Firms in International Trade

Lecture 2: The Melitz Model

Stephen Redding

London School of Economics

Essential Reading

- Melitz, M. J. (2003) "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," *Econometrica*, 71, 1695-1725.
- Chaney, Thomas (2008) "Distorted Gravity: The Intensive and Extensive Margins of International Trade," *American Economic Review*, 98(4), 1707-1721.
- Arkolakis, Costas, Klenow, Peter, Demidova, Svetlana and Andres Rodriguez-Clare (2009) "The Gains from Trade with Endogenous Variety," *American Economic Review*, Papers and Proceedings, 98 (4), 444-450.

What Does This Paper Do?

- Dynamic Industry Model with heterogeneous firms where opening to trade leads to reallocations of resources within an industry
- Opening to trade leads to
  - Reallocations of resources across firms
  - Low productivity firms exit
  - High productivity firms expand so there is a change in industry composition
  - High productivity firms enter export markets
  - Improvements in aggregate industry productivity
  - No change in firm productivity
- Consistent with empirical evidence from trade liberalizations?

Theory and Evidence

- The theoretical model is consistent with a variety of other stylized facts about industries
  - Heterogeneous firm productivity
  - Ongoing entry and exit
    - Co-movement in (gross) entry and exit due to sunk entry costs
    - Exiting firms are low productivity (selection effect)
  - Explains why some firms export within industries and others do not
    - Contrast with traditional theories of comparative advantage
    - Exporting firms are high productivity (selection effect)
    - No feedback from exporting to productivity

Where Does the Paper Fit in the Literature?

- Theoretical
  - Dynamic industry models of heterogeneous firms under perfect competition
    - Jovanovic (1982) and Hopenhayn (1992)
  - Models of trade under imperfect competition
    - Krugman (1980)
  - Other frameworks for modeling firm heterogeneity
    - Bernard, Eaton, Jensen and Kortum (2003)
    - Yeaple (2003)
- Empirical
  - Empirical literature on heterogeneous productivity, entry and exit
    - Davis and Haltiwanger (1991)
    - Dunne, Roberts and Samuelson (1989)
    - Bartelsman and Doms (2000)
  - Empirical literature on exports and productivity
    - Bernard and Jensen (1995, 1999)
    - Roberts and Tybout (1996, 1997)
    - Clerides et al. (1998)
  - Empirical literature on trade liberalization
    - Levinsohn (1999)
    - Pavcnik (2002) and Tybout and Westbrook (1995)

Road Map

- Overview of Model Structure
- Equilibrium in a Closed Economy
- Equilibrium in an Open Economy
- The impact of the opening of trade
- What did we learn?

Overview of Model Structure

- Single factor: labor (numeraire, $w = 1$)
- Firms enter market by paying sunk entry cost (f_e)
- Firms observe their productivity (φ) from distribution $g(\varphi)$
- Productivity is fixed thereafter
- Once productivity is observed, firms decide whether to produce or exit
- Firms produce horizontally-differentiated varieties, with a fixed production cost (f_d) and a constant variable cost that depends on their productivity
- Firms face an exogenous probability of death (δ) due to *force majeure* events

Closed Economy
Demand

- CES "love of variety" preferences:

$$C = \left[\int_{\omega \in \Omega} q(\omega)^{\rho} d\omega \right]^{\frac{1}{\rho}}, \qquad 0 < \rho < 1,$$

- Dual price index:

$$P = \left[\int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}, \qquad \sigma = \frac{1}{1-\rho} > 1,$$

- Equilibrium firm revenue:

$$r(\omega) = R \left(\frac{p(\omega)}{P} \right)^{1-\sigma},$$

Production

- Production technology:

$$l = f + \frac{q}{\varphi},$$

- Firms of all productivities behave symmetrically and therefore we can index firms by productivity alone
- Profit maximization problem:

$$\max_{p(\varphi)} \left\{ p(\varphi)q(\varphi) - w\left(f + \frac{q(\varphi)}{\varphi}\right)\right\},$$

- The first-order condition yields the equilibrium pricing rule:

$$p(\varphi) = \left(\frac{\sigma}{\sigma - 1}\right)\frac{w}{\varphi} = \frac{1}{\rho\varphi},$$

- where we choose the wage for the numeraire, $w = 1$

Firm Revenue

- Substituting the pricing rule into equilibrium revenue:

$$r(\varphi) = (\rho\varphi)^{\sigma-1} RP^{\sigma-1}, \qquad \pi(\varphi) = \frac{r(\varphi)}{\sigma} - f.$$

- Therefore the relative revenue of any two firms within the same market depends solely on their relative productivities:

$$r(\varphi'') = \left(\frac{\varphi''}{\varphi'}\right)^{\sigma-1} r(\varphi'), \tag{1}$$

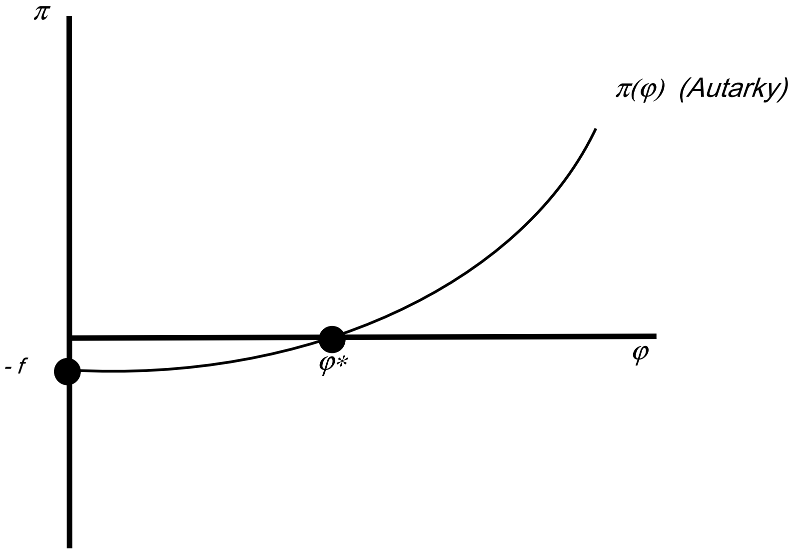
- The presence of a fixed production cost implies a *zero-profit cutoff productivity* below which firms exit:

$$\pi(\varphi^*) = 0, \qquad \Leftrightarrow \qquad r(\varphi^*) = \sigma f, \tag{2}$$

- The revenue of any firm can therefore be written as:

$$r(\varphi) = \left(\frac{\varphi}{\varphi^*}\right)^{\sigma-1} \sigma f,$$

Profits and Productivity



Firm Entry and Exit

- The *ex post* productivity distribution conditional on successful firm entry is therefore:

$$\mu(\varphi) = \begin{cases} \frac{g(\varphi)}{1-G(\varphi^*)} & \text{for } \varphi \geq \varphi^* \\ 0 & \text{otherwise} \end{cases},$$

- The value of a firm with productivity φ is:

$$v(\varphi) = \max\left\{0, \frac{\pi(\varphi)}{\delta}\right\},$$

- In equilibrium, the free entry condition requires the expected value of entry to equal the sunk entry cost

$$v_e = \frac{1 - G(\varphi^*)}{\delta}\bar{\pi} = f_e, \tag{3}$$

  - where $[1 - G(\varphi^*)]$ is the probability of successful entry
  - where $\bar{\pi}$ is expected profits conditional on successful entry

Free Entry

- Expected profits conditional on successful entry are:

$$\bar{\pi} = \int_{\varphi^*}^{\infty} \pi(\varphi) \frac{g(\varphi)}{1 - G(\varphi^*)} d\varphi,$$

- which using the relationship between variety revenues and the zero-profit cutoff condition (2) can be written as:

$$\bar{\pi} = f \int_{\varphi^*}^{\infty} \left[\left(\frac{\varphi}{\varphi^*} \right)^{\sigma-1} - 1 \right] \frac{g(\varphi)}{1 - G(\varphi^*)} d\varphi,$$

- Therefore the free entry condition becomes:

$$v_e = \frac{f}{\delta} \int_{\varphi^*}^{\infty} \left[\left(\frac{\varphi}{\varphi^*} \right)^{\sigma-1} - 1 \right] g(\varphi) d\varphi = f_e, \tag{4}$$

- which is monotonically decreasing in φ^*
- Therefore the model has a recursive structure where φ^* can be determined from the free entry condition alone

Aggregate Variables

- Define a weighted average of firm productivity:

$$\tilde{\varphi} = \left[\int_{\varphi^*}^{\infty} \varphi^{\sigma-1} \frac{g(\varphi)}{1 - G(\varphi^*)} d\varphi \right]^{\frac{1}{\sigma-1}} . \tag{5}$$

- Aggregate variables, such as dual price index P, can be written as functions of mass of firms M and weighted average productivity:

$$P = \left[\int_{\varphi^*}^{\infty} p(\varphi)^{1-\sigma} M \frac{g(\varphi)}{1 - G(\varphi^*)} d\varphi \right]^{\frac{1}{1-\sigma}} ,$$

$$P = \left[\int_{\varphi^*}^{\infty} (\rho\varphi)^{\sigma-1} M \frac{g(\varphi)}{1 - G(\varphi^*)} d\varphi \right]^{\frac{1}{1-\sigma}} ,$$

- where there is a mass of firms with each productivity $Mg(\varphi)/[1 - G(\varphi^*)]$

$$P = M^{\frac{1}{1-\sigma}} p(\tilde{\varphi}) = M^{\frac{1}{1-\sigma}} \frac{1}{\rho\tilde{\varphi}} . \tag{6}$$

Closed Economy General Equilibrium

- The closed economy general equilibrium is referenced by the triple $\{\varphi^*, P, R\}$
- All other endogenous variables can be written in terms of this triple
- The steady-state equilibrium is characterized by a constant mass of firms entering each period, M_e, a constant mass of firms producing, M, and a stationary *ex post* distribution of firm productivity, $g(\varphi) / [1 - G(\varphi^*)]$
- To determine general equilibrium, we use the recursive structure of the model
- Equilibrium φ^* follows from the free entry condition (4) alone

Closed Economy General Equilibrium, R

- To determine R, we use the steady-state stability condition that the mass of successful entrants equals the mass of exiting firms

$$[1 - G(\varphi^*)]\, M_e = \delta M$$

- Using this steady-state stability condition to subsitute for $1 - G(\varphi^*)$ in the free entry condition (3), competitive entry implies that total payments to labor used in entry equal total firm profits:

$$L_e = M_e f_e = M\bar{\pi} = \Pi,$$

- Total payments to labor used in production equal total revenue minus total firm profits:

$$L_p = R - M\bar{\pi} = R - \Pi.$$

- Therefore total revenue equals total labor payments and the labor market clears:

$$R = L = L_p + L_e.$$

Closed Economy General Equilibrium, P

- To determine P in (6), we need to solve for $\tilde{\varphi}$ and M
- Having determined φ^*, $\tilde{\varphi}$ follows immediately from (5)
- The mass of firms can be determined from:

$$M = \frac{R}{\bar{r}} = \frac{L}{\sigma(\bar{\pi} + f)},$$

- where $\bar{r}$ and $\bar{\pi}$ can be written as a function of φ^* and $\tilde{\varphi}$, which have both been determined:

$$\bar{r} = \int_{\varphi^*}^{\infty} r(\varphi) \frac{g(\varphi)}{1 - G(\varphi^*)} d\varphi = r(\tilde{\varphi}) = \left(\frac{\tilde{\varphi}}{\varphi^*}\right)^{\sigma - 1} \sigma f,$$

$$\bar{\pi} = \int_{\varphi^*}^{\infty} \pi(\varphi) \frac{g(\varphi)}{1 - G(\varphi^*)} d\varphi = \pi(\tilde{\varphi}) = \left[\left(\frac{\tilde{\varphi}}{\varphi^*}\right)^{\sigma - 1} - 1\right] f,$$

Open Economy Model

- Consider a world of symmetric countries
- Suppose that each country can trade with $n \geq 1$ other countries
- Choose the wage in *one* country as the numeraire, which with country symmetry implies $w = w^* = 1$
- To export a firm must incur a fixed export cost of f_x units of labor
- In addition, exporters face iceberg variable costs of trade such that $\tau > 1$ units of each variety must be exported for 1 unit to arrive in the foreign country
- As firms face the same elasticity demand in both markets, export prices are a constant multiple of domestic prices due to the variable costs of trade:

$$p_x(\varphi) = \tau p_d(\varphi) = \frac{\tau}{\rho\varphi},$$

- Consumer optimization implies that export market revenue is a constant fraction of domestic market revenue:

$$r_x(\varphi) = \tau^{1-\sigma} r_d(\varphi) = \tau^{1-\sigma} R(P\rho\varphi)^{\sigma-1},$$

Firm Exporting Decision

- Total firm revenue depends on whether or not a firm exports:

$$r(\varphi) = \begin{cases} r_d(\varphi) & \text{not export} \\ r_d(\varphi) + n r_x(\varphi) = (1 + n\tau^{1-\sigma}) r_d(\varphi) & \text{export} \end{cases}$$

- Consumer love of variety and fixed production costs $\Rightarrow$ no firm will ever export without also serving the domestic market
- Therefore we can apportion the fixed production cost to domestic market and the fixed exporting cost to export market
  - When deciding whether to export, firms compare export market profits to the fixed exporting costs
  - Equivalently, we could compare the sum of domestic and export market profits to the sum of the fixed production and exporting costs
- Given fixed exporting costs, there is an exporting cutoff productivity φ_x^* such that only firms with $\varphi \geq \varphi_x^*$ export:

$$r_x(\varphi_x^*) = \sigma f_x. \tag{7}$$

Selection into Export Markets

- A large empirical literature finds evidence of selection into export markets (e.g. Bernard and Jensen 1995, Roberts and Tybout 1997)
  - Only some firms export
  - Exporters are more productive than non-exporters
- From the relative revenues of firms with different productivities in the same market (1), and from relative revenue in the domestic and export markets (18), we have:

$$r_d(\varphi_x^*) = \left(\frac{\varphi_x^*}{\varphi^*}\right)^{\sigma-1} r_d(\varphi_d^*), \qquad r_x(\varphi_x^*) = \tau^{1-\sigma} r_d(\varphi_x^*).$$

- Therefore using the zero-profit and exporting cutoff conditions, (2) and (7), we obtain the following relationship between the productivity cutoffs:

$$\varphi_x^* = \tau \left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}} \varphi^*, \tag{8}$$

- where selection into export markets, $\varphi_x^* > \varphi^*$, requires $\tau^{\sigma-1} f_x > f$

Free Entry

- The free entry condition in the open economy becomes:

$$v_e = [1 - G(\varphi^*)] \frac{[\bar{\pi}_d + \chi \bar{\pi}_x]}{\delta} = f_e,$$

- where $[1 - G(\varphi^*]$ is the probability of successful entry, $\bar{\pi}_d$ is expected domestic profits conditional on successful entry, $\chi = [1 - G(\varphi_x^*)]/[1 - G(\varphi_d^*)]$ is the probability of exporting conditional on successful entry, and $\bar{\pi}_x$ is expected export profits conditional on exporting
- Using the relationship between variety revenues and the zero-profit and exporting cutoff conditions, we obtain:

$$\begin{aligned} v_e = \frac{f}{\delta} \int_{\varphi^*}^{\infty} \left[\left(\frac{\varphi}{\varphi^*} \right)^{\sigma - 1} - 1 \right] g(\varphi) d\varphi \\ + \frac{f_x}{\delta} \int_{\varphi_x^*}^{\infty} \left[\left(\frac{\varphi}{\varphi_x^*} \right)^{\sigma - 1} - 1 \right] g(\varphi) d\varphi = f_e, \end{aligned} \tag{9}$$

Average Firm Revenue and Profits

- Average firm revenue and profits are now:

$$\bar{r} = r_d(\tilde{\varphi}) + \chi n r_x(\tilde{\varphi}_x), \qquad \bar{\pi} = \pi_d(\tilde{\varphi}) + \chi n \pi_x(\tilde{\varphi}_x),$$

- where average revenue in each market is:

$$\bar{r}_d = r_d(\tilde{\varphi}) = \left(\frac{\tilde{\varphi}}{\varphi^*}\right)^{\sigma-1} \sigma f, \qquad \bar{r}_x = r_x(\tilde{\varphi}_x) = \left(\frac{\tilde{\varphi}_x}{\varphi_x^*}\right)^{\sigma-1} \sigma f_x,$$

- and average profits in each market are:

$$\bar{\pi}_d = \pi_d(\tilde{\varphi}) = \left[\left(\frac{\tilde{\varphi}}{\varphi^*}\right)^{\sigma-1} - 1\right] f,$$

$$\bar{\pi}_x = \pi_x(\tilde{\varphi}_x) = \left[\left(\frac{\tilde{\varphi}_x}{\varphi_x^*}\right)^{\sigma-1} - 1\right] f_x,$$

Aggregate Variables

- Define weighted average productivity for the export market:

$$\tilde{\varphi}_x = \left[\int_{\varphi_x^*}^{\infty} \varphi^{\sigma-1} \frac{g(\varphi)}{1 - G(\varphi_x^*)} d\varphi \right]^{\frac{1}{\sigma-1}}. \qquad (10)$$

- The dual price index P can be written as a function of the mass of firms supply each market M_t and overall weighted average productivity $\tilde{\varphi}_t$:

$$P = M_t^{\frac{1}{1-\sigma}} p(\tilde{\varphi}_t) = M_t^{\frac{1}{1-\sigma}} \frac{1}{\rho \tilde{\varphi}_t},$$

$$\tilde{\varphi}_t = \left\{ \frac{1}{M_t} \left[M \tilde{\varphi}^{\sigma-1} + n M_x \left(\tau^{-1} \tilde{\varphi}_x \right)^{\sigma-1} \right] \right\}^{\frac{1}{\sigma-1}},$$

$$M_t = M + n M_x, \qquad M_x = \chi M,$$

Open Economy General Equilibrium

- The open economy general equilibrium is referenced by the quadruple $\{\varphi^*, \varphi_x^*, P, R\}$
- All other endogenous variables can be written in terms of this quadruple
- The steady-state equilibrium is characterized by a constant mass of firms entering each period, M_e, constant masses of firms producing and exporting, M and M_x, and stationary *ex post* distributions of firm productivity in the domestic and export markets, $g(\varphi)/\left[1 - G(\varphi^*)\right]$ and $g(\varphi)/\left[1 - G(\varphi_x^*)\right]$
- To determine general equilibrium, we use the recursive structure of the model
- Equilibrium φ^* can be determined from the free entry condition (10), substituting for φ_x^* using the relationship between the cutoffs (8)
- Having determined φ^*, φ_x^* follows immediately from the relationship between the cutoffs (8)

Open Economy General Equilibrium, R

- To determine R, we use the steady-state stability condition that the mass of successful entrants equals the mass of exiting firms

$$[1 - G(\varphi^*)]\, M_e = \delta M$$

- Using this steady-state stability condition to subsitute for $1 - G(\varphi^*)$ in the free entry condition (3), competitive entry implies that total payments to labor used in entry equal total firm profits:

$$L_e = M_e f_e = M\,[\bar{\pi}_d + \chi \bar{\pi}_x] = \Pi,$$

- Total payments to labor used in production are:

$$L_p = R - M\,[\bar{\pi}_d + \chi \bar{\pi}_x] = R - \Pi.$$

- Therefore total revenue equals total labor payments and the labor market clears:

$$R = L = L_p + L_e.$$

- Labor used in production includes fixed production, fixed exporting and variable production costs

Open Economy General Equilibrium, P

- To determine P, we can use the expressions for $\tilde{\varphi}_t$ and M_t above
- Having pinned down φ^* and φ_x^*, we can determine $\chi = \left[1 - G(\varphi_x^*)\right] / \left[1 - G(\varphi^*)\right]$, $\tilde{\varphi}$ and $\tilde{\varphi}_x$
- Having pinned down the probability of exporting and weighted average productivities, we can determine $\bar{r}$ and $\bar{\pi}$
- We can therefore also determine the mass of firms serving the domestic market and exporting

$$M = \frac{R}{\bar{r}} = \frac{L}{\sigma(\bar{\pi} + f + \chi n f_x)}, \qquad M_x = \chi M,$$

- Having pinned down M and M_x, we have determined M_t
- Having pinned down M_t, M, M_x and weighted average productivities, we have determined $\tilde{\varphi}_t$
- We have therefore determined the price index P

Trade Liberalization and Within-Industry Reallocation

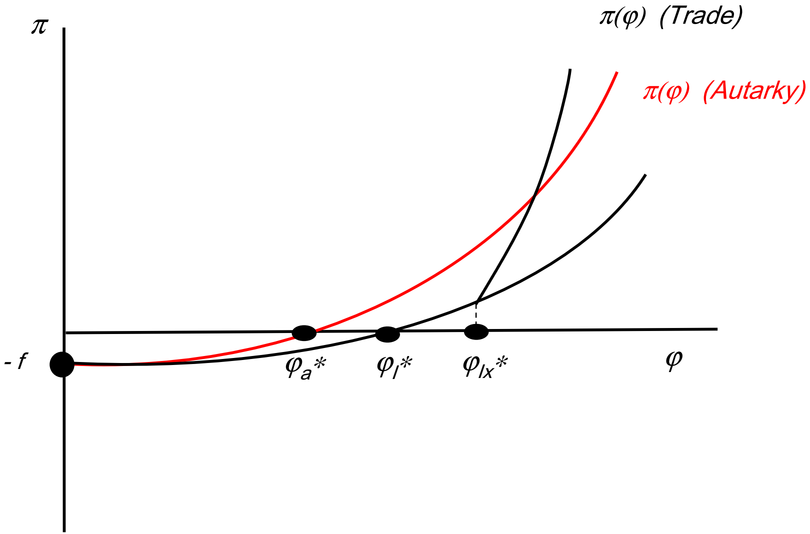
- The open economy free entry condition provides a downward-sloping relationship between the productivity cutoffs φ^* and φ_x^*

$$v_e = \frac{f}{\delta} \int_{\varphi^*}^{\infty} \left[\left(\frac{\varphi}{\varphi^*} \right)^{\sigma-1} - 1 \right] g(\varphi) d\varphi + \frac{f_x}{\delta} \int_{\varphi_x^*}^{\infty} \left[\left(\frac{\varphi}{\varphi_x^*} \right)^{\sigma-1} - 1 \right] g(\varphi) d\varphi = f_e,$$

- The closed economy free entry condition can be obtained by considering the case where trade costs become prohibitive and $\varphi_x^* \to \infty$

- Proposition 1: The opening of trade raises the zero-profit productivity cutoff below which firms exit, φ^*

Profits, Entry and Exit



The Effects of Trade

- The opening of trade leads to:
  - Rise in the zero profit cutoff productivity
  - Rise in average firm revenue and profit
- Low productivity firms between φ_A^* and φ_I^* exit
  - Increased exit by low productivity firms
- Intermediate productivity firms between φ_I^* and φ_{xI}^*
  - Contraction in revenue at domestic firms
- Only firms with productivities greater than φ_{xI}^* enter export markets
  - Selection into export markets
  - Expansion in revenue at exporting firms
- All of the above lead to a change in industry composition that raises aggregate industry productivity
- As the zero profit cutoff productivity and average revenue rise:
  - Mass of domestically produced varieties falls: $M_I < M_A$
  - Total mass of varieties available for consumption typically rises: $(1 + n\chi)M_I > M_A$
  - Welfare necessarily rises due to aggregate productivity gains

What Did we Learn?

- The opening of trade leads to reallocations of resources across firms within industries
  - Low productivity firms exit
  - Intermediate productivity surviving firms contract
  - High productivity surviving firms enter export markets and expand
  - Change in industry composition
- Improvements in aggregate industry productivity
- No change in firm productivity
- Selection into export markets but no feedback from exporting to firm productivity

Subsequent Research

- Helpman, Melitz and Rubinstein (2004) "Export Versus FDI with Heterogeneous Firms," *American Economic Review*, 94, 300-316.
  - Introduces both exports and FDI as alternative means of serving a foreign market
  - Introduces an outside sector to tractably characterize equilibrium with many asymmetric sectors
- Antras and Helpman (2004) "Global Sourcing," *Journal of Political Economy*, 112(3), 552-580.
  - Combines the Melitz model with the Antras (2003) model of incomplete contracts and trade
- Bernard, Redding and Schott (2007) "Comparative Advantage and Heterogeneous Firms," *Review of Economic Studies*, 73(1), 31-66.
  - Incorporates the Melitz model into the framework of integrated equilibrium of Helpman and Krugman (1985)

Subsequent Research

- Chaney, Thomas (2008) "Distorted Gravity: the Intensive and Extensive Margins of International Trade," *American Economic Review*, September.
  - Provides a simplified static version of the Melitz model without ongoing firm entry and with an outside sector
  - Examines the model's implications for the extensive and intensive margins of international trade
- Arkolakis, Costas, Klenow, Peter, Demidova, Svetlana and Andres Rodriguez-Clare (2009) "The Gains from Trade with Endogenous Variety," *American Economic Review*, Papers and Proceedings, 98 (4), 444-450.
  - Solves the Chaney version of the model without an outside sector
  - Derives a sufficient statistic for welfare of the same form as that in Eaton and Kortum (2002)

Subsequent Research

- Bernard, Andrew B., Peter K. Schott and Stephen J. Redding (2006) "Multi-product Firms and Trade Liberalization," *NBER Working Paper*, 12782
  - Motivated by the empirical importance of multi-product firms, uses the Melitz (2003) framework to develop a general equilibrium model of multi-product firms
  - The model accounts for key observed features of the distribution of exports across firms, products and countries
  - Trade liberalization gives rise to measured within-firm productivity growth by inducing firms to focus on their core competencies