

Multiple-Product Firms and Product Switching

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Product switching (the adding and/or dropping of products by surviving firms) is frequent, widespread and substantial

- Half of firms change their mix of products every five years
- Switched products constitute a substantial share of firms' output
- Product switching is correlated with changes in firm outcomes
- 1/6 of product output is accounted for by recently-added or recently-dropped products of surviving firms

Explanations for product switching:

- Patterns of product switching are difficult to reconcile with explanations based purely on net reallocation
- We find evidence supporting a natural extension of existing models of industry dynamics to allow selection within firms as well as across firms

Outline of Talk

Related Literature

Data

Multiple-Product Firms

Product Switching and Aggregate Manufacturing

Product Switching and Firm Behaviour

Why Do Firms Switch Products?

Related Literature

- Empirical evidence on firm entry/exit and within-industry reallocation
 - Baily et al. (1992), Davis & Haltiwanger (1992), Dunne et al. (1989a,b), Foster, Haltiwanger & Krizan (2001) among many others
- Theoretical models of industry dynamics
 - Firm creation and destruction: Jovanovic (1982), Hopenhayn (1992), Ericson & Pakes (1995), Melitz (2003)
 - Innovation: Klette & Kortum (2005), Chatterjee & Rossi-Hansberg (2007), Luttmer (2007)
- Multi-product firms and the breadth of product scope
 - Baumol (1977), Panzar & Willig (1977), Brander & Eaton (1984), Sutton (1990), Eaton & Schmidt (1994), Bernard et al. (2006)
- Welfare implications of the introduction of new goods
 - Nevo (2001), Petrin (2002), Broda & Weinstein (2007)

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Why Do Firms Switch Products?

Data : US Census of Manufactures

- Collected at the level of the establishment or plant
 - 1987-1997, every fifth year
- We aggregate up to the level of the firm
 - Administrative records are dropped (firms w/small plants)
- We observe
 - Number of products produced
 - Product-level output
 - Firm-level input intensities (capital, labor, skill)
 - Birth and death

Table 1: Products within an Industry

	SIC	Description
Sector →	33	Primary Metal Industries
Industry →	3357	Nonferrous Wiredrawing and Insulating
Products {	33571	Aluminum Wire
	33572	Copper Wire
	33573	Other Nonferrous Metal Wire
	33575	Nonferrous Wire Cloth
	33576	Apparatus Wire and Cord Sets
	33577	Magnet Wire
	33578	Power Wire
	3357A	Electronic Wire
	3357B	Telephone Wire
	3357C	Control Wire
	3357D	Building Wire
3357E	Other Wire NES	
	33579	Fiber Optic Cable

Source: U.S. Census Bureau (1996).

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Multiple-Product Firms

Prevalence

Characteristics

Product Switching and Aggregate Manufacturing

Product Switching and Firm Behaviour

Why Do Firms Switch Products?

TABLE 1—PREVALENCE OF FIRMS PRODUCING MULTIPLE PRODUCTS, INDUSTRIES AND SECTORS IN 1997

Type of firm	Percent of firms	Percent of output	Mean products, industries, or sectors per firm
Multiple product	39	87	3.5
Multiple-industry	28	81	2.8
Multiple-sector	10	66	2.3

Notes: The table categorizes firms according to whether they produce multiple products (five-digit SIC categories), industries (four-digit SIC categories), or sectors (two-digit SIC categories). Columns 1 and 2 summarize the distribution of firms and output, respectively. The final column reports the mean number of products, industries, and sectors across firms producing more than one of each.

The 39% of firms that produce multiple products account for the vast majority of US manufacturing output

MP firms produce an average of 3.5 products

**MP firms
are larger**

TABLE 2—1997 MULTIPLE-PRODUCT VERSUS SINGLE-PRODUCT FIRM CHARACTERISTICS

Firm characteristic	Multiple product	Multiple industry	Multiple sector
Output	0.66	0.67	0.92
Employment	0.58	0.61	0.86
Probability of export	0.12	0.12	0.16
Labor productivity	0.08	0.06	0.06
TFP	0.02	0.02	0.00

Notes: Results are from OLS regressions of log characteristics on a dummy variable indicating the firms' status as well as main industry fixed effects, i.e., the industry in which firms have the highest value of shipments. Regressions are restricted to the 110,414 observations for which all firm characteristics are available. All differences are statistically significant at the 1 percent level based on standard errors clustered by main industry except for multiple-sector firms' TFP.

**MP firms
are more
productive**

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Why Do Firms Switch Products?

Decomposition of Product Output

- Product output at year t can be decomposed looking backward:
 - Surviving firms that continue to produce the product (B)
 - Surviving firms that add the product (A)
 - Entering firms (N)

$$Y_{tp} = \sum_{j \in B_{tp}} Y_{tpj} + \sum_{j \in A_{tp}} Y_{tpj} + \sum_{j \in N_{tp}} Y_{tpj}$$

- Product output at year t can be decomposed looking forward:
 - Surviving firms that continue to produce the product (B)
 - Surviving firms that drop the product (D)
 - Exiting firms (X)

$$Y_{tp} = \sum_{j \in B_{tp}} Y_{tpj} + \sum_{j \in D_{tp}} Y_{tpj} + \sum_{j \in X_{tp}} Y_{tpj}$$

TABLE 6—DECOMPOSITION OF PRODUCT OUTPUT BY PRODUCER TYPE, 1987 TO 1997

	Average Share (percent) of product output in year t produced by:					
	Backward-looking			Forward-looking		
	Firms producing product in years $t - 5$ and t (1)	Firms that add the product between years $t - 5$ and t (2)	Firms born between years $t - 5$ and t (3)	Firms producing product in years t and $t + 5$ (4)	Firms that drop the product between years t and $t + 5$ (5)	Firms that die between years t and $t + 5$ (6)
<i>Panel A</i>						
1987	—	—	—	65	16	19
1992	67	14	19	67	15	18
1997	70	15	15	—	—	—
<i>Panel B</i>						
1987	—	—	—	44	27	29
1992	40	23	37	44	25	32
1997	45	26	29	—	—	—

Notes: The table reports average percentage decomposition of product output (panel A) and number of firms producing a product (panel B) according to firm activity. Columns 1–3 summarize backward-looking firm activities while columns 4–6 summarize forward-looking firm activities. Each row represents the average across all five-digit SIC products in the noted year. Decompositions cover the 1987 to 1997 censuses.

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Prevalence

Firm Outcomes

Method

Why Do Firms Switch Products?

54% of surviving firms
change their product mix
over a five year period

TABLE 3—PRODUCT SWITCHING BY US MANUFACTURING FIRMS, 1987 TO 1997

Firm activity	All firms	Multi-product firms	Exporters	Large firms	Multi-plant firms
<i>Panel A. Percent of firms</i>					
None	46	20	38	39	25
Drop product(s) only	15	12	18	17	21
Add product(s) only	14	32	14	16	15
Both add and drop	25	36	31	28	38
<i>Panel B. Output-weighted percent of firms</i>					
None	11	6	6	10	5
Drop product(s) only	10	8	9	10	10
Add product(s) only	10	12	9	10	9
Both add and drop	68	75	76	70	77

Notes: Panel A displays average percent of surviving US manufacturing firms engaging in each type of product-changing activity across five-year intervals from 1987 to 1997. Panel B provides a similar breakdown but weighting each firm by its output. Products refer to five-digit SIC categories. The four firm activities are mutually exclusive. “Large firms” are defined as firms whose output is above the seventy-fifth percentile.

These firms produce
89% of output

TABLE 4—PRODUCT SWITCHING AND CHANGES IN FIRM CHARACTERISTICS, 1987 TO 1997

	Net drop	Net add	Observations	R^2
Log change in real output	−0.078*** (0.0093)	0.096*** (0.0076)	94,012	0.05
Log change in employment	−0.085*** (0.0100)	0.078*** (0.0075)	94,012	0.03
Log change in real output/worker	0.007** (0.0038)	0.018*** (0.0043)	94,012	0.03
Change in TFP	−0.041*** (0.0070)	0.031*** (0.0076)	94,012	0.08

Notes: Table summarizes OLS regression results of log change in firm characteristics over five-year intervals according to whether firms net add or net drop products. Each row summarizes the regression for the noted dependent variable. Standard errors in parentheses are adjusted for clustering by product mix. Regressions include product mix by year fixed effects.

Method of Product Switching

(Average across 5-year intervals, 1987-97)

	Share of Firms	
Method of Sector Adding	Unweighted	Value-
Existing Plant(s) Only	0.824	0.257
Acquired Plant(s) Only	0.041	0.190
New Plant(s) Only	0.080	0.194
Combination with M&A	0.034	0.286
Combination without M&A	0.020	0.073
	Share of Firms	
Method of Sector Dropping	Unweighted	Value-
Existing Plant(s) Only	0.802	0.225
Divested Plant(s) Only	0.032	0.173
Closed Plants Only	0.102	0.239
Combination with M&A	0.040	0.307
Combination without M&A	0.024	0.055

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Why Do Firms Switch Products?

Product or Firm-Specific
Explanations

Interactions of Products
and Firms

Potential Explanations

A. Product-Specific Explanations

B. Firm-Specific Explanations

C. Interactions of Products and Firms

- Natural extension of existing models of industry dynamics to allow selection within firms as well as across firms

A. Product-Specific Explanations

- Specific to products but common across firms
- Changes in relative demand or supply
 - Add “hot” products and drop “cold” products based on demand
 - Technological change and international trade lead firms to drop uncompetitive products and add competitive products
- However, net reallocation across products implies a negative correlation between product add and drop rates
- In fact, we find a **positive** correlation between product add and drop rates

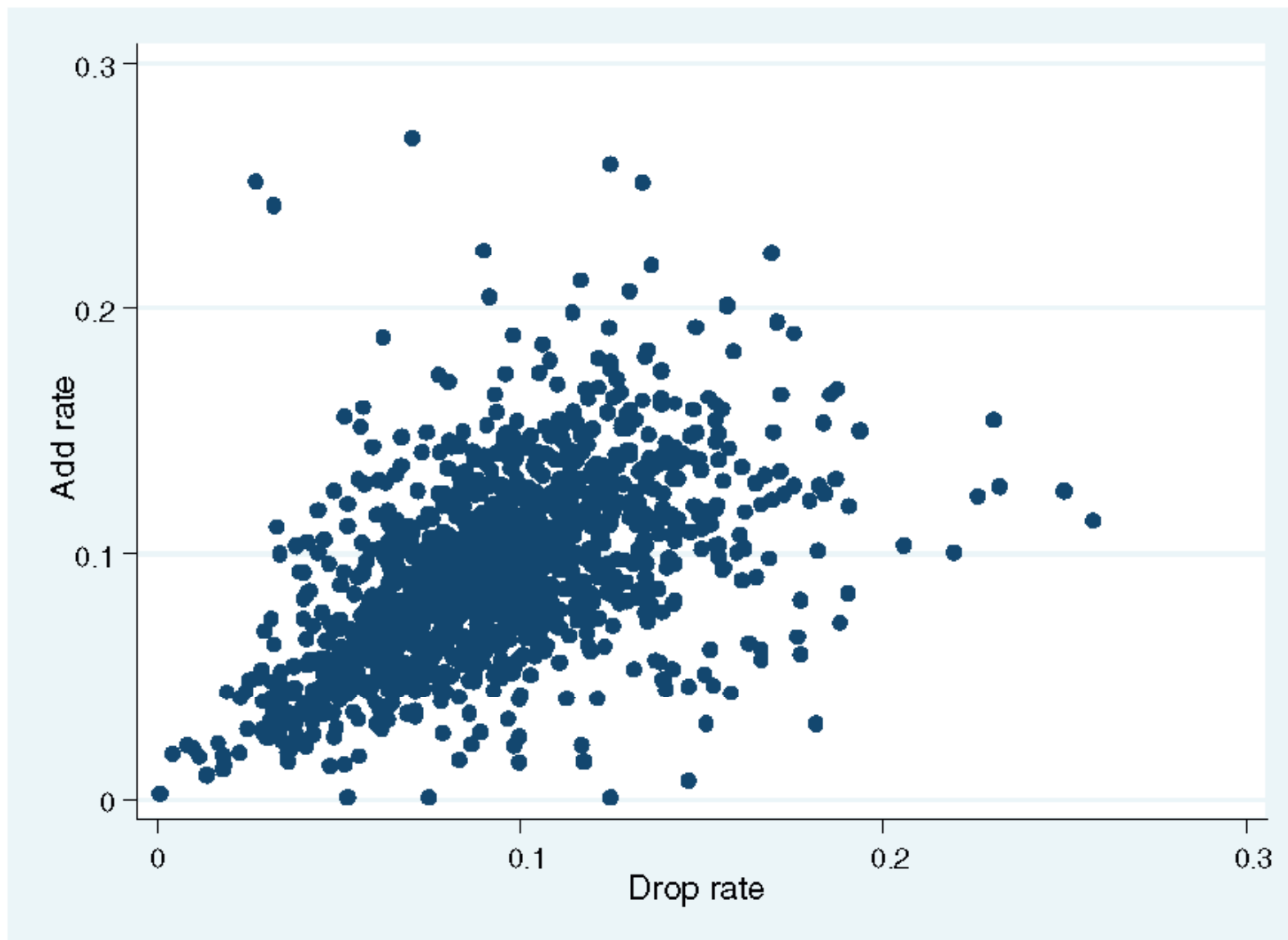


FIGURE 1. AVERAGE PRODUCT ADD AND DROP RATES, 1987 TO 1997

Note: Add (drop) rates are defined as the number of firms adding (dropping) the product between census years divided by the average number of firms producing the product in both years.

B. Firm-Specific Explanations

- Specific to firms but common across products
- Technological progress that reduces a firm's costs for all products
 - Enhances the profitability of all products
 - Induces the firm to add previously unprofitable products
- However, there is frequent simultaneous adding and dropping of products by firms
- More fundamentally, firm-product characteristics matter for product switching in addition to firm characteristics

TABLE 8—1992 TO 1997 FIRM-PRODUCT OLS DROP REGRESSIONS

	Drop _{tt+5}	Drop _{tt+5}	Drop _{tt+5}
ln(relative product size) _t	−0.059*** (0.001)	−0.086*** (0.001)	−0.077*** (0.001)
ln(relative product tenure) _t	−0.189*** (0.006)	−0.219*** (0.008)	−0.223*** (0.008)
Fixed effects	None	Firm	Firm, product
R ²	0.48	0.48	0.47
Observations	80,371	80,371	80,371

Notes: Table summarizes OLS regression results of a dummy variable indicating a firm-product drop between 1992 and 1997 on 1992 firm-product attributes and fixed effects. Firm-product size and tenure are relative to their average values across firms for the product in a given year. The regression sample is surviving multiple-product firms. Robust standard errors in parentheses are adjusted for clustering by product.

$$Drop_{ji} = \alpha_i + \eta_j + \beta_1 Size_{ji} + \beta_2 Tenure_{ji} + \varepsilon_{ji}$$

C. Firm-Product-Specific Explanations

- The empirical evidence so far has documented
 - Pervasive product switching within firms
 - A positive correlation across products between the rate at which products are added and dropped
 - The probability that a product is dropped by a firm is decreasing in firm-product shipments and tenure
- These features of the data are consistent with a natural extension of existing models of industry dynamics to incorporate multi-product firms and selection within firms

C. Firm-Product Selection Model

- Labor is the sole factor of production with wage w (numeraire)
- Continuum of products $i \in [0,1]$ within which firms manufacture differentiated varieties ω
- Incurring a sunk entry cost (f_e) creates a firm brand and an associated blueprint for a differentiated variety of each product
- Productivity and demand are uncertain prior to entry. Once the sunk cost entry cost is paid, firms observe initial values of
 - **Productivity φ** : independently distributed $g_e(\varphi)$, common across products; cumulative $G_e(\cdot)$
 - **Consumer tastes $\lambda(i)$** : distributed $z_{ie}(\lambda(i))$, product-specific and independent across products; cumulative $Z_{ie}(\cdot)$
- If the firm enters, productivity and demand evolve stochastically
 - **Productivity φ** : with probability θ a productivity shock occurs and productivity is redrawn from $g_c(\varphi'|\varphi)$
 - **Consumer tastes $\lambda(i)$** : with probability ε_i a demand shock occurs and consumer tastes are redrawn from $z_{ic}(\lambda(i)'|\lambda(i))$

C. Firm-Product Selection Model

- Once the sunk entry cost is paid, firms decide whether to enter and which products to make based on initial consumer tastes and productivity and their known stochastic processes
- Fixed headquarters cost (f_h); fixed production cost (f_{pi}) and constant variable cost for each product i
- As consumer tastes evolve stochastically for a firm's variety of a product, some previously unprofitable products are added, while other previously profitable products are dropped
 - Selection within firms across products
- As firm productivity evolves stochastically, product range expands or contracts, and some previously profitable firms may decide to exit
 - Selection across firms
- In addition to endogenous firm exit, we allow for an exogenous probability δ of firm death due to force majeure events

Positive Correlation Between Adding and Dropping

- In steady-state, there is a stationary distribution of consumer tastes and firm productivity
 - The flow of new and existing firms that enter a product equals the flow of existing firms that exit a product
 - Positive correlation between the rates at which a product is added and dropped
 - Rate of adding and dropping co-vary across products with product uncertainty ε_i

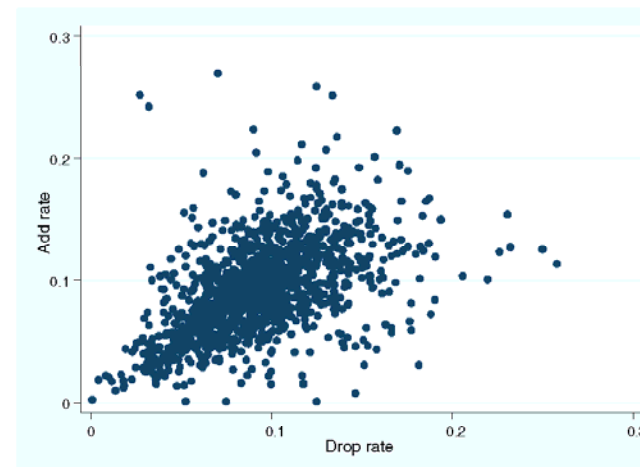


FIGURE 1. AVERAGE PRODUCT ADD AND DROP RATES, 1987 TO 1997

Note: Add (drop) rates are defined as the number of firms adding (dropping) the product between census years divided by the average number of firms producing the product in both years.

Firm-Product Selection

- In the model, the strength of consumer tastes for a firm's variety of a product λ_i is positively correlated with
 - Firm-product shipments (revenue)
 - The length of time the firm has produced the product (tenure)
- The probability a firm drops a product should be decreasing in
 - Firm-product shipments
 - Firm-product tenure

Product Co-production 1987-1997

Sector	20	31	30	22	23	39	24	26	25	27	28	29	32	33	34	35	36	37	38
20 Food	158	2	9	5	16	5	6	9	6	11	42	3	4	6	15	16	8	5	4
31 Leather	2	1	1	1	3	1	1	1	1	1	1	0	0	1	2	1	1	0	0
30 Rubber & Plastic	9	1	47	8	9	7	8	10	6	6	24	2	6	13	33	41	22	12	11
22 Textile	5	1	8	25	18	3	3	4	3	4	11	1	2	4	8	8	5	3	3
23 Apparel	16	3	9	18	64	6	6	6	6	9	12	1	3	7	20	21	9	5	5
39 Miscellaneous	5	1	7	3	6	10	3	3	2	9	6	0	1	3	10	9	6	3	3
24 Lumber	6	1	8	3	6	3	79	16	16	5	9	1	4	5	16	10	6	3	3
26 Paper	9	1	10	4	6	3	16	19	3	23	13	1	3	5	12	12	6	2	4
25 Furniture	6	1	6	3	6	2	16	3	23	3	4	0	1	4	12	10	7	3	3
27 Printing & Pub	11	1	6	4	9	9	5	23	3	401	10	1	2	5	13	15	10	4	5
28 Chemicals	42	1	24	11	12	6	9	13	4	10	74	13	10	16	25	29	20	10	18
29 Petroleum	3	0	2	1	1	0	1	1	0	1	13	4	2	3	4	3	2	1	1
32 Stone & Concrete	4	0	6	2	3	1	4	3	1	2	10	2	10	6	11	11	7	3	3
33 Primary Metal	6	1	13	4	7	3	5	5	4	5	16	3	6	27	48	47	29	15	10
34 Fabricated Metal	15	2	33	8	20	10	16	12	12	13	25	4	11	48	124	117	53	30	22
35 Industrial Mach	16	1	41	8	21	9	10	12	10	15	29	3	11	47	117	119	72	38	31
36 Electronic	8	1	22	5	9	6	6	6	7	10	20	2	7	29	53	72	70	25	37
37 Transportation	5	0	12	3	5	3	3	2	3	4	10	1	3	15	30	38	25	18	10
38 Instruments	4	0	11	3	5	3	3	4	3	5	18	1	3	10	22	31	37	10	14

Conclusions

A primary concern of economics is whether resources are allocated to their most efficient use

- Existing research on reallocation has focused on the entry of plants/firms and compositional changes across plants/firms

In this paper, we examine reallocation within surviving firms through product market entry and exit

- One half of firms change their product mix every five years
- The recently added and dropped products account for a substantial proportion of firm and product output
- We find empirical support for a natural extension of existing models of industry dynamics to allow selection within firms as well as across firms

Thank You

TABLE 5—SECTOR AND INDUSTRY SWITCHING BY US MANUFACTURING FIRMS, 1987 TO 1997

Firm activity	Percent of firms		
	Product activity	Industry activity	Sector activity
None	46	59	84
Drop only	15	14	6
Add only	14	13	6
Both add and drop	25	14	3

Notes: Table displays average share of surviving firms that engage in product, industry, and sector switching across five-year intervals from 1987 to 1997. Product, industry, and sector activity refers to the adding and/or dropping of five-digit, four-digit, and two-digit SIC categories, respectively. The four firm activities are mutually exclusive.

TABLE 9—AVERAGE DECOMPOSITION OF FIRM OUTPUT BY TYPE OF PRODUCT, 1987 TO 1997

	Average share (percent) of firm output in year t accounted for by:			
	Backward-looking		Forward-looking	
	Products produced in years $t - 5$ and t	Products added between years $t - 5$ and t	Products produced in years t and $t + 5$	Products dropped between years t and $t + 5$
1987	—	—	71	29
1992	74	26	74	26
1997	69	31	—	—

Notes: Table reports the average percentage decomposition of firm output according to whether products were previously (left panel) or subsequently (right panel) produced. Each row represents the average across all surviving firms in the noted year.

TABLE 10—MEAN DISTRIBUTION OF WITHIN-FIRM OUTPUT SHARES, 1987 TO 1997

		Number of products produced by the firm									
		1	2	3	4	5	6	7	8	9	10
Average percent of output	1	100	80	70	63	58	54	52	50	48	46
	2		19	21	22	21	21	21	20	20	20
	3			7	9	10	11	11	11	11	12
	4				4	5	6	7	7	7	7
	5					2	3	4	4	5	5
	6						2	2	3	3	3
	7							1	2	2	2
	8								1	1	2
	9									1	1
	10										1

Notes: Columns indicate the number of products produced by the firm. Rows indicate the share of the products in firm output, in descending order of size. Each cell is the average across the relevant set of firm-products in the sample. Sample includes all firms producing at least ten products in the 1987 to 1997 censuses.

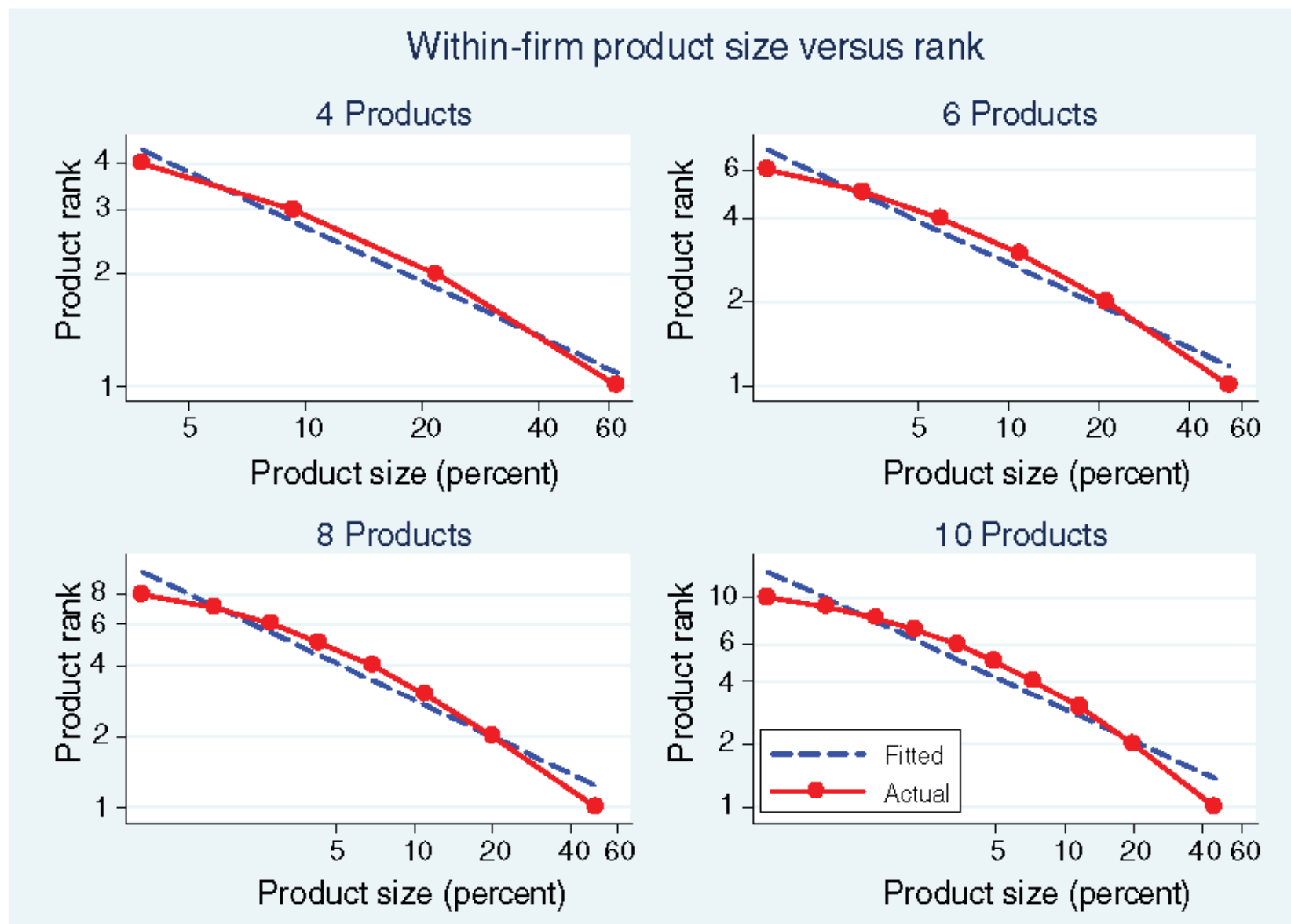


FIGURE 2. DISTRIBUTION OF PRODUCT SHIPMENTS WITHIN FIRMS, 1987 TO 1997

Notes: The solid line plots within-firm product rank against within-firm product size. Dashed lines are the result of an OLS regression of log product rank on log product size.

Consumer Preferences

- The consumer derives utility from a continuum of products $i \in [0,1]$, with elasticity of substitution $\kappa > 1$

$$U = \left[\int_0^1 (a_i C_i)^\nu di \right]^{\frac{1}{\nu}}$$

- Within products, firms manufacture differentiated varieties ω , with elasticity of substitution $\sigma > \kappa > 1$

$$C_i = \left[\int_{\omega \in \Omega} (\lambda_i(\omega) c_i(\omega))^\rho d\omega \right]^{\frac{1}{\rho}}$$

Production

- Labor demanded by a firm with productivity φ

$$l(\varphi) = \underbrace{f_h}_{\text{Headquarters cost}} + \int_0^1 I_i \left[\underbrace{f_{pi}}_{\text{Fixed production cost}} + \frac{q_i(\varphi, \lambda_i)}{\varphi} \right] di$$

- Since a firm manufactures one of a continuum of varieties of a product, it is unable to influence the price indices
- The firm's profit maximization problem reduces to choosing the price of each product variety separately to maximize the profits from that product variety

$$p_i(\varphi, \lambda_i) = \frac{1}{\rho} \frac{w}{\varphi}$$

Firm-Product and Firm Profitability

- There is a zero-profit cutoff for consumer tastes $\lambda_i^*(\varphi)$ for each firm productivity φ such that a firm produces the product for $\lambda_i \geq \lambda_i^*(\varphi)$

$$r_i(\varphi, \lambda_i^*(\varphi)) = \sigma f_{pi}$$

- There is a zero-value cutoff for firm productivity φ^* such that a firm exits for $\varphi < \varphi^*$

$$v(\varphi^*) = \frac{\pi(\varphi^*) + \theta \left[\int_{\varphi^*}^{\bar{\varphi}} [v(\varphi') - v(\varphi^*)] g_c(\varphi' | \varphi^*) d\varphi' \right]}{\delta + \theta G_c(\varphi^* | \varphi^*)} = 0$$

Stationary Productivity Distribution

- The stationary productivity distribution is defined by the requirement that the inflow of firms to each productivity equals the corresponding outflow for all $\varphi \geq \varphi^*$

$$\gamma_g(\varphi) = \frac{g_e(\varphi)M_e}{(\delta+\theta)M} + \left(\frac{\theta}{\delta+\theta}\right) \int_{\varphi^*}^{\bar{\varphi}} g_c(\varphi|\varphi')\gamma_g(\varphi')d\varphi'$$

$$f(x) = (\Omega(f)(x)) \equiv \psi(x) + \mu \int_{\varphi^*}^{\bar{\varphi}} K(x,y)f(y)dy$$

where $\mu\bar{K}|\bar{\varphi} - \varphi^*| < 1$

Product Characteristics

Sector	Industries	Products	Products per Industry	Capital Intensity		Skill Intensity	
				Mean	Std Dev	Mean	Std Dev
20 Food	49	156	3.2	169	127	0.21	0.11
22 Textile	23	82	3.4	171	220	0.18	0.11
23 Apparel	31	76	2.4	74	70	0.19	0.13
24 Lumber	17	59	3.5	273	158	0.19	0.06
25 Furniture	13	37	2.8	52	39	0.20	0.09
26 Paper	17	55	3.2	293	145	0.21	0.06
27 Printing & Publishing	14	70	5.1	102	97	0.48	0.27
28 Chemicals	29	100	3.5	438	359	0.34	0.13
29 Petroleum	5	16	3.0	878	418	0.34	0.08
30 Rubber & Plastics	15	61	4.2	185	205	0.26	0.10
31 Leather	11	12	1.1	52	23	0.20	0.10
32 Stone & Concrete	26	47	1.8	206	169	0.21	0.10
33 Primary Metal	26	90	3.4	176	109	0.22	0.08
34 Fabricated Metal	38	131	3.6	125	95	0.26	0.13
35 Industrial Machinery	51	178	3.7	123	80	0.29	0.16
36 Electronic	37	111	3.0	140	153	0.30	0.16
37 Transportation	18	61	3.6	128	70	0.31	0.18
38 Instruments	17	45	2.6	142	96	0.35	0.17
39 Miscellaneous	18	53	2.9	103	76	0.28	0.11
	455	1440	3.2				

Multiple-Product Firms By Two-digit Sector

Sector	Multiple Product Firms	
	Average	Output-Weighted Average
20 Food	48	90
22 Textile	33	84
23 Apparel	22	74
24 Lumber	38	69
25 Furniture	42	82
26 Paper	32	89
27 Printing & Publishing	60	83
28 Chemicals	46	92
29 Petroleum	28	98
30 Rubber & Plastics	42	76
31 Leather	26	67
32 Stone & Concrete	22	67
33 Primary Metal	46	91
34 Fabricated Metal	37	71
35 Industrial Machinery	30	73
36 Electronic	33	77
37 Transportation	40	94
38 Instruments	28	86
39 Miscellaneous (e.g. Toys)	34	59

Note: Table reports the average and output-value weighed average share of multi-product firms in each two-digit SIC sector across the 1987 to 1997 Census years.

Product Switching By Two-digit Sector

Sector	Percent of Firms Altering Their Product Mix
20 Food	51
22 Textile	53
23 Apparel	47
24 Lumber	53
25 Furniture	60
26 Paper	42
27 Printing & Publishing	71
28 Chemicals	59
29 Petroleum	34
30 Rubber & Plastics	65
31 Leather	39
32 Stone & Concrete	33
33 Primary Metal	53
34 Fabricated Metal	52
35 Industrial Machinery	49
36 Electronic	52
37 Transportation	59
38 Instruments	52
39 Miscellaneous	46

Note: Table reports average share of firms engaging in some form of product switching (adding, dropping or both) across five-year intervals from 1987 to 1997.

Is Co-Production Random?

- Under the null hypothesis that products are produced with independent probabilities

$$\frac{o_{rc} - e_{rc}}{\sqrt{e_{rc}}}$$

Distributed standard normal

$$\sum_{cells} \frac{(o_{rc} - e_{rc})^2}{e_{rc}}$$

Distributed chi-squared with $(R-1)*(C-1)/2$ degrees of freedom

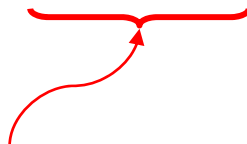
o: observed frequency

e: expected frequency

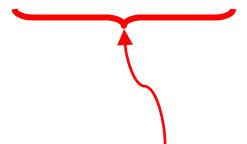
Example

- Expected frequency of producing (20-23)

$$\frac{17,389}{448,168} \times \frac{35,889}{448,168} \times 448,168 = 1,392$$



Probability of
produce 20



Probability of
produce 23



Total number
of firm-cell
observations