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Optimal timing of defections from price-setting cartels in volatile markets

Shakill Hassan[1]

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[1] School of Economics, University of Cape Town

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Shakill Hassan*

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Abstract

ABSTRACT: We model cartel defection in markets with stochastic demand fluctuations as an investment timing problem. We show that (i) the optimal timing of cartel defection is pro-cyclical, suggesting higher probability of competitive pricing during booms; and (ii) the defection trigger is a positive function of demand variability, and larger than its deterministic demand counterpart, implying that market volatility facilitates collusion. The first result is consistent with the counter-cyclical pricing prediction originally due to Rotemberg and Saloner (1986), but not dependant on lack of persistence in demand fluctuations. The analysis reveals insights on implications of co-variation between volatility and demand shocks.

JEL Classification numbers: G13, L13

KEYWORDS: cartel defections, volatility, real options

1. Introduction

1.1. Introductory discussion

How does market volatility affect collusive behaviour and, in particular, the timing of cartel defections? Theoretical research on collusion in oligopolistic industries where the evolution of market demand is characterised by unpredictable shocks can be broadly categorised into two branches. The first, originated by Green and Porter (1984), concentrates on the effect of imperfect monitoring of cartel members' actions on collusive behaviour, in settings where monitoring and demand variability are inseparable problems. The second assumes perfect observability of demand shocks, and concentrates on the cyclical properties of collusive prices. The central question in this branch has been whether collusive behaviour involves higher or lower prices in periods of high and low demand, generally associated with booms and recessions, respectively. The answer to this question hinges on whether

*School of Economics, University of Cape Town, Private Bag, Rondebosch. 7701. South Africa

cartels become more or less stable, in the sense of maintaining the incentive to collude, as demand changes over time.

The seminal contribution to this branch of the literature is Rotemberg and Saloner's (1986) analysis of repeated interaction in markets where market demand is subject to identically and independently distributed shocks over time. Their analysis indicates that the temptation to defect (by price undercutting) from a cartel is stronger in periods of high demand, which are associated with booms. The motivation is simple. As observed demand approaches an upper bound, it becomes increasingly likely that future realisations will be lower, as demand returns to 'normal' levels, and hence that the expected cost of defection in terms of (lost) future collusive profits net of punishment phase profits becomes smaller, relative to the immediate gain from undercutting the cartel and enjoying temporary monopoly profits. Hence the surprising prediction that, in contrast to Green and Porter (1984) [1] and 'conventional wisdom,'[2] competitive pricing is more likely in booms than recessions, if we associate high (low) demand with booms (recessions). Effective collusion may thus require that the cartel promote price reductions during booms in order to attenuate the destabilising effect of strong positive demand shocks.

Empirical support for this prediction is significant.[3] It has been observed that in certain collusive industries cartels collapse after large increases in demand. Tirole (1988, p. 250) notes the example of

[1] With imperfectly observable demand uncertainty, Green and Porter (1984) show that price-wars occur in low demand periods, implying that collusive pricing is pro-cyclical.

[2] The following quote from The Economist (2002), referring to the series of cartel investigations launched by the European Commission in the late 1990's and early 2000's, suggests the existence of differing views of what constitutes conventional wisdom: "The current attack on cartels comes at a time when the temptation to strike cosy deals with competitors is stronger than ever, as pricing power remains weak and excess capacity abounds in the wake of the global economic slowdown."

[3] Of course, collusion is not the only theoretic explanation for this result. See Rotemberg and Woodford (1992), and Chevalier, Kashyap and Rossi (2003), for surveys and critiques of alternative reasons for counter-cyclical mark-ups.

the market for an antibiotic, in which cartel discipline broke down after a particularly large Government order.[4] A possible manifestation of cartel breakdowns after increases in demand is the occurrence of price wars during booms, well-known evidence of which includes Porter (1983), for US railroad pricing, and Bresnahan (1987), for the US automobile industry. In addition, a number of empirical studies have documented evidence of counter-cyclical mark-ups in imperfectly competitive industries - which can be the result of price wars during booms. Industry specific studies include Rotemberg and Saloner (1986), for (real) cement prices, and Chevalier, Kashyap, and Rossi (2003), for supermarket retail prices. Rotemberg and Woodford (1992) present cross-sectional evidence for United States industry, and Fedderke, Kularatne and Mariotti (2004) for a cross-section of South African manufacturers.

One widely recognised limitation of the counter-cyclical pricing prediction in the Rotemberg-Saloner study, noted here for subsequent reference, concerns its reliance on the assumption that demand follows an identically and independently distributed process. As noted by Haltiwanger and Harrington (1991, p. 90), "a result of the i.i.d. assumption is that firms' expectations on future demand are independent of current demand, and one implication of this is that firms never expect demand to be stronger tomorrow if it is relatively strong today." A natural question then, is whether the result holds when strong shocks affect current expectations of future demand. In recognition of this limitation, Haltiwanger and Harrington (1991), Kandori (1991), and Bagwell and Staiger (1997) build on the Rotemberg-Saloner setup to accommodate the possibility of persistence in the time-varying demand process. Haltiwanger and Harrington (1991) present an alternative to Rotemberg-Saloner for markets in which demand movements are deterministic and subject to seasonal fluctuations. In Bagwell and Staiger (1997) demand alternates stochastically between phases of low and high growth. In both these studies the Rotemberg-Saloner prediction is reversed - except (in Bagwell-Staiger) for the case of *negatively*

[4]See also Scherer and Ross (1990).

correlated demand growth. In Kandori (1991) the Rotemberg-Saloner result is shown to be robust to serially correlated shocks, provided the discount factor is either above but close to (n-1)/n, with n being the number of firms, or it tends to unity.

1.2. Present paper's contribution

Absent from all the above theories of collusion in markets subject to stochastic demand shocks is an explicit treatment of the effect of the magnitude of market volatility, or the degree of uncertainty that characterises the market, on cartel behaviour. However, the behaviour of at least two of the world's most prominent cartels, the DeBeers controlled international diamond cartel and the Organisation of Petroleum Exporting Countries (henceforth, OPEC), suggest that volatility may play an important role in cartel behaviour. Indeed, in both cases, explicit collusion has been motivated by the desirability of reducing (in the case of OPEC) or preventing (in the case of DeBeers) volatility.[5] The central aim in this paper is the presentation of a simple extension of the literature on collusion under observable but random demand shocks which can provide an explicit treatment of the effect of market volatility on the stability of price fixing cartels. The contribution is twofold.

First, a simple modification of the Rotemberg-Saloner and Bagwell-Staiger basic setups is shown to produce a simple version of the countercyclical pricing result while allowing for (positive) serial correlation. In contrast to Haltiwanger and Harrington (1991) and partly in line with Bagwell and Staiger (1997), we allow for persistence while assuming unpredictable changes. We obtain a result which is consistent with Rotemberg-Saloner in predicting that, under uncertainty, observable cartel defections are more tempting in periods of high demand than downturns. In particular, there is a threshold level of demand beyond which the cartel breaks. This threshold is analytically characterized

[5] See Kretschmer (1998), on the international diamond cartel, and Plourde and Watkins (1998), on crude oil prices and OPEC.

as a function of a set of observable parameters, and it gives us a timing rule for defection. Hence, price-fixing cartels have an incentive to attenuate the destabilising effect of strong positive demand shocks through a degree of counter-cyclical pricing.

Second, we analyse the defection timing decision as a simple stopping problem, and propose that the strategic decision problem faced by collusive firms contemplating cheating on the cartel in a market faced by volatile demand be viewed as analogous to the decision of whether to exercise a non-tradable exchange option. The argument is most evident in the context of trigger strategies with prohibitively costly renegotiation. When deciding whether to cheat on a cartel operating in a volatile market, the oligopolist is deciding whether to exchange the discounted share of expected collusive profits (a stream of uncertain and non-tradable cash flows) for the temporary monopoly windfall plus discounted expected non-collusive profits (another stream of uncertain and non-tradable cash flows). Firms are not, of course, under an obligation to defect. But they have the opportunity to do so at any stage in the repeated interaction. The criterion for optimal defection corresponds to an optimal option exercise (or investment) rule. This analogy, and associated framing of cartel decisions, permits us to derive an explicit and intuitive relationship between volatility and the defection trigger, and quantify the effect of changes in the magnitude of market volatility on the temptation to cheat on the cartel by undercutting the collusive price.

Although anchoring the basic setup in the literature on collusion in markets subject to stochastic demand shocks, the analytic apparatus employed in the present paper draws on (indeed, can be seen as a simple application of) the continuous-time methods in the investment timing problems analysed by McDonald and Siegel (1986), Dixit and Pindyck (1994), Grenadier (1996), and Lambrecht (2004). The remainder of the paper proceeds as follows. The assumptions and setup are stated in section 2. Section 3 contains the analysis and results, with all proofs in the appendix. Section 4 concludes.

2. The Model

2.1. Description of the strategic interaction

We model the indefinitely repeated interaction between a fixed set of $n \geq 2$ symmetric firms with no capacity constraints, constant marginal costs, and producing a homogenous good. In any decision period in $\{1, 2, \ldots \infty\}$, firms play a standard Bertrand game, where the stage-game action spaces consist of the set of non-negative prices: $\forall i \in \{1, 2, \ldots, n\}, A^i = [0, \infty)$,and each firm chooses its price without observing the others' contemporary choices. (Outputs are chosen to clear the market.) The firm charging the lowest price captures the whole market. If all firms charge the price p, each firm's stage-game payoff *per consumer* is $\pi^i(p) \equiv (p-c)D(p)$, where c is positive and represents the constant unit cost, and $D(p)$ is the (identical) consumers' common demand function, assumed to be differentiable and decreasing in p. The function $\pi^i(p)$, representing a firm's profit per consumer, is assumed to satisfy the following properties: for $p > 0$,$\pi^i(p)$ has a unique maximizer, p^m, and $d\pi^i(p)/dp > 0$ whenever $p < p^m$. Hence p^m is the monopoly price.

It is a standard result that the unique *pure*-strategy (Bertrand-) Nash equilibrium of the pricing stage game involves all firms pricing at marginal cost. However, it can be shown that this need not be the case with unbounded monopoly profits, where a continuum of prices above marginal cost can be supported in symmetric mixed Nash equilibria of the one-shot Bertrand game.[6] We assume that the stage game equilibrium involves all firms choosing a price which may be marginally higher than marginal cost, and henceforth refer to this as the "competitive" price.[7] Over time, past and current demand

[6] See Baye and Morgan (1999) and Klemperer (2003).

[7] This choice may seem somewhat arbitrary but the analysis is equally applicable in the more familiar case of marginal cost pricing as the unique Nash equilibrium of the pricing stage game. Allowing for a non-collusive price just slightly above marginal cost reflects the idea that some small level of profit may be consistent with non-collusive price competition in real world oligopoly.

levels as well as rivals' past prices are perfectly observable, so dynamic strategies can be conditional on price histories.

Following Rotemberg-Saloner and Bagwell-Staiger, we model the possibility of collusive pricing through symmetric history-dependent Nash-reversion trigger strategies, where one firm's deviation from the collusive price triggers the oligopoly's subsequent reversion to competitive pricing, the simplest collusion inducing mechanism. When a firm under-cuts the collusive price, it is able to capture the whole market for a limited period by incurring a cost m where $0, < m < \infty$, before rivals retaliate.[8] The possibility of renegotiation following defection is not precluded but it is assumed to be "prohibitively costly", in the sense that the "innocent" firms will be unwilling to renegotiate, so that the possibility of renegotiation does not impede the sustainability of collusion through simple Nash-reversion trigger strategies.[9]

Future payoffs are discounted at a common constant rate, denoted by r, with $r \in (0, 1)$ and $r > \alpha > 0$. If we ignore demand variability, collusion is strategically sustainable if the present value of temporary monopoly profits plus subsequent competitive pricing does not exceed the present value of collusive profits.

2.2. Evolution of demand shocks

We equate the continuously evolving level of market demand, at $t \in [0, \infty)$, with the contemporary mass of consumers which we denote x_t . The evolution of x_t is described by the following stochastic differential equation:

$$dx_t = \alpha x_t dt + \sigma x_t dW_t \tag{1}$$

where dW_t is an increment of a standard Wiener process. The constants α and σ can be interpreted, respectively, as the instantaneous

[8] Alternatively, the parameter m could be interpreted as a monitoring cost which has to be incurred to ensure that market demand and/or rivals' prices are perfectly observable.

[9] See McCutcheon (1997).

conditional expected percentage change in x_t , and the instantaneous conditional standard deviation, per unit time. Intuitively, demand grows at a constant long-term rate, and its evolution is constantly perturbed by a random shock. The size of the parameter σ reflects the magnitude of demand oscillations. Note a well known property of the geometric Brownian motion described by (1), with reference to the introductory discussion: it exhibits positive persistence - not in the sense that a rise in today's demand signals a further increase tomorrow, which is always uncertain, but in the weaker sense that the cumulative probability distribution of future realisations of demand shifts uniformly to the right when current demand increases. Hence, the model captures the idea that demand as well as expectations of future demand, change over time.

3. Analysis

3.1. Analysis and results

We start by characterising the payoffs which determine the central trade-off between collusion and defection. At any point in time, say zero, the discounted expected value of $1/n$ of the stream of collusive profits, denoted by K, is given by

$$K = E\left(\int_0^\infty e^{-rt}\pi^i\left(p^m\right)\left(1/n\right)x_t dt \mid x_0\right) \tag{2}$$

where $\pi^i\left(p^m\right) = \left(p^m - c\right)D\left(p^m\right)$, and the expectation is conditional on the observed current level of demand. Noting first, that the assumed evolution of demand shocks implies that for any $t > 0$ we have $E\left(x_t \mid x_0\right) = x_0 e^{\alpha t}$, and second, that the properties assumed to be satisfied by $\pi\left(p\right)$ imply that p^m, the monopoly (and fully collusive) price, is invariant to changes in the mass of consumers in the market, equation (2) can be easily solved to obtain: [10]

$$K = \frac{\pi^i(p^m)}{r-\alpha}\frac{1}{n}x_0 \tag{3}$$

Let τ denote the duration of the period between defection and retaliation, so the case of immediate retaliation corresponds to $\tau = 0$. (Notice that the evolution of all flow variables between decision dates is continuous.) During the period between $t = 0$ and $t = \tau$ the defector earns a defection rent, corresponding to the monopolist profit. At $t = \tau$ the rivals retaliate and the cartel breaks down, with consequent competitive pricing. Since renegotiation costs are prohibitive, renegotiation does not occur and retaliation is followed by an infinite stream of competitive profits. We emphasise that what the level of demand will be at any moment subsequent to defection is not known when the decision to defect is made. Hence, the discounted expected gain from defection is determined by the discounted sum of two uncertain streams, denoted by V:

$$\begin{aligned} V &= E\left(\int_0^\tau \pi^i\left(p^m\right)x_t e^{-rt}dt \mid x_0\right) \\ &\quad -m + e^{-r\tau}E\left\{E\left[\int_\tau^\infty \pi^i\left(p^c\right)\left(1/n\right)x_t e^{-r(t-\tau)}dt\right] \mid x_0\right\} \end{aligned} \tag{4}$$

where $\pi^i(p^c) = (p^c - c)D(p^c)$.Using elementary results from stochastic calculus to solve the conditional expectations gives:[11]

$$V = \frac{\pi^i(p^m)x_0}{r-\alpha}\left(1 - e^{-(r-\alpha)\tau}\right) - m + \frac{\pi^i(p^c)}{r-\alpha}\frac{1}{n}x_0 \tag{5}$$

[10] To verify the invariance of the fully collusive (monopoly) price to changes in consumer mass (or demand shocks), fix the level of demand at an arbitrary level X_1. By definition, the monopolist's profit at price p^m will be $\pi^m\left(p^m\right)X_1$.Suppose demand changes to X_2 where X_2 is positive but can be larger or smaller than X_1, and the monopolist responds by reducing the price to p^2 where $p^2 < p^m$. Since $\pi^i(p)$ is increasing in p for any p below p^m, $p^2 < p^m$ if and only if $\pi^m(p^2) < \pi^m(p^m)$ and therefore $\pi^m(p^2)X_2 < \pi^m(p^m)X_2$, which contradicts profit maximisation. (The uniqueness of p^m obviates the case where the monopolist responds to the change in demand by raising the price.)

[11] See Bjork (1998).

The incremental gain from defection, in expected present value terms, is given by $V - K$. It is obtained by subtracting (3) from (5) and can be re-arranged and simplified to yield:

$$\frac{x_0}{r-\alpha}\left\{\pi^i\left(p^m\right)\gamma+\pi^i\left(p^c\right)\eta\right\}-m \tag{6}$$

where

$$\gamma=\left(1-e^{-(r-\alpha)\tau}\right)-1/n, \text{and } \eta=1/n \tag{7}$$

The following familiar result follows directly from (6).

Proposition 1 *Defection is never optimal (the incentive constraint for collusion is always satisfied) if the following condition holds:*
$\tau < \frac{-LN\left(\frac{n-1}{n}\right)}{r-\alpha} \equiv \underline{\tau}$.

Basically, a minimum retaliation lag during which the "cheater" receives monopoly profits is necessary for defection to be potentially attractive. Proposition 1 simply tells us exactly how long that period must be, as a function of the number of firms and the rates of discount and expected growth in demand. Our interest is in the analysis of the effect of unpredictable demand fluctuations on the incentive to cheat on a collusive agreement, and the cartel's response to this effect, and particularly, a characterisation of optimal defection timing. Hence we restrict attention to the case where $\tau \geq \underline{\tau}$, in which defections may be optimal depending on the level of demand. The following result is now evident.

Proposition 2 *For $\tau \geq \underline{\tau}$ the relative strength of the incentive to deviate from collusive pricing is strictly increasing in the current level of demand.*

Intuitively, this result suggests that boom periods with associated high demand reduce the attractiveness of collusion. Hence, in the absence of cartel behaviour involving a degree of counter-cyclical pricing to attenuate the pro-cyclical incentive to defect, price wars may follow large increases in demand. Provided the negation of the condition *in Proposition 1 is satisfied, the cartel is increasingly fragile as demand rises. There will be a range of values of demand within which the incentive to defect is insufficiently strong to trigger defection. Beyond this collusive range, which represents the continuation region in the implicit optimal stopping problem, the cartel breaks. Collusion can only be sustained provided demand does not reach a threshold level,* x^**, which will trigger defection.*[12] *Moreover, note that the optimal stopping rule in this problem is identical to an optimal exercise rule for the analogous (non-tradable) option to exchange one (non-tradable) risky asset for another. This intuition is formally summarised through the characterisation of the threshold* x^**, and associated timing rule.*

Proposition 3 *The timing of cartel defections in markets characterised by stochastic demand fluctuations is pro-cyclical - defection becomes optimal at the first moment that the demand process hits the threshold* x^**from below:*

$$t^* = \inf \{t \geq 0 : x_t \geq x^*\}$$

where the defection threshold is given, for retaliation periods $\tau \geq \underline{\tau}$ *, by:*

$$x^* = \left(\frac{\beta}{\beta - 1}\right) \frac{(r - \alpha)\, m}{(\pi^i\,(p^m)\,\gamma + \pi^i(p^c)\eta)}$$

and β *is an explicit function of* r*,* α *and* σ*.*

It is worth noting that the derivation of this threshold is consistent with the belief that the threshold will be hit at some point (see the appendix). The explicit dependence of the threshold on the volatility parameter σ *is of particular interest. It leads to the results collected in Proposition 4.*

Proposition 4 *(i)The threshold* x^* *is increasing in* σ *; (ii) with immediate retaliation and positive* σ *, the defection threshold consistent*

[12] The result in Proposition 2 and persistence in (1) ensure uniqueness of x^*

with optimal timing of defection always exceeds the trigger implied by the "defect as soon as the discounted expected benefit exceeds the discounted expected loss" rule; (iii) the difference between this trigger and the derived threshold x^* is increasing in the level of the volatility parameter σ .

3.2. Discussion of the effect of volatility

Intuitively, when a firm defects it faces the possibility that demand will subsequently fall, and post-retaliation non-collusive prices plus short-term gains prior to retaliation will not compensate for the lost, and also uncertain, collusive income stream. This point implies a fundamental distinction between optimal defection rules under deterministic and stochastic market demand environments. In the first environment, there is no possibility of "regret". In the latter, constant uncertainty about future demand implies an opportunity cost to defection. Our analysis shows that this cost imposes a wedge between expected gains from defection and expected gains from collusion, such that equality between these two quantities does not delineate the collusive and competitive regimes. It is clear from the proof of part (ii) of Proposition 4 (see the appendix) that in environments of uncertain demand, the expected payoff from defection has to exceed the expected gain from collusion by a strictly positive amount.

Most interestingly, this difference, which as we show is not captured by the decision rule commonly used to operationalize the idea of sequential rationality in models of collusion, is increasing in the magnitude of volatility. As volatility (or uncertainty) increases, β decreases, so $\beta/(\beta - 1)$ increases, and the difference between x^* and the "defect as soon as the discounted expected benefit exceeds the discounted expected loss" trigger becomes wider. The implied prediction is that the greater the degree of uncertainty that characterises an industry, the easier it will be to sustain collusive agreements - and consequently the less informative (or 'reliable', in terms of predictive power) any trigger not capturing this effect will be.

Notice that since changes in volatility cause changes in the demand threshold in the same direction, a strong positive demand shock which is concurrent with an increase in volatility may well have no impact on the incentive to cheat. The combined effect of demand and volatility changes can be an increased, decreased, or unaffected temptation to defect, depending on the relative strengths of these effects. In brief: increases in demand uncertainty absorb at least part of the destabilising effect of large increases in demand.

Lastly notice that x^* is increasing in α and decreasing in r. The first effect is due to β being decreasing in α . Intuitively, it reflects the larger impact that higher long-term growth has on infinitely earned collusive profits than it has on short-term monopoly rents. The second effect is due to β being increasing in r, and it is consistent with standard results. It captures the effect of patience on the facility of collusion.

3.3. Varying the collusive price

It is tempting to conjecture that the cartel may attempt to prevent defections and ensure collusion across all demand states by reducing the collusive price as demand passes the threshold. Suppose for example that the collusive price is adjusted so as to keep industry profits constant and prevent increases in the incremental gain from defection, as demand rises past the threshold.[13] To be more precise, let $\pi^* \equiv (x^* - \varsigma)(p^m - c) D(p^m)$where ς is an arbitrarily small constant, denote industry profit at the monopoly price when market demand is marginally below the threshold, and let G be an increasing real valued price adjustment function satisfying the following properties:

No 1

$$G(x_t) \leq 0 \text{ when } x_t < x^* \text{ and } G(x_t) > 0 \text{ when } x_t \geq x^*$$

No 2

$$x_t (p^m - G(x_t) - c) D(p^m - G(x_t)) = \pi^* when \text{ x}_t \geq x^*$$

[13] I am thankful to an anonymous referee for this suggestion.

The first property is easily satisfied by monotonic transformations of the incremental gain from defection. The second property simply ensures that price adjustments made to mitigate the temptation to cheat are consistent with maximisation of collusive profits in the following collusive pricing scheme:

$$p^v(x_t) = \max\left[p^m - \max\left(G(x_t), 0\right), p^c\right] \qquad (8)$$

Thus, if demand is below the threshold we have that $G(x_t) \leq 0$ and $p^v = p^m$, so that the collusive price corresponds to the monopoly price. Once demand breaks through the threshold we have that by construction $G(x_t) > 0$ and the collusive price moves to the interval $[p^c, p^m]$, between the perfectly competitive and monopoly prices. The further demand moves past the threshold, the closer the collusive price will have to be to the competitive price.

Letting $(p^c - c) D(p^c) = 0$ the trade-off between payoffs from defection and continued collusion is now the same at all levels of demand above the threshold, since the payoff stream from collusion is a fraction $1/n$ of an indefinitely earned constant flow, while the payoff from defection consists of n times this constant stream earned between defection and retaliation. (For simplicity we ignore the possibility that between defection and retaliation demand does not stay above the threshold.) The incentive constraint for continued collusion then assumes the standard textbook form, namely $e^{-rt} \geq (n-1)/n$. Recall that under the simpler scheme with a non-varying collusive price (at the demand invariant monopoly price), we have from a re-statement of Proposition 1 that the incentive constraint is satisfied for any level of demand provided $e^{-(r-a)\tau} \geq (n-1)/n$. From $r > \alpha \geq 0$ it follows that collusion across all states is feasible under the varying collusive price and fixed profits scheme only when it is also feasible with the non-varying collusive price and variable profits scheme.

4. Conclusion

This paper presented an analysis of optimal cartel defection timing, where members of the cartel are price-setting Bertrand competitors and the evolution of demand shocks is represented by Brownian motion. The results reveal that increases (respectively, decreases) in demand variability increase (decrease) the threshold level of demand which separates the collusion and defection regions. Hence, volatility serves as a hitherto unidentified collusion facilitating factor.

Moreover, it was shown that the temptation to cheat on a price-manipulating agreement is monotonically increasing in the level of demand, so the timing of cartel defection is pro-cyclical. Given our restriction of the analysis to the case where defections may occur depending on the observed state of demand, this result implies the possibility of price wars in periods of particularly high demand. Specifically, the fully collusive price corresponds to the monopoly price for levels of demand below the derived threshold, and falls discontinuously to the competitive price once demand hits this threshold. Although a more detailed exploration of collusive pricing could be a valuable extension of our analysis, this prediction is consistent with the interesting results in Rotemberg and Saloner (1986), and suggests that the crucial assumption behind the theoretic possibility of price wars during booms is the unpredictability of future demand and not, as previously contended in the theoretic literature, the lack of persistence in the evolution of demand shocks.

Combining the effects of demand fluctuations and volatility in the analysis of collusion suggests that to determine whether collusive pricing is pro- or counter-cyclical in industries faced by volatile demand, it is necessary to isolate the effect of changes in volatility. If, for example, there is a tendency for changes in demand to be positively associated with changes in volatility, decreases in demand would be accompanied by decreases in the defection threshold. Hence we could observe price wars in periods of low demand, despite the positive relationship between levels of demand and the incentive to defect.

Unfortunately, the existing empirical literature on price wars and collusive pricing over the business cycle alluded to in the introduction cannot be used to test our predictions at this stage, since the effects in the preceding paragraph cannot be disentangled. It would be instructive to re-examine the evidence to determine whether volatility plays a significant role in explaining the reported differences in observed behaviour and pricing patterns across industries. Another direction for further work, currently being pursued by the author, is to extend the analysis to the case of an output-fixing cartel, and confront the adapted model's predictions with the behaviour of members of the OPEC cartel, and the evolution of oil prices.

5. Appendix

Proof of Proposition 1 This follows immediately by noting that by assumption: $(i) n, m, (r-\alpha)$, and monopoly profits are strictly positive; and (iii) $\pi^i\left(p^c\right)\eta \approx 0$.It follows that (6) will be strictly negative at all demand states if:

$$\left(1-e^{-(r-\alpha)\tau}\right) < (1/n) \tag{9}$$

Re-arranging and taking logs gives the result.■

Proof of Proposition 2 The result follows directly from (6) and Proposition 1, since $\tau > \underline{\tau} \iff \gamma > 0$, with the consequence that:

$$\frac{1}{r-\alpha}\left\{\pi^i\left(p^m\right)\gamma + \pi^i\left(p^c\right)\eta\right\} > 0 \tag{10}$$

implying:

$$\overline{x} > \underline{x} \iff \tfrac{\overline{x}}{r-\alpha}\left(\pi^i\left(p^m\right)\gamma + \pi^i\left(p^c\right)\eta\right) > \tfrac{\underline{x}}{r-\alpha}\left(\pi^i\left(p^m\right)\gamma + \pi^i\left(p^c\right)\eta\right) \blacksquare$$

Proof of Proposition 3 First note that the claimed pro-cyclicality of defection timing is a direct consequence of Proposition 2, where we show that the incremental gain from defection is increasing in x. Hence collusion is sustainable for x between zero and a positive threshold x^*. It follows that defection becomes optimal once x_t hits the threshold x^* from below. Second, the uniqueness of x^* is ensured by the result in Proposition 2 (incremental gain increases monotonically with x) and the persistence in the dynamic evolution of x_t.[14] Let $F(x_t)$ denote the value of the hypothetical option to defect, assumed to be a twice continuously differentiable function of x_t. In the continuation (i.e. collusive) region the Bellman equation can be written as:

$$rF\left(x_t\right) = E\left(dF\left(x_t\right)\right) \tag{11}$$

and standard application of Ito's lemma gives:

$$dF\left(x_t\right) = \frac{\partial F}{\partial x}dx_t + \frac{1}{2}\frac{\partial^2 F}{\partial x^2}\left(dx_t\right)^2 \tag{12}$$

Substituting (1) for dX_t and using the fact that $E\left(dW_t\right) = 0$ gives the condition,

$$\frac{1}{2}\sigma^2 x^2 \frac{\partial^2 F}{\partial x^2} + \alpha x \frac{\partial F}{\partial x} - rF = 0 \tag{13}$$

with general solution of the form:

$$F\left(x\right) = A_1 x^{\lambda} + A_2 x^{\beta} \tag{14}$$

This solution is applicable over the range of prices for which it is optimal to collude - or not to defect. The Threshold can be

[14] See the appendix to chapter 4 of Dixit and Pindyck (1994) for a particularly relevant discussion of optimal stopping.

obtained from the procedure for the valuation of the implicit option F. This is done by imposing two boundary conditions on (13). First, that the payoff from defection goes to zero as demand goes to aero: $F(x_t) \longrightarrow 0$ as $x_t \longrightarrow 0$; second, a value matching condition specifying the payoff from defection (option exercise) when the timing of defection is optimal. Expression (6) gives the surplus from defecting. The value-matching condition requires that at the defection threshold:

$$F\left(x^*\right) = \frac{x^*}{r-\alpha}\left[\pi^i\left(p^m\right)\gamma + \pi^i\left(p^c\right)\eta\right] - m \tag{15}$$

It can be shown that this implies that the expected discounted value of the surplus gained if defection occurs when threshold level x^* is hit is:

$$F\left(x_t, x^*\right) = \left[\frac{x^*}{r-\alpha}\left[\pi^i\left(p^m\right)\gamma + \pi^i\left(p^c\right)\eta\right] - m\right]\left(\frac{x_t}{x^*}\right)^\beta \tag{16}$$

where β is an explicit function of r, α,and σ (in particular, it is the positive root of the quadratic $\frac{1}{2}\sigma^2\xi\left(1-\xi\right)+\alpha\xi - r = 0$),and the factor $\left(x_t/x^*\right)^\beta$ can be interpreted as the probability with which the level of demand hits the threshold x^*,conditional on the current level of deman, x_t.[15]

The defection threshold x^* can now be obtained from the (necessary and sufficient) first order condition for the maximisation of (16):

$$\frac{\partial F\left(x_t, x^*\right)}{\partial x^*} = 0 \Longleftrightarrow x^* = \left(\frac{\beta}{\beta-1}\right)\frac{\left(r-\alpha\right)m}{\left(\pi^i\left(p^m\right)\gamma + \pi^i\left(p^c\right)\eta\right)} \blacksquare$$

[15] See Grenadier (1996) and Lambrecht (2004)

Proof of proposition 4 Part (i). Standard - It follows from the facts that $\partial\beta/\partial\sigma < 0$ and $\partial\left(\beta/\left(\beta-1\right)\right)/\partial\sigma > 0$. (See Dixit and Pindyck, 1994).

Part (ii). With immediate retaliation ($\tau = 0$) we have from (6):

$$\gamma = (1/n) \text{ and (as before) } \eta = 1/n \tag{17}$$

The 'defect as soon as discounted expected benefits exceed discounted expected costs' rule is satisfied for values of x such that $V - K \geq 0$, determined for $\tau = 0$ by:

$$\frac{\pi^i\left(p^c\right)}{r-\alpha}\frac{1}{n}x - m \geq \frac{\pi^i\left(p^m\right)}{r-\alpha}\frac{1}{n}x \tag{18}$$

and the associated trigger, or cut-off level of demand, denoted by x^{**}, is thus:

$$x^{**} = \frac{\left(r-\alpha\right)mn}{\pi^i\left(p^c\right) - \pi^i\left(p^m\right)} \tag{19}$$

However, the defection threshold x^*,when evaluated at $\tau = 0$, is given by:

$$x^* = \left(\frac{\beta}{\beta-1}\right)\frac{\left(r-\alpha\right)mn}{\pi^i\left(p^c\right) - \pi^i\left(p^m\right)} = \left(\frac{\beta}{\beta-1}\right)x^{**} > x^{**} \tag{20}$$

where the inequality holds for $\sigma > 0$ since $\left(\frac{\beta}{\beta-1}\right) > 1$(see Dixit and Pindyck, 1994)

Part (iii). This follows from part (i) and the non-dependence of x^{**} on σ.■

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