwise: stop; all agents in the region become bank owners with equity levels $e_i^* = e_i^{1*}$

⋮

Iteration n: Assume all agents in the region except the first n (with the n highest θs) choose to become equity holders. This means that the first n agents are now a loan applicants and the loan approval process occurs as in 3.7.2
- Find the vector $\mathbf{e}^{n*}$ (with $e_i^{n*} = 0$ imposed $\forall i < n$)

$$\mathbf{e}^{n*} = \arg\max_{\mathbf{e}} \left[\sum_{i=n}^{N_k} s_i U_i\left(e_i \left|\mathbf{e}\right.\right)\right]$$

- If $U_n\left(e_n^{n*}\left|\mathbf{e}^{n*}\right.\right) < V_n^A$, conclude that agent with productivity θ_n prefers to be a loan applicant/entrepreneur and move to the next iteration.
- Otherwise: stop: the n agents with highest θ become loan applicants/entrepreneurs, all other agents become bank owners with equity levels $e_i^* = e_i^{n*}$

3.9 Central Bank Operations

While the substructure of the model is being developed, the Central Bank decisions are kept strictly exogenous. I.e. I ask of the model what are how the evolution of the economy differs for different Central Bank spreads and reserve interest rates.

4 Results and Discussion

I am currently in the last phase of implementing corrections to my code and will be updating this document with results and discussion when they are available. A typical output of the model will be a time path of a simulated economy where one can analyze the impacts of various regulations on welfare and systemic risk.

References

Angeloni, I. and Faia, E. (2013). Capital regulation and monetary policy with fragile banks. *Journal of Monetary Economics* **60**(3), pp. 311–324. [online] available from http://dx.doi.org/10.1016/j.jmoneco.2013.01.003

Ashraf, Q., Gershman, B. and Howitt, P. (2011). Banks, Market Organization, and Macroeconomic Performance: An Agent-Based Computational Analysis. Working Paper 17102, National Bureau of Economic Research. [online] available from http://www.nber.org/papers/w17102

Cúrdia, V. and Woodford, M. (2009). Credit frictions and optimal monetary policy (278). [online] available from http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1440289

Delis, M. D. and Kouretas, G. P. (2011). Interest rates and bank risk-taking. *Journal of Banking & Finance* **35**(4), pp. 840–855. [online] available from http://dx.doi.org/10.1016/j.jbankfin.2010.09.032

du Plessis, S. (2012). *Collapse. The story of the international financial crisis, its causes and policy consequences*. Barbara Budrich Publishers, Berlin.

Dubey, P., Geanakoplos, J. and Shubik, M. (2005). Default and Punishment in General Equilibrium1. *Econometrica* **73**(1), pp. 1–37. [online] available from http://dx.doi.org/10.1111/j.1468-0262.2005.00563.x

Gertler, M. and Karadi, P. (2011). A model of unconventional monetary policy. *Journal of Monetary Economics* **58**(1), pp. 17–34. [online] available from http://dx.doi.org/10.1016/j.jmoneco.2010.10.004

Goodfriend, M. and McCallum, B. T. (2007). Banking and interest rates in monetary policy analysis: A quantitative exploration. *Journal of Monetary Economics* **54**(5), pp. 1480–1507. [online] available from http://dx.doi.org/10.1016/j.jmoneco.2007.06.009

Goodhart, C., Sunirand, P. and Tsomocos, D. (2006). A model to analyse financial fragility. *Economic Theory* **27**(1), pp. 107–142.
[online] available from http://dx.doi.org/10.1007/s00199-004-0572-7

Howitt, P. and Clower, R. (2000). The emergence of economic organization. *Journal of Economic Behavior & Organization* **41**(1), pp. 55–84.
[online] available from http://dx.doi.org/10.1016/S0167-2681(99)00087-6

Kiyotaki, N. and Moore, J. (1997). Credit cycles. *Journal of Political Economy* **105**(2), pp. 211–248.

Shin, H. S. (2009). Securitisation and Financial Stability. *The Economic Journal* **119**(536), pp. 309–332.
[online] available from http://dx.doi.org/10.1111/j.1468-0297.2008.02239.x

Stock, J. and Watson, M. (2003). Has the business cycle changed and why? In M. Gertler and K. Rogoff (Eds.) *NBER Macroeconomics Annual 2002, Volume 17*, vol. 17, MIT Press.
[online] available from http://www.nber.org/chapters/c11075.pdf

Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary policy*. Princeton University Press.