Arbitrage, Cointegration and Efficiency in Financial Markets in the Presence of Financial Crisis

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The authors gratefully acknowledge the assistance of Genesis Analytics in providing both financial and above all data support to the present project. The results were obtained for the South African Financial Services Board. The views expressed in this paper are those of the authors alone, and should not be taken to necessarily reflect those of either Genesis Analytics or the Financial Services Board in any form.

The present paper examines the link between South African stock index futures markets and the underlying stock market index over the 1960-98 period. Analysis proceeds by means of Johansen VECM and ARDL cointegration analysis. The paper finds strong evidence of the cost-of-carry arbitrage relationship between the two markets. We conclude that the futures market in South Africa is zero-arbitrage efficient even across the period of financial instability that characterised the 1997-98 period.

JEL Classification: G12, G13.
1 Introduction

Derivatives have been regularly vilified by regulators and the media. They have been associated with increased stock market volatility and leverage and have been blamed for mortally wounding some of the world’s largest banks. Moreover they have been under particularly heavy attack in the context of emerging markets, as being a potential source of instability to financial markets in such economies, with potentially serious real consequences for the economies.

Despite the bad press, world derivative volumes have continued to grow by an impressive 10% to US $ 2.2bn during 1998. Similarly, trading volumes are up in the South African derivatives market, boosted by the strong rise in Johannesburg Stock Exchange trade and the increased participation of foreigners in South Africa’s security markets. This paper examines the role of futures markets in the financial markets of one emerging economy, South Africa, during a period of considerable volatility on world financial markets.

Derivatives have been hailed by practitioners, analysts and theorists, as they play a valuable role in financial risk management and price discovery. Of particular interest to these market participants has been the temporal relationship between stock index futures prices and spot prices as this issue has implications for some fundamental concepts in financial theory, notably market efficiency and arbitrage.

The literature on the relationship between futures and spot markets recognizes the centrality of the role and function of arbitrageurs who transmit information into both futures and spot markets by taking advantage of risk free profitable opportunities between futures and spot markets. A linkage between spot market and the futures market is maintained by arbitrageurs, such that a no-arbitrage pricing relationship between the futures price and the underlying spot price is determined by the net cost of holding the asset relative to taking a futures position. In attempting to exploit relative mispricing across the spot and futures market, arbitrageurs ensure that this fair value is relatively quickly reestablished - though the presence of transactions costs may allow some divergence of the actual price from its fair price.

This understanding of the impact of arbitrage leads to a specific conception of efficient futures and spot markets. An efficient market is defined as one in which there are no risk-free returns above opportunity costs and transaction costs, given investors’ information i.e. there are no profitable arbitrage
opportunities - see Dwyer and Wallace (1992).\footnote{1} For the sake of clarity, it is worth pointing out that this conception of efficiency in markets differs from that which has informed earlier discussions in the literature, which denied the possibility of cointegration between prices in two efficient markets.\footnote{2} Since the no-arbitrage definition of efficiency presumes that all profitable arbitrage opportunities come to be exploited and reflected in spot and futures prices, it comes to be consistent with the presence of cointegration, as long as the impact of transactions and opportunity costs are appropriately accounted for. Futures prices and spot prices are linked by the arbitrage relationship, hence making cointegration possible. The point is essentially that it would be strange to consider a market with no expected utility increasing profit opportunities due to expected utility maximizing acquisition of information as inefficient. While the earlier understanding of the relationship between cointegration and efficiency would have insisted on a verdict of inefficiency for such a market, this does not follow for the no-arbitrage definition of efficiency.

While the theoretical discussion below provides greater detail, we can illustrate the point readily by way of preamble. We have two possibilities in the relationship between the forward price, \( F \), and the spot price, \( S \), mediated by the expected net cost of carry or “differential”, denoted \( D \).\footnote{3}

Where:

\[
F > S (1 + D)
\]

\[ (1) \]

we anticipate that arbitraging would trigger spot purchases and forward sales.

\footnote{1}{And also the work of Levich (1985) and Ross (1987).}

\footnote{2}{See for instance Granger (1986), Baillie and Bollerslev (1989), MacDonald and Taylor (1989), Coleman (1990), Booth and Mustafa (1991), Copeland (1991), Ghosh (1993), Ferret and Page (1998) and some comments in Hakkio and Rush (1989) by way of example. Dwyer and Wallace (1992) point out that this is effectively based on the Fama (1970) conception of efficient markets, in which the prices of assets would “fully reflect” all available relevant information. As a consequence, it should not be possible to predict a price series in one of the futures or spot markets on the basis of the other, precluding the possibility of cointegration. If two price series are cointegrated it implies that Granger-causality must exist in at least one direction, and possibly in both directions. Hence the presence of any cointegrating relationship between futures and spot prices was held to contradict the Fama (1970) definition of efficiency because it would imply that one price series can help in forecasting the other. As Fama (1991) and others have now pointed out, stock prices can be predictable even in efficient markets.}

\footnote{3}{We will discuss the question of what is to be included in \( D \) below.}
Arbitrage, Cointegration and Efficiency

until the inequality is eliminated. Conversely, where:

$$F < S (1 + D)$$

we anticipate that arbitraging would trigger spot sales and forward purchases, until the inequality is eliminated.

The attraction of the arbitraging mechanism is that it serves as a means of price discovery. If the arbitraging mechanism works effectively, we would anticipate that for the most part markets would be characterized by:

$$F = S (1 + D)$$

provided only that we have correctly specified $D$.

What this implies is that the spot and futures prices should co-move over time, provided that the appropriate spot and futures prices are employed, and provided that the net cost-of-carry has been correctly specified. Alternatively, we might say that $F, S, D \sim CI$, i.e. the three series should be cointegrated where the arbitrage mechanism is operative, since the arbitraging mechanism is forcing the different series to “move together” over time. Any residual that remains should therefore be $\sim I (0)$.

In testing for the presence of cost-of-carry arbitrage in the South African spot and futures markets, we employ end-of-day data for South African futures and spot markets over the 1996-98 period.

2 Arbitrage, Asset Markets and Cointegration

We can be more precise about the anticipated relationship between futures prices, spot prices and the cost of carry differential - our discussion here closely follows Brenner and Kroner (1995).

Assume that the continuously compounded returns from spot prices are normally distributed with $\mu$ mean and variance $\sum_{i=1}^{n} \gamma_i^2$. Then we can have the spot price evolving in terms of an $n$-factor geometric Brownian motion, given by:

$$dS_t = \mu S_t dt + \sum_{i=1}^{n} \gamma_i S_t dW_{i,t}$$

where $W_{i,t}$, $n = 1, ..., n$ are $n$ sources of standard Brownian motions representing $n$ sources of uncertainty, $\mu$ now represents the instantaneous rate
of return, and the $\gamma_i$ the diffusion coefficients. Where $n = 1$ we have the Black-Scholes diffusion process employed in their option pricing process, $dS_t = \mu S_t dt + \gamma_1 S_t dW_{1,t}$, in which case $\mu$ and $\gamma_1$ are the mean and standard deviation of the continuously compounded returns respectively. Solving equation 4 and log transforming we obtain:

$$\ln S_t - \ln S_{t-k} = \left(\mu - \frac{1}{2} \sum_{i=1}^{n} \gamma_i^2\right) k + \sum_{i=1}^{n} \gamma_i (W_{i,t} - W_{i,t-k})$$

(5)

implying that the growth rate in the spot price reduces to a constant and residual, where the residual $\sim IIN (0, \sum_{i=1}^{n} \gamma_i^2)$. Thus we have $\ln S_t \sim I (1)$.

Assume next that: (a.) equities have known dividends, (b.) investors are indifferent between purchasing a stock and collecting the dividends, and buying a risk-free bond to purchase the equity later at a previously contracted forward price, and (c.) that there is a non-zero correlation between the risk-free interest rate, the continuous dividend yield, and the spot price of equities. Let $P^d_{t/t-k} = e^{-k r_{t/t-k}}$, where $r_{t/t-k}$ represents the domestic risk-free $k$-period interest rate at time $t - k$. Hence $P^f_{t/t-k}$ represents the compounded rate of return on risk-free alternatives to equity. Also, let $P^f_{t/t-k} = e^{-k V_{t/t-k}}$ where $V_{t/t-k}$ represents the dividend yield on the stock between $t - k$ and $t$. As such $P^f_{t/t-k}$ represents the rate of return on the stock. Under these conditions, and as long as arbitrage-based trading strategies enforces the cost-of-carry relationship, we expect:

$$F_{t/t-k} = S_{t-k} \cdot \frac{P^f_{t/t-k}}{P^d_{t/t-k}} e^{Q_{t/t-k}}$$

$$= S_{t-k} \cdot D_{t/t-k} e^{Q_{t/t-k}}$$

(6)

$$\therefore \ln F_{t/t-k} = \ln S_{t-k} + \ln D_{t/t-k} + Q_{t/t-k}$$

(7)

where $F_{t/t-k}$ denotes the price of the futures contract at time $t - k$ to expire at time point $t$, and $Q_{t/t-k}$ is an adjustment term for marking-to-market features of futures contracts. The adjustment term depends on the volatility of the interest rate and spot processes, and $\to 0$ as $k \to 0$. $Q_{t/t-k}$ is obtained from:

$$Q_{t/t-k} = \sum_{i=t-k}^{t} \int_{t-k}^{t} a_{di} (v, T) \left[ a_{di} (v, T) - a_{fi} (v, T) - \delta_{di} (v) \right]$$

(8)
where \( a_d(v, T) \) denotes volatility of the domestic bond prices, \( a_f(v, T) \) volatility of equity prices, and \( \delta_d(v) \) volatility of the net return on equity versus bonds. For the purposes of estimation, fortunately the expression of equation 8 may be treated as a constant, and will thus not have to be separately calculated.

Note also note that \( D_{t-k} = e^{-k(v_{t-k} - r_{t-k})} \), or the net rate of return on stocks relative to the risk-free alternative offered by bonds.

Equation 7 carries some immediate implications. We already know \( \ln S_{t-k} \sim I(1) \), and since \( Q_{t-k} \rightarrow 0 \) as \( k \rightarrow 0 \), unless \( \ln S_{t-k} \sim CI (1, -1) \) (i.e. are cointegrated such as to form a stationary linear combination of variables), it follows that therefore \( \ln F_{t-k} \sim I(0) \). Note that a sufficient condition for this to be satisfied would be that \( \ln D_{t-k} \sim I(0) \). Further, we note that the forward premium (basis) \( \ln S_{t-k} - \ln F_{t-k} \), would be serially correlated as long as the cost-of-carry is serially correlated (as is likely given its composition in terms of \( r, V \)), which would serve to explain persistence in forward premia.

Finally, a combination of equations 5 and 7 allows us to specify:

\[
\ln S_t - \ln F_{t-k} = \left( \mu - \frac{1}{2} \sum_{i=1}^{n} \gamma_i^2 + Q_{t-k} \right) k + \ln D_{t-k} + \sum_{i=1}^{n} \gamma_i (W_{i,t} - W_{i,t-k})
\]

such that the difference between the spot and futures price has three components:

1. A constant term, \( \left( \mu - \frac{1}{2} \sum_{i=1}^{n} \gamma_i^2 + Q_{t-k} \right) k \). This represents an expected change term, which may be either positive or negative, as determined by the magnitude of the relevant parameters. Note also that the \( Q_{t-k} \) term may be subsumed into the constant, since it is non-stochastic.

2. The cost-of-carry term, or the differential over the remaining life of the futures contract, and which reflects the relative rates of return on risk-free assets and the dividend returns from stocks. In particular, \( \ln D_{t-k} = -k (V_{t-k} - r_{t-k}) \).

3. The random noise term, \( \sum_{i=1}^{n} \gamma_i (W_{i,t} - W_{i,t-k}) \), due to the influence of the underlying economic factors, \( W_{i,t} \), and which is \( \sim IIN (0, \sum_{i=1}^{n} \gamma_i^2) \).
It is now apparent why:

- as long as \( \ln D_{t/t-k} = -k (V_{t/t-k} - r_{t/t-k}) \sim I (0) \), then \( (\ln S_t, \ln F_{t/t-k}) \sim CI (1, -1) \).

- if \( \ln D_{t/t-k} = -k (V_{t/t-k} - r_{t/t-k}) \sim I (1) \), then as long as long as \( \ln D_{t/t-k}, \ln S_{t-k} \) are not cointegrated, then we have \( (\ln S_t, \ln F_{t/t-k}, \ln D_{t/t-k}) \sim CI(1, -1) \), since \( \sum_{i=1}^{n} \gamma_i (W_{i,t} - W_{i,t-k}) \sim IN(0, \sum_{i=1}^{n} \gamma_i^2) \). But it does mean that in order for cointegration in the basis to be found, requires that the cost-of-carry be included in the test for the presence of cointegration.

But we note that the nature of the cointegrating relationship amongst the variables is critically dependent on the integrating characteristics of \( \ln D_{t/t-k} \). In order to construct a legitimate test of cointegration, it is vital that the correct choice is made with respect to the prices chosen in terms of the time at which the prices are measured. In terms of the above exposition, what is critical:

- is not whether the current forward and realized spot prices are employed (i.e. \( \ln S_t, \ln F_{t/t-k} \)), or whether contemporaneous forward and spot prices are chosen (i.e. \( \ln S_{t-k}, \ln F_{t/t-k} \)); the reason for this is that the proposition provides for cointegration at any lead or lag

- but what is critical, is that the time to expiration \( k \) is kept constant, while the time to expiration \( t \) is changing.

The last point is crucial, since if we had a constant \( t \) and a \( k \) moving toward zero, then it is inevitably the case that the spot and futures prices have to converge on one another. Any test for cointegration would thus potentially be misled into finding an absence of cointegration since the residual from equation 7 would have a variance which would approach zero over time, precluding the possibility of covariance stationarity. To avoid this difficulty in the empirical work which is to follow, however, we exclude the possibility of expiry on the futures contract from any data employed in estimation. Instead, we roll over to the next futures contract as the date of expiration approaches. While the variance of residuals will strictly speaking still be time-varying, in practice this is unlikely to be critical, since the power of cointegration tests to identify time varying variance in residuals is limited.
As a last point here, we note that to the best of our knowledge, there appear to be no studies internationally that examine whether the interest rate-dividend yield spread is stationary. Again, this information is crucial in formulating the correct test for cointegration in the current context. Stationarity implies that we may move to a test of the stationarity of the (i.e. ln \( S_t \), ln \( F_{t-k} \)) linear combination directly - where it is not, we have to test for stationarity of the (i.e. ln \( S_t \), ln \( F_{t-k} \), ln \( D_{t-k} \)) linear combination.

3 The Data

The study employed end-of-day data containing spot and futures prices of the ALSI 40 index, and end-of-day information on the risk free interest rate on government bonds, the dividend yield on shares, and the transactions costs of cost of carry trades (set at 1.5 percent). The sample period covered the 1996 to 1998 time frame, therefore including the period of market turbulence associated with the emerging market crises in 1997 and 1998. The data set provided a total of 751 observations.

This allows for the construction of the following cost-of-carry term:

\[
\ln COC_t = \ln \left( r_{t-k} - V_{t-k} + \tau \right)
\]

where \( r, V \) are defined as above as the risk free rate of return on government bonds and the dividend yield on shares, and \( \tau \) denotes the transactions costs of cost of carry trades (set at 1.5 percent).

In Table 1, we report standard augmented Dickey-Fuller test statistics on the univariate time series characteristics of our data. In terms of the evidence, we note at the outset that not only the spot and futures prices are \( I(1) \), but that \( \ln COC_t \sim I(1) \). Thus, in terms of the preceding discussion, it follows that the appropriate specification to test for the presence of cointegration would be:

\[
\ln S_t = \alpha + \beta_1 \ln F_t + \beta_2 \ln COC_t + u_t
\]

where \( \ln S_t \) and \( \ln F_t \) denote the log of spot and futures prices respectively.

4The latter was determined on the basis of interviews with market participants.
Table 1: Augmented Dickey Fuller Statistics. Order of augmentation chosen by Akaike’s Information Criterion. * denotes rejection of the null of a unit root.

<table>
<thead>
<tr>
<th>Null</th>
<th>Alternative</th>
<th>Eigenvalue Statistic</th>
<th>95%Critical Value</th>
<th>Trace Statistic</th>
<th>95%Critical Value</th>
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<tbody>
<tr>
<td>r = 0</td>
<td>r = 1</td>
<td>39.78*</td>
<td>21.12</td>
<td>49.51*</td>
<td>31.54</td>
</tr>
<tr>
<td>r ≤ 1</td>
<td>r = 2</td>
<td>5.30</td>
<td>14.88</td>
<td>9.73</td>
<td>17.86</td>
</tr>
</tbody>
</table>

Table 2: Cointegration Based on Maximal Eigenvalue of the Stochastic Matrix: * denotes statistical significance; 618 observations; VAR order = 8; list of variables included in cointegrating vector: lnS, lnF, lnCOC; List of eigenvalues is descending order: 0.06, 0.009, 0.007,

4 Empirical Methodology and Estimation Results

Estimation proceeds by both Johansen FIML VECM’s, and Pesaran, Shin & Smith ARDL cointegration techniques. Both techniques of estimation are well established, so that can proceed directly.

We test for the number of cointegrating vectors present between variables in terms of Johansen trace and maximal eigenvalue test statistics or in terms of PSS F-tests (which determine the direction of association between variables). We begin with estimations that do not control for the impact of the Asian crisis. Table 2 reports the Johansen trace and maximal eigenvalue test statistics, while Table 3 reports the PSS F-statistics. Both imply the presence of a single cointegrating relationship between the variables, and the PSS F-tests further imply that lnS<sub>t</sub> is the outcome, and lnF<sub>t</sub> and lnCOC<sub>t</sub> the forcing variables, as would be implied by the cost of carry arbitrage relationship.

The cointegrating vector estimated by means of Johansen FIML is given


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Table 3: PSS F-statistics; * denotes a value above the upper bound

<table>
<thead>
<tr>
<th></th>
<th>F – statistic</th>
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</thead>
<tbody>
<tr>
<td>ln S</td>
<td>4.53*</td>
</tr>
<tr>
<td>ln F</td>
<td>2.18</td>
</tr>
<tr>
<td>ln COC</td>
<td>1.59</td>
</tr>
</tbody>
</table>

Table 3: PSS F-statistics; * denotes a value above the upper bound

by:

\[ \ln S_t = 0.95 \ln F_t - 0.06 \ln COC_t \]

(12)

(figures in parentheses denote standard errors), while that estimated by means of PSS ARDL is given by:7

\[ \ln S_t = 0.96 \ln F_t - 0.05 \ln COC_t + 0.48 \]

(13)

implying a high degree of consistency between the two estimation methods.8 For both estimation methods the coefficients of the equilibrium relationship prove to be statistically significant.9 Moreover, for both estimation methods, the anticipated negative \( \ln COC \) coefficient emerges, and the null of proportionality between spot and futures prices cannot be rejected for either methodology.

Tables 1 and 2 report the associated error correction models for the two long run equilibrium relationships estimated by the VECM and ARDL cointegration approaches respectively. The implication is of a set of long run parameters that conform to the prior expectations generated by the cost-of-carry relationship, while the error correction model confirms the presence of a long term stable relationship between variables.

Finally, we test for the possibility that the emerging market crises of 1997 and 1998 may have disrupted either the existence of the arbitraging relationship between spot and futures markets.

The impact of the emerging markets crises were tested for in a number of respects:

7The optimal lag length for the ARDL was 4, 2, 8 for lnS, lnF, lnCOC respectively.
8This is as should be, since PSS ARDL amounts to a single equation application of Johansen.
9In the case of Johansen FIML, this requires the application of overidentifying restrictions on the cointegration space. Test statistics are distributed \( \chi^2 \), and are available from the authors.
**Diagnostic Tests**

<table>
<thead>
<tr>
<th>Test Type</th>
<th>LM Version</th>
<th>F Version</th>
</tr>
</thead>
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<td>Test Statistic</td>
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<td>*</td>
</tr>
<tr>
<td>DW Statistic</td>
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<td>0.02981</td>
</tr>
<tr>
<td>S.E. of Regression</td>
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<td>0.7814</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.58784</td>
<td>0.58784</td>
</tr>
<tr>
<td>R-Bar-Squared</td>
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**ECM for variable LNS estimated by OLS based on cointegrating VAR(3)**

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**List of additional temporary variables created:**
- dLNCOC7 = LNCOC(-7) - LNCOC(-8)
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- dLNCOC5 = LNCOC(-5) - LNCOC(-6)
- dLNCOC4 = LNCOC(-4) - LNCOC(-5)
- dLNCOC3 = LNCOC(-3) - LNCOC(-4)
- dLNCOC2 = LNCOC(-2) - LNCOC(-3)
- dLNCOC1 = LNCOC(-1) - LNCOC(-2)
- dLNS = LNS - LNS(-1)
- dLNS1 = LNS(-1) - LNS(-2)
- dLNS2 = LNS(-2) - LNS(-3)
### Error Correction Representation for the Selected ARDL Model
ARDL(4,2,8) selected based on Akaike Information Criterion

**Dependent variable is dLNS**

**618 observations used for estimation from 114 to 731**

<table>
<thead>
<tr>
<th>Regressor</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>T-Ratio [Prob]</th>
</tr>
</thead>
<tbody>
<tr>
<td>dLNS1</td>
<td>-.16493</td>
<td>.039000</td>
<td>-4.2289 [.000]</td>
</tr>
<tr>
<td>dLNS2</td>
<td>.032459</td>
<td>.013033</td>
<td>2.4906 [.013]</td>
</tr>
<tr>
<td>dLNS3</td>
<td>-.022695</td>
<td>.013044</td>
<td>-1.7400 [.082]</td>
</tr>
<tr>
<td>dLNF</td>
<td>.81162</td>
<td>.010567</td>
<td>76.8055 [.000]</td>
</tr>
<tr>
<td>dLNF1</td>
<td>.12970</td>
<td>.034037</td>
<td>3.8105 [.000]</td>
</tr>
<tr>
<td>dLNCOC</td>
<td>.039254</td>
<td>.022390</td>
<td>1.7532 [.080]</td>
</tr>
<tr>
<td>dLNCOC1</td>
<td>-.070247</td>
<td>.022419</td>
<td>-3.1533 [.002]</td>
</tr>
<tr>
<td>dLNCOC2</td>
<td>-.5555E-3</td>
<td>.022423</td>
<td>-2.4505 [.014]</td>
</tr>
<tr>
<td>dLNCOC3</td>
<td>-.51932</td>
<td>.021633</td>
<td>-1.7131 [.086]</td>
</tr>
<tr>
<td>dLNCOC4</td>
<td>-.070247</td>
<td>.022419</td>
<td>-3.1533 [.002]</td>
</tr>
<tr>
<td>dLNCOC5</td>
<td>.039254</td>
<td>.022390</td>
<td>1.7532 [.080]</td>
</tr>
<tr>
<td>dLNCOC6</td>
<td>-.01193</td>
<td>.021574</td>
<td>-1.7131 [.086]</td>
</tr>
<tr>
<td>dLNCOC7</td>
<td>.054360</td>
<td>.021788</td>
<td>2.4505 [.014]</td>
</tr>
<tr>
<td>dCONST</td>
<td>.079014</td>
<td>.024304</td>
<td>3.2511 [.001]</td>
</tr>
<tr>
<td>ecm(-1)</td>
<td>.1470</td>
<td>.022282</td>
<td>-7.3938 [.000]</td>
</tr>
</tbody>
</table>

**List of additional temporary variables created:**

- dLNS = LNS - LNS(-1)
- dLNS1 = LNS(-1) - LNS(-2)
- dLNS2 = LNS(-2) - LNS(-3)
- dLNS3 = LNS(-3) - LNS(-4)
- dLNF = LNF - LNF(-1)
- dLNF1 = LNF(-1) - LNF(-2)
- dLNCOC = LNCOC - LNCOC(-1)
- dLNCOC1 = LNCOC(-1) - LNCOC(-2)
- dLNCOC2 = LNCOC(-2) - LNCOC(-3)
- dLNCOC3 = LNCOC(-3) - LNCOC(-4)
- dLNCOC4 = LNCOC(-4) - LNCOC(-5)
- dLNCOC5 = LNCOC(-5) - LNCOC(-6)
- dLNCOC6 = LNCOC(-6) - LNCOC(-7)
- dLNCOC7 = LNCOC(-7) - LNCOC(-8)
- dCONST = CONST - CONST(-1)
- ecm = LNS - .96011*LNF + .053625*LNCOC -.47974*CONST

**R-Squared and R-Bar-Squared measures refer to the dependent variable dLNS and in cases where the error correction model is highly restricted, these measures could become negative.**
Four shocks were controlled for during the course of 1997 and 1998 (July 1997, October 1997, May 1998 and August 1998)

Three different “durations” of the crisis impact was tested for, for each frequency of data: two week, four week and eight week durations.

In each case, the crisis was not found to affect the presence of the arbitraging relationship between the futures and spot markets. Moreover, the characteristics of the arbitraging relationship was not found to be significantly affected by controlling for the effects of the crisis, in the sense that the parameter values of the arbitraging relationship proved robust to the inclusion of crisis dummies into estimation.

5 Evaluation and Discussion

Our empirical evidence is very clear in terms of its implications. We find evidence for the presence of no more than a unique cointegrating vector between the futures price, spot price and the cost of carry term.

The estimated long run relationship between the futures price, the spot price and the cost of carry term conform to the prior expectations that the cost of carry arbitrage relationship implies. In particular, regardless of whether we employ Johansen VECM cointegration analysis, or PSS ARDL estimations we find a statistically significant unitary elasticity between futures and spot prices, and the anticipated statistically significant negative association between the cost of carry term and the spot price. Moreover, for both the VECM and the ARDL cointegration estimations, error correction is present, further confirming the presence of an equilibrium relationship between the variables. While the dynamics are such as to imply relatively slow convergence to equilibrium, this is not surprising. The cost of carry relationship, while important for price discovery between futures and spot markets, is not the only source of price discovery. Firm specific information is relevant to the spot market, and may thus be the source of additional price movement not captured by our model.

An examination of impulse responses to shocks to the cointegrating vector further confirms the presence of a stable relationship between the three

\[ \text{Full results from estimation incorporating crises are available from the authors on request.} \]
We note in passing that the unique cointegrating vector in the data, which conforms to the COC relationship, precludes the possibility of a negative feedback loop from futures to spot markets in South Africa.

The results of this paper thus confirm the presence of the cost-of-carry arbitrage relationship between South African futures and spot markets. Moreover, we have confirmed that the impact of the cost-of-carry relationship is such as to provide a long run equilibrium relationship between futures and spot markets. Our results thus imply that futures markets act as a means of price discovery in South African financial markets, improving information in markets, and allowing them to settle on equilibrium prices more rapidly.

Our findings imply that the South African futures and spot markets conform to the no-arbitrage definition of efficiency - confirming the expectations set out in our introduction.

References


\footnote{11}{Full results are available from the authors on request.}


