Monetary policy and commodity terms of trade shocks in emerging market economies

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Abstract

Commodity terms of trade shocks have continued to drive macroeconomic fluctuations in most emerging market economies. The volatility and persistence of these shocks have posed great challenges for monetary policy. This study employs a New Keynesian Dynamic Stochastic General Equilibrium (DSGE) model to evaluate the optimal monetary policy responses to commodity terms of trade shocks in commodity dependent emerging market economies. The model is calibrated to the South African economy. The study shows that CPI inflation targeting performs relatively better than exchange rate targeting and non-traded inflation targeting both in terms of reducing macroeconomic volatility and enhancing welfare. However, macroeconomic stabilisation comes at a cost of increased exchange rate volatility. The results suggest that the appropriate response to commodity induced exogeneous shocks is to target CPI inflation.

Keywords: Commodity terms of trade, monetary policy; DSGE
JEL classification: E52, G28

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1 Introduction

For many years, commodity terms of trade shocks have been shaping the macroeconomic outlook of most emerging market economies (EMEs) (Mendoza, 1995; Kose, 2002). These shocks have proved to be very volatile and persistent, especially in commodity exporting emerging countries, resulting in significant macroeconomic volatility. The recurrence of large commodity terms of trade shocks has called into question the ability of alternative monetary policy frameworks to stabilise emerging market economies and enhance welfare.

Several studies have analysed the macroeconomic implications of alternative monetary policy regimes under domestic and external shocks in small open economies (see e.g. Laxton and Pesenti, 2003; Gali and Monacelli, 2005; Medina and Soto, 2005; Devereux et al., 2006). They largely focus on shocks such as productivity shocks, interest rate shocks and demand shocks. While these shocks are important for EMEs, an important channel of fluctuations in EMEs has to do with the fact that their exports are undiversified and dominated by a few primary commodities. As such, most studies did not explain the case for country specific commodity terms of trade shocks. Because of the high volatility and persistence of commodity terms of trade shocks, their consideration may help to account for the high volatility of exchange rates and other macroeconomic variables observed in most EMEs (Chen and Rogoff, 2003). Also, the consideration of commodity terms of trade shocks in the presence of sticky prices may significantly change the conventional wisdom on optimal policy in commodity dependent EMEs. Chen and Rogoff (2003) emphasise that rigidities may prevent standard terms of trade from adequately incorporating contemporaneous shocks that would induce immediate exchange rate and macroeconomic responses. More importantly, commodity terms of trade shocks may induce unfavourable trade-offs between inflation and output gap variability. The risks they pose in EMEs call for policy intervention.

On policy responses to external shocks, the literature has given much prominence to the role of flexible exchange rates (see e.g. Friedman, 1953; Chia and Alba, 2006). But the challenge is that flexible exchanges without activist monetary policy may not adjust in the right direction to achieve the desired outcomes, resulting in negative welfare effects (Devereux, 2004). Also, the fluctuations of the flexible exchange rates may generate inefficient relative price movements in the short-run (Ortiz and

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1See e.g Gali and Monacelli, 2005; Devereaux et. al., 2006; Broda, 2004, Mendoza, 1995
Sturzenegger, 2007). Therefore, policy responses should go beyond exchange rate choice and consider monetary policy as well. Indeed, recent studies have shown that monetary policy regimes such as inflation targeting can play a role in dampening cyclical macroeconomic fluctuations and improve welfare in small open economies (see e.g. Svensson, 2000; Cuche-Curti et al., 2008). The choice of monetary policy regimes also matters because wages and prices of non-tradable goods are sticky in the short-run and the speed at which relative prices adjust depends crucially on the monetary policy regime. Although the role of monetary policy can be ascertained, the question that remains is, which monetary policy regime is effective in dealing with commodity terms of trade shocks in EMEs.

This study develops a multi-sector New Keynesian dynamic stochastic general equilibrium (DSGE) model to evaluate the appropriate monetary policy responses to commodity terms of trade shocks in commodity dependent EMEs. Precisely, the paper analyses the relative merits of CPI inflation targeting (CIT) rule compared with non-traded inflation targeting (NTIT) rule and exchange rate targeting (ET) rule in the face of commodity terms of trade shocks in commodity dependent EMEs. Within the same framework, the paper also examines the monetary policy implications of productivity shocks in the commodity sector. The model is framed in the new open economy macroeconomics (NOEM), which integrates nominal rigidities and monopolistic competition.\(^2\) It builds closely on the work of Devereux et al. (2006) and Gali and Monacelli (2005) and extends their models by incorporating the commodity sector to account for country specific commodity terms of trade shocks in a broader monetary model using the framework of Cashin et al. (2004). The model is calibrated to South Africa, a typical commodity dependent emerging market economy. This economy is ideal for this analysis because it has a significant portion of trade (about 30% of GDP) which is concentrated in primary commodities such as gold, platinum and diamonds. In South Africa, commodity exports account for about half of export earnings (Stokke, 2008), while the Rand is considered as a commodity currency because of its sensitivity to the movement of commodity prices (Cashin et al., 2004).

The multi-sector DSGE set up allows the distinction between non-traded inflation and CPI inflation which provides a richer framework for analysing dynamic macroeconomic responses to commodity shocks. The choice of a DSGE model is motivated by several factors. For instance, DSGE models are micro-founded in the sense that they are explicitly derived from the constrained optimising behavior of households.

\(^2\)See Lane (2001) for a detailed survey of the new open economy macroeconomics.
and firms in the economy (Tovar, 2008). Further, their structural nature permits clear identification, interpretation and discussion of alternative policy interventions and their transmission mechanisms (Smets and Wouters, 2003). Finally, as argued by Woodford (2003), DSGE models help to overcome the Lucas critique because the estimated deep structural parameters are less likely to change when policies change.

This paper contributes to the literature in two main ways. First, it incorporates the commodity sector in the multi-sector DSGE model of a small open commodity dependent emerging market economy. This allows explicit examination of the country-specific commodity terms of trade shocks and their implications for monetary policy. This characterisation especially in a dynamic equilibrium setting is not common to many small open economy models. The study demonstrates that the incorporation of the commodity sector in the model changes the conventional wisdom on optimal monetary policy in EMEs. The view that non-traded inflation targeting (a version of domestic inflation targeting) is the optimal monetary policy in small open economies does not hold. The chapter argues that CPI inflation targeting is the appropriate monetary policy for commodity dependent emerging market economies because it stabilises both output and inflation. Its stabilising power is attributed to its forward looking nature, credibility and a flexible exchange rate which help to insulate the economy from external shocks.

Secondly, using the central bank loss functions, the paper evaluates the welfare implications of alternative monetary policy regimes to determine the optimal monetary policy in countries which are prone to commodity shocks. This is important because different monetary policy rules contain important trade-offs which affect welfare. Most work done on commodity dependent emerging economies such as South Africa do not evaluate the welfare implications of alternative monetary policy rules (see e.g Steinbach et al., 2009; Alpanda et al., 2010). Thus, this paper offers guidance for the formulation of monetary policy in South Africa; a commodity dependent emerging market economy.

The analysis shows that commodity terms of trade shocks have less impact on some macroeconomic variables under CIT than under NTIT and ET rules. However, the stabilisation of the economy by CIT comes at the expense of high real and nominal exchange rate fluctuations. The analysis also shows that the economy achieves less volatility in aggregate and sectoral output, consumption and CPI inflation under CIT rule. On the other hand, NTIT rule delivers less volatility in non-tradable inflation. The comparison of welfare shows that CIT rule results in less welfare loss than other
rules when the central bank prefers to stabilise inflation, interest rates and exchange rates. However, when the central bank cares more about output stabilisation, it achieves less welfare loss by targeting non-traded inflation, but the difference with CPI inflation targeting is very small.

The rest of the paper is structured as follows. Section 2 provides some review of related literature. Section 3 develops the model while section 4 describes the calibration of parameters and solution of the model. Section 5 analyses the results. Section 6 provides the sensitivity analysis and section 7 concludes and provides policy recommendations.

2 Review of the literature

There are many studies that have examined the design of monetary policy in small open economies using DSGE models. Much previous work has focused on the analysis of monetary policy rules in the face of several shocks. Given the diverse conclusions and the specificity of focus of individual studies, it is important to briefly review some relevant studies.

Aoki (2001) develops a two sector model in which different price rigidities exist, to analyse the effects of relative price changes on inflation outcomes. He finds that the optimal monetary policy is the one that targets the sticky price rather than broad inflation measure. Laxton and Pesenti (2003) use a small open economy model and show that inflation-forecast-based rules perform better than conventional Taylor rules in stabilising output and inflation because of their forward looking nature. Also, in a DSGE model, Parrado (2004) shows that domestic inflation targeting yields better outcomes than CPI targeting, while flexible exchange rates performs better than fixed exchange rates in terms of stabilising the economy. However, these studies did not consider some peculiar features of small open economies such as exposure to commodity terms of trade shocks.

Gali and Monacelli (2005) also study the macroeconomic implications of CPI inflation targeting, domestic inflation targeting and exchange rate targeting in a small open economy under productivity shocks. They show that domestic inflation targeting yields better stabilisation outcomes than CPI and exchange rate targeting especially with respect to inflation and output gap. But this good performance of domestic inflation targeting comes at the expense of higher nominal and real exchange rate volatility. In terms of welfare, they show that domestic inflation targeting out-
performs CPI inflation targeting and exchange rate targeting. While their study is an important contribution to the understanding of optimal monetary policy rules in EMEs, the model lacks a multi-sector dimension which distinguishes between traded and non-traded goods. Also, it does not evaluate alternative monetary policy responses to commodity terms of trade shocks, which are an important source of economic fluctuations in EMEs. The present study argues that the sectoral structure can significantly affect monetary policy outcomes and welfare because of different propagation mechanisms which they imply for the model.

Santacreu (2005) develops a multi-sector Bayesian DSGE model for New Zealand, and shows that if the central bank cares more about inflation stabilisation, it should react to CPI inflation, but if it is more concerned about output stabilisation, it should react to non-traded inflation. Nevertheless, the model does not take explicit account of the commodity sector which is important for many EMEs. The present study takes this into account.

Similarly, Devereux et al. (2006) examine alternative monetary policy responses to terms of trade and foreign interest rate shocks in a small open economy model calibrated to the Asian economies. They find that CPI inflation targeting is better than non-traded inflation targeting and exchange rate targeting in stabilising output, but it stabilise the economy at the expense of high exchange rate fluctuations. They also find that financial constraints propagate external shocks but do not alter the ranking of monetary policy rules. In terms of welfare, they show that non-traded inflation targeting performs better that CPI inflation targeting and exchange rate targeting. However, they neither incorporate the commodity sector nor consider commodity terms of trade shocks which may have significant implications for macroeconomic dynamics in EMEs.

In the context of South Africa, DSGE models are very limited and have been developed to analyse different issues. For example Steinbach et al. (2009), Liu et al. (2009), Alpanda et al. (2010) and Alpanda et al. (2011) develop DSGE models to evaluate business cycle characteristics, forecasting and the role of the exchange rate in shaping the South African business cycle. A common shortfall of these models is that they do not incorporate the commodity sector, despite the importance of this sector in shaping South Africa’s macroeconomic dynamics. Moreover, none of these studies are in a multi-sectoral setting which arguably helps to address some questions and debates which cannot be tackled by one sector based models.
3 The model

3.1 Basic outline of the model

The model describes a small open commodity exporting emerging market economy which has three domestic economic actors: consumers, firms and monetary authorities. There is one external sector which is the rest of the world. There are two production sectors in the domestic economy: traded and non-traded sectors. The traded sector (commodity export sector) produces primary commodities which are completely exported. This sector is meant to characterise the production and export of commodities, especially minerals in South Africa. The non-traded sector produces final goods which are consumed domestically. The commodity export sector is perfectly competitive while the non-traded goods sector faces monopolistic competition. The asymmetric consideration of the two sectors allows deeper analysis of their linkages in the presence of commodity terms of trade shocks. The external traded sector supplies imports to the domestic economy.

The model also features nominal rigidities in the form of Calvo (1983), staggered price setting in the non-traded sector. The nominal friction allows the model to reproduce realistic inflation dynamics and makes the framework suitable for the evaluation of monetary policy (Clarida et al., 1999). Capital stock is assumed to be constant. Consumers own firms and supply labour to the firms in return for profits and wages. Labour is assumed to be perfectly mobile across sectors which implies that nominal wages are similar in traded and non-traded sectors. The economy is assumed to be small relative to the rest of the world. Monetary policy is modelled as a Taylor rule that incorporates interest rate smoothing. The basic structure of the economy is described in Figure 1.

3.2 Consumers

There is a representative household who maximises its intertemporal utility subject to an intertemporal budget constraint. The household utility function is:

\[
U = E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\sigma}}{1-\sigma} - \eta \frac{L_t^{1+\psi}}{1+\psi} \right)
\]

McCallum and Nelson (1999) argue that the capital stock may be irrelevant for the dynamics of the small open economy because its variation contributes very little to the business cycle fluctuations, at least in the US. Also, the inclusion of capital may make the analysis complex.
where $\beta$ is the subjective discount factor, $\eta$ is the marginal disutility of work, $\sigma$ is the inverse of the elasticity of substitution between consumption and labour and $\psi$ is the inverse of wage elasticity of labour supply. $\sigma$, and $\psi$ are strictly positive while $0 < \beta < 1$. $L_t$ is the total labour supply in both traded and non-traded sectors. $C_t$ is a composite consumption index composed of non-tradable goods and tradable goods (imports) which takes the constant elasticity of substitution (CES) function of the form:

$$C_t = \left[ \frac{1}{\alpha} C_{N_t}^{\frac{\rho-1}{\rho}} + (1 - \alpha) \frac{1}{\rho} C_{T_t}^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}$$  \quad (2)$$

where $\alpha$ and $1 - \alpha$ are shares of non-traded and imported goods in total consumption respectively. Implicitly, it is a measure of the degree of openness. $C_{N_t}$ is the consumption of non-traded goods, $C_{T_t}$ is the consumption of imports, $\rho > 0$, is the elasticity of substitution between traded and non-traded goods. The implied consumer price index is:

$$P_t = (\alpha P_{N_t}^{1-\rho} + (1 - \alpha) P_{T_t}^{1-\rho})^{\frac{1}{1-\rho}}$$  \quad (3)$$

Figure 1: Flow chart of the economy
where \( P_{Nt} \) and \( P_{Tt} \) are prices of non-traded and import goods respectively. When \( \rho = 1 \), the CPI takes the Cobb-Douglas form:

\[
P_t = P_{Nt}^{\alpha} P_{Tt}^{1-\alpha}
\]

Thus, the consumer price index is a weighted sum of the prices of traded and non-traded goods.

Consumption of non-traded goods and imports is differentiated and the elasticity of substitution across varieties is \( \lambda \). The consumption indices are represented by the following Dixit and Stiglitz (1977) aggregator:

\[
C_{Nt} = \left[ \int_0^1 C_{Nt}(i) \frac{\lambda}{\lambda - 1} di \right]^{\lambda - 1} \\
C_{Tt} = \left[ \int_0^1 C_{Tt}(i) \frac{\lambda}{\lambda - 1} di \right]^{\lambda - 1}
\]

where \( \lambda > 1 \). The consumer’s intertemporal budget constraint is:

\[
P_tC_t \leq W_t L_t + \Pi_t + D_t - E_t(Q_{t+1}D_{t+1})
\]

where \( W_t \) is wages, \( \Pi_t \) is profits and \( D_t \) is the portfolio of assets. \( D_{t+1} \) is the nominal payoff of period \( t+1 \) of the portfolio held at the end of time \( t \) and \( Q_{t+1} \) is the stochastic discount factor. Minimising expenditure on the total composite demand, the optimal allocations give the following demand functions for non-traded goods and imports.

\[
C_{Nt} = \alpha \left( \frac{P_{Nt}}{P_t} \right)^{-\rho} C_t \\
C_{Tt} = (1 - \alpha) \left( \frac{P_{Tt}}{P_t} \right)^{-\rho} C_t
\]

The household optimisation problem gives the following first order conditions:

\[
C_t^\sigma \eta L_t^\psi = \frac{W_t}{P_t}
\]

\[
\beta R_t E_t \left( \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) \right) = 1
\]

where \( R_t \) is thus the gross interest rate of the bond. Equation (10) is the intratemporal optimal condition which shows the equalisation of marginal utility of consumption to the marginal value of labour. Equation (11) is the consumption Euler equation which
represents the trade-off to the economy of moving consumption across time. Log-linearising equation (10) and (11), gives:

\[ \sigma c_t + \psi l_t = w_t - p_t \]  
(12)

\[ c_t = E_t c_{t+1} - \frac{1}{\sigma} (r_t - E_t \pi_{t+1}) \]  
(13)

where small letters denote log deviation from steady state.\(^4\)

### 3.3 Firms

#### 3.3.1 Domestic production

There are two sectors in the domestic economy; the traded sector (commodity export sector) and the non-traded sector. The domestic traded sector produces primary commodities which are all exported. Firms in the traded sector operate under perfect competition and use the following linear technology:

\[ Y_{X_t} = A_{X_t} L_{X_t} \]  
(14)

where \( A_{X_t} \) is a productivity variable and \( L_{X_t} \) is labour in the commodity export sector. \( A_{X_t} \) follows an AR(1) process such that in logarithms, it is:

\[ \ln A_{X_t} = \rho X \ln A_{X_{t-1}} + \epsilon_{X_t} \]  
(15)

where \( \epsilon_{X_t} \sim N(0,1) \). Cost minimisation in the export commodity sector gives the following marginal cost:

\[ MC^R_{X_t} = \frac{W_t}{A_{X_t} P_{X_t}} \]  
(16)

where \( MC^R_{X} \) is the real marginal cost in the commodity export sector. Log-linearising equation (16) gives:

\[ mc^R_{X_t} = w_t - a_{X_t} - p_{X_t} \]  
(17)

Equation (16) shows the choice of employment which achieves cost minimisation in the commodity export sector.

Firms in the non-traded sector face monopolistic competition and produce differentiated non-traded goods using the linear production technology:

\(^4\)Going forward, small letters will be used to denote log deviation from steady state and the log-linearisation is around the steady state.
\[ Y_{Nt} = A_{Nt}L_{Nt} \]

where \( A_{Nt} \) is a productivity variable and \( L_{Nt} \) is labour in the non-traded sector. \( A_{Nt} \) follows an AR(1) process such that in logarithms, it is:

\[ \ln A_{Nt} = \rho \ln A_{Nt-1} + \epsilon_{Nt} \]

where \( \epsilon_{Nt} \sim N(0, 1) \). Cost minimisation in the non-traded sector leads to the following optimality condition:

\[ MC_{Nt}^R = \frac{W_t}{A_{Nt}P_{Nt}} \]

where \( MC_{Nt}^R \) is the real marginal cost in the non-traded sector. Log-linearising the marginal cost in the traded sector gives:

\[ mc_{Nt}^R = w_t - a_{Nt} - p_{Nt} \]

Because of perfect competition in the traded sector, the price of tradable goods can be expressed as a function of wages and productivity only. Also, the price of non-traded goods can be expressed as a function of wages, productivity and marginal costs.

\[ P_{Xt} = \frac{W_t}{A_{Xt}} \]

\[ P_{Nt} = \frac{W_t}{A_{Nt}MC_{Nt}^R} \]

Since wages are equalised between sectors, the relative price of non-traded goods to traded goods can be expressed as follows:

\[ P_{Nt} = \frac{A_{Xt}}{A_{Nt}MC_{Nt}^R} P_{Xt} \]

This shows that the relative price of non-traded goods to primary commodities is determined by technological factors and marginal cost.

### 3.3.2 Foreign production

Following Cashin et al. (2004), the foreign economy is assumed to be composed of three production sectors that is non-traded sector, intermediate goods sector, and final goods sector. All foreign production sectors operate under perfect competition. Labour is mobile across sectors such that wages are equalised across sectors. Firms
in the foreign non-traded goods sector use linear production technologies as follows:

\[ Y_{Nt}^* = A_{Nt}^* L_{Nt}^* \]  (25)

where \( A_{Nt}^* \) is a productivity variable and \( L_{Nt}^* \) is labour in the foreign non-traded sector.\(^5\) \( A_{Nt}^* \) follows an AR(1) process such that in logarithms, it is:

\[ \ln A_{Nt}^* = \rho_{Nt} \ln A_{Nt-1}^* + \epsilon_{Nt}^* \]  (26)

where \( \epsilon_{Nt}^* \sim N(0,1) \). Firms in the foreign intermediate goods sector also use the following linear production technology:

\[ Y_{It}^* = A_{It}^* L_{It}^* \]  (27)

where \( A_{It}^* \) is a productivity variable and \( L_{It}^* \) is labour in the foreign intermediate sector. \( A_{It}^* \) follows an AR(1) process such that in logarithms, it is:

\[ \ln A_{It}^* = \rho_{It} \ln A_{It-1}^* + \epsilon_{It}^* \]  (28)

where \( \epsilon_{It}^* \sim N(0,1) \). Because all foreign sectors are assumed to be perfectly competitive, the price of foreign non-traded goods can be expressed as a function of relative productivity and the price of foreign intermediate goods as:

\[ P_{Nt}^* = \frac{A_{It}^*}{A_{Nt}^*} P_{It}^* \]  (29)

where \( P_{It}^* \) is price of intermediate inputs. The final good is assumed to be a tradable good and its production uses two intermediate inputs. The first input is the intermediate good which is produced in the foreign economy. The second is the commodity which is exported by the domestic economy and other primary commodity producing countries. The traded good is thus produced using the following Cobb-Douglas technology:

\[ Y_{Tt}^* = \theta (Y_{It}^*)^\nu (Y_{Xt}^*)^{1-\nu} \]

Cost minimisation leads to the following per unit cost:

\[ P_{Tt}^* = (P_{It}^*)^\nu (P_{Xt}^*)^{1-\nu} \]  (30)

\(^5\)Going forward, the foreign variables will be indicated by asterisks.
Foreign consumption is assumed to be similar to that in the domestic economy, such that the implied consumer price index is:

$$P_t^* = P_{Nt}^* P_{Tt}^{1-\alpha^*}$$  \hspace{1cm} (31)

### 3.4 Real exchange rate, commodity terms of trade and inflation

The real exchange rate is defined as the domestic price of a basket of consumption relative to foreign price of a basket of consumption:

$$Q_t = \frac{\varepsilon_t P_t}{P_t^*}.$$  \hspace{1cm} (32)

The law of one price is assumed to hold for both exports and imports such that:

$$P_{Xt} = P_{Xt}^*$$  \hspace{1cm} (33)

$$P_{Tt} = P_{Tt}^*$$  \hspace{1cm} (34)

The real exchange rate is decomposed so that it has a commodity terms of trade component. From equation (32), and after some algebra, the following version of the real exchange rate can be derived:

$$Q_t = \left( \frac{A_{Xt}^* A_{Nt}^* P_{Xt}^*}{A_{It}^* A_{Nt}^* P_{It}^*} \right)^\alpha \left( \frac{1}{MC_{Nt}^R} \right)^\alpha$$  \hspace{1cm} (35)

where $P_{Xt}^* / P_{Tt}^*$ is the commodity terms of trade index, defined as the price of primary commodity with respect to the intermediate foreign good. $A_{Xt} / A_{Nt}^*$ shows the productivity differential between the export and import sectors and $A_{Xt}^* / A_{Nt}^*$ is the productivity differential between domestic and foreign non-traded sectors. The two productivity ratios capture the Balassa-Samuelson effect where an increase in productivity in the tradable sector (commodity sector) tends to increase wages in both the tradable and non-tradable sectors and results in an increase in the price of non-traded goods relative to tradables and an appreciation of the real exchange rate (Cashin et al., 2004). This is a version of real exchange rate which is almost similar to the real exchange rate decomposition of Cashin et al. (2004). The difference is that this expression contains the marginal cost term which follows from the assumption of monopolistic

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$^6$See Appendix for derivation
Commodity terms of trade is defined as:

\[ F_t = \frac{P_{Xt}^*}{P_{It}^*} \quad (36) \]

which can be log-linearised to give:

\[ f_t = p_{Xt}^* - p_{It}^*. \quad (37) \]

Lagging and taking the difference of equation (37) results in:

\[ f_t = f_{t-1} + \pi_{Xt}^* - \pi_{It}^*. \quad (38) \]

Substituting equation (37) into (35), the real exchange rate can be written as:

\[ Q_t = \left( \frac{A_{Xt}^* A_{Nt}^*}{A_{It}^* A_{Nt}} F_t \right)^\alpha \left( \frac{1}{MC_{Nt}} \right)^\alpha \quad (39) \]

in which the log-linearised version is:

\[ q_t = \alpha (a_{Xt} - a_{It}^* + a_{Nt}^* - a_{Nt} + f_t - mc_{Nt}). \quad (40) \]

Foreign traded inflation can be derived from equation (30) by taking the lag and the difference:

\[ \pi_{I_t}^* = \nu \pi_{I_t}^* + (1 - \nu) \pi_{Xt}^*. \quad (41) \]

From equation (4), CPI inflation in the domestic economy can be derived as:

\[ \pi_t = \alpha \pi_{Nt} + (1 - \alpha) \pi_{It}. \quad (42) \]

From equation (34), imported inflation equation can be derived as:

\[ \pi_{It} = \pi_{Tt}^* - \Delta e_t. \quad (43) \]

Substituting \( \pi_{Tt}^* \) from equation (41) into equation (43) gives the modified imported inflation equation:

\[ \pi_{It} = \nu \pi_{It}^* + (1 - \nu) \pi_{Xt}^* - \Delta e_t. \quad (44) \]
3.5 International risk sharing and uncovered interest parity

Complete international markets are assumed where domestic agents have access to foreign securities. This means that the expected nominal return from riskless bonds in the home currency terms is the same as the expected domestic currency return from foreign bonds.\(^7\) This permits the derivation of the international risk sharing condition where consumption risk is perfectly shared between domestic and foreign agents as follows:

\[
\beta E_t \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) = \beta E_t \left( \frac{C^*_{t+1}}{C^*_t} \right)^{-\sigma} \left( \frac{\varepsilon_t P^*_t}{\varepsilon_{t+1} P^*_{t+1}} \right).
\] (45)

As in Gali and Monacelli (2005), solving and iterating equation (45) gives \(^8\)

\[
C_t = \Omega Q_t^{\frac{1}{\sigma}} C^*_t
\] (46)

where \(\Omega\) is a constant that represents initial asset positions. Log-linearising results in:

\[
c_t = c^*_t + \frac{1}{\sigma} q_t
\] (47)

Under complete international markets, the uncovered interest parity condition can be derived as:

\[
E_t Q_{t+1} (R_t - R^*_t \frac{\varepsilon_{t+1}}{\varepsilon_t}) = 0
\] (48)

Log-linearising around the steady state gives:

\[
r_t - r^*_t = E_t \Delta e_{t+1}.
\] (49)

Equation (49) is the uncovered interest parity condition which relates expected variations of nominal exchange rates to interest rate differentials.

3.6 Domestic price setting

Non-traded good firms follow Calvo (1983) price setting where they adjust their prices with only some probability. That is in period \(t\), \(1 - \theta_N\) firms set prices optimally and \(\theta_N\) keep prices unchanged, where \(\theta_N \in (0, 1)\). \(\theta_N\) measures the degree of nominal rigidity. The larger this parameter, the less frequently prices are adjusted. The

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\(^7\)The assumption of complete international markets helps to eliminate foreign asset movements from the open economy dynamics. This makes steady state unique, where consumption is independent of past shocks (Parrado 2004)

\(^8\)See the derivation in the Appendix
general price index at each period evolves according to:

\[ P_{Nt} = \left\{ (1 - \theta_N)P_{Nt}^{new} + \theta_N P_{Nt-1}^{new} \right\}^{1-\varepsilon} \]  

(50)

where \( P_{Nt}^{new} \) is the price level of an optimising firm. The firms that reoptimise their prices at time \( t \) maximize their current values of dividend streams subject to sequences of demand constraints:

\[
\begin{align*}
\max & \sum_{t=0}^{\infty} (\theta_N)^k E_t \{ Q_{t+k}Y_{t+k} \left( P_{Nt}^{new} - MC_{Nt+k}^{m} \right) \} \\
\text{s.t} & \quad Y_{t+k} \leq \left( \frac{P_{Nt}^{new}}{P_{Nt+k}} \right)^{-\varepsilon} \left( C_{Nt+k} + C_{Nt+k}^{m} \right)
\end{align*}
\]

(51)

where \( MC_{Nt+k}^{m} \) is the nominal marginal cost and \( \theta_N^k E_t Q_{t+k} \) is the effective stochastic discount factor. The first order condition for the problem is:

\[
\sum_{t=0}^{\infty} (\theta_N)^k E_t \left\{ Q_{t+k}Y_{t+k} \left( P_{Nt}^{new} - \frac{\varepsilon}{\varepsilon - 1} MC_{Nt+k}^{m} \right) \right\} = 0
\]

(52)

Further computations lead to the following New Keynesian Phillips curve equation for the non-traded sector:

\[
\pi_{Nt} = \beta E_t \pi_{Nt+1} + \lambda_{Nt} m_R^{e} \]  

(53)

where \( \lambda_{Nt} = \frac{(1-\theta_N)(1-\theta_N)}{\theta_N} \). The equation shows that inflation is a function of next period’s expected inflation and the real marginal cost.

### 3.7 Monetary policy rules

The model is closed by describing how monetary policy is conducted. Recent work has modelled monetary policy as an interest rate feedback rule of a Taylor (1993) type where the central bank adjusts policy interest rates in response to economic conditions (see e.g. Clarida et al., 2000; Benigno, 2004). Taylor rules have become so popular in describing monetary policy for several reasons. First, they are consistent with the principles of optimal monetary policy and capture well the behavior of monetary policy in many countries (see e.g. Clarida et al., 2000; Woodford, 2003; Lubik and Schorfheide, 2007). These rules are robust and consistent with the main principles of

---

\(^9\)See Appendix for derivation.
optimal monetary policy (Clarida et al., 1999; Woodford, 2003). Second, Taylor rules have been found to provide determinacy, implying that they ensure a unique stationary rational expectations equilibrium of the model (Clarida et al., 1999). Third, they are flexible in nesting a wide range of alternative monetary policy strategies. As in Ortiz and Sturzenegger (2007) and Lubik and Schorfheide (2007), the following generalised Taylor rule is considered:

$$R_t = R_{t-1}^\rho \left\{ \left( \frac{Y_t}{\bar{Y}} \right)^{\omega_1} \left( \frac{\pi_t}{\bar{\pi}} \right)^{\omega_2} \left( \frac{\pi_{Nt}}{\bar{\pi}_N} \right)^{\omega_3} \left( \frac{\varepsilon_t / \varepsilon_{t-1}}{\bar{\varepsilon}} \right)^{\omega_4} \right\}^{1-\rho_r}. \quad (54)$$

The log-linearised version of the monetary policy rule is:

$$r_t = \rho_r r_{t-1} + (1 - \rho_r) (\omega_1 y_t + \omega_2 \pi_t + \omega_3 \pi_{Nt} + \omega_4 \varepsilon_t) + \varepsilon_{r,t} \quad (55)$$

where $\omega_1$, $\omega_2$, $\omega_3$ and $\omega_4$ allow monetary authorities to control output, CPI inflation, non traded inflation and nominal exchange rate respectively. To allow for the comparison of different monetary policy regimes, the parameters are changed so that one monetary policy regime can be specified at a time.\(^{10}\) $\rho_r$ is the smoothing parameter. The smoothing parameter is included in this specification to capture inertia in interest rates, as observed by several empirical studies (see e.g. Clarida et al., 2000; Sack and Wieland, 2000). Clarida et al. (2000) argue that policy reaction functions without interest rate smoothing are too restrictive to describe the actual interest rate changes in most central banks. Also, Sack and Wieland (2000) note that the presence of uncertainty about the relevant model parameters, the structure of the economy and concerns about the soundness of the financial system may motivate central banks to have interest rate smoothing.

### 3.8 Equilibrium

In equilibrium, the markets for non-traded goods, traded goods and labour must clear. The goods market clearing condition in the domestic economy requires that total domestic production which is made up of non-traded output and exported output is equal to total demand. That is:

$$Y_t = Y_{Nt} + Y_{Xt} \quad (56)$$

\(^{10}\) For CPI targeting rule, it is considered that $\omega_3 = \omega_4 = 0$, for non-traded inflation targeting rule $\omega_2 = \omega_4 = 0$ and for exchange rate targeting rule, $\omega_2 = \omega_3 = 0$. 

16
where \( Y_{Nt} = C_{Nt} \) and \( Y_{Xt} = C_{Xt} \). Log-linearising (56):

\[
y_t = y_{Nt} \left( \frac{Y_{Nt}}{Y} \right) + y_{Xt} \left( \frac{Y_{Xt}}{Y} \right)
\]

Using \( Y_{Nt} = C_{Nt} \) and combining with equation (8) results in:

\[
y_{Nt} = -\rho (1 - \alpha) [p_{Nt} - e_t - p_{Mt}^*] + c_t.
\]

Equation (58) is the equilibrium condition for the non-traded sector. The equilibrium condition for the commodity export sector is given by:

\[
Y_{Xt} = Y_{Xt}^* = C_{Xt}^*.
\]

Using the equation for the consumption of exports, it can be shown that:

\[
Y_X = \left( \frac{1 - \nu}{\nu} \right) Y_{Tt}^* \left( \frac{P_{Xt}^*}{P_{It}^*} \right)\]

and the log-linearised version of (60) is:

\[
y_{Xt} = y_{Tt}^* + \nu (p_{Xt}^* - p_{It}^*)
\]

Thus, the equilibrium condition depicting the IS equation for the domestic economy is given by\(^{11}\):

\[
y_t = \left( \frac{Y_{Nt}}{Y} \right) (-\rho (1 - \alpha) [p_{Nt} - e_t - p_{Mt}^*] + c_t) + \left( \frac{Y_{Xt}}{Y} \right) (y_{Tt}^* + \nu (p_{Xt}^* - p_{It}^*))
\]

where \( \frac{Y_{Nt}}{Y} \) and \( \frac{Y_{Xt}}{Y} \) are steady state ratios of labour in the non-traded goods and exports to total income.

The supply side of the model is given by the marginal cost equations in both the commodity export sector and the non-traded sectors. For the commodity export sector, marginal cost is given by:

\[
mc_{Xt}^R = w_t - p_X - a_X
\]

Combining equation (63) with (12), (4) and (34) gives the new expression for the

\(^{11}\)IS equation depicts the locus of all combinations of income and interest rate for which the goods market is in equilibrium.
marginal cost in the commodity export sector:

\[ mc^R_X = \sigma c_t + \psi l_t + (1 - \alpha)(p^*_T - e_t) + \alpha p_{Nt} - p^*_X + e_t - a_X \]  

(64)

In the non-traded sector, the marginal cost is given by:

\[ mc^R_{Nt} = w_t - p_{Nt} - a_{Nt} \]  

(65)

Combining equation (65) with (12), (4) and (34) gives the final marginal cost function:

\[ mc^R_X = \sigma c_t + \psi l_t + \alpha p_{Nt} + (1 - \alpha)(p^*_T - e_t) - p_{Nt} - a_{Nt} \]  

(66)

The labour market must clear. The labour market clearing condition is:

\[ L_t = L_{Xt} + L_{Nt} \]  

(67)

Log-linearising equation (67) and substituting (18) and (14) into (67) gives:

\[ l_t = \frac{L_X}{L}(y_{Xt} - a_{Xt}) + \frac{L_N}{L}(y_{Nt} - a_{Nt}) \]  

(68)

where \( \frac{L_X}{L} \) and \( \frac{L_N}{L} \) are steady state labour in the traded and non-traded sectors respectively. Finally, some equations characterising the foreign economy are:

\[ y^*_t = \alpha^* y^*_{Nt} + (1 - \alpha^*) y^*_{Tt} \]  

(69)

\[ y^*_N = -\rho^* y^*_N + \rho^*(\alpha^* y^*_{Nt} + (1 - \alpha^*) y^*_{Tt}) + y^* \]  

(70)

\[ y^*_T = \nu y^*_t + (1 - \nu) y^*_{Xt} \]  

(71)

\[ r^*_t = \rho r^*_{t-1} + \epsilon^*_{r,t} \]  

(72)

The general equilibrium is characterised by a sequence of \( y_t, y_{nt}, y_{Xt}, c_t, r_t, \pi_t, \pi_{Nt}, \pi_{Tt}, a_{Nt}, a_{Xt}, p_{Nt}, e_t, q_t, mc_{Nt}, mc_{Xt}, \beta, l_t, y^*_t, y^*_N, y^*_T, y^*_It, r^*_t, \pi^*_t, \pi^*_{It}, \pi^*_{Xt}, \pi^*_{Tt}, p^*_N, p^*_T, p^*_It, a^*_It, a^*_Xt \) that gives the solution to equations describing the domestic and foreign economies.

4 Calibration and solution

In order to solve the model, the values of parameters need to be determined. The model is calibrated to match the key features of the South African quarterly data for the period 1990-2008. Other parameters are obtained from previous studies on the
South African economy and business cycle literature. The benchmark parameters are described in Table 4.1.

The elasticity of substitution between traded goods and non-traded goods is set at 1 following Devereux et al. (2006) and Santacreu (2005). In line with real business cycles literature, the discount factor is set at 0.99 implying that the steady state real interest rate is about 4% per annum. Following Alpanda et al. (2010), the inverse of the elasticity of labour supply is assumed to be 6 which reflects that in steady state, the gross wage mark-up is about 1.2 over the marginal rate of substitution. As suggested by Steinbach et al. (2009), the elasticity of substitution between consumption and labour for South Africa is set at 1.

The import share in consumption is set at 0.2, consistent with estimates by Steinbach et al. (2009) for South Africa for the period 2002-2007. They find that the import penetration ratio to total GDP is about 30% and the import penetration in consumption is about 7.5% during this period. This implies that the share of non-traded goods in consumption is about 0.8. However, these parameters are changed through experiments in sensitivity analysis. The productivity parameter in the non-traded goods sector is set at 0.74 based on estimates by Alpanda et al. (2010). Following Ricci et al. (2008), the productivity parameter in the commodity export sector is set at 0.85 while the foreign productivity parameter in the non-traded goods and the intermediate goods sectors are both set at 0.8.

As in Alpanda et al. (2010) and Steinbach et al. (2009), the degree of nominal price rigidity is set at 0.75, which suggest that prices are adjusted on average after 4 quarters in South Africa. This allows the model to generate realistic impulse responses. The weight on CPI inflation and non-traded inflation in the Taylor rule, $\omega_2$ and $\omega_3$ are both set initially at 1.5 while the weight on output, $\omega_1$ is set at 0.5 following Steinbach et al. (2009). As in Ortiz and Sturzenegger (2007), the weight on the exchange rate $\omega_4$ and smoothing parameter $\rho_r$ are calibrated at 0.05 and 0.73 respectively. The value of the smoothing parameter is expected to capture recent efforts by the South African Reserve Bank (SARB) to reduce interest rate volatility and make monetary policy more predictable. Most steady state parameters are obtained from equilibrium relations in the model.

The linearised model is solved using DYNARE programme which uses the Blanchard and Kahn (1980) algorithm (see Juillard, 1996). The analysis focuses mainly

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12 The DYNARE programme can derive the reduced-form representation of the model and then provides standard moments based on assumptions about the stochastic processes. Blanchard and
Table 4.1: Calibration of parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>Elasticity of substitution between traded and non-traded goods</td>
<td>1</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Share of non-traded goods in consumption</td>
<td>0.8</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Subjective discount factor</td>
<td>0.99</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Inverse of the elasticity of substitution between consumption and labour</td>
<td>1</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Inverse of the elasticity of labour supply</td>
<td>6</td>
</tr>
<tr>
<td>$\theta_N$</td>
<td>Stickiness parameter in the non traded sector</td>
<td>0.75</td>
</tr>
<tr>
<td>$\rho_{\pi_f}$</td>
<td>Persistence parameter for foreign inflation</td>
<td>0.5</td>
</tr>
<tr>
<td>$\rho_r$</td>
<td>Persistence parameter for foreign interest rate</td>
<td>0.8</td>
</tr>
<tr>
<td>$\rho_r^*$</td>
<td>Smoothing parameter for Taylor rule</td>
<td>0.73</td>
</tr>
<tr>
<td>$\rho_N$</td>
<td>Persistence parameter of labour productivity in the non traded sector</td>
<td>0.74</td>
</tr>
<tr>
<td>$\rho_X$</td>
<td>Persistence parameter of labour productivity in the commodity export sector</td>
<td>0.85</td>
</tr>
<tr>
<td>$\rho_{\pi_f}$</td>
<td>Persistence parameter of labour productivity in the foreign intermediate sector</td>
<td>0.8</td>
</tr>
<tr>
<td>$\rho_{N_f}$</td>
<td>Persistence parameter of labour productivity in the foreign non-traded sector</td>
<td>0.8</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Share of exported commodity (by domestic economy) in foreign production</td>
<td>0.26</td>
</tr>
<tr>
<td>$\rho_{pX}$</td>
<td>Persistence parameter of world price of export commodity</td>
<td>0.8</td>
</tr>
<tr>
<td>$\rho_{pI}$</td>
<td>Persistence parameter of foreign price of intermediate good</td>
<td>0.8</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Elasticity of substitution across varieties</td>
<td>10</td>
</tr>
<tr>
<td>$\omega_1$</td>
<td>Weight on output in the Taylor rule</td>
<td>0.5</td>
</tr>
<tr>
<td>$\omega_2$</td>
<td>Weight on CPI inflation in the Taylor rule</td>
<td>1.5</td>
</tr>
<tr>
<td>$\omega_3$</td>
<td>Weight on domestic inflation in the Taylor rule</td>
<td>1.5</td>
</tr>
<tr>
<td>$\omega_4$</td>
<td>Weight on exchange rate in the Taylor rule</td>
<td>0.05</td>
</tr>
</tbody>
</table>
on commodity terms of trade shocks but later considers productivity shocks in the commodity export sector for comparison purposes.

5 Results Analysis

This section analyses the impulse responses of selected macroeconomic variables to commodity terms of trade shocks and productivity shocks in the export sector under the following alternative monetary policy rules: CPI inflation targeting (CIT) rule, non-traded inflation targeting (NTIT) rule and exchange rate targeting (ET) rule. These monetary policy rules are assessed based on the degree to which they minimise volatility of selected macroeconomic variables as reflected by their impulse response functions. It also proceeds to provide analysis of volatility and welfare under different monetary policy rules.

5.1 Impulse response analysis

5.1.1 Commodity terms of trade shocks

Figure 2 presents impulse responses to a one standard deviation positive shock to commodity terms of trade. The figure shows that the commodity terms of trade shock results in higher output in the export sector. Output in the non-traded sector falls initially, but later rises. The fall in the output of the non-traded sector could be attributed to the movement of resources to the booming commodity export sector. Aggregate output also falls initially but rises steadily over time. While the response of traded output raises aggregate output, the response of non-traded output acts to decrease initial output. Thus the pattern of aggregate output reflects the bigger impact coming from the non-traded sector. Over time, the commodity terms of trade shock generates a wealth effect which increases demand for non-traded goods. The increase in demand for non-traded goods results in overheating of the economy which puts pressure for non-traded goods prices to increase. The increase in prices of non-traded goods results in increase in CPI inflation and non-traded inflation. Central banks respond to the rise in inflation by raising interest rates. Due to monetary contraction, aggregate consumption decreases on impact but grow back to steady state over time, possibly reflecting intertemporal consumption smoothing. As expected, the increase in commodity terms of trade induces appreciation of the nominal exchange

Kahn (1980) show that if the number of eigen values outside the unit circle is equal to the number of non-predetermined variables, then there exists a unique rational expectations solution to the system.

21
Figure 2: Impulse responses to commodity terms of trade shock
rate that translates into an appreciation in the real exchange rate. The appreciation of the exchange rate reflects the fact that the substitution effect of domestic demand towards foreign goods which could potentially offset the appreciation is very small.

The response patterns of most variables depend on the monetary policy regime in place. The shock leads to a fall in aggregate output, and the largest fall occurs under ET rule and smallest under CIT rule. For sectoral output, non-traded output falls on impact while export output increases in response to booming commodity prices. The response of non-traded output is greater under NTIT rule and smallest under CIT. This is contrary to conventional wisdom that a rule that places large weight on a price index that is sensitive to exchange rate movements (CIT rule) is likely to induce large fluctuations in sectoral output.

The commodity terms of trade shock results in an increase in traded output under NTIT rule with positive and mild responses under CIT. The effect on non-traded output is also persistent under NTIT rule possibly due to nominal rigidities in the non-traded sector. In all rules, labour supply increases. The dynamic response pattern of the labour supply function traces the pattern of the exported output, possibly reflecting the existence of the resource movement effect. Since labour is mobile between sectors, the boom in the commodity export sector raises the value of the marginal product of labour, resulting in the increase in labour in that sector. However, the response of labour supply is strong under NTIT rule, moderate under ET rule and weak under CIT rule. This can be explained by the resource movement effect where the booming commodity export sector attracts labour from the non-traded sector.

The commodity terms of trade shock also causes a large appreciation of the nominal exchange rate under NTIT and ET. As pointed out by Obstfeld and Rogoff (1996), this is intuitive because under flexible exchange rate, the presence of sticky prices makes the adjustment to terms of trade shocks to take place through changes in the nominal exchange rates. As expected, the commodity terms of trade shock triggers an initial appreciation of real exchange rates in all regimes. Possibly this reflects the "commodity currency" effect which was highlighted by Cashin et al. (2004). In their study of commodity currencies and real exchange rates, they find strong evidence of a long-run relationship between real exchange rate and commodity terms of trade for commodity exporting countries. The larger impact of the commodity terms of trade shock on the real exchange rate is much larger and persistent under CIT than under NTIT and ET. The larger impact on real exchange rate under CIT could be attributed to the presence of the floating exchange rates under CIT regime
which implies active use of the exchange rate channel to stabilise variables such as CPI inflation and output (Svensson, 2000). As emphasised by Bouakez (2005), the greater real exchange rate persistence may be rationalised by the presence of the marginal cost (which is the inverse of the mark up) in the real exchange rate equation, which amplifies the volatility and persistence of the real exchange rate. Under ET, the real exchange rate is less volatile due to fixed nominal exchange rate. According to Mussa (1986), this reflects excess smoothness of the exchange rate. The stability of the exchange rate under ET can also be a result of the central bank absorbing part of the proceeds of export revenue by building reserves. Devereux et al. (2006) and Parrado (2004) find similar results where CIT exhibit greater contemporaneous real exchange rate and nominal exchange rate responses than in exchange rate pegs.

Non-traded inflation increases contemporaneously and falls thereafter especially under NTIT and ET rules following a commodity terms of trade shock. The response of non-traded inflation is greater under NTIT but moderate under CIT. The contemporaneous response of CPI inflation is greater under NTIT and ET but more muted under CIT. Since in the baseline calibration 80% of CPI inflation comes from non-traded inflation, the shape and profile of the former follows that of the latter. The low CPI inflation response under CIT rule can partly be explained by the presence of flexible exchange rates which dampen the direct effects of commodity terms of trade shocks on inflation. Another possible explanation is that the CIT rule is credible to the extent that inflation expectations are well anchored.

In all cases, central banks respond to the rise in inflation by increasing interest rates. Under the NTIT rule, the central bank responds to the shock more aggressively while under CIT, the response is moderate. The ET rule displays very small and less persistent interest rate responses. But under the NTIT rule, stabilising non-traded inflation requires a much sharper rise in interest rates than in other regimes. This result underlines the conventional wisdom that the interest rate channel dominates under the NTIT rule (Svensson, 2000). Under CIT, the moderate response can be explained by the presence of interest rate smoothing aimed at making monetary policy more predictable and credible. The weak interest rate responses under ET can possibly imply that ET central banks do not use interest rates, but instead use reserves as instruments of monetary policy (Benes et al., 2008).

Overall, the dynamic adjustment of most variables shows that CIT rule is superior to the NTIT and ET rules because it generally stabilises most variables. It is followed by ET and lastly NTIT. However, the stabilisation of these variables under CIT
comes at the cost of increased real exchange rate fluctuations. The responses of most variables are generally consistent with the structural characteristics of the South African economy, for example its volatile exchange rates.

5.1.2 Export productivity shock

Although the main aim of this paper is to evaluate the responses to commodity terms of trade shocks, the introduction of the commodity sector in the model makes the analysis of responses to productivity shocks in the commodity export sector interesting. Since the total commodity is exported, the productivity shock in the commodity export sector is closely related to the commodity terms of trade shock. The evaluation of the monetary policy implications of a productivity shock in the commodity export sector also helps to examine the presence of the Balassa-Samuelson effect. Figure 3 presents impulse responses of variables to an export sector productivity shock.

The export sector productivity shock increases aggregate output and exported output while non-traded output decreases. In the presence of the two opposing effects on aggregate output, the expansionary effects of the former seem to be greater than the contractionary effects of the latter. The expanding export sector also generates nominal and real exchange rate appreciations. However, the real exchange rate may also appreciate due to an increase in interest rates. Because of labour mobility across sectors, wages are equalised, thus the increase in productivity in the traded sector raises wages also in the non-traded sector. This results in higher costs which push up prices of non-traded goods and increase inflation especially under ET and CIT rules. Central banks respond by raising interest rates. The dynamic responses of these variables suggest the presence of the Balassa-Samuelson effect where an increase in productivity in the traded sector appreciates the real exchange rate and increases prices of non-tradable goods through wage equalisations (Obstfeld and Rogoff, 1996).

The adjustment patterns of aggregate output are similar under ET and CIT rules. Non-traded output decreases in all regimes, but NTIT rule exhibits the largest fall. This suggests that non-traded output is very sensitive to productivity shocks in the export sector. The productivity shock also raises output in the export sector, with the strongest response being experienced under NTIT rule. Consumption decreases on impact following a productivity shock with the greatest decline being observed under NTIT rule and smallest responses under CIT and ET. The greater fall in consumption may be explained by the substitution effects between traded and non-traded goods which are stronger than the income effects. The largest fall in consumption under
Figure 3: Impulse responses to an export sector productivity shock
NTIT is not a surprise given that there is a large proportion of non-traded goods in the consumption basket. The commodity export sector productivity shock raises the marginal productivity of labour and wages in the traded sector, resulting in increase in labour supply to this sector. The response of labour is stronger and more persistent under NTIT than under CIT and ET, implying that a NTIT regime does not stabilise aggregate labour supply.

The shock also induces more volatility of nominal and real exchange rates under CIT than in other regimes. They are lowest under NTIT and more muted under ET since the adjustment to the shock takes place through relative prices. This is in line with other studies, for example that of Obstfeld and Rogoff (2000) which shows that under productivity shocks, the optimal monetary policy involves some exchange rate fluctuations.

Under ET and CIT, the productivity shock immediately raises the non-traded goods prices which generate higher non-traded inflation and CPI inflation. The responses of both non-traded inflation and CPI inflation under NTIT are more muted, both on impact and along the transition. This implies that NTIT rule succeeds in stabilising both non-traded inflation and overall inflation. The volatility of non-traded and CPI inflation is quite high under ET rule, suggesting that the fixed exchange rate is a weak shock absorber. As in the case of commodity terms of trade shocks, the stabilisation of inflation by CIT following productivity shocks results in substantial movements in real and nominal exchange rates.

5.2 Volatility analysis

In order to compliment impulse response analysis, Table 5.1 presents standard deviations (volatilities) of selected macroeconomic variables under alternative monetary policy regimes. The results show that total output and non-traded output exhibit lowest volatility under CIT and highest under ET regimes. The intuition for higher volatility under ET is that in the presence of sticky prices at least in some sectors, the adjustment entails higher volatility in the real sector. Exported output and consumption are less volatile under CIT. This is consistent with Santacreu (2005) and Devereux et al. (2006) who also conclude that responding to CPI inflation generates less volatility in output and consumption. By design, ET delivers substantially lower volatility of both nominal exchange rate and real exchange rate, while CIT involve higher volatility of these variables. This result is a confirmation of earlier impulse responses where the exchange rates display larger responses under CIT. The possible
reason is that the process of stabilising output and consumption by CIT rule involves substantial movements in the exchange rates. The volatility of labour supply is lowest under CIT but highest under NTIT. However, responding to non-traded inflation generates lower volatility under NTIT rule since this regime focuses on non-traded inflation stabilisation. As expected, CIT delivers more stability in CPI inflation, but substantially higher instability in interest rates than occurs under NTIT and ET. Intuitively, CIT rule actively uses the interest rate to stabilise the economy.

<table>
<thead>
<tr>
<th></th>
<th>CIT</th>
<th>NTIT</th>
<th>ET</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>0.06</td>
<td>0.07</td>
<td>0.07</td>
</tr>
<tr>
<td>Non-traded output</td>
<td>0.02</td>
<td>0.02</td>
<td>0.03</td>
</tr>
<tr>
<td>Exported output</td>
<td>0.25</td>
<td>0.39</td>
<td>0.28</td>
</tr>
<tr>
<td>Labour</td>
<td>0.10</td>
<td>0.19</td>
<td>0.11</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.09</td>
<td>0.41</td>
<td>0.33</td>
</tr>
<tr>
<td>Nominal exchange rate</td>
<td>0.33</td>
<td>0.24</td>
<td>0.01</td>
</tr>
<tr>
<td>Real exchange rate</td>
<td>0.22</td>
<td>0.11</td>
<td>0.10</td>
</tr>
<tr>
<td>Non-traded inflation</td>
<td>0.19</td>
<td>0.09</td>
<td>0.25</td>
</tr>
<tr>
<td>CPI inflation</td>
<td>0.52</td>
<td>1.10</td>
<td>2.36</td>
</tr>
<tr>
<td>Interest rates</td>
<td>1.44</td>
<td>0.04</td>
<td>0.03</td>
</tr>
</tbody>
</table>

CIT is CPI inflation targeting, NTIT is non-traded inflation targeting and ET is exchange rate targeting. The numerical values are standard deviations.

5.3 Welfare implications of alternative monetary policy regimes

Much of the literature on monetary policy and welfare assumes that the central bank minimises a loss function which translates the behavior of policy targets into some aggregate welfare measure (see e.g Clarida et al., 1999; Laxton and Pesenti, 2003). Following this literature, this section assumes that the objective of the central bank is to minimise welfare losses from deviations of output, inflation, interest rates and exchange rates from their steady state values.

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13 The alternative is to estimate welfare using utility function of the representative consumer.
14 Woodford (2003) shows that under certain conditions, it is possible to motivate a quadratic loss function as a second order Taylor series approximation of the expected utility of the economy’s representative household which is equal to the expected discounted sum of period losses for certain coefficients. He also shows that a linear approximation to the policy function is sufficient to accurately approximate welfare up to a second order if the second order approximation to the welfare function contains quadratic terms, where the welfare loss is proportional to the expected discounted
Loss functions are appealing as ways of characterising welfare for several reasons. Firstly, they allow the incorporation of some aspects of interest-rate smoothing, consistent with the principle of optimal monetary policy since interest rate smoothing captures policy inertia often observed in the data (Clarida et al., 1999). Secondly, as noted by Adolfson et al. (2011), loss functions can be formulated in terms of observable macroeconomic variables which help to provide simple and reasonable welfare analysis especially in economies which are characterised by massive economic fluctuations. Thirdly, Clarida et al. (1999) argue that loss functions capture the major cost of inflation that is uncertainty generated from inflation variability. The welfare loss function considered is:

\[ L_t = \lambda_\pi \pi_t^2 + \lambda_y y_t^2 + \lambda_r r_t^2 + \lambda_e e_t^2 \]  

(73)

Taking unconditional expectations, the loss function can be expressed as:

\[ E(L_t) = \lambda_\pi Var(\pi_t) + \lambda_y Var(y_t) + \lambda_r Var(r_t) + \lambda_e Var(e_t) \]  

(74)

where \( \pi_t \) is a measure of inflation depending on the choice of inflation under consideration (CPI or non-traded inflation), \( y_t \) is output, \( e_t \) is nominal exchange rates, \( r_t \) is the interest rate. \( Var(\pi_t), Var(y_t), Var(r_t) \) and \( Var(e_t) \) are the unconditional variances of inflation, output, nominal exchange rates and interest rates. The loss function includes the stabilisation of the exchange rate since the model is that of an open economy. Kirsanova et al. (2006) show that there is a case for including exchange rate in the welfare function in small open economies. Interest rate variability term is included to capture the central bank’s desire to avoid both extreme interest rate volatility and hitting the zero lower bound (Woodford, 2003). The coefficients in the policy rule are optimally chosen to minimise the loss function.\(^{15}\)

To avoid the arbitrary choice of weights in the loss function, the study follows Laxton and Pesenti (2003) and Alpanda et al. (2010) and sets alternative values of relative loss function weights from 0.5 to 2 in increments of 0.5. The relative weights on each variable in the loss function reflects central bank’s preference in terms of stabilising those variables.

Table 5.2 reports the results of the welfare losses of different monetary policy

\(^{15}\)The optimal parameters are computed using Dynare’s optimal simple rule algorithm. In this computation, Dynare searches numerically the parameters of the policy function that minimise the weighted variance of variables and provides the value of the objective function (welfare loss).
Table 5.2: Welfare losses of alternative monetary policy regimes

<table>
<thead>
<tr>
<th>Weights on loss function</th>
<th>Welfare losses</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_\pi$ $\lambda_y$ $\lambda_\tau$ $\lambda_e$</td>
<td>CIT NTIT ET</td>
</tr>
<tr>
<td>1 0.5 0.5 0.5</td>
<td>0.04 0.05 0.28</td>
</tr>
<tr>
<td>1 1 0.5 0.5</td>
<td>0.01 0.09 0.18</td>
</tr>
<tr>
<td>1 1.5 0.5 0.5</td>
<td>0.03 0.02 0.11</td>
</tr>
<tr>
<td>1 2 0.5 0.5</td>
<td>0.04 0.03 0.33</td>
</tr>
<tr>
<td>1 0.5 1 0.5</td>
<td>0.27 1.54 1.57</td>
</tr>
<tr>
<td>1 0.5 1.5 0.5</td>
<td>0.52 1.84 1.72</td>
</tr>
<tr>
<td>1 0.5 2 0.5</td>
<td>0.62 2.12 2.93</td>
</tr>
<tr>
<td>1 0.5 0.5 1</td>
<td>0.05 0.06 0.05</td>
</tr>
<tr>
<td>1 0.5 0.5 1.5</td>
<td>0.33 0.63 0.52</td>
</tr>
<tr>
<td>1 0.5 0.5 2</td>
<td>0.34 0.98 0.58</td>
</tr>
</tbody>
</table>

CIT is consumer price index inflation targeting,
NTIT is non tradable inflation targeting and
ET is exchange rate targeting.

The results show that when the central bank places a weight of 1 on inflation variability and equal relative weight of 0.5 on output, interest rates and exchange rates in the loss function, CIT achieves the lowest welfare losses followed by NTIT and lastly ET. This implies that CIT rule is welfare enhancing possibly due to its broad based features. In fact, stabilising CPI inflation can help to remove market distortions caused by price stickiness, resulting in enhanced welfare, while NTIT rule generates significant welfare losses through higher relative price dispersions. Using utility based welfare measures, Devereux et al. (2006) rank NTIT first, followed by CPI targeting and lastly ET in a model with financial constraints and complete exchange rate pass-throughs. Similarly, Aoki (2001) finds that the optimal monetary policy is the one which targets sticky price inflation that is, non-traded inflation targeting. The intuition is that monetary policy try to prevent disequilibrium in the market for goods with stickiest price. Gali and Monacelli (2005) also find that domestic inflation targeting is the optimal monetary policy followed by CPI inflation targeting and lastly, exchange rate targeting.

When the relative weight on output is increased, while the weights on other variables are kept constant, NTIT rule delivers less welfare while large welfare loss are observed under ET with CIT being the intermediate case. However, the difference between welfare losses under NTIT and CIT is relatively small. The intuition for the less welfare loss under NTIT is that targeting non-traded inflation avoids excessive
volatility in interest rates which subsequently reduces volatility in output. The result is consistent with Santacreu (2005) who concludes that the central bank which is concerned about output stabilisation should target non-traded inflation. When the central bank responds to overall CPI inflation, it attempts to offset the direct effects of exchange rate movements which are largely temporary. Thus, by responding to non-traded inflation, the central bank ignores the direct exchange rate impact on CPI, but focuses on the direct effect through the output gap. The finding that NTIT performs better when more weight is put on output suggests the presence of a trade-off between inflation and output stabilisation. Alpanda et al. (2010) also observe this trade-off in South Africa.

When more weight is put on interest rates, CIT rule is superior, followed by NTIT rule and lastly ET rule. The intuition is that higher interest rate stabilisation enhances credibility and reputation which helps to reduce welfare losses. The ET rule continues to perform poorly in terms of welfare because in the face of real shocks, it does not allow smooth adjustment of macroeconomic variables. As the central bank becomes more concerned about exchange rate stabilisation, CIT dominates other regimes because the reduction of exchange rate volatility quickly translates into output and inflation stabilisation which enhances welfare.

6 Sensitivity analysis

This section investigates the sensitivity of the reported results to changes in openness, price stickiness and the elasticity of substitution between traded goods and non-traded goods.

6.1 Sensitivity tests on impulse response functions

Firstly, the degree of openness is changed from 0.2 to 0.6. Figure A.1 presents the impulse response functions. The response patterns of non-traded output, exported output, non-traded inflation, real exchange rate and non-traded goods prices are generally similar to the benchmark calibration case. However, in terms of magnitude, the impulse responses of most variables are marginally greater under high openness than in the benchmark case, suggesting that increased openness magnifies the impact of commodity terms of trade shocks. This is expected since openness results in increases in demand for foreign goods and reduction in demand for domestic goods. Openness also increases the volatility and persistence of nominal and real exchange rates under
CIT because of increased exposure to shocks. This confirms the results of Chia and Alba (2006) who observe that increasing openness doubles the volatility of real exchange rates under flexible exchange rates than under fixed exchange rates. Higher openness also propagates the impact of commodity terms of trade shocks on non-traded inflation and CPI inflation especially under CIT because of increased traded goods in the CPI basket.

Secondly, the degree of stickiness is changed to a special case where there is no stickiness of non-traded goods prices. This case is similar to the framework of Cashin et al. (2004). Figure A.2 shows the impulse responses when there is no stickiness. In this case, aggregate output, non-traded output, exported output consumption, labour and exchange rate responses are generally similar to the baseline calibration where stickiness is high. In terms of magnitude, the responses are larger when prices are flexible. This suggests that price stickiness dampens the effects of commodity terms of trade shocks on the economy. This indicates that price stickiness acts as a constraint to firms in the non-traded sector and when the constraint is removed, the variables become more volatile. The responses of nominal and real exchange rates are similar to the baseline case, consistent with the findings of Cashin et al. (2004). However, under NTIT, CPI inflation rises sharply and is more persistent, while under CIT rule, it is relatively stable, suggesting that CIT central banks pay more attention to the mechanism of price adjustment. If prices are flexible, central banks are more aggressive in ensuring that inflation expectations remain anchored.

Thirdly, the sensitivity test entails decreasing the elasticity of substitution between traded and non-traded goods from 1 to 0.4. Figure A.3 shows the impulse responses. Aggregate output increases and is more persistent following commodity terms of trade shocks because economic agents cannot easily substitute more expensive domestic goods for less expensive foreign goods. As a result, a higher income effect will generate greater demand for non-traded goods. The dynamic adjustment of exported output, nominal exchange rates, real exchange rates, non-traded good prices and interest rates are similar to the baseline calibration. Apart from this similarity, the responses in the new parameterisation framework (where the elasticity of substitution is low) are weaker than when the elasticity of substitution is high. This shows that low elasticity of substitution induces smaller responses of variables to international relative price movements and weaker feedback effects because of fewer substitution possibilities (Gali and Monacelli, 2005).
6.2 Sensitivity tests on volatility and welfare evaluations

The performance of alternative monetary policy rules with respect to volatility and welfare is also analysed when parameter values of openness, stickiness and elasticity of substitution of traded and non-traded goods are changed. Table A.1 in the Appendix presents the results. As in the baseline case, when the degree of openness is high, responding to non-traded inflation generates higher volatility in total output and non-traded output while responding to CIT results in greater nominal, real exchange rate and interest rate volatility. This shows that greater openness increases the sensitivity of output to shocks. On the other hand, labour supply, non-traded inflation, CPI inflation and consumption are more volatile under ET. Overall, CIT still performs better than NTIT and ET in terms of stabilising most macroeconomic variables.

When the value of the stickiness parameter is reduced, ET generates volatile aggregate output, exported output, consumption, non-traded prices and CPI inflation as in the benchmark calibration. The exchange rates and interest rates still display more volatility under CIT, suggesting that even when all sectoral prices are flexible, exchange rates continue to fluctuate more. Also, when the elasticity of substitution between traded and non-traded goods is reduced, volatility patterns of most variables remain broadly similar to the baseline case.

Table A.2 shows the effects of changing the degrees of openness, stickiness and elasticity of substitution on welfare. When the weight on all other variables are the same relative to inflation, greater openness results in less welfare losses under CIT. This suggests that CIT rule simultaneously makes use of the multiple channels of monetary policy transmission to reduce economic volatility. However, if more weight is put on output stabilisation, openness improves the welfare effects of NTIT rule followed by CIT rule and lastly ET rule. Also, when the central bank prefers to stabilise interest rates and exchange rates under high openness, CIT delivers less welfare loss as in the baseline case. This is consistent with the intuition that, as the economy becomes more open, the reduction in interest rate and exchange rate fluctuations tends to lower excess volatility in economic activity in the face of external shocks which in turn reduces welfare losses.

Under no stickiness, the CIT rule still performs better than other regimes especially when more weight is put on inflation, interest rate and exchange rate stabilisation. However, greater preference for output and exchange rate stabilisation generates less welfare losses under NTIT. Although the ranking of the regimes is broadly similar to the baseline case, the magnitude of welfare loss is greater under flexible prices than
under price stickiness, suggesting that to some extent, welfare losses depend on the
degree of nominal rigidity.

On the other hand, when the elasticity of substitution between traded goods and
non-traded goods is low and the relative weight on inflation variability is greater than
other variables, NTIT delivers less welfare losses. The NTIT rule also dominates
when the central bank increases preference for output stabilisation. The possible
reason for the better welfare performance of NTIT rule is that under low elasticity of
substitution, consumers cannot easily substitute the more expensive non-traded goods
for cheaper imports. Also, as in the baseline case, greater preference for interest rate
and exchange rate stabilisation results in less welfare loss under CIT rule because low
elasticity of substitution generates less volatility in exchange rates which enhances
welfare.

7 Conclusion

This paper develops a multi-sector New Keynesian DSGE model to examine the
appropriate monetary policy responses to commodity shocks. Particularly, the study
examines whether CPI inflation targeting performs better than NTIT and ET regimes.
It also evaluates the optimal monetary policy implications of these shocks using the
central bank loss function. The model features the commodity sector in a multi-sector
setting and incorporates nominal price rigidities and monopolistic competition in the
non-traded sector. It is calibrated to the South Africa economy.

The analysis shows that the dynamic effects of commodity terms of trade shocks
and productivity shocks on the commodity export sector depends to a large extent on
the monetary policy rule in place. The results show that CIT stabilises most variables
such as output, consumption, CPI inflation and non-traded inflation. However, this
stabilisation is at the cost of high real and nominal exchange rate volatility. The
analysis of welfare shows that the central bank achieves less welfare loss under a CIT
regime when CPI inflation has higher relative weight in the loss function. However, if
the central bank cares more about output stabilisation, targeting non-traded inflation
reduces welfare losses. Also, the stabilisation of interest rates and exchange rates is
welfare enhancing under CIT. The results are generally robust to changes in some
parameters such as openness, price stickiness and elasticity of substitution. Increasing
openness tends to increase real exchange rate fluctuations under ET regimes. On the
other hand, reducing price stickiness has the effect of propagating higher real exchange
rate volatility under ET than in other regimes. Reducing the elasticity of substitution weakens the responses of variables because of weaker feedback effects generated by fewer substitution possibilities. The ranking of monetary policy rules however remains unchanged.

The results generally suggest that the central bank can reduce macroeconomic volatility by targeting CPI inflation. However, this stabilisation comes at the cost of higher exchange rate volatility. This implies that when the central bank responds to external shocks, it should consider the economy’s greater vulnerability to exchange rate fluctuations. The evidence from welfare analysis suggests that a small open economy exposed to volatile commodity terms of trade shocks can reduce welfare losses by targeting CPI inflation. The implication of the analysis is that the consideration of the commodity sector in the model changes the conventional wisdom on monetary policy making that domestic inflation targeting is more optimal. For economies which are prone to commodity terms of trade shocks, the analysis shows that it pays to respond to CPI inflation rather than for non-traded inflation and exchange rates.
References


A Appendix

A.1 Household optimisation

\[
\begin{align*}
\text{Max } U &= E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\sigma} - \eta L_t^{1+\psi}}{1-\sigma} \right) \\
&\text{s.t. } P_tC_t = W_tL_t + \Pi_t + D_t - E_t(Q_{t+1}D_{t+1}) \\
&\text{(A.1.1)}
\end{align*}
\]

First order conditions:

\[
\frac{dL}{dC_t} = C_t^{-\sigma} = -\lambda_tP_t \\
\text{(A.1.2)}
\]

\[
C_{t+1}^{-\sigma} = -\lambda_{t+1}P_{t+1} \\
\text{(A.1.3)}
\]

Dividing equation (A.1.3) by (A.1.2) gives:

\[
\left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} = \frac{-\lambda_{t+1}P_{t+1}}{-\lambda_tP_t} \\
\text{(A.1.4)}
\]

\[
\frac{dL}{dL_t} = -\frac{\eta L_t^\psi}{W_t} = \lambda_t \\
\text{(A.1.5)}
\]

Substituting equation (A.1.2) into (A.1.5):

\[
C_t^{-\sigma} \eta L_t^\psi = \frac{W_t}{P_t} \\
\text{(A.1.6)}
\]

\[
\frac{dL}{dD_t} = \frac{E_t(Q_{t+1})}{\beta} = \frac{\lambda_{t+1}}{\lambda_t} \\
\text{(A.1.7)}
\]

Combining (A.1.7) and (A.1.4) gives:

\[
\beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) = E_t(Q_{t+1}) \\
\text{(A.1.8)}
\]

Using \( E_t(Q_{t+1}) = R_t^{-1} \) and substituting in equation (A.1.8) gives:

\[
\beta R_tE_t \left( \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) \right) = 1 \\
\text{(A.1.9)}
\]

A.2 Real exchange rate and commodity terms of trade

Real exchange rate is given by:
\[ Q_t = \frac{\varepsilon_t P_t}{P_t^*} \]  

(A.2.1)

The law of one price is assumed to hold for both imports and exports such that:

\[ P_{Xt} = \frac{P_{Xt}^*}{\varepsilon_t} \]  

(A.2.2)

\[ P_{Tt} = \frac{P_{Tt}^*}{\varepsilon_t} \]  

(A.2.3)

Substituting \( P_t \) and \( P_t^* \) from equations (4) and (31) into (A.2.1) gives:

\[ Q_t = \frac{\varepsilon_t P_{Nt}^* P_{Tt}^*}{P_{Nt}^* P_{Tt}^*} \]  

(A.2.4)

Substituting \( P_{Nt}, P_{Tt} \) and \( P_{Nt}^* \) from equations (24), (29) and (34) into gives:

\[ Q_t = \frac{A_{Xt} A_{Nt}^* P_{Xt}^*}{P_{Nt}^* A_{Nt} P_{Tt}^*} \left( \frac{1}{MC_{Nt}^*} \right)^\alpha \]  

(A.2.5)

Substituting \( \varepsilon_t P_{Xt} = P_{Xt}^* \), into equation (A.2.5) and assuming that \( \alpha = \alpha^* \), gives:

\[ Q_t = \left( \frac{A_{Xt}}{A_{Nt}^*} \right)^\alpha \left( \frac{1}{MC_{Nt}^*} \right)^\alpha \]  

(A.2.6)

### A.3 International risk sharing and uncovered interest parity

\[ \beta E_t \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) = E_t (Q_{t+1}) \]  

(A.3.1)

Since \( E_t (Q_{t+1}) = \frac{1}{R_t} \), equation (A.3.1) can be written as:

\[ \beta E_t \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) = \frac{1}{R_t} \]  

(A.3.2)

International risk sharing implies that:

\[ \beta E_t \left( \frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \left( \frac{\varepsilon_t P_t^*}{\varepsilon_{t+1} P_{t+1}^*} \right) = \frac{1}{R_t} \]  

(A.3.3)

Equating domestic and foreign consumption and solving gives:

\[ C_t^{-\sigma} = \left[ \frac{C_{t+1} C_t^*}{C_{t+1}^* C_t} \right]^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) \left( \frac{\varepsilon_{t+1} P_{t+1}^*}{\varepsilon_t P_t^*} \right) \]  

(A.3.4)
But \( Q_t = \frac{\varepsilon_t P_t}{P^*_t} \) and \( Q_{t+1} = \frac{\varepsilon_{t+1} P_{t+1}}{P^*_{t+1}} \), substituting into A.3.4:

\[
C_t = \left( \frac{C^*_t}{C^*_t} \right) \frac{1}{Q^*_t} Q^*_t C^*_t
\]  
(A.3.5)

As shown in Gali and Monacelli (2005), iterating equation (A.3.5) results in:

\[
C_t = \Omega Q_t \frac{1}{C^*_t} C^*_t
\]  
(A.3.6)

where \( \Omega \) is a constant that represents initial asset positions.

Under complete markets, the uncovered interest parity condition can be derived as follows:

\[
\beta E_t \left( \frac{C^*_{t+1}}{C^*_t} \right) = E_t Q_{t+1} = \frac{1}{R_t} = \beta E_t \left( \frac{C^*_{t+1}}{C^*_t} \right) = \frac{1}{R_t}
\]  
(A.3.7)

Also,

\[
\beta E_t \left( \frac{C^*_{t+1}}{C^*_t} \right) = \frac{1}{R_t}
\]  
(A.3.8)

Substituting (A.3.7) and \( \Omega = \frac{1}{R_t} \) for \( E_t Q_{t+1} \) into (A.3.8) gives:

\[
E_t Q_{t+1} = \frac{1}{R_t} \frac{\varepsilon_t}{E_t \varepsilon_{t+1}}
\]  
(A.3.9)

A.3.9 can be written as:

\[
E_t Q_{t+1} R_t \frac{\varepsilon_{t+1}}{\varepsilon_t} = 1
\]  
(A.3.10)

Subtracting \( E_t Q_{t+1} R_t = 1 \) from A.3.10 leads to:

\[
E_t Q_{t+1}(R_t - R_t \frac{\varepsilon_{t+1}}{\varepsilon_t}) = 0
\]  
(A.3.11)

Log-linearising around the steady state:

\[
r_t - r^*_t = E_t \Delta e_{t+1}
\]  
(A.3.12)

Combining equation (A.3.12) with the log-linearised version of A.2.1 gives:

\[
E_t \Delta q_{t+1} = (r_t + E_t \pi_{t+1}) - (r^*_t + E_t \pi^*_{t+1})
\]  
(A.3.13)
A.4 Domestic price setting

Optimisation problem for optimising firms in the non-traded sector is:

$$\text{Max} \sum_{t=0}^{\infty} \left( \theta_N \right)^k E_t \left\{ Q_{t+k} Y_{t+k} \left( P_{Nt}^{\text{new}} - MC_{Nt+k}^n \right) \right\}$$

subject to

$$Y_{t+k} \leq \left( \frac{P_{Nt}^{\text{new}}}{P_{Nt+k}^{\text{new}}} \right)^{-\varepsilon} \left( C_{Nt+k} + C_{Nt+k}^{\text{new}} \right)$$ (A.4.1)

where $MC_{Nt+k}^n$ is the nominal marginal cost and $\theta_N E_t Q_{t+k}$ is the effective stochastic discount factor. Substituting $Y_{t+k}$ and expanding:

$$\mathcal{L} = \sum_{t=0}^{\infty} \left( \theta_N \right)^k E_t \left\{ Q_{t+k} \left( \frac{P_{Nt}^{\text{new}}}{P_{Nt+k}^{\text{new}}} \right)^{-\varepsilon} \left( C_{Nt+k} + C_{Nt+k}^{\text{new}} \right) - \left( \frac{P_{Nt}^{\text{new}}}{P_{Nt+k}^{\text{new}}} \right)^{-\varepsilon} \left( C_{Nt+k} + C_{Nt+k}^{\text{new}} \right) MC_{Nt+k}^n \right\}$$ (A.4.2)

$$\frac{d\mathcal{L}}{dP_{Nt}^{\text{new}}} = \sum_{t=0}^{\infty} \left( \theta_N \right)^k E_t \left\{ Q_{t+k}(1-\varepsilon) \left( \frac{P_{Nt}^{\text{new}}}{P_{Nt+k}^{\text{new}}} \right)^{-\varepsilon} \left( C_{Nt+k} + C_{Nt+k}^{\text{new}} \right) + \varepsilon \left( \frac{P_{Nt}^{\text{new}}}{P_{Nt+k}^{\text{new}}} \right)^{-\varepsilon-1} \left( C_{Nt+k} + C_{Nt+k}^{\text{new}} \right) MC_{t+k}^n \right\} = 0$$ (A.4.3)

Substituting the value of $Y_{t+k}$, and factorising gives the first order condition:

$$\sum_{t=0}^{\infty} \left( \theta_N \right)^k E_t \left\{ Q_{t+k} Y_{t+k} \left( P_{Nt}^{\text{new}} - \frac{\varepsilon}{\varepsilon-1} MC_{Nt+k}^n \right) \right\} = 0$$ (A.4.4)

Using the fact that $E_t (Q_{t+1}) = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right)$ which implies that $(Q_{t+k}) = \beta^k E_t \left( \frac{P_t}{P_{t+k}} \right) \left( \frac{C_{t+k}}{C_t} \right)^{-\sigma}$ and applying to (A.4.4) results in:

$$\sum_{t=0}^{\infty} \left( \beta \theta_N \right)^k E_t \left\{ P_{Nt} C_{t}^{-\sigma} P_{Nt+k}^{-1} C_{Nt+k}^{-\sigma} Y_{t+k} \left( P_{Nt}^{\text{new}} - \frac{\varepsilon}{\varepsilon-1} MC_{Nt+k}^n \right) \right\} = 0$$ (A.4.5)

Since $P_{Nt} C_{t}^{-\sigma}$ are known at time $t$, they can be taken off. Thus (A.4.5) becomes:

$$\sum_{t=0}^{\infty} \left( \beta \theta_N \right)^k E_t \left\{ P_{Nt+k}^{-1} C_{Nt+k}^{-\sigma} Y_{t+k} \left( P_{Nt}^{\text{new}} - \frac{\varepsilon}{\varepsilon-1} MC_{Nt+k}^n \right) \right\} = 0$$ (A.4.6)

Using the definition of real marginal costs $MC_{t+k}^R = MC_{Nt+k}^n / P_{Nt+k}$ and substituting into equation (A.4.6), dividing by $P_{Nt-1}$ and factorising $P_{Nt-1}$ gives:
\[
\sum_{t=0}^{\infty} (\beta \theta_N)^k E_t \left\{ C_{t+k} Y_{t+k} P_{Nt-1} \left( \frac{P_{Nt}^{new}}{P_{Nt+k}} - \frac{\varepsilon}{\varepsilon - 1} MC_{Nt+k} \left( \frac{P_{Nt+k}}{P_{Nt-1}} \right) \right) \right\} = 0 \quad (A.4.7)
\]

where \( \frac{\varepsilon}{\varepsilon - 1} \) is the markup. Using the geometric sum formula \( \sum_{t=0}^{\infty} (\beta \theta_N)^k = \frac{1}{1-\theta_N \beta} \) and log-linearising, gives:

\[
p_{Nt}^{new} = p_{Nt-1} + \sum_{t=0}^{\infty} (\beta \theta_N)^k \left\{ E_t \pi_{Nt+k} + (1 - \beta \theta_N) E_t mc_{Nt+k} \right\} \quad (A.4.8)
\]

Rewriting equation (A.4.8) after splitting the equation into two parts \( t \) and \( t+1 \) to \( \infty \):

\[
p_{Nt}^{new} = p_{Nt-1} + \pi_{Nt} + (1 - \beta \theta_N) mc_{t} + (\theta_N \beta) \sum_{t=0}^{\infty} (\theta \beta)^k \left\{ E_t \pi_{Nt+k} + (1 - \beta \theta_N) E_t mc_{Nt+k} \right\} \quad (A.4.9)
\]

Using equation (A.4.8) to substitute the last term of equation (A.4.9) and rearranging:

\[
p_{Nt}^{new} - p_{Nt-1} = \left( \theta \beta \right) \left\{ E_t \pi_{Nt+1} - p_{Nt+1}^{new} \right\} + \pi_{Nt} + (1 - \beta \theta_N) mc_{Nt} \quad (A.4.10)
\]

Substituting \( p_{Nt}^{new} - p_{Nt-1} \) of equation (A.4.9) into equation (A.4.10) and solving for \( \pi_{Nt} \):

\[
\pi_{Nt} = \beta E_t \pi_{Nt+1} + \lambda_{Nt} mc_{Nt} \quad (A.4.11)
\]

where \( \lambda_{Nt} = \frac{(1 - \beta \theta_N)(1 - \theta_N)}{\theta_N} \) and \( \theta_N \) is the stickiness parameter.

**A.5 Equations characterising the model**

\[
c_t = E_t c_{t+1} - \frac{1}{\sigma}(r_t - E_t \pi_{t+1}) \quad (A.5.1)
\]

\[
\pi_t = \alpha \pi_{Nt} + (1 - \alpha) \pi_{Tt} \quad (A.5.2)
\]

\[
p_{Nt} = a_{Xt} - a_{Nt} + p_{Xt}^* - e_t \quad (A.5.3)
\]

\[
\pi_{Nt} = \beta E_t \pi_{Nt+1} + \lambda_{Nt} mc_{Nt} \quad (A.5.4)
\]

\[
\pi_{Tt} = \nu \pi_{Tt}^* + (1 - \nu) \pi_{Xt}^* - e_t \quad (A.5.5)
\]

\[
y_{Nt} = -\rho (1 - \alpha) \left[ p_{Nt} + e_t - p_{Tt}^* \right] + c_t \quad (A.5.6)
\]

\[
y_{Xt} = y_{Tt}^* + \nu (p_{Xt}^* - p_{Tt}^*) \quad (A.5.7)
\]
\[ y_t = y_{Nt} \left( \frac{Y^N}{Y} \right) + y_{xt} \left( \frac{Y^X}{Y} \right) \]  
(A.5.8)

\[ l_t = \frac{L_X}{L} (y_{xt} - a_{Xt}) + \frac{L_N}{L} (y_{Nt} - a_{Nt}) \]  
(A.5.9)

\[ q_t = \alpha (a_{Xt} - a_{Xt}^* + a_{Nt}^* - a_{Nt} + f_t - m_{cNt}^R) \]  
(A.5.10)

\[ e_t = q_t - \alpha p_{Nt} - (1 - \alpha) (p_{Tt}^* - e_t) + \alpha^* p_{Nt}^* + (1 - \alpha^*) p_{Tt}^* \]  
(A.5.11)

\[ f_t = f_{t-1} + \pi_{Xt}^* - \pi_{Xt}^* + \epsilon_{ft} \]  
(A.5.12)

\[ r_t = \rho^r r_{t-1} + (1 - \rho^r) (\omega_1 y_t + \omega_2 \pi_t + \omega_3 y_{Nt} + \omega_4 \Delta e_t) + \epsilon_{r,t} \]  
(A.5.13)

\[ m_{cX}^R = \sigma c_t + \psi l_t + (1 - \alpha) (p_{Tt}^* - e_t) + \alpha p_{Nt} - p_{Xt}^* + e_t - a_{Xt} \]  
(A.5.14)

\[ m_{cNt}^R = \sigma c_t + \psi l_t + \alpha p_{Nt} + (1 - \alpha) (p_{Tt}^* - e_t) - p_{Nt} - a_{Nt} \]  
(A.5.15)

\[ a_{Xt} = \rho_X a_{Xt-1} + \epsilon_{Xt} \]  
(A.5.16)

\[ a_{Nt} = \rho_N a_{Nt-1} + \epsilon_{Nt} \]  
(A.5.17)

\[ a_{Nt}^* = \rho_{Nt} a_{Nt-1}^* + \epsilon_{Nt}^* \]  
(A.5.18)

\[ a_{lt} = \rho^l a_{lt-1} + \epsilon_{lt} \]  
(A.5.19)

\[ p_{Nt}^* = a_{Nt}^* - p_{Nt}^* + \pi_{Nt}^* \]  
(A.5.20)

\[ y_{Tt}^* = \nu (y_{lt}^*) + (1 - \nu) y_{Xt}^* \]  
(A.5.21)

\[ p_{Tt}^* = \nu (p_{lt}^*) + (1 - \nu) p_{Xt}^* \]  
(A.5.22)

\[ \pi_{Tt}^* = \nu \pi_{lt}^* + (1 - \nu) \pi_{Xt}^* \]  
(A.5.23)

\[ r_t - r_t^* = \delta_t \Delta e_{t+1} \]  
(A.5.24)

\[ r_t^* = \rho^r r_{t-1}^* + \epsilon_{r*,t} \]  
(A.5.25)

\[ y_t^* = \alpha^* y_{Nt}^* + (1 - \alpha^*) y_{Tt}^* \]  
(A.5.26)

\[ p_{lt}^* = \rho_{pt}^* p_{lt-1}^* + \epsilon_{pt_t}^* \]  
(A.5.27)

\[ \pi_{lt}^* = \rho_{pt}^* \pi_{lt-1}^* + \epsilon_{xt_t}^* \]  
(A.5.28)

\[ p_{Xt}^* = \rho_{px}^* p_{Xt-1}^* + \epsilon_{px_t}^* \]  
(A.5.29)

\[ \pi_{Xt}^* = \rho_{px}^* \pi_{Xt-1}^* + \epsilon_{xxt_t}^* \]  
(A.5.30)

\[ y_{Xt}^* = y_{lt}^* + (p_{Xt}^* - p_{lt}^*) \]  
(A.5.31)

\[ y_{Nt}^* = -\rho^* p_{Nt}^* + \rho^* (\alpha^* p_{Nt}^* + (1 - \alpha^*) p_{Tt}^*) + y_t^* \]  
(A.5.32)
Figure A.1: Impulse responses to commodity terms of trade shocks: high openness
Figure A.2: Impulse responses to commodity terms of trade shock: no price stickiness
Figure A.3: Impulse responses to commodity terms of trade shock: low elasticity of substitution between traded and non traded goods
### Table A.1: Sensitivity tests on volatility

<table>
<thead>
<tr>
<th></th>
<th>High openness</th>
<th></th>
<th>No price stickiness</th>
<th></th>
<th>Low elasticity of substitution</th>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>CIT</td>
<td>NTIT</td>
<td>ET</td>
<td></td>
<td>CIT</td>
<td>NTIT</td>
</tr>
<tr>
<td>Output</td>
<td>0.11</td>
<td>0.35</td>
<td>0.22</td>
<td></td>
<td>0.06</td>
<td>0.05</td>
</tr>
<tr>
<td>Non-traded output</td>
<td>0.13</td>
<td>0.32</td>
<td>0.15</td>
<td></td>
<td>0.04</td>
<td>0.11</td>
</tr>
<tr>
<td>Exported output</td>
<td>0.51</td>
<td>0.19</td>
<td>0.61</td>
<td></td>
<td>0.30</td>
<td>0.31</td>
</tr>
<tr>
<td>Labour</td>
<td>0.13</td>
<td>0.23</td>
<td>0.30</td>
<td></td>
<td>0.03</td>
<td>0.05</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.25</td>
<td>0.43</td>
<td>0.45</td>
<td></td>
<td>0.17</td>
<td>0.18</td>
</tr>
<tr>
<td>Nominal exchange rate</td>
<td>0.20</td>
<td>0.18</td>
<td>0.03</td>
<td></td>
<td>0.21</td>
<td>0.03</td>
</tr>
<tr>
<td>Real exchange rate</td>
<td>0.14</td>
<td>0.09</td>
<td>0.07</td>
<td></td>
<td>0.22</td>
<td>0.11</td>
</tr>
<tr>
<td>Non-traded inflation</td>
<td>0.32</td>
<td>0.04</td>
<td>0.34</td>
<td></td>
<td>0.38</td>
<td>0.37</td>
</tr>
<tr>
<td>CPI inflation</td>
<td>0.19</td>
<td>0.25</td>
<td>2.34</td>
<td></td>
<td>0.15</td>
<td>0.27</td>
</tr>
<tr>
<td>Interest rates</td>
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<td>0.02</td>
<td>0.09</td>
<td></td>
<td>1.28</td>
<td>0.09</td>
</tr>
</tbody>
</table>

**Note:** CIT is CPI inflation targeting, NTIT is non-traded inflation targeting and ET is exchange rate targeting.

### Table A.2: Sensitivity tests on welfare evaluations

<table>
<thead>
<tr>
<th>Weights on central bank loss function</th>
<th>High openness</th>
<th>Welfare losses</th>
<th>Low elasticity of substitution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CIT</td>
<td>NTIT ET</td>
<td>CIT  NTIT ET</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda_y$ $\lambda_y$ $\lambda_r$ $\lambda_e$</td>
<td>0.07 0.13 1.32</td>
<td></td>
<td>0.06 0.48 0.68</td>
</tr>
<tr>
<td>1 0.5 0.5 0.5</td>
<td>0.03 0.14 0.72</td>
<td>0.02 0.02 0.69</td>
<td>0.13 0.12 0.27</td>
</tr>
<tr>
<td>1 1 0.5 0.5</td>
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<td>0.04 0.05 0.72</td>
<td>0.13 0.22 0.30</td>
</tr>
<tr>
<td>1 1.5 0.5 0.5</td>
<td>0.62 0.15 0.44</td>
<td>0.03 0.02 0.69</td>
<td>0.15 0.10 0.45</td>
</tr>
<tr>
<td>1 1 0.5 0.5</td>
<td>0.01 0.01 1.00</td>
<td>0.15 0.20 0.65</td>
<td>0.15 0.11 0.27</td>
</tr>
<tr>
<td>1 1 0.5 0.5</td>
<td>0.17 0.64 1.44</td>
<td>0.22 0.24 0.68</td>
<td>0.17 0.12 0.44</td>
</tr>
<tr>
<td>1 1 0.5 0.5</td>
<td>0.17 0.64 1.03</td>
<td>0.98 0.99 1.02</td>
<td>0.19 0.03 0.29</td>
</tr>
<tr>
<td>1 0.5 0.5 1</td>
<td>0.02 0.03 0.07</td>
<td>0.07 1.42 1.62</td>
<td>0.17 0.32 0.27</td>
</tr>
<tr>
<td>1 0.5 0.5 1.5</td>
<td>0.03 0.04 0.04</td>
<td>0.18 0.44 1.68</td>
<td>0.21 0.23 0.28</td>
</tr>
<tr>
<td>1 0.5 0.5 2</td>
<td>0.03 0.04 0.09</td>
<td>0.24 0.90 1.79</td>
<td>0.24 0.29 0.26</td>
</tr>
</tbody>
</table>

**Note:** CIT is consumer price index inflation targeting, NTIT is non tradable inflation targeting and ET is exchange rate targeting.