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Social capital as an engine of growth: multisectoral modelling and implications

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Abstract

We propose a multisector endogenous growth model incorporating social capital. Social capital only serves as an input in the production of human capital and it involves a cost in terms of the final good. We show that in contrast to existing alternative specifications, this setting assures that social capital enhances productivity gains by playing the role of a timing belt driving the transmission and propagation of *all* productivity shocks throughout society whatever the sectoral origin of the shocks. Further econometric work is conducted in order to estimate the contribution of social capital to human capital formation. We find that depending on the measure of social capital considered, the elasticity of human capital to social capital varies from 6% to 10%. Finally we investigate the short-term dynamics and imbalance effects properties of the models depending on the value of this elasticity (taking the Lucas-Uzawa model as a limit case). In particular, it's shown that when the substitutability of social capital to human capital increases, the economy is better equipped to surmount initial imbalances as individuals may allocate more working time in the final goods sector without impeding economic growth.

Keywords: social capital, human capital, economic growth, imbalance effects.

JEL Classification: C61, E20, E22, E24, O41.

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1 Introduction

The concept of social capital has recently received rising acceptance in economics research. It has been pointed as a potential source of economic growth and economic performance (Putnam *et al.*, 1993; Knack and Keefer, 1997; Beugelsdijk and van Schaik, 2005). Like other sociological concepts, social capital encompasses several different meanings. According to Knack and Keefer (1997), trust, cooperative norms, and associations within groups represent the essence of the definition of that concept. Putnam *et al.* (1993, p. 167) also supports this view by defining social capital as “those features of social organisation, such as trust, norms, and networks, that can improve the efficiency of society by facilitating co-ordinated actions”.

It may however be difficult to pick out a definition suitable for a tractable economic model (Routledge and Amsberg, 2003). To address that issue, one may have to determine the most relevant effect of social capital for modelling. By which channel social capital affects economic performance? Social capital increases economic performance by increasing the number of mutually beneficial trades, by solving collective action problems, by reducing and monitoring and transactions costs and by improving the flow of information (Knowles, 2005). The beneficial effects of social capital arise mostly through its capacity of improving cooperation and confidence between economic agents. Economic activities that require some agents to rely on the future actions of others are realized at lower cost in higher trust environments (Knack and Keefer, 1997). Individuals in high trust societies are also likely to divert fewer resources to protecting themselves — through tax payments, bribes, or private security services and equipment — from unlawful (criminal) violations of their property rights.

One approach to modelling social capital is to focus on the “capital” aspect of social capital (Routledge and Amsberg, 2003).¹ Within this approach, several modelling strategies may be used. One of them is to consider by considering social capital as a factor of production (Chou, 2006; Sequeira and Ferreira-Lopes, 2008). The intuition driving this modelling strategy is that spending time to develop a personal network may increase the income of specific professions like medical practitioners, solicitors etc.² In that context,

¹Some researchers still express some reluctance to consider it as a “capital”. For instance, Solow (1995) suggests that social capital may hardly be considered as capital since the measurement of its stock “seems very far away”. However, an increasing number of economists now admit that social capital shares at least some similarities with physical and human capital namely its intertemporal dimension and its ability to generate a stream of future benefits (Chou, 2006).

²We are grateful to Jean-Pierre Laffargue for this insight.

the effect of social capital is to enhance the output of final good.

However, an alternative modelling strategy may be considered where social capital affect the accumulation of other factors of production rather than social capital being a new factor of production in its own right. For example, if social capital leads to the establishment of informal credit markets, this will ease the accumulation of physical, and perhaps human, capital (Knowles, 2005).

In this contribution we choose specifically to account for the influence of social capital in human capital accumulation. To do so, we assume that human and social capital are both necessary inputs for human capital accumulation and that social capital does not play any role in the production of goods and services. The literature on social capital has acknowledged the fact that trusting societies, in addition to have stronger incentives to innovate and to accumulate physical capital, are also likely to have higher returns to the accumulation of human capital. Where trust improves access to credit for the poor, enrollment in secondary education — which, unlike primary education, has a high cost in forgone income — may be higher (Galor and Zeira, 1993; Knack and Keefer, 1997). The issue of the interaction between human and social capital and their joint effects on economic growth, though less developed by the literature, is very important (Sequeira and Ferreira-Lopes, 2008). Glaeser *et al.* (2002) find a strong empirical relationship between human capital and membership of a given social organization (the proxy used to measure social capital). Regarding the cost social capital accumulation, we assume that it is incurred in terms of final goods production. This assumption allows to capture the fact that maintaining social networks may be costly in terms of resources that could otherwise be allocated to consumption or physical capital accumulation.³

Previewing our main results, we theoretically show that the impact of social capital on long-term growth (in the range of admissible parameterizations) as measured by the elasticity of human capital with respect to social capital in the education sector is ambiguous. One effect is obvious: since social capital (increased through final good expenditures) adds to human capital as a growth engine, long-term growth should be increased thanks to this additional channel. Furthermore, one would conclude that according to the latter reasoning, the more important is social capital in the education sector, the larger the long-term growth attainable. However, this property is not true for all values of the elasticity of human capital to social capital: in particular when this elasticity is close to zero (or equivalently, if we are close enough to the benchmark Lucas-Uzawa model), the long-term growth rate

³In Appendix E, we explore an alternative setup where the cost of investing in social capital is incurred in terms of time.

first decreases. It only raises when this elasticity becomes large enough. Therefore, there is another mechanism counter-balancing the one just mentioned, the interaction of both being responsible of that non-monotonic pattern. This opposite mechanism may be explained as follows: as the elasticity rises, the education sector relies less on human capital and more on social capital, which leads to having a smaller share of human capital in the education sector (and more in the final good sector), ultimately pushing long-term growth down.

Moreover, because the previous finding points at the necessity to estimate the elasticity of human capital with respect to social capital in the education sector, we do undertake this challenge. Given data limitation concerning social capital, we could only run simple econometric regressions using the World Values Survey for social capital measures and GDP per capita growth (PPP-adjusted) within the period 1980-2000. Our main finding in this respect is that the elasticity of human capital to social capital varies from 6% to 10% depending on the measure of social capital selected.

Last but not least, we provide with a complete study of the dynamic implications of social capital. To this end, we carefully calibrate the model and simulate the resulting dynamic systems. Two sets of exercises are considered: technological shocks and imbalance effects analysis (that is dynamics induced when initial conditions are not equal to the corresponding long-term values). For the exercises to be insightful, we consider three different structures depending on the value of elasticity of human capital with respect to social capital: The limit Lucas-Uzawa case (without social capital), the “realistic” parameterization using the result of the priori econometric step, and a last fictive case where social capital is as important as human capital in the education sector. It is shown that this elasticity parameter plays a crucial role in the short-term dynamics and imbalance effects generated by the model. In particular, it is shown that when the substitutability of social capital to human capital increases, the economy is better equipped to surmount initial imbalances as individuals may allocate more working time in the final goods sector without impeding economic growth.

The paper is organized as follows. The second section presents a brief review of the related literature. The third section describes the model. The fourth section presents some numerical simulations. Then, section 5 provides some concluding remarks.

2 Relation to literature

The impact of civil society, and social networks on the education and raising of children has long been recognized (Chou, 2006). This has been well encapsulated in the old African

proverb, “It takes a village to raise a child”. Coleman (1994) argues that “social capital is the set of resources that inhere in family relations and in community social organization and that are useful for the cognitive or social development of a child or young person” and Teachman *et al.* (1997) point out that social capital is important in the creation of human capital. These last authors found that a wide range of social capital indicators determine school continuation (Sequeira and Ferreira-Lopes, 2008). Becker (1993), the great pioneer in the study of the economics of human capital, also recognizes the importance of family relations on human capital accumulation. He acknowledges that “no discussion of human capital can omit the influence of families on the knowledge, skills, values, and habits of children”.

Empirical evidence backs the alleged interaction between human and social capital. Using the number of siblings as a proxy for social capital,⁴ Coleman (1988) provides empirical evidence that the presence of social capital within the family is important in determining whether a child drops out of school. Coleman also uncovers that social capital outside the family has a significant impact on the dropping out decision. Students who have changed schools because their parents moved are more likely to drop out than others. For families that often changed locations, the social relations that constitute social capital are broken at each move. Furthermore, he finds that dropout rates are lower in religiously based private schools than in public or secular private schools. He also discovered that whether parents devote time to religious activities affects human capital accumulation in their offspring. Grootaert and Van-Bastelaer (2002) provide evidence that community participation in parent teacher associations in Burkina Faso is associated with substantially higher rates of school attendance. Other relevant empirical findings are provided by Costa and Kahn (2001), where the rise in women’s labor force participation rates explains the observed decline in social capital produced within the home, and by Glaeser *et al.* (2000), who find that people who invest in human capital also invest in social capital.

From all this empirical literature, we may conclude that there are complementarities between social and human capital in human capital accumulation and a positive correlation between both possibly caused by double-sided causality (Sequeira and Ferreira-Lopes, 2008).

To date, only few papers have jointly considered social and human capital in models of economic growth. This has been done in Bisin and Guaitoli (2006) in an overlapping generations model (OLG) framework. They are concerned with the different roles human and social capital have in rural and urban societies. Sequeira and Ferreira-Lopes (2008)

⁴The number of siblings measures the dilution of adult attention to a child.

consider a three sector endogenous growth model where human capital and social capital are complementary in the production of the final good and substitutes in the production of each other. They assume that there is an opportunity cost of accumulating social capital in terms of human capital and foregone earnings.

In his formal three sector model of bonding social capital, Chou (2006) departs from Sequeira and Ferreira-Lopes (2008) in two respects: firstly, he assumes that social capital has no direct effect on final goods production but impacts it only indirectly through human capital accumulation. Secondly, he considers that human and social capital are complementary in the production of each other. We will follow Chou (2006) in two respects. Firstly, as we believe that social capital matters essentially in the education sector, we assume that social capital has no direct effect on final goods production. Secondly, as we aim to remain consistent with the aforementioned evidence that human and social capital are complementary in human capital accumulation, we depart from Sequeira and Ferreira-Lopes (2008) assumption that human and social capital are substitutes in human capital accumulation. However, conversely to Chou (2006) and Sequeira and Ferreira-Lopes (2008) we do not consider that accumulation of social capital requires a specific technology. We rather assume that the output of the final goods sector can be used on a one-for-one basis for consumption, investment in physical capital and investment in social capital. Such an assumption implies that accumulating social capital implies an opportunity cost only in terms of consumption and physical capital accumulation. By doing so, we aim to capture two main features of social capital.

1. Essentially, our multisectoral modelling attempts at reflecting the main property of social capital as advocated by its early proponents: “...that social networks have value. Just as a screwdriver (physical capital) or a university education (human capital) can increase productivity (both individual and collective), so do social contacts affect the productivity of individuals and groups...” (Putnam, 2000). That is social capital enhances productivity gains by playing the role of a timing belt driving the transmission and propagation of productivity improvements throughout society. By modelling investment in social capital in terms of the final good, and not in terms of specific human capital, we can obtain the latter outcomes. In particular, we will show that any productivity shock in any sector, including the final good sector, will affect *all* the sectors and the long-run growth rate of the economy, in contrast to the benchmark Lucas-Uzawa model where only productivity shocks in the education sector matter for long-term growth. If social investment were integrally built with human capital as in Sequeira and Ferreira-Lopes (2008), we would obtain exactly the same

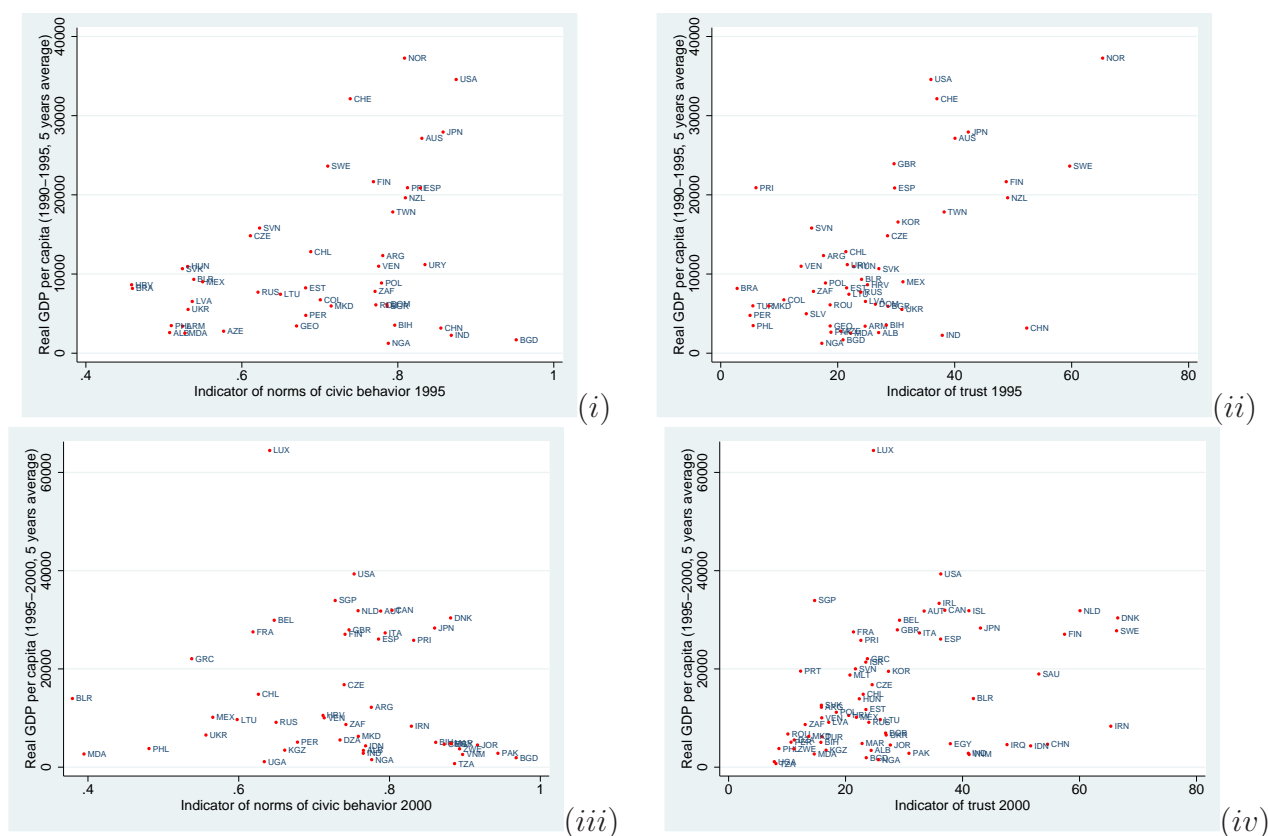


Figure 1: Relationship between social capital and economic development

Sources: The sources of the data used in those figures are indicated in Appendix F.

shock propagation mechanism as in the Lucas-Uzawa model. This is demonstrated explicitly in our Appendix E.

2. An immediate implication of our modelling of social capital is that it should be larger in richer countries. Though such an implication could be discussed on many grounds, specially given the variety of ways to measure social capital, we believe that it does not go at odds with cross-country evidence as reflected in Figure 1: using 2 different measure of social capital (norms of civic behavior and trust indicators) in 1995 and 2000, we show that social capital measures grow in average with real GDP per capita.

3 The model

We present here our endogenous growth model with many identical infinitely lived agents and two sectors. The final goods sector production technology relies on a Cobb-Douglas

production function using two types of input: physical and human capital. The second model is devoted to human capital accumulation thanks to a Cobb-Douglas human capital technology, using human and social capital as inputs. As just mentioned, the output of the final goods sector can be used either for consumption, investment in physical capital or for investment in social capital. Therefore, in our model each agent faces a trade-off between devoting human capital to final goods production and to human capital accumulation, and between allocating final goods production to either consumption, investments in physical and social capital.

Therefore, our formal model implies the following assumptions: (1) the building or accumulation of social capital requires resources to be diverted from final goods production; (2) social capital decays over time without new “investment” in social capital; (3) social capital has a positive impact of human capital accumulation but no direct effect on final goods production; (4) human capital has positive intertemporal spillovers in its accumulation; and (5) human capital is an important input in final goods production.

3.1 Production of final goods and capital accumulation

3.1.1 First sector: final goods production, physical and social capital accumulation

The final good sector produces a homogeneous good that is used either to consume or to invest in either physical or social capital. The investment in social capital may increase social interaction. It is detrimental to the physical capital accumulation since it reduces the amount of resources devoted to the physical capital investment. Therefore, it is potentially harmful to the growth of final good production. Moreover, it implies an opportunity cost in terms of foregone consumption. However, as shown in the next subsection, those adverse effects may be compensated by its positive impact on human capital growth. Individuals allocate a fraction $u(t)$ of their time to the production of final goods. Under the Cobb-Douglas technology, the production function takes the following form:

$$Y(t) = A(K(t))^\alpha (u(t)H(t))^{1-\alpha} = C(t) + I_K(t) + I_S(t) \quad (1)$$

The remaining fraction of time is allocated to human capital accumulation. Equation (1) shows that production of the final goods enable current consumption, and investment in either physical and social capital. Physical and social capital laws of accumulation are

respectively:

$$K(t+1) = I_K(t) + (1 - \delta)K(t) = Y(t) - C(t) - I_S(t) + (1 - \delta)K(t) \quad (2)$$

$$S(t+1) = I_S(t) + (1 - \delta)S(t) \quad (3)$$

We consider that all forms of capital depreciate at the same rate δ .

3.1.2 Second sector: human capital accumulation

Individuals allocate the complementary fraction of their time, i.e. $1 - u(t)$, to the accumulation of human capital

$$H(t+1) = B((1 - u(t))H(t))^\beta (S(t))^{1-\beta} + (1 - \delta)H(t) \quad (4)$$

The law of motion depicted in (4) is consistent with the assumption that final goods sector is more intensive in physical capital while the education sector is more intensive in human and social capital. Social and human capital are to a certain extent complementary in the educational sector production function. This captures the aforementioned observation that social capital is important in the creation of human capital and that both interact in human capital accumulation. Further, one may ensure the usual requirement that the final good sector is less intensive in human capital than the education sector (putting social capital aside). This requires the restriction:

$$1 - \alpha < \beta,$$

to hold. We shall have it in mind throughout this paper but we also consider theoretically possible situations where the education sector output relies more on social capital than on “pure” human capital. This possibility is certainly consistent with a much less academic view of human capital. In this case, the elasticity parameter β might be below $1 - \alpha$.

Eventually, this model exhibits an asymmetry that is quite standard in two sectors endogenous growth model (Mulligan and Sala-i-Martin, 1992): we have on one hand physical and social capital whose accumulations are perfect substitute for consumption, and human capital on the other hand whose accumulation proceeds from a different technology.

3.2 Firms

The final good sector produces a composite good that is used either to consume or to invest in physical capital or in social capital. Following the classical Ramsey, Cass and Koopmans model (Barro and Sala-i-Martin, 1995), we make the standard assumption that

firms produce final goods, pay wages for human capital input and make rental payments for physical capital input. Given the Cobb–Douglas production function already described in (1), the discounted profits are given by:

$$\Pi(t) = \sum_{t=0}^{\infty} [Y(t) - r(t)K(t) - w_Y(t)(u(t)H(t))] R(t) \quad (5)$$

where $R(0) = 1$ and $R(t) = \prod_{\tau=0}^t \left(\frac{1}{1+r(\tau)} \right)$ is the discount factor at time t .

The representative firm chooses physical capital and human capital in order to maximize its discounted profits taking prices as given and subject to its technological constraint:

$$\max_{\{K(t)_{t=0}, (u(t)H(t))_{t=0}\}} \Pi(t) \quad (6)$$

Because the firm rents capital and labor services and do not face any adjustment costs, there are no intertemporal elements in the firm's optimization problem. This implies that the problem of maximizing the present value of profits reduces to a problem of maximizing profits in each period without considering the outcomes in other periods as in the Ramsey, Cass and Koopmans model (Barro and Sala-i-Martin, 1995). Therefore, the first-order conditions characterizing an interior maximum for $\Pi(t)$ are the following:

$$r(t) = \alpha A (K(t)/u(t)H(t))^{\alpha-1} \quad (7)$$

$$w_Y(t) = (1 - \alpha) A (K(t)/(u(t)H(t)))^{\alpha} \quad (8)$$

(7) and (8) indicate that the firm chooses the ratio of physical to human capital in order to equate the rental price of physical capital (i.e. the interest rate) to the marginal product of capital and the wage rate to the marginal product of labor. This implies for the firm zero profit in each period since factor payments exhaust total output.

3.3 Households behavior

We consider a closed economy inhabited by a constant population normalized to one. This population is composed by identical infinitely-lived households that maximize the following intertemporal utility function:

$$\sum_{t=0}^{\infty} \frac{C(t)^{1-\sigma} - 1}{1-\sigma} \rho^t, \quad \rho > 0, \quad \sigma > 0 \quad (9)$$

subject to the household flow budget constraint and human and social capital laws of accumulation (4) and (3):

$$A(t+1) = r(t)A(t) + w_Y(t)u(t)H(t) - C(t) - I_S(t) + (1-\delta)A(t) \quad (10)$$

$$H(t+1) = B((1-u(t))H(t))^\beta (S(t))^{1-\beta} + (1-\delta)H(t) \quad (11)$$

$$S(t+1) = I_S(t) + (1-\delta)S(t) \quad (12)$$

where $A(t)$ is the stock of assets and ρ is a psychological discount factor that is inversely related to the rate of time preference.⁵ The representative household must end up with 0 net debt. Therefore, since the economy is closed we have $A(t) = K(t)$. This implies that the household flow budget constraint (10) reduces to expression (2), the law of motion of physical capital.

The first-order necessary conditions for this problem are the following:

$$\left(\frac{C(t+1)}{C(t)}\right)^\sigma = \rho(1+r(t+1)-\delta) \quad (13)$$

$$u(t) = u(t+1) \frac{K(t)H(t+1)}{K(t+1)H(t)} \left(\left(\frac{1+r(t+1)-\delta}{1+w_H(t+1)-\delta} \right) \left(\frac{1-u(t)}{1-u(t+1)} \frac{S(t+1)H(t)}{S(t)H(t+1)} \right)^{(1-\beta)} \right)^{\frac{1}{\alpha}} \quad (14)$$

$$\frac{K(t)}{S(t)} = \left(\frac{\alpha}{1-\alpha} \right) \left(\frac{\beta}{1-\beta} \right) \left(\frac{u(t)}{1-u(t)} \right) \quad (15)$$

where $w_H(t) = B\beta(S(t)/(1-u(t))H(t))^{1-\beta}$ is the marginal productivity of human capital in the educational sector. Equations (13) and (14) describe respectively the optimal consumption and time allocation to final goods consumption. Equation (15) gives the physical to social capital ratio at equilibrium.

3.4 Equilibrium

We now characterize the equilibrium of this economy. This is done in the following proposition.

Proposition 1 (Equilibrium) *Given the initial conditions $K(-1)$, $H(-1)$, and $S(-1)$, an equilibrium is a path $\{Y(t); I_K(t); K(t); C(t); u(t); H(t); I_S(t); S(t); r(t); w_H(t)\}_{t \geq 0}$*

⁵The precise expression of the psychological discount factor ρ in terms of the rate of time preference ξ is the following $\rho = \frac{1}{1+\xi}$.

that satisfies the following conditions:

$$\left(\frac{C(t+1)}{C(t)}\right)^\sigma = \rho(1+r(t+1)-\delta) \quad (16)$$

$$u(t) = u(t+1) \frac{K(t)H(t+1)}{K(t+1)H(t)} \left(\left(\frac{1+r(t+1)-\delta}{1+w_H(t+1)-\delta} \right) \left(\frac{1-u(t)}{1-u(t+1)} \frac{S(t+1)H(t)}{S(t)H(t+1)} \right)^{(1-\beta)} \right)^{\frac{1}{\alpha}} \quad (17)$$

$$r(t) = \alpha A (K(t)/H(t))^{\alpha-1} u(t)^{1-\alpha} \quad (18)$$

$$w_H(t) = B\beta (S(t)/(1-u(t))H(t))^{1-\beta} \quad (19)$$

$$\frac{K(t)}{S(t)} = \left(\frac{\alpha}{1-\alpha}\right) \left(\frac{\beta}{1-\beta}\right) \left(\frac{u(t)}{1-u(t)}\right) \quad (20)$$

$$Y(t) = A (K(t))^\alpha (u(t)H(t))^{1-\alpha} \quad (21)$$

$$Y(t) = C(t) + I_K(t) + I_S(t) \quad (22)$$

$$K(t+1) = I_K(t) + (1-\delta)K(t) \quad (23)$$

$$H(t+1) = B((1-u(t))H(t))^\beta (S(t))^{1-\beta} + (1-\delta)H(t) \quad (24)$$

$$S(t+1) = I_S(t) + (1-\delta)S(t) \quad (25)$$

Equations (23), (24) and (25) are the accumulation rules of respectively physical, human and social capital. Equations (16)–(25), together with the usual transversality conditions:

$$\lim_{T \rightarrow \infty} K(T) \rho^T = 0 \quad (26)$$

$$\lim_{T \rightarrow \infty} H(T) \rho^T = 0 \quad (27)$$

$$\lim_{T \rightarrow \infty} S(T) \rho^T = 0, \quad (28)$$

are sufficient for an optimum.

The proof of proposition 1 is given in Appendix A. In this proof, the household intertemporal optimization problem depicted by (9), (10), (11) and (12) is solved explicitly. Solving this problem implies the optimization of the following expression of the current value Hamiltonian:

$$J = \frac{(C(t))^{1-\sigma} - 1}{1-\sigma} \rho^t + \lambda_1(t+1) (AK(t)^\alpha (u(t)H(t))^{1-\alpha} - C(t) - I_S(t) - \delta K(t)) \\ + \lambda_2(t+1) \left(B((1-u(t))H(t))^\beta S(t)^{1-\beta} - \delta H(t) \right) + \lambda_3(t+1) (I_S(t) - \delta S(t))$$

We can draw two standard results from this optimization: firstly, household's intertemporal optimization with respect to consumption and investment in social capital (equations (45)

and (46)) implies that

$$C(t)^{-\sigma} = \lambda_1(t+1) = \lambda_3(t+1).$$

This entails the fact that on the margin goods must be equally valuable in their three uses: consumption, physical capital accumulation and social capital accumulation. Secondly, the household's intertemporal optimization with respect to the control variable $u(t)$ (equation (47)) yields:

$$p(t+1) = \frac{\lambda_2(t+1)}{\lambda_1(t+1)} = \frac{A(1-\alpha)(K(t)/(u(t)H(t)))^\alpha}{B\beta(S(t)/((1-u(t))H(t)))^{1-\beta}} \Leftrightarrow \quad (29)$$

$$\lambda_2(t+1)B\beta(S(t)/((1-u(t))H(t)))^{1-\beta} = \lambda_1(t+1)A(1-\alpha)(K(t)/(u(t)H(t)))^\alpha \quad (30)$$

where $p(t+1)$ denotes the shadow price of human capital in terms of final goods.

(29) ensures that the marginal productivity of human capital is equalized across sectors. (30) expresses the condition that time is on the margin equally valuable in its two uses, production and human capital accumulation. Therefore, this is equivalent to equalization of returns of human capital across sectors.

3.5 Balanced growth paths

We now come to the study of balanced growth path (BGP) regimes. As usual, a balanced growth path is a particular solution to the equilibrium dynamics system displayed above where all variables grow at a constant rate except $r(t)$, $w_H(t)$ and $u(t)$ which should be constant along this path. For human capital we have $H(t+1) = H(t)(1 + \gamma_H)$. The growth rates of the variables $Y(t)$, $I_K(t)$, $K(t)$, $C(t)$, $I_S(t)$, $S(t)$ are respectively γ_Y , γ_{I_K} , γ_K , γ_C , γ_{I_S} , and γ_S . We first show that in our model with social capital all growing variables along the BGP should have the same growth path.

Proposition 2 *If $H(t)$ grows at a rate $\gamma_H^* > 0$, then all the other variables $I_K(t)$, $K(t)$, $C(t)$, $I_S(t)$, $S(t)$ grow at strictly positive rates with:*

$$\gamma_Y^* = \gamma_H^* = \gamma_K^* = \gamma_{I_K}^* = \gamma_C^* = \gamma_S^* = \gamma_{I_S}^* \quad (31)$$

The proof of this proposition is given in Appendix B.

Next we have to determine γ_H . To this end we need to impose restrictions on the long-run levels. Computing these restrictions from the dynamic system (16)–(25) we end up with

8 equations for 9 unknowns $\{\bar{Y}; \bar{K}; \bar{C}; \bar{u}; \bar{H}; \bar{I}_K; \bar{I}_S; \bar{S}; \gamma_H\}$. Therefore, the system in terms of levels is undetermined, which is a usual property of endogenous growth models. However, it is possible to stationarize this system to get rid of this indeterminacy. To do so we rewrite the dynamic system (13)–(25) as a function of the following six stationary variables: $y(t) = Y(t)/H(t)$, $k(t) = K(t)/H(t)$, $i_k(t) = I_K(t)/H(t)$, $c(t) = C(t)/H(t)$, $s(t) = S(t)/H(t)$, $i_s(t) = I_S(t)/H(t)$, $u(t)$ and γ_H . The stationarized dynamic system is given in Appendix D. With such a stationarized system, we are able to discuss the existence and uniqueness of the steady state growth rate, and its main determinants as well. As we shall see hereafter, the introduction of social capital crucially changes the comparative statics of the steady state growth rate relative to the Lucas-Uzawa benchmark. In particular, (permanent) technological shocks in the final good sector affect the growth rate in the presence of social capital (as modeled in our paper) while it definitely does not in the Lucas-Uzawa case. This is discussed in the following section.

3.6 Steady state growth rate: the role of social capital

Solving the aforementioned stationarized system, and after many tedious computations (see Appendix C), it is possible to identify a closed-form solution to the steady state growth rate. Specifically, one gets the following result.

Proposition 3 (Existence and uniqueness) *Let $B > \left(\frac{1}{\Psi}\right)^{1-\beta} \left(\frac{1}{\rho} + \delta - 1\right)^{\frac{2-(\alpha+\beta)}{1-\alpha}}$. Then, an unique positive steady state growth rate exists and is characterized by the following stable and positive long-run value:*

$$\gamma_H^* = \left(\rho \left(1 - \delta + \left(B^{\frac{1}{1-\beta}} \Psi \right)^{\frac{(1-\alpha)(1-\beta)}{2-(\alpha+\beta)}} \right) \right)^{\frac{1}{\sigma}} - 1 \quad (32)$$

with $\Psi = (A\alpha)^{\frac{1}{1-\alpha}} \frac{(1-\alpha)}{\alpha} \frac{(1-\beta)}{\beta} \beta^{\frac{1}{1-\beta}}$

Proposition 3 suggests that, if the education sector is productive enough, there exists a unique steady state growth rate. Because the sectors are heavily inter-related, the expression of the growth rate is much more complicated than the counterpart in the benchmark Lucas-Uzawa model. But in both cases, the growth rate is positive if the productivity in the education sector is large enough. In the model with social capital, the growth rate is a complicated function of many parameters, including the technology parameters in the final good sector: this makes a significant difference with the Lucas-Uzawa model (see algebraic

details below). This is a desirable property of social capital which essential role is to facilitate the connection between the different activity sectors. As modeled here, social capital is the vehicle through which productivity improvements in the final good sector may also have a positive impact in the education sector: the resulting positive wealth effect is likely to increase investment in social capital, therefore boosting growth in the education sector.

We may perform comparative statics to check out the impact of the model's parameters on the balanced growth path. Unfortunately, the expressions involved are so complex that it turns out to be impossible to obtain the comparative statics analytically except for the psychological discount factor (standard negative effects of impatience on the long-term growth rate). The same can be claimed on the other expressions obtained in the BGP, like the equilibrium allocation of human capital to the final good sector, the ratios physical to human capital and social to human capital respectively:

$$u^* = 1 - \frac{\beta(\gamma^* + \delta)}{(1 + \gamma^*)^\sigma \frac{1}{\rho} - 1 + \delta} \quad (33)$$

$$k^* = u^* \left(\frac{A\alpha}{(1 + \gamma^*)^\sigma \frac{1}{\rho} - 1 + \delta} \right)^{\frac{1}{1-\alpha}} \quad (34)$$

$$s^* = (\gamma^* + \delta) \left(\frac{\left((1 + \gamma^*)^\sigma \frac{1}{\rho} - 1 + \delta \right)^\beta}{B\beta^\beta} \right)^{\frac{1}{1-\beta}}. \quad (35)$$

We shall obtain the comparative statics numerically in Section 3 once the model conveniently calibrated. At this stage, we move for comparison with the benchmark Lucas-Uzawa model.

3.7 Comparison with the Lucas–Uzawa' model

With $\beta \rightarrow 1$, there is no payoff of accumulating social capital since it contributes neither to physical capital nor to human capital accumulation. Therefore, the model reduces to Lucas–Uzawa' framework without externality as there is no social capital accumulation and the production functions from (1) and (4) simplify to:

$$Y(t) = A(K(t))^\alpha (u(t)H(t))^{1-\alpha} = C(t) + I_K(t) \quad (36)$$

$$H(t+1) = B(1 - u(t))H(t) + (1 - \delta)H(t) \quad (37)$$

It can be readily verified that the dynamic optimization of Lucas–Uzawa' model yields

the following expressions of the steady state growth rate, the share of human capital allocated to physical capital accumulation and the physical to human capital ratio:

$$\gamma_{lu}^* = (\rho(1 - \delta + B))^{\frac{1}{\sigma}} - 1 \quad (38)$$

$$u_{lu}^* = 1 - \frac{\gamma_{lu}^* + \delta}{B} \quad (39)$$

$$k_{lu}^* = u_{lu}^* \left(\frac{A\alpha}{B} \right)^{\frac{1}{1-\alpha}} \quad (40)$$

In contrast to our model with social capital, the steady state growth rate is unaffected by parameters of the final goods production function such as the productivity constant A (Mulligan and Sala-i-Martin, 1992) and the physical capital elasticity α . In our model productivity shocks in any of the two sectors raise the economy's ability to accumulate more social capital, therefore fostering human capital accumulation and ultimately economic growth. In the classical Lucas–Uzawa's model, only **direct** productivity shocks in the education sector can do the job.

Proposition 4 (Impact of A and α on γ_H) *In contrast to the Lucas–Uzawa special case, the BGP growth rate γ_H^* is sensitive to productivity shocks in the final goods sector and to changes in the physical capital elasticity.*

The same remark can be done on the share of human capital in the final good sector and physical to human capital ratios along the BGP. Comparison of equations (38) and (39) with equations (33) and (34) speaks by itself. Only the productivity parameter in the education sector, B is relevant in the long-run for the latter magnitudes in the Lucas-Uzawa case while the presence of social capital in our model provides the necessary vehicle for technology improvements in the final good sector to matter in the long-run for these magnitudes. In this sense, our modelling exemplifies the role of social capital in the development process.

Note that this result is sensitive to the assumption made about the cost of social capital accumulation. Assuming that social capital accumulation implies an opportunity cost in terms of foregone consumption or physical capital investment entails a direct link between the two sectors which materialized in the expression of the steady state growth rate. An alternative modelling strategy implying that the cost of human capital accumulation is incurred in terms of human capital would entail a thoroughly different result as shown in Appendix E. Indeed, as can be seen in (84) the expression of the growth rate in this alternative model does not depend of the parameters of the final goods production function A and α .

3.8 BGP and factor intensity of social capital in the education sector

An interesting side product of our BGP analysis is the evolution of the growth rate when the education sector becomes more and more intensive in social capital, that's when β decreases. The analysis of the function form of γ_H^* shows that it is continuous in β on the interval $]0, 1[$. Moreover, we have:

$$\begin{aligned}\lim_{\beta \rightarrow 0} \gamma_H^* &= \left(\rho \left(\left((1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} B \left(A^{\frac{1}{1-\alpha}} \right) \right)^{\frac{1-\alpha}{2-\alpha}} - \delta + 1 \right) \right)^{\frac{1}{\sigma}} - 1 \\ \lim_{\beta \rightarrow 1} \gamma_H^* &= (\rho(1-\delta+B))^{\frac{1}{\sigma}} - 1 = \gamma_{lu}^*\end{aligned}$$

A careful examination of the steady state growth rate first and second derivatives yields the following results:

$$\lim_{\beta \rightarrow 0} \frac{d\gamma_H}{d\beta} = -\infty, \quad \lim_{\beta \rightarrow 1} \frac{d\gamma_H}{d\beta} = +\infty, \quad \lim_{\beta \rightarrow 0} \frac{d^2\gamma_H}{d^2\beta} = +\infty, \quad \text{and} \quad \lim_{\beta \rightarrow 1} \frac{d^2\gamma_H}{d^2\beta} = +\infty$$

Since $\frac{d\gamma_H}{d\beta}(\beta)$ is also continuous in β on $]0, 1[$, then the equation $\frac{d\gamma_H}{d\beta}(\beta) = 0$ should admit at least one solution in that interval. Therefore, the steady state growth rate is a non-monotonic function of the elasticity of human capital in the education sector. One can easily find numerical examples when the first derivative is always increasing, i.e. $\frac{d^2\gamma_H}{d^2\beta} > 0$ for $\beta \in]0, 1[$. In such cases, the human capital steady growth rate displays an inverted-U shape curve. ⁶

Three comments are in order here. First of all, it is important to notice that having the education sector more intensive in social capital is good for long-run growth: in our model, as social capital is produced from the final good and not from human capital (reflecting the hypothesis that it builds more on time diverted from production than on specific human capital), the economy has two ways to stimulate growth, either through social capital or human capital, instead of one in the Lucas-Uzawa model. The more important is social capital in the education sector, the larger the long-term growth attainable. Second, this property is not true when β is close to one: when one starts departing from the Lucas-Uzawa model, the growth rate first decreases. It only increases (when β keeps decreasing) when β is low enough. As a consequence, there must be another mechanism counter-balancing the one mentioned just above, the interaction of both being responsible of the non-monotonic picture encountered. A potential opposite mechanism is the following: as β goes down, the

⁶Figures 2 and 3 illustrate such an example with the following baseline parameters: $\delta=0.05$, $\rho=0.98$, $\sigma=2$, $\alpha=0.3$, $A=1$, $B=0.12273$.

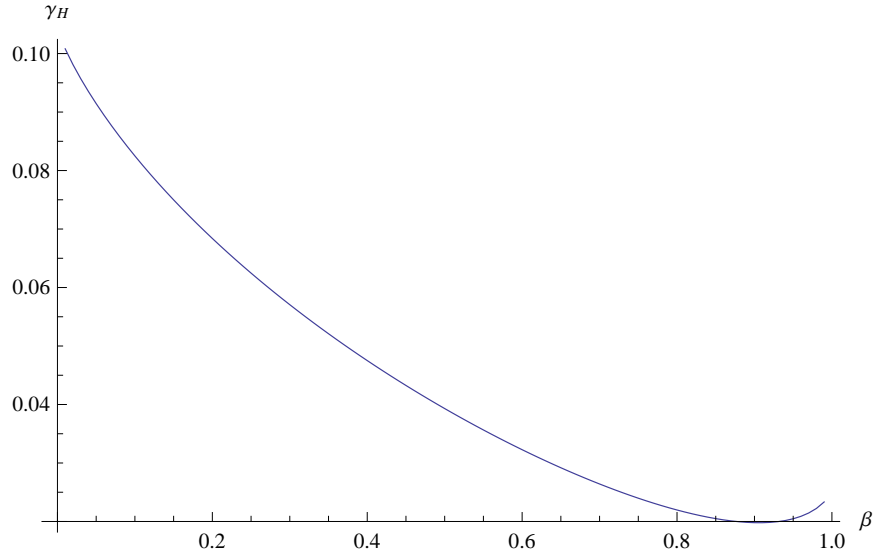


Figure 2: Human capital's growth rate as a function of β .

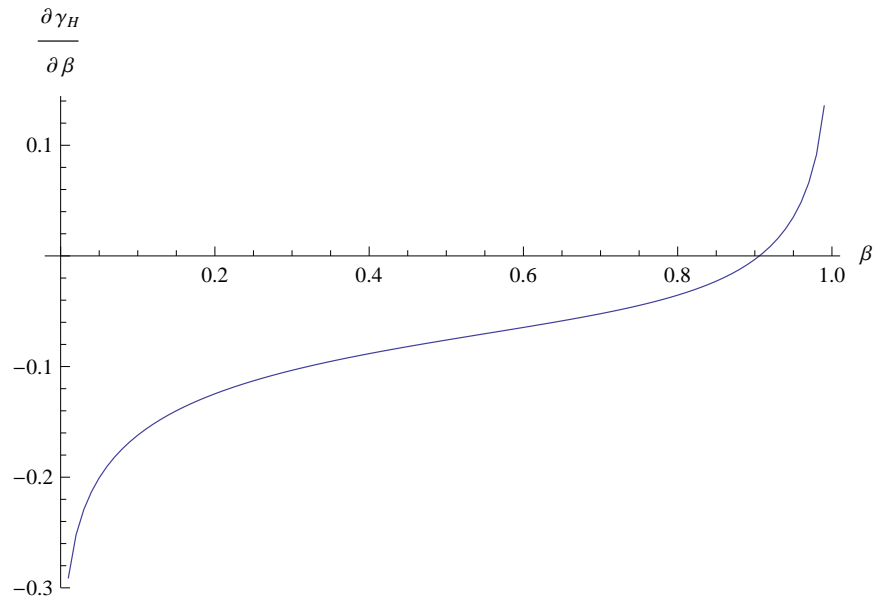


Figure 3: Human capital's growth rate derivative as a function of β .

education sector relies less on human capital (and more on social capital), therefore the share of human capital in this sector is likely to decrease,⁷ which causes the growth rate to drop. In the neighborhood of the Lucas-Uzawa model, that when β is not too distant from 1, the latter mechanism dominates and the BGP growth rate drops when β goes down. As β continues to decrease, this mechanism gets dominated by the first one (the availability of a second powerful growth engine, social capital). Such a monotonic pattern may also

⁷This specific effect is corroborated in the next numerical section.

be observed in the model presented in Appendix E (cfr Figure 7), where the cost of social capital accumulation is expressed in terms of human capital. Last but not least, even under the restriction $1 - \alpha < \beta$, ensuring that the education sector is more intensive in “pure” human capital than the final good sector, the non-monotonicity property still holds.⁸

4 Numerical exercises

Let us consider the following calibration of the model. A first set of parameters is fixed a priori to what we view as reasonable values given the available empirical evidence (see Table 1). The rate of depreciation of all the forms of capital is set to 5%. The psychological discount factor is 0.98. The absolute value of the elasticity of marginal utility is 2. A , the total factor productivity of the goods and services sector, is normalized to 1. B , the productivity parameter of the education sector is set to 0.12273, in order to obtain a growth rate of 2%. While the values of most of the parameters are calibrated on the basis of the

Table 1: Parameters

Parameter	Symbol	Value
Rate of depreciation of capital	δ	0.05
Psychological discount factor	ρ	0.98
Absolute value of the elasticity of marginal utility	σ	2
Physical capital share in the final sector	α	0.3
Total productivity in the final sector	A	1
Total productivity in the education sector	B	0.12273
Human capital share in the education sector	β	0.9

existing empirical studies, it is quite impossible to calibrate β in that way. Indeed, one can hardly find in the literature information about the elasticity of either social or human capital in the education sector. To circumvent that difficulty, we perform a structural estimation of (4), the law of accumulation of human capital.

⁸In Figure 2, $\alpha = 0.3$, non-monotonicity arises in the β -interval,]0.7 1].

4.1 Estimation of the elasticity of social capital

To simplify our specification, we assume full depreciation of human capital, that is $\delta = 1$ and obtain

$$g_H(t) = B(1 - u(t))^\beta \left(\frac{S(t)}{H(t)} \right)^{1-\beta} \quad (41)$$

where $g_H(t) = 1 + \gamma_H(t)$ is the growth factor. Taking the logs of both sides of (41), assuming that $B = \bar{B}e^\epsilon$ and that the economies are in the steady state so that Proposition (2) implies $\gamma_Y^* = \gamma_H^* = \gamma_K^* = \gamma_{I_K}^* = \gamma_C^* = \gamma_S^* = \gamma_{I_S}^* = \gamma^*$, we may write:

$$\log(g^*) = \log(\bar{B}) + \beta \log(1 - u^*) + (1 - \beta) \log\left(\frac{S^*}{H}\right) + \epsilon \quad (42)$$

The specification (42) implies that the sum of the coefficients of the regressors is equal to 1. To be consistent with our theoretical model we may assume that this restriction holds. In that case, the estimation of (42) is equivalent to estimating:

$$\log\left(\frac{g^*H}{S}\right) = \log(\bar{B}) + \beta \log\left(\frac{(1 - u^*)H}{S}\right) + \epsilon \quad (43)$$

Therefore, we may estimate β through a simple regression model by regressing the logarithm of the product of the steady state growth rate and the human to social capital ratio on a constant and the logarithm of the product of the fraction of time the human capital factor devotes to educational sector and the inverse of the normalized social capital.⁹

The steady state growth rate is proxied by data on GDP per-capita growth adjusted for purchasing power parity (PPP, expressed in constant 2000 US Dollars). Those growth rates are computed on the basis on data on real GDP per capita from the Penn World Tables 6.2 between 1980 and 2000 (More information on the data sources is provided on Appendix F).¹⁰

Data for social capital are obtained from the World Values Surveys.¹¹ According to previous researches on social capital at macro level, social capital can be measured through

⁹It is obvious that it would have been better to use more sophisticated methods, such that those proposed by Panel data econometrics, to estimate the human capital elasticity. But, the significant amount of missing data in the social capital indicators prevents it.

¹⁰Since yearly data on GDP per capita may incorporate short-run disturbances, real GDP per capita rates averaged over five-year periods and growth rates are computed. More precisely, we compute growth rates for the following period: 1980–1985, 1985–1990, 1990–1995 and 1995–2000.

¹¹Detailed information on the World Values Survey may be obtained on <http://www.worldvaluessurvey.org/>

different indicators: the levels of generalized trust, associational activity and norms of civic behavior. Trust is coded from WVS data as the percentage of respondents answering that most people can be trusted when asked “Generally speaking, would you say that most people can be trusted, or that you can’t be too careful in dealing with people?” (Inglehart *et al.*, 2000; Knack and Keefer, 1997; Paxton, 1999; Paxton, 2002; Uslaner, 1999; Alesina and La Ferrara, 2000; Putnam, 2000; Whiteley, 2000; Zak and Knack, 2001; Delhey and Newton, 2005).

Associational activity is the percentage of people involved in the following organizations or activities: social welfare services for elderly; handicapped or deprived people; education, arts, music or cultural activities; local community action on issues like poverty, employment, housing, racial equality; third world development or human rights; youth work; religious or church organizations; sports or recreation; Peace movement; Voluntary organizations concerned with health.¹²

Following Knack and Keefer (1997), the strength of the indicator of norms of civic behavior is evaluated from responses to question about whether each of the following behaviors: “claiming government benefits to which you are not entitled”, “avoiding a fare on public transport”, “cheating on taxes if you have a chance”, “keeping money that you have found”, “failing to report damage you’ve done accidentally to a parked vehicle” can always be justified, never be justified or something in between.

There are several ways to construct an indicator of social capital: we may either consider separately measure of trust, of norms and of participation in networks or we may combine different measures of social capital in a unique social capital index. This unique social capital index can be built from the different measures of social capital through principal component analysis. Then, we retain the first principal component which account for the highest share of the total variance of a set of social capital variables. The problem with principal components is that they take negative values which are not convenient for logarithmic transformation. We circumvent this difficulty by considering a monotonic transformation of the first principal component: the Cumulative Normal Distribution Function. This allows us to obtain a social capital index with values between 0 and 1.

Our model needs an approximation for the time spent by individuals to build up human capital accumulation, $(1 - u^*)$. For this, we use the ratio of the average years of schooling to life expectancy. For average years of schooling data we take Barro and Lee (2000)’ data

¹²Other associations or activities like political parties, labor unions or professional organizations are discarded since they seem to refer predominantly to organizations generally oriented towards redistributive goals at the exclusive benefit of their members.

about the educational attainment of the total population aged 15 and over. Data for life expectancy are taken from the World Bank. We consider data for 1980, 1985, 1990, 1995, and 2000. Following Földvari and Van Leeuwen (2009), we consider that $(1 - u^*)$ is roughly equal to the share of time allocated to education and learning. Thus, “dividing this by the life expectancy yields the share of the representative agent’s life that is devoted to human capital formation by means of education” (Földvari and Van Leeuwen, 2009).

We get the following results: when social capital is measured as trust, or as a combined measure of trust and civic norms, we obtain an estimate of β , of roughly 100%. Such results appear as a confirmation of the Lucas–Uzawa model where human capital is the only factor that plays a role in its own accumulation. They seem to suggest that social capital do not impact human capital formation, although this measure of social capital have been shown to impact positively the growth rate of the economy (Knack and Keefer, 1998; Temple, 2000; Fukuyama, 2002).

When social capital is measured by the indicator of norms and civic behavior, we obtain a value of the elasticity of human capital in the education of 94% which implies an elasticity of social capital of roughly 6%. Those ways of measuring social capital allow to enough degrees of freedoms in the estimation (68 observations for trust, 62 observations for norms and for the combined measure of trust and norms). However, considering only norms and trust alone does not seem to be the most intuitive way to capture social capital. While norms and trust are pertinent dimensions of social, they are not sufficient to capture all the aspects of the polymorphous concept of social capital. Considering associational activity may allow us to broaden the perception of that concept. Yet, this implies a severe drawback: the degree of freedoms decreases sharply.

Measuring social capital exclusively associational activity implies a value of β of 0.89 and a number of used observations equal to 29. Combining trust, norms and associational activity in a single indicator, we obtain 0.90 as the OLS estimate of the elasticity of human capital in the education sector. In the simulation, we will consider 0.90 as our value of β . This implies an elasticity of social capital in the educational sector of 10%. There is obviously an issue of endogeneity with this OLS estimate. Indeed, one may expect that the level of output growth rate may affect the choice of the inputs of the educational sector. To tackle that problem, we use values of regressors measured at the beginning of the period on which the growth rates are evaluated (Knack and Keefer, 1997).

4.2 Numerical comparative statics of the BGPs

As mentioned in subsection 2.6, the comparative statics of the BGPs cannot be obtained analytically except for the psychological discount factor. For this parameter, we find the standard and expected result that the higher is the psychological discount factor ρ (i.e. the lower the rate of time preference or equivalently the more people value future consumption), the higher is the long-run growth rate. Table 2 includes the numerical comparative statics with respect to other parameters and for other variables than the BGP growth rate. The computations are performed on the baseline described above.

Table 2: Comparative statics: general model.

	γ_H^*	u^*	k^*	s^*
A	+	+	+	+
B	+	+	+	-
α	-	+	+	+
β	-	-	+	-
ρ	+	-	-	+
δ	-	-	-	+
σ	⁻¹	+	+	-
Baseline	0.0200	0.4335	1.7893	0.6062

Notes: ¹ Holds if $\rho \left(1 - \delta + \left(B \frac{1}{1-\beta} \Psi \right)^{\frac{(1-\alpha)(1-\beta)}{2-(\alpha+\beta)}} \right) > 1$. The red signs are displayed when it is impossible to uniquely sign the derivatives through an analytic inspection. Then, they are determined from the evaluation of the derivatives of the steady state values arrayed vertically with respect to the parameters arrayed horizontally.

Some comments are in order. In first place, and as suggested in Proposition 4, technological shocks in the final good sector do foster long-term growth in our model with social capital. An increase in A does raise the BGP share of human capital in the final good sector and the ratios physical to human capital, and social to human capital as well, which is the intuitive outcome. An increase in B , that's a productivity boom in the education sector, has the same qualitative properties except that it raises human capital more than social capital, which is again intuitively acceptable. The obtained impact of B -shocks on the ratio physical to human capital is standard (see the Lucas-Uzawa case below). Second, a decrease in β is found to increase growth: we are in a parametric region where making the education sector more intensive in social capital triggers long-term growth. Note also that

decreasing β increases the share of human capital in the final good sector. This corroborates our interpretation of the non-monotonicity feature in Section 2.8, and more precisely, our identification of the reverse mechanism playing through the human capital share decision variable. In this sense, the obtained numerical comparative statics are completely consistent with the intuitions presented so far.

The benchmark Lucas-Uzawa case In order to highlight the role of social capital in the findings above, we present here the counterpart comparative statics on the Lucas-Uzawa model.

Table 3: Comparative statics: Lucas–Uzawa model.

	γ_H^*	u^*	k^*
A	0	0	+
B	+	+	-
α	0	0	+ ³
ρ	+	-	-
δ	-	- ²	- ²
σ	- ¹	+ ¹	+ ¹
Baseline	0.0255	0.3846	1.3791

Notes: ¹ Holds if $\rho(1 - \delta + B) > 1$. ² Holds if $\rho(1 - \delta + B) > 1$ and $\sigma > 1$. ³ Holds if $(1 - \alpha + \alpha \log(\frac{\alpha A}{B})) > 0$.

As in table 2 the red signs are determined from the evaluation of the derivatives of the steady state values arrayed vertically with respect to the parameters arrayed horizontally evaluated with the following baseline parameters: $\delta=0.05$, $\rho=0.98$, $\sigma=2$, $\alpha=0.3$, $A=1$, $B=0.12273$.

Comparison of the two tables confirm the main and essential difference between the two models: while technology shocks in the final good sector do not play any role in the BGP in the Lucas-Uzawa model, they do matter when social capital is modelled. Other than this, the comparative statics of the two models are qualitatively similar.

4.3 Productivity shocks

By propositions 3 and 4, we know that productivity parameters impact positively the growth rate of the economy. This is one of the important differences between our model and the standard Lucas-Uzawa model where productivity parameters of the final good sector have no impact in the long run growth. In what follow, we analyze how an economy

responds to shocks to the parameters, A (the productivity in the final good sector), and B (the productivity in the education sector). All the shocks considered are permanent (from $t = 0$) and have an intensity equal to 1%. Shocks to these parameters can arise due to changes in education policy, to education subsidies, to changes in policy regimes, to innovation, etc...

4.3.1 Productivity shocks in the goods and services sector

In response to a productivity shock in the final goods sector, standardized physical capital increases to take advantage of the increased efficiency of the productive sector (Figure 4). Depending on the value of the human capital elasticity in the education sector, the share of human capital allocated in the goods and services sector may initially increase (for $\beta = 1$ or $\beta = 0.9$, Figure 8 in Appendix G) or decrease (for $\beta = 0.5$, Figure 9 in Appendix G). In the first case this implies a reallocation of human capital from the education to the productive sector, while this would entail a reassignment of human capital from the final goods to the educative sector in the second case.

As the marginal productivity of human capital in the education sector rises with social capital, human and social capitals are complements in human capital accumulation. Therefore, the demand for social capital in the education sector decreases in the first case, while it raises in the second case (Figure 4). But as the education sector is the driver of economic growth, it also entails a smaller human capital accumulation, an initial decline of the economic growth rate in the first case (Figure 9 in Appendix G) and an initial increase in the second case (Figure 10 in Appendix G). In the first case, the resulting paucity of human capital induces agents to devote less time in the productive sector. This entails a reverse reallocation of human capital from the productive to the education sector and a subsequent rise of the demand of social capital and of the economic growth rate. A reverse mechanism occurs in the second case.

The intensity of the elasticity of human capital in the education sector is of fundamental importance in the way the economy adjust in case of productivity shocks. For values of β close to one, human and social capital are less substitutable as inputs of the education sector. Therefore, in case of a raise of A the reallocation of human capital from the education to the final goods sector entails an initial drop-off of the growth rate. With $\beta \ll 1$, the substitutability between the two inputs of the education sector increases.¹³ Therefore, the

¹³A property of the Cobb-Douglas production function of the education sector is that the inputs involved are complements since no human capital accumulation is possible when the stock of either of them is zero. But, at the same time they are substitutable since human capital accumulation may be kept constant while

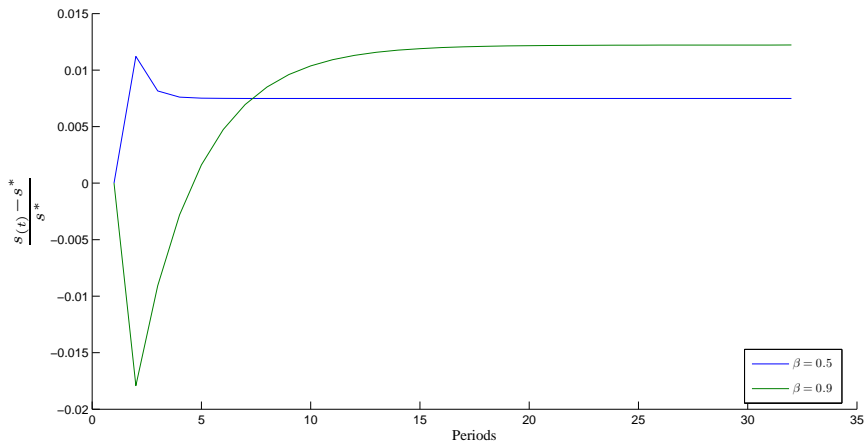
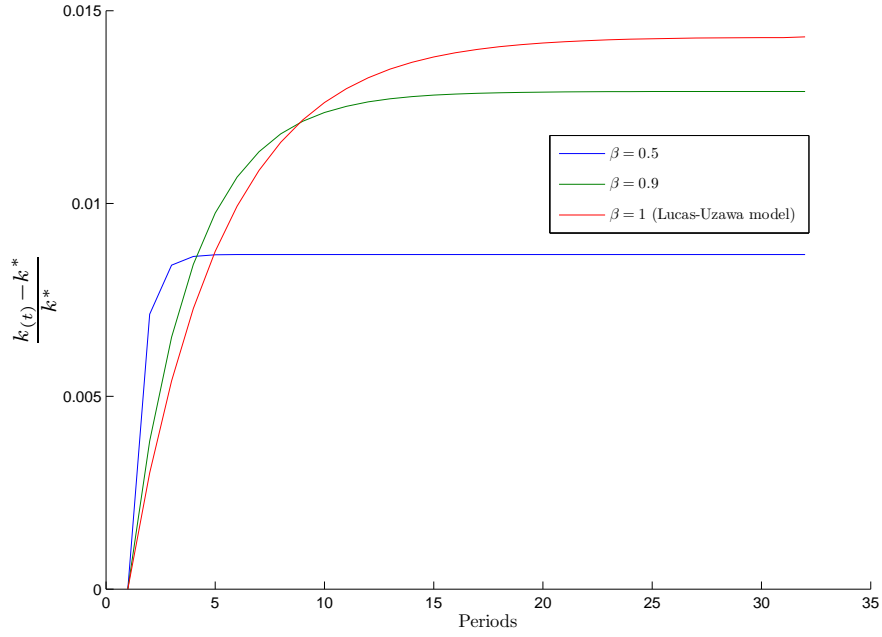


Figure 4: Effects of productivity shocks on k and s .

reallocation of human capital to the final goods sector can be accommodated by an increase of the social to human capital ratio.

the decrease in the stock of one of the input is compensated by the increase of the stock of the other. The substitutability between human and social capital increases when $\beta \ll 1$.

4.3.2 Productivity shocks in the education sector

Let us now consider the impact of a shock in the education sector. As before, depending on the intensity of the elasticity of human capital, we may distinguish two cases. If β is close to 1, then human capital growth rate increases (Figures 12 and 13 in Appendix H). As the education sector is more efficient, workers reallocate their working time in its favor (Figure 11 in Appendix H). Since human and physical capital are complements, this lowers the physical to human capital ratio (Figure 5).

Human capital and social capital are complements as well, therefore standardized social capital also increases in a first step (Figure 5). But, the reallocation of working time in favor of the education sector ends up by causing a shortage of physical capital investment. To avoid detrimental effects on output and economic growth rate, agents opt to subsequently spend more time in the productive sector.

In the case $\beta \ll 1$, once again the substitutability between social and human capital increases. Therefore, agents take advantage of the income effect generated by the increased productivity of the education sector by increasing their working time in the final goods sector (Figure 12). As human and social capital are complements, this induces an initial decrease of the social to human capital ratio (Figure 5). This has initially a detrimental effect on the human capital growth rate. However, it subsequently increases as agents increase their working time in the education sector.

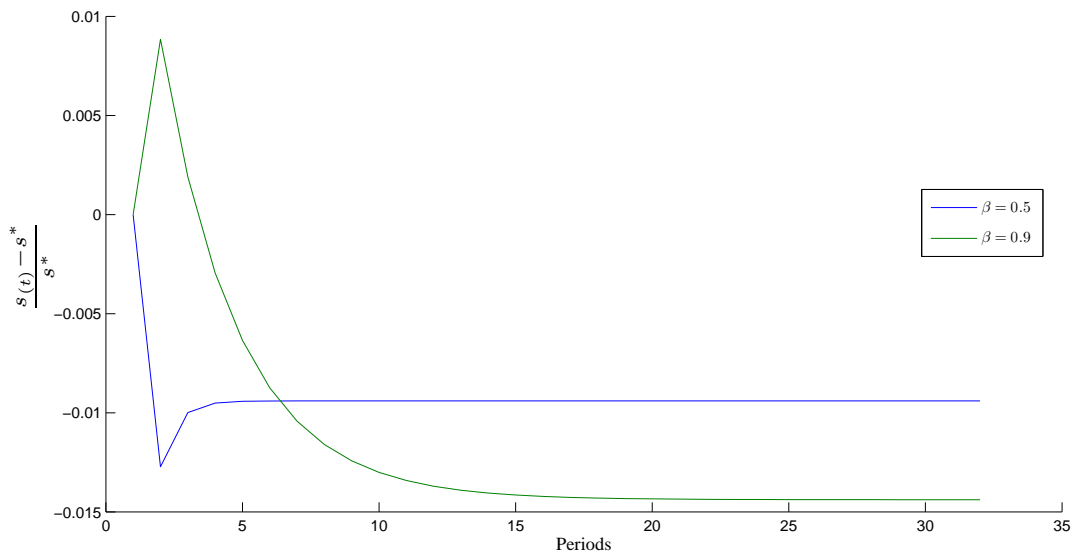
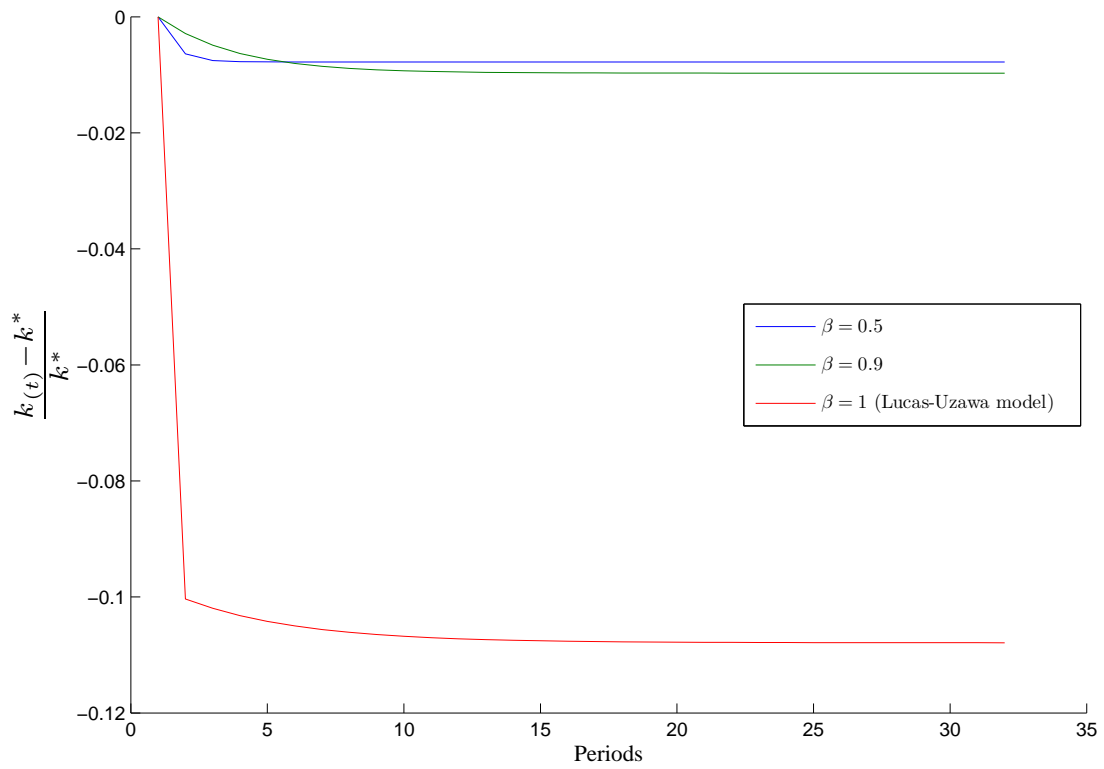


Figure 5: Effects of productivity shocks on k and s .

4.4 Imbalance effects

The analysis of imbalance effects represent an important line of research in endogenous growth models with human capital. They are due either to the relative abundance of physical capital¹⁴ or inversely to the relative abundance of human capital. The most important result that can be derived in analyzing the imbalance effects in the Lucas-Uzawa model is that a shortage of human capital motivates an allocation of resources to production of goods rather than education. This will decrease the accumulation of human capital, lowering the economy's growth rate (Boucekkine and Ruiz-Tamarit, 1995). Thus, the model predicts that an economy should experience faster recovery after an event that destroys physical capital, than if it had destroyed human capital. It also suggests that the economies which are growing faster are those with higher ratios of human capital to physical capital.

Contrary to the one sector model with the same technology to produce physical and human capital, the two-sector model does not give rise to symmetric U-shaped imbalance effects. The rationale behind this finding is quite simple: since the education sector is more intensive in human capital, its operation cost is larger in case of a shortfall of human capital because of the induced higher wage. This motivates people to allocate human capital to the final good sector, rather than to the education sector (Boucekkine and Ruiz-Tamarit, 2004 and 2008, Boucekkine *et al.*, 2008).

Imbalances effects are depicted in Figure 6. Figure 6 shows that the relationship between the human capital growth rate and the physical to human capital ratio is always monotonic. However, depending on the human capital elasticity β , it can have a negative (when $\beta = 0.5$) or a positive slope (when $\beta = 0.9$ or $\beta = 1$). Indeed, as stated before when the elasticity of human capital in the education sector is high, social capital is less substitutable to human capital. Therefore, individuals allocate more working time in the education sector which allows a higher accumulation of human capital and therefore a higher economic growth rate. Consequently, since there is less human capital available in the final goods sector, firms use proportionally more physical capital which explains the positive relationship between the economic growth rate and the physical to human capital ratio.

A contrario, when $\beta = 0.5$, the substitutability between social and human capital increases, therefore individuals may allocate more working time in the final goods sector without impeding economic growth. Therefore, lower physical to human capital ratio co-exist with higher economic growth rates as suggests by the negative slope of the curve in Figure 6 corresponding to $\beta = 0.5$.

¹⁴or equivalently to the shortage of human capital.

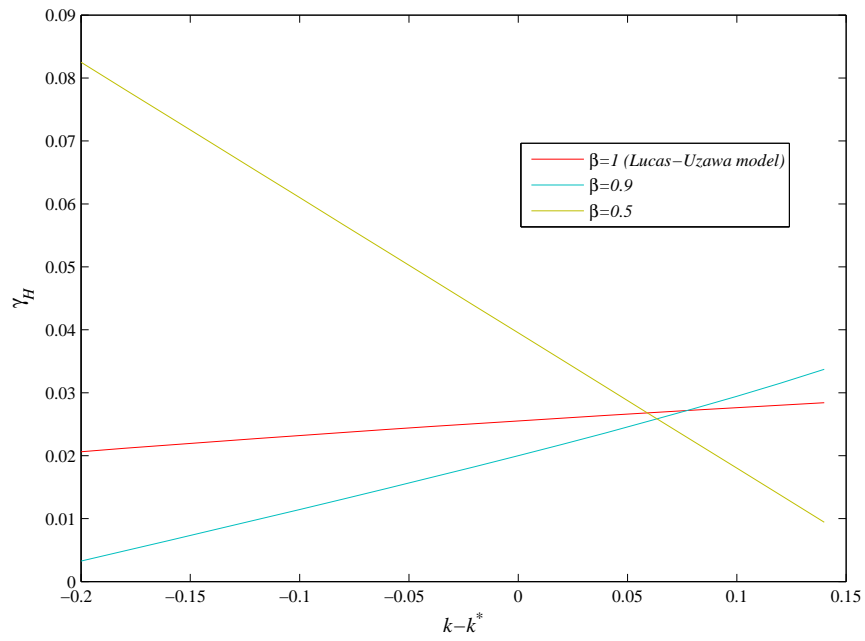


Figure 6: Comparison of different Physical to Human capital ratio imbalance effects for different values of the elasticity of the human capital in the education sector ($\beta = 1$ (Lucas-Uzawa model), $\beta = 0.9$, $\beta = 0.5$).

5 Conclusion

In this paper, we build an endogenous two-sector model where the interaction between human and social capital drive the accumulation of human capital. First of all, we choose a multi-sector modelling such that social capital plays the advocated role of timing belt propagating shocks through the macroeconomy. As a result, and in contrast to the seminal Lucas-Uzawa model, the steady state growth rate depends on productivity parameters of ALL the sectors, not only those arising from the education sector. Assuming that any investment in social capital implies an opportunity cost in terms of foregone physical capital accumulation and consumption creates a direct link between the two sectors which materializes in the expression of the steady state growth rate.

Three types of findings are obtained in this paper. First of all, it is shown that the presence of social capital has an ambiguous effect on long-term growth. Indeed, we obtain a U-shape pattern for the steady state growth rate with respect to social capital elasticity in the education sector. When the education sector is intensive in social capital, the latter may act as a substitute of human capital. This allows a higher allocation of human capital in the final goods sector and therefore enable the economy to achieve higher output, higher consumption level, and higher investment in social capital which recursively may sustain a higher economic growth rate. However, in the neighborhood of the Lucas-Uzawa model, when the social capital elasticity is much lower, the education sector relies less on social capital. Therefore, in such a case the only way to foster economic growth is through increases of the human to social capital ratio. Such a parameter region is consistent with a strong empirical result uncovered by Putnam (2000) on the US economy: the concomitance of a growing economy with a declining social to human capital ratio. Second, we try to provide with some estimate for the “weight” of social capital in the process of human capital formation. Our main finding in this respect is that the elasticity of human capital to social capital varies from 6% to 10% depending on the measure of social capital selected. Last but not least, through some numerical examples, we show that the magnitude of social capital elasticity may have a strong impact on transitory dynamics. A higher social capital elasticity may induce a decreasing pattern of the steady state growth with respect to the physical to human capital ratio, while lower values may entail an increasing pattern.

It goes without saying that our analysis has the advantages and limits of multi-sector endogenous growth models, which build on stylized laws of motion for aggregate variables. The process of social capital formation is probably much trickier at the micro level. However we believe that our model highlights in a transparent and accurate way the role of

social capital in the growth process, and this role is indeed ambiguous. We have uncovered the sources of this ambiguity and provided a numerical assessment of the impact of social capital on long-term growth and short-term dynamics using available data on social capital measures. More work is needed on the microfoundations of social capital and on its measurement.

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Appendix A: Proof of Proposition 1

The households' intertemporal optimization problem yields the following discrete time current value Hamiltonian:

$$\begin{aligned}
 J = & \frac{(C(t))^{1-\sigma} - 1}{1-\sigma} \rho^t + \lambda_1(t+1) (AK(t)^\alpha (u(t)H(t))^{1-\alpha} - C(t) - I_S(t) - \delta K(t)) \\
 & + \lambda_2(t+1) \left(B((1-u(t))H(t))^\beta S(t)^{1-\beta} - \delta H(t) \right) + \lambda_3(t+1) (I_S(t) - \delta S(t)) \quad (44)
 \end{aligned}$$

where $\lambda_1(t)$, $\lambda_2(t)$, $\lambda_3(t)$ respectively denote are the $K(t)$, $H(t)$ and $S(t)$ co-state variables. We consider that the households fully internalize the benefit of social capital. The necessary first order conditions are:

$$\frac{\partial J}{\partial C(t)} = 0 \Rightarrow C(t)^{-\sigma} = \lambda_1(t+1) \quad (45)$$

$$\frac{\partial J}{\partial I_S(t)} = 0 \Rightarrow \lambda_1(t+1) = \lambda_3(t+1) \quad (46)$$

$$\frac{\partial J}{\partial u(t)} = 0 \Rightarrow \frac{\lambda_1(t+1)}{\lambda_2(t+1)} = \frac{AK(t)^\alpha (1-\alpha) u(t)^{-\alpha} H(t)^{1-\alpha}}{B\beta (1-u(t))^{\beta-1} H(t)^\beta S(t)^{1-\beta}} \quad (47)$$

$$\begin{aligned}
 \frac{\partial J}{\partial K(t)} &= -(\lambda_1(t+1) - \lambda_1(t)) \Rightarrow \\
 \frac{\lambda_1(t)}{\lambda_1(t+1)} &= 1 + A\alpha K(t)^{\alpha-1} (u(t)H(t))^{1-\alpha} - \delta \quad (48)
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial J}{\partial H(t)} &= -(\lambda_2(t+1) - \lambda_2(t)) \Rightarrow \\
 \frac{\lambda_2(t)}{\lambda_2(t+1)} &= 1 + \frac{\lambda_1(t+1)}{\lambda_2(t+1)} AK(t)^\alpha u(t)^{1-\alpha} (1-\alpha) H(t)^{-\alpha} \\
 &+ B\beta (1-u(t))^\beta H(t)^{\beta-1} S(t)^{1-\beta} - \delta \quad (49)
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial J}{\partial S(t)} &= -(\lambda_3(t+1) - \lambda_3(t)) \Rightarrow \\
 \frac{\lambda_3(t)}{\lambda_3(t+1)} &= 1 + \frac{\lambda_2(t+1)}{\lambda_3(t+1)} B((1-u(t))H(t))^\beta (1-\beta) S(t)^{-\beta} - \delta \quad (50)
 \end{aligned}$$

Our boundaries conditions are standard. They imply the initial values $K(-1)$, $H(-1)$, $S(-1)$ and the following transversality conditions:

$$\lim_{T \rightarrow \infty} K(T) \rho^T = 0$$

$$\lim_{T \rightarrow \infty} H(T) \rho^T = 0$$

$$\lim_{T \rightarrow \infty} S(T) \rho^T = 0$$

After some manipulation on equation (45), we obtain the following Euler equation on the consumption control variable:

$$\left(\frac{C(t+1)}{C(t)}\right)^\sigma = \rho(1 + A\alpha K(t+1)^{\alpha-1}(u(t+1)H(t+1))^{1-\alpha} - \delta)$$

Equating the growth rates of $\lambda_1(t)$ and $\lambda_3(t)$ (cfr. equations (46), (48) and (50)), we obtain after further manipulations the following relationship between $K(t)$ and $S(t)$:

$$\frac{K(t)}{S(t)} = \left(\frac{\alpha}{1-\alpha}\right) \left(\frac{\beta}{1-\beta}\right) \left(\frac{u(t)}{1-u(t)}\right)$$

From (47) giving the ratio of costates variables $\lambda_1(t)$ and $\lambda_2(t)$, we can compute the ratio of their growth rates; this yields the second Euler equation on $u(t)$:

$$u(t) = u(t+1) \frac{k(t)}{k(t+1)} \left(\left(\frac{1 + A\alpha k(t+1)^{(\alpha-1)}u(t+1)^{(1-\alpha)} - \delta}{1 + B\beta(1-u(t+1))^{(\beta-1)}s(t+1)^{(1-\beta)} - \delta} \right) \left(\frac{1-u(t)}{1-u(t+1)} \frac{s(t+1)}{s(t)} \right)^{(1-\beta)} \right)^{\frac{1}{\alpha}}$$

with $k(t) = \frac{K(t)}{H(t)}$ and $s(t) = \frac{S(t)}{H(t)}$. The other conditions of the equilibrium are merely accumulation rules of physical, human and social capital or final goods equilibrium conditions.

Appendix B: Proof of Proposition 2

If a balanced growth path should satisfy the eight Equations (16)–(25), then one should have the following eight restrictions among the various growth rates:

$$(1 + \gamma_C^*)^\sigma = \rho(1 + r^* - \delta) \tag{51}$$

$$\frac{1 + \gamma_K^*}{1 + \gamma_H^*} = \left(\frac{1 + \gamma_S^*}{1 + \gamma_H^*}\right)^{\frac{1-\beta}{\alpha}} \tag{52}$$

$$\gamma_K^* = \gamma_S^* \tag{53}$$

$$1 + \gamma_Y^* = (1 + \gamma_K^*)^\alpha (1 + \gamma_H^*)^{1-\alpha} \tag{54}$$

$$\gamma_Y^* = \gamma_C^* = \gamma_{I_K}^* = \gamma_{I_S}^* \tag{55}$$

$$1 + \gamma_K^* = \frac{I_K^*}{K} + 1 - \delta \tag{56}$$

$$1 + \gamma_H^* = w_H^*/\beta + 1 - \delta \tag{57}$$

$$1 + \gamma_S^* = \frac{I_S^*}{S} + 1 - \delta \tag{58}$$

(52) and (53) implies that $\gamma_K^* = \gamma_H^*$. As γ_K^* and γ_H^* are constant we may conclude from (56) and (58) that K and I_K on one hand and S and I_S on another hand grow at the same rate. Therefore, we have the following relationship:

$$\gamma_Y^* = \gamma_H^* = \gamma_K^* = \gamma_{I_K}^* = \gamma_C^* = \gamma_S^* = \gamma_{I_S}^* = \gamma^*$$

Appendix C: Proof of Proposition 3

At the steady state the growth rate of the costate variables $\lambda_1(t)$ and $\lambda_2(t)$ are equal. From this equality, we derive that the marginal productivity of $K(t)$ in the final goods sector, r^* and the margin productivity of $H(t)$ in the education sector are equal in the steady state $r^* = w_H^*$. Then, from (51) and (57) we may derive the steady state value of time allocation of human capital to the final goods sector:

$$u^* = 1 - \frac{\beta(\gamma^* + \delta)}{(1 + \gamma^*)^\sigma \frac{1}{\rho} - 1 + \delta} \quad (59)$$

Replacing this expression in (51) and (57), we obtain the following steady state values of stationnarized physical and social capital:

$$\frac{K^*}{H} = u^* \left(\frac{A\alpha}{(1 + \gamma^*)^\sigma \frac{1}{\rho} - 1 + \delta} \right)^{\frac{1}{1-\alpha}} \quad (60)$$

$$\frac{S^*}{H} = (\gamma^* + \delta) \left(\frac{\left((1 + \gamma^*)^\sigma \frac{1}{\rho} - 1 + \delta \right)^\beta}{B\beta^\beta} \right)^{\frac{1}{1-\beta}} \quad (61)$$

Replacing the expression of the ratio of (60) and (61) in the leftside of (51) yields, after some straightforward computations, (32) the expression of the steady state growth rate.

Appendix D: The stationarized dynamic system

The dynamic system (16)–(25) can be rewritten in terms of stationary variables as:

$$\left(\frac{c(t+1)(1+\gamma(t+1))}{c(t)}\right)^\sigma = \rho(1+r(t+1)-\delta) \quad (62)$$

$$u(t) = u(t+1) \frac{k(t)}{k(t+1)} \left(\left(\frac{1+r(t+1)-\delta}{1+w_H(t+1)-\delta} \right) \left(\frac{1-u(t)}{1-u(t+1)} \frac{s(t+1)}{s(t)} \right)^{(1-\beta)} \right)^{\frac{1}{\alpha}} \quad (63)$$

$$r(t) = \alpha A (k(t))^{\alpha-1} u(t)^{1-\alpha} \quad (64)$$

$$w_H(t) = B\beta (s(t)/(1-u(t)))^{1-\beta} \quad (65)$$

$$\frac{k(t)}{s(t)} = \left(\frac{\alpha}{1-\alpha} \right) \left(\frac{\beta}{1-\beta} \right) \left(\frac{u(t)}{1-u(t)} \right) \quad (66)$$

$$y(t) = A (k(t))^\alpha (u(t))^{1-\alpha} \quad (67)$$

$$y(t) = c(t) + i_K(t) + i_S(t) \quad (68)$$

$$k(t+1)(1+\gamma(t+1)) = i_K(t) + (1-\delta)k(t) \quad (69)$$

$$(1+\gamma(t+1)) = B(1-u(t))^\beta (s(t))^{1-\beta} + (1-\delta) \quad (70)$$

$$s(t+1) = i_S(t) + (1-\delta)s(t) \quad (71)$$

Appendix E: Alternative model

In our model, we assume that, while increasing social interaction, investment in social capital implies an opportunity cost in terms of foregone physical capital accumulation and consumption. There is an alternative way to model the cost of social capital accumulation; we may assume that individuals devote a fraction $l(t)$ of their time to build their social networks. Such an assumption implies the following laws of motion for human and social capital accumulation:

$$H(t+1) = B((1-u(t)-l(t))H(t))^\beta (S(t))^{1-\beta} + (1-\delta)H(t) \quad (72)$$

$$S(t+1) = Cl(t) + (1-\delta)S(t) \quad (73)$$

With the intertemporal utility function (9) and the physical capital accumulation law (10), they imply the following equilibrium conditions:

$$\left(\frac{C(t+1)}{C(t)}\right)^\sigma = \rho(1+r(t+1)-\delta) \quad (74)$$

$$\frac{u(t)}{u(t+1)} = \frac{K(t)H(t+1)}{K(t+1)H(t)} \left(\left(\frac{1+r(t+1)-\delta}{1+w_H(t+1)-\delta} \right) \left(\frac{(1-u(t)-l(t))S(t+1)H(t)}{(1-u(t+1)-l(t+1))S(t)H(t+1)} \right)^{(1-\beta)} \right)^{\frac{1}{\alpha}} \quad (75)$$

$$r(t) = \alpha A (K(t)/u(t)H(t))^{\alpha-1} \quad (76)$$

$$w_H(t) = B\beta (S(t)/(1-u(t)-l(t))H(t))^{1-\beta} \quad (77)$$

$$\frac{1-u(t)-l(t)}{1-u(t-1)-l(t-1)} = \frac{S(t)H(t-1)}{S(t-1)H(t)} \left(\frac{1+C\frac{(1-\beta)}{(\beta)}\frac{(1-u(t)-l(t))H(t)}{S(t)}-\delta}{1+w_H(t)-\delta} \right)^{\frac{1}{1-\beta}} \quad (78)$$

$$Y(t) = A(K(t))^\alpha (u(t)H(t))^{1-\alpha} \quad (79)$$

$$Y(t) = C(t) + I_K(t) + I_S(t) \quad (80)$$

$$K(t+1) = I_K(t) + (1-\delta)K(t) \quad (81)$$

$$H(t+1) = B((1-u(t)-l(t))H(t))^\beta (S(t))^{1-\beta} + (1-\delta)H(t) \quad (82)$$

$$S(t+1) = Cl(t) + (1-\delta)S(t) \quad (83)$$

As before after solving the corresponding stationarized system, we are able to identify a closed-form solution to the steady state growth rate:

$$\gamma_H^* = \left(\rho \left(1 - \delta + \left(B\beta^\beta ((1-\beta)C)^{1-\beta} \right)^{\frac{1}{2-\beta}} \right)^{\frac{1}{\sigma}} - 1 \right) \quad (84)$$

Conversely to expression (32), this function does not depend on parameters A and α . However, as shown in Figure 7 for baseline parameters $\delta=0.05$, $\rho=0.98$, $\sigma=2$, $\alpha=0.3$, $A=1$, $B=0.12273$ and $C=1.07105$, it displays an inverted-U shape as in Figure 2.

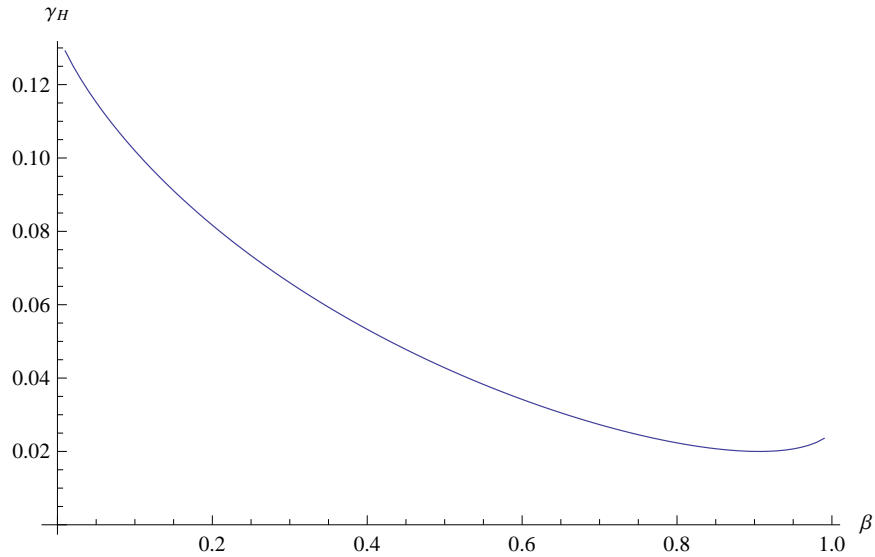


Figure 7: Human capital's growth rate as a function of β (alternative model).

Appendix F: Data and descriptives

The dataset used in the empirical subsection consists of 74 countries:

Country list: Albania, Argentina, Australia, Austria, Bangladesh, Belarus, Belgium, Bosnia and Herzegovina, Brazil, Bulgaria, Canada, Chile, China, Colombia, Croatia, Czech Republic, Denmark, Egypt, Estonia, Finland, France, Georgia, East Germany, West Germany, Great Britain, Hungary, Iceland, India, Indonesia, Iran, Iraq, Ireland, Italy, Japan, Jordan, Republic of Korea, Latvia, Lithuania, Macedonia, Malta, Mexico, Moldova, Montenegro, Morocco, Netherlands, New Zealand, Nigeria, Northern Ireland, Norway, Pakistan, Peru, Philippines, Poland, Portugal, Puerto Rico, Romania, Russia, Rwanda, Serbia, Slovakia, Slovenia, South Africa, Spain, Sweden, Switzerland, Taiwan, Tanzania, Turkey, Ukraine, United States, Uruguay, Venezuela, Vietnam.

The dataset draws on publicly available data only. The following variables are available online with the following web links:

Variables:

1. GDP per capita growth rate adjusted by for Purchasing Power Parity (PPP, expressed in constant 2000 US Dollars) between 1980 and 2000 computed from Alan Heston, Robert Summers and Bettina Aten, Penn World Table Version 6.2, Center for International Comparisons of Production, Income and Prices at the University of Penn-

- sylvania, September 2006 available online at http://pwt.econ.upenn.edu/php_site/pwt62/pwt62_form.php.
2. Social capital indicators — levels of generalized trust, associational activity and norms of civic behavior — obtained from the World Value Surveys (European and World Values Surveys four-wave integrated data file, 1981-2004, v.20060423, 2006) available online at <http://www.wvsevsdb.com/wvs/WVSDData.jsp>.
 3. Human capital indicator — average years of schooling of the total population aged 15 and over from Barro and Lee (2000) available online at <http://www.cid.harvard.edu/ciddata/ciddata.html>.
 4. Life expectancy data obtained from World Bank online data available at <http://data.worldbank.org/topic/social-development>.

Appendix G: Effects of productivity shocks in the goods and services sector on $u(t)$ and $\gamma_H(t)$

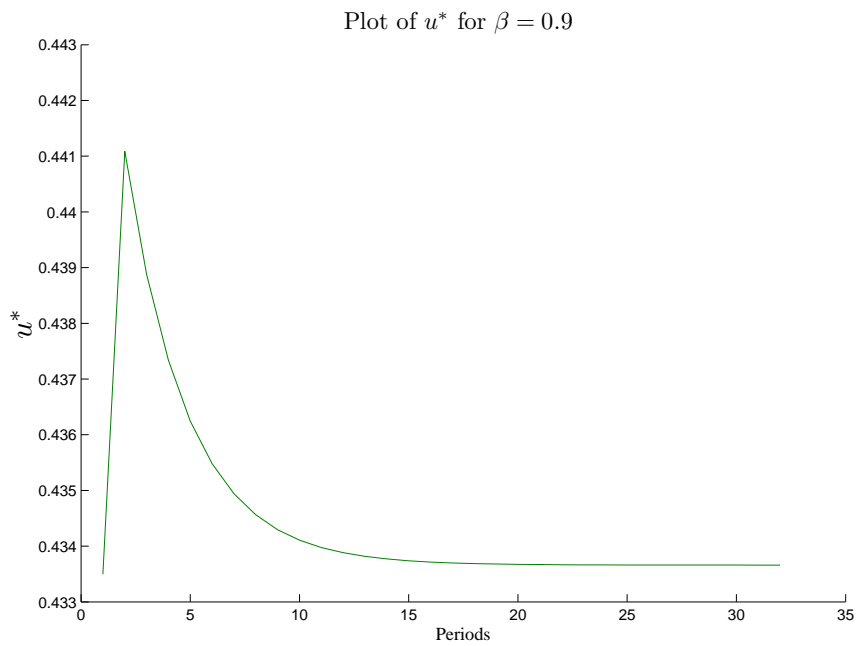
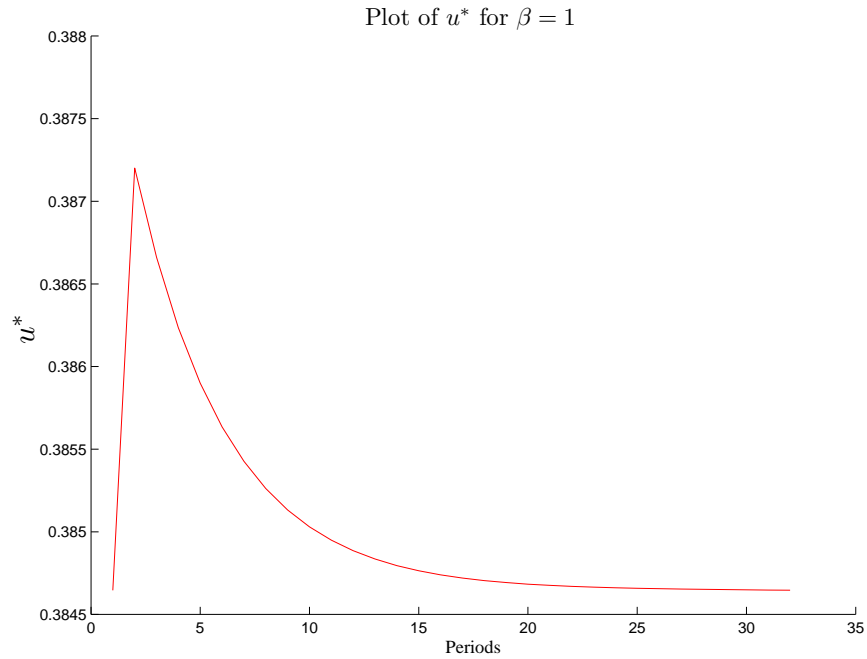


Figure 8: Effects of A productivity shocks on u for $\beta = 1$ and $\beta = 0.9$.

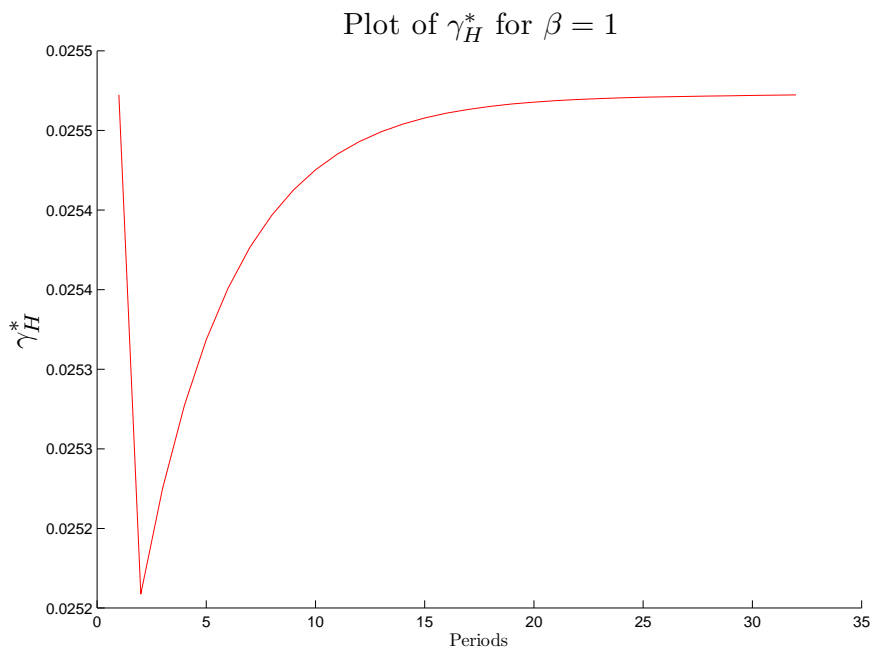
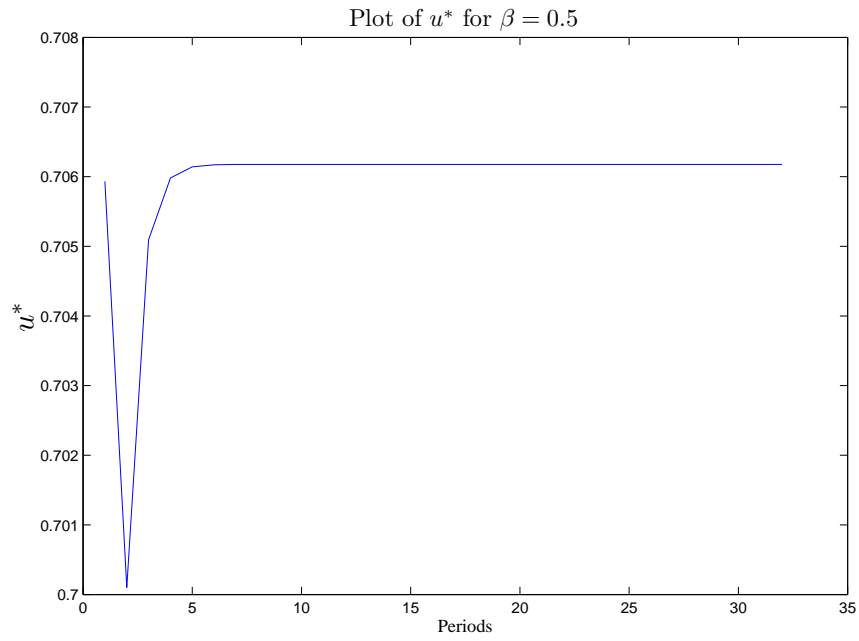


Figure 9: Effects of A productivity shocks on u for $\beta = 0.5$ and on γ_H for $\beta = 1$.

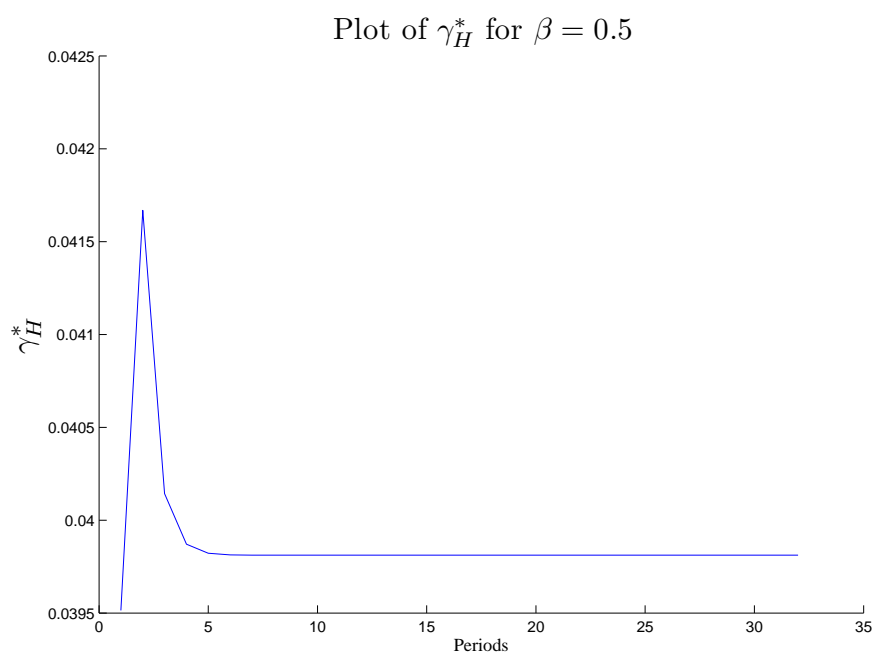
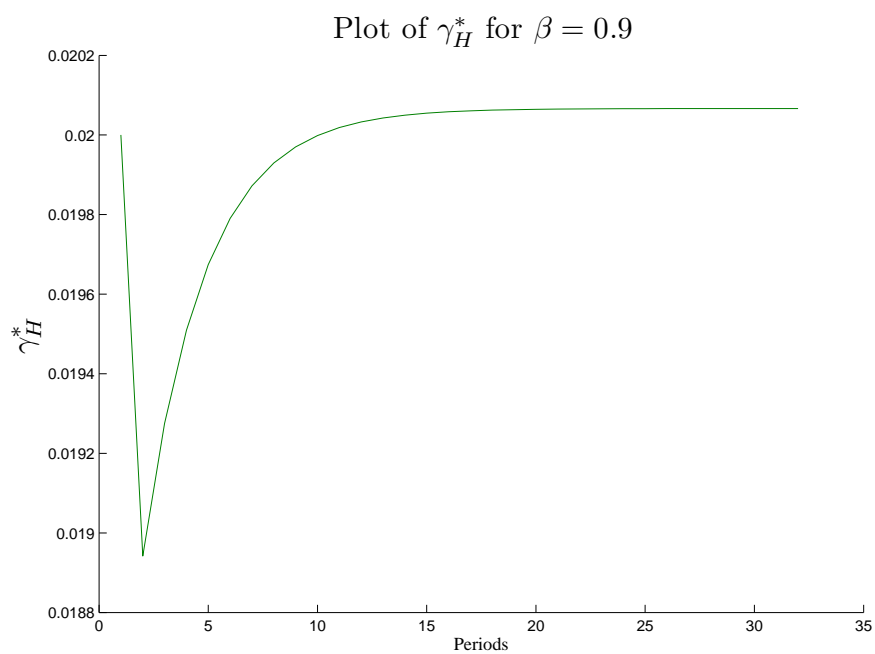


Figure 10: Effects of A productivity shocks on γ_H for $\beta = 0.9$ and $\beta = 0.5$.

Appendix H: Effects of productivity shocks in the education sector on $u(t)$ and $\gamma_H(t)$.

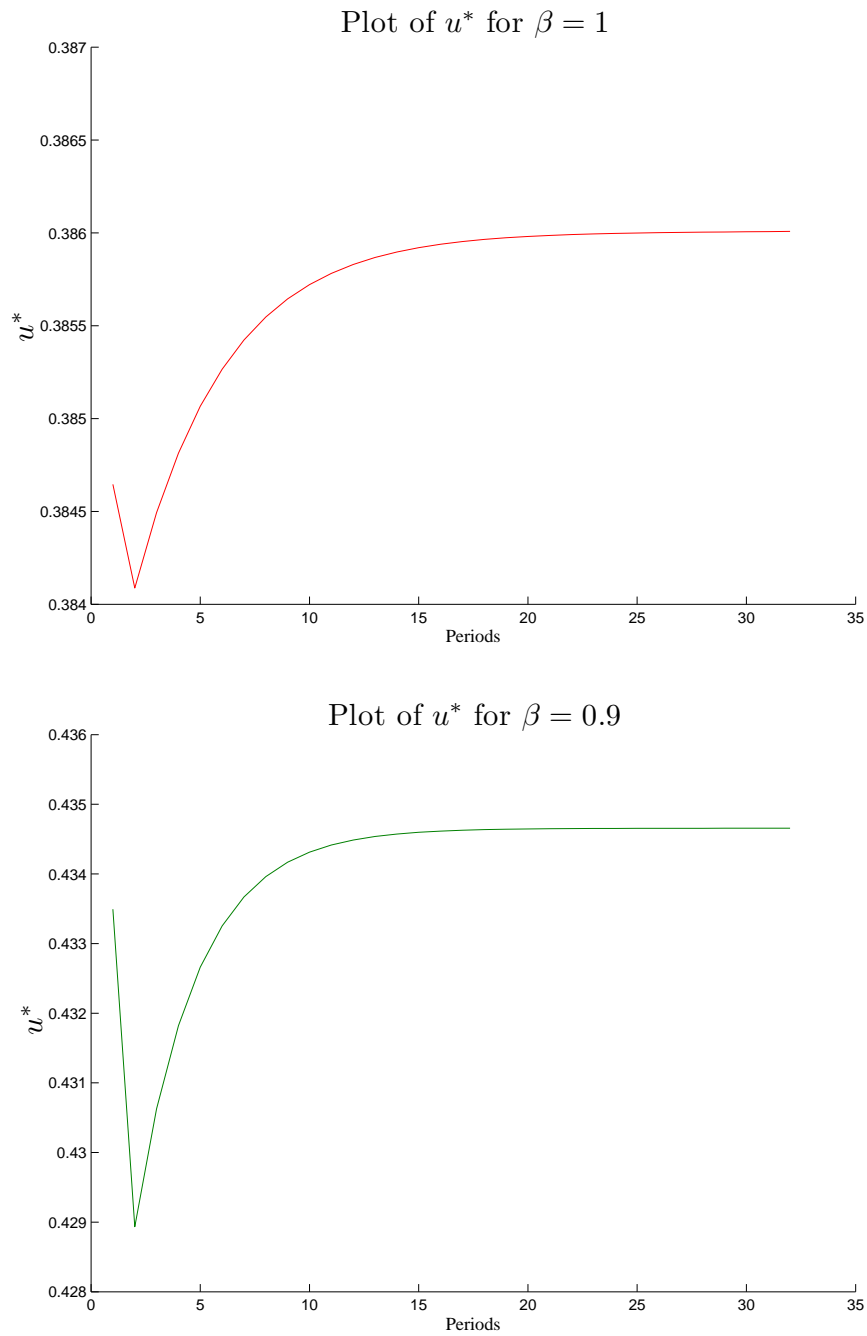


Figure 11: Effects of B productivity shocks on u for $\beta = 1$ and $\beta = 0.9$.

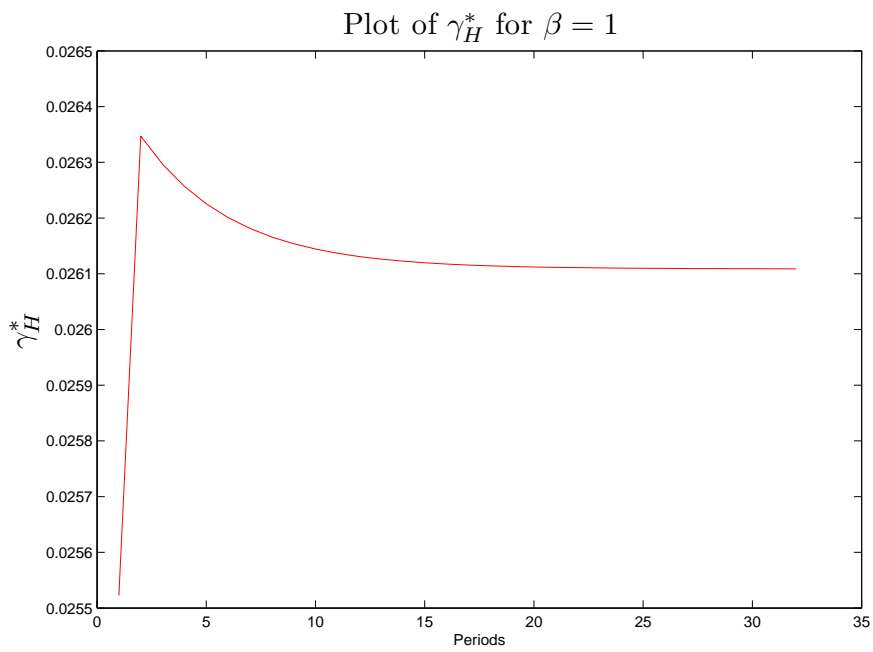
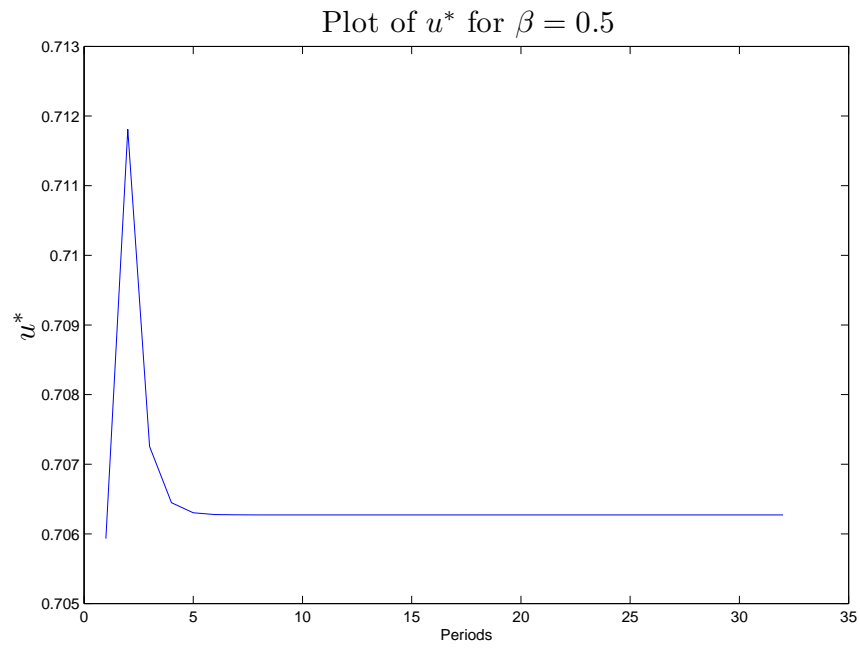


Figure 12: Effects of B productivity shocks on u for $\beta = 0.5$ and on γ_H for $\beta = 1$.

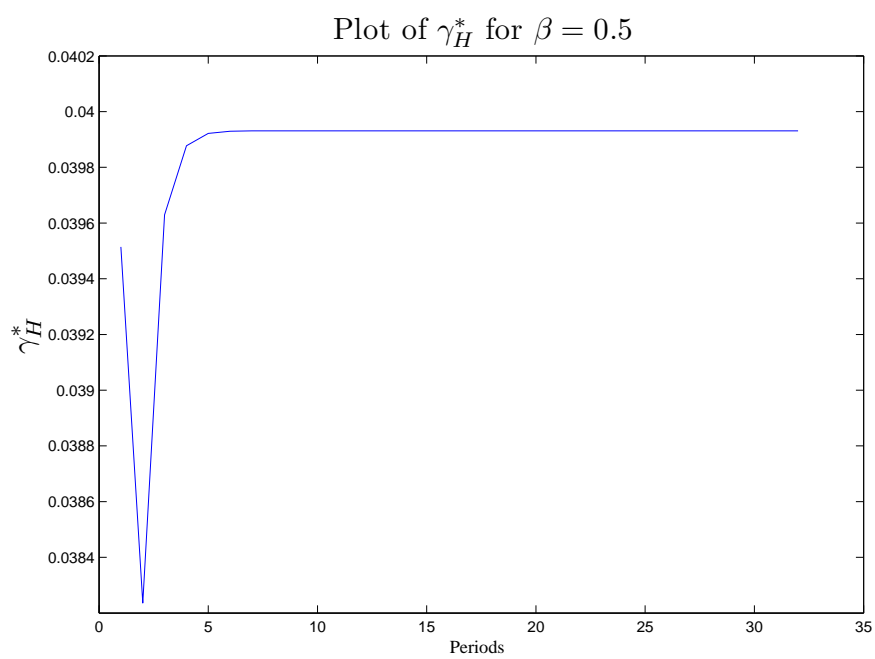
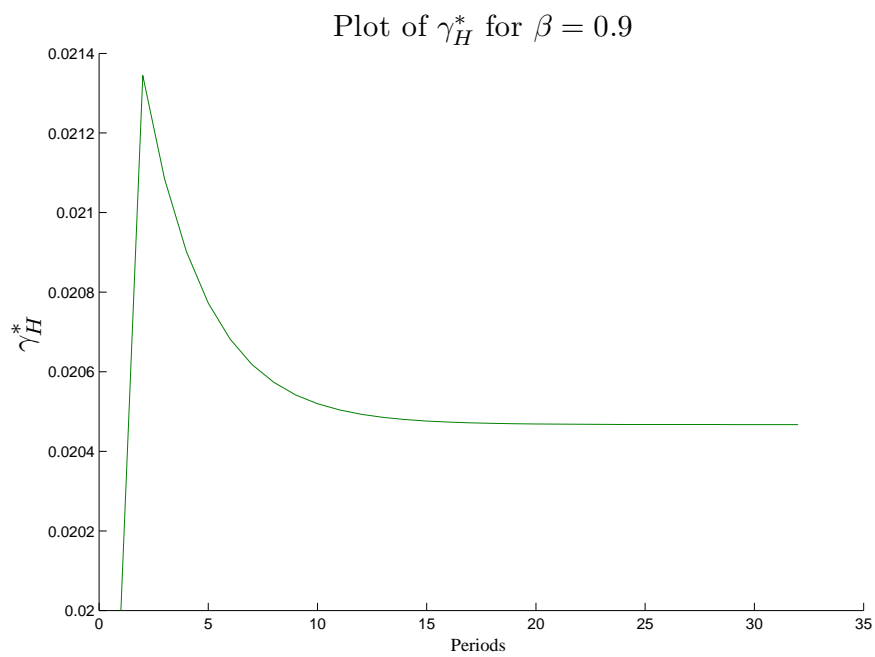


Figure 13: Effects of B productivity shocks on γ_H for $\beta = 0.9$ and $\beta = 0.5$.