Productivity estimates for South Africa from CES production functions

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Abstract

This paper provides estimates of the elasticity of substitution and total factor productivity (TFP) for South Africa. Estimates are based on constant elasticity of substitution (CES) production functions. Estimates of potential output and the output gap implied by different CES model specifications are also compared to those from other models.

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1 Introduction[1]

This paper provides estimates of the elasticity of substitution and total factor productivity (TFP) for the South African economy. Estimates are based on constant elasticity of substitution (CES) production functions. These provide a richer narrative about the drivers of the economy's growth performance than the commonly used Cobb-Douglas form. The advantage of the approach used in this paper is that estimates of structural parameters such as the elasticity of substitution that affect relationships between TFP, factor shares and relative prices are produced alongside estimates of TFP, and the approach simultaneously explains any variation in the labour share and changes to the capital-to-labour ratio. As a result, this paper contributes to our understanding of the nature of technical change in South Africa and the implications of economic growth for welfare and inequality.

The estimates of productivity produced are slightly higher than those produced using a Cobb-Douglas approximation or alternative estimates for South Africa. The elasticity of substitution is generally found to be below one (i.e. factors are gross complements), which is in line with other estimates for South Africa. This suggests that labour-augmenting technical change will tend to increase the labour share, all else equal.

To understand the implications of the TFP estimates obtained for the appropriate stance of monetary policy, this paper also compares the estimates of potential and the output gap implied by different CES model specifications to those from alternative models. Given the large estimation uncertainty around the output gap, this paper aims to help develop our understanding of the implications of the assumptions made when estimating different production functions on the one hand, and understanding how structural changes in the economy interact with estimates of TFP and potential, on the other. The results from this paper imply that the output gap has recently been lower than the alternative estimates presented and that accumulation of capital and labour played a larger role than productivity in the growth of potential.

2 CES production functions

Cobb-Douglas production functions are commonly used in theoretical models and for the empirical estimation of TFP. When assuming constant returns to scale, only two factors of production and perfect competition, a Cobb-Douglas function can be written as:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{1}$$

[1] Thanks to Franz Ruch and Rudi Steinbach for comments and alternative model estimates.

where Y_i is output volume, K_t is capital volume, L_t is labour volume, α is the share of capital used in production and A is an efficiency parameter (ie. the proportional difference between actual and potential output). Most approaches for estimating A apply a logarithmic transformation to a Cobb-Douglas specification and either use regression techniques or calculate it as a residual after expressing the function in terms of index numbers (ie. compared to a base value) or growth rates (as in the Solow growth model, for example).

While TFP is easy to calculate in this way, the disadvantage of using such techniques is that one does not learn about the nature of technical change or its relationships to income shares or relative prices. A more general production function that can be estimated is the constant elasticity of substitution (CES) form:

$$Y_t = [\alpha(\Gamma_t^K K_t)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(\Gamma_t^L L_t)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}} \tag{2}$$

where σ (the elasticity of substitution between K and L) is not assumed to be one, as in the Cobb-Douglas case. The elasticity of substitution captures the effect of changes in the relative cost of capital compared to labour on the demand for capital relative to labour.[2] The advantage of using a more general form is that the contributions of each factor to technical change can be directly estimated (in equation 2 as Γ_t^K and Γ_t^L, which denote K and L-augmenting technological change, respectively, with both in percentage growth terms). As TFP is not explicitly estimated, it is backed out using what is known as the Kmenta approximation (see Appendix section A for details).

A key advantage of using a CES specification is that σ is estimated jointly with TFP and different restrictions on the parameters in the model can be tested empirically. Another reason why CES production functions have recently become more popular is that deviating from a Cobb-Douglas production function (i.e. the assumption that the elasticity of substitution between capital and labour of exactly one, $\sigma = 1$) can help explain variation in the labour share. In the context of a growth model, the use of a Cobb-Douglas production function means that changes in the capital-labour ratio will be accompanied by a proportional change in their income shares so that income shares should be constant over time. Deviating from the Cobb Douglas assumption (and allowing for interaction between capital and labour), instead results in different implications for relationships between TFP and factor shares. Though still substitutes, when $\sigma < 1$ (capital and labour are closer to being complements so that the demand for labour, for example, rises when the cost of capital falls), a higher capital labour ratio would be associated with a fall in the capital income share, while $\sigma > 1$ (capital and labour are

[2]Formally, the elasticity is defined as $\sigma = \frac{d(ln(K/L))}{d(logln(F_L/F_K))}$ and depends on the percentage change in capital intensity and the marginal rate of substitution (relative marginal products of capital and labour $\frac{\partial(Y/L)}{\partial(Y/K)}$ which in perfect markets depends on the real rental rates of capital and labour r_t and w_t such that $d(ln(F_L/F_K) = d(ln(w/r))$.

more substitutable) would imply a higher capital income share.

3 Estimation and data

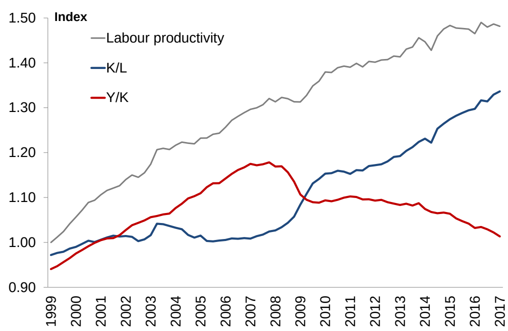
Estimation is based on the 'supply-side system' approach of Klump et al. (2007) that involves estimating equation 2 alongside the first order conditions for capital and labour shares implied by the standard neo-classical model. Estimates of 'deep' parameters such as the elasticity of substitution are sensitive to the units in which capital, labour and output are measured in. To make sure that the parameter estimates obtained are independent of units of measurement, the series used are 'normalised' by expressing them as index numbers (in this case relative to baseline values for the factor shares and other variables in the system of equations). A non-linear estimation approach is used and is described in more detail in Appendix section A.

Output is measured using value added at basic prices excluding agriculture, capital is total fixed capital stock, labour is based on employment in total non-agricultural industries and the labour share is the share of compensation of employees in gross value added at basic prices of all industries. The same data is used to produce estimates of aggregate TFP for South Africa by Botha et al. (2018).[3]

Unlike the experience in several western economies over recent years, South Africa's labour share has been relatively stable: averaging around 50 percent since 1999, but rising slightly post-GFC (Figure 1). Figure 2 shows that the data suggest that South Africa's labour productivity (the output-labour ratio) grew strongly over the sample (1999Q1-2017Q1). In spite of a strong rise in capital intensity (the capital-labour ratio) after the GFC, output per unit of capital fell. The estimation approach used in this paper will simultaneously explain the movements in the labour share, capital-labour ratio and output over time.

Figure 1: Labour share

Figure 2: Key series



[3]All data sourced from the Reserve Bank of South Africa.

4 Estimation results

The approach used here to estimate CES functions is useful for explaining both changes in the labour share and output over time, and produces estimates of 'deep' parameters such the elasticity of substitution. The system approach applied in this paper involves imposing restrictions on the CES function that allow for the economic identification of σ, while permitting a variety of forms of technological change.[4]

Table 1 summarises the results for several specifications for the South African economy.[5] TFP estimates are similar when imposing different restrictions on the model, i.e: Factor augmenting (where capital and labour can have different rates of technical change), Hicks neutrality (where technical progress is assumed to be the same for both labour and capital), Solow neutrality (labour augmenting technical progress is zero and capital augmenting technical progress is positive) or Harrod neutral technical change (capital augmenting technical progress is zero and labour augmenting technical progress is positive). Under a factor augmenting specification, implied TFP growth is 1.7 percent per annum when technical change is time-varying and an unrealistic 6.2 percent when it is assumed to be constant. Implied TFP growth averages 1.6 and 1.7 percent under the two Hicks neutral specifications, 1.6 under Harrod neutral and 1.3 percent under a Solow neutral model.

The scale parameter (ξ) is close to unity in all cases, as expected. The elasticity of substitution σ is estimated to be 0.44 in the general specification allowing for time variation in technical growth (Column 1), 0.97 when assuming constant growth in the general specification (Column 2), 0.56 when assuming time-varying Hicks neutral technical change (Column 4), 0.77 in the constant growth Hicks neutral case (Column 5), 1.2 under a Harrod neutral specification (Column 7) and 0.91 under a Solow neutral specification (Column 8). Columns 3 and 6 show the impacts of fixing the capital share α in estimation at its calibrated level to maximise the number of data points per parameter to be estimated. For all specifications, Wald tests reject a unity restriction to σ.

Under a general factor-augmenting specification in Column 1, the estimate for capital-augmenting growth ($\hat{\gamma}_K$) is 0.1 percent, while labour-augmenting growth ($\hat{\gamma}_L$) is 0.4 percent. This implies net labour augmentation, and together with $\hat{\sigma} < 1$ is consistent with the stability in the labour share over the sample. While capital-augmenting technical progress is estimated to be exponential but decelerating, labour-augmenting technical progress is hyperbolic but asymptotes toward zero.

[4]This paper is focused on estimating TFP and the elasticity of substitution for the aggregate South African economy using CES production functions. The specification used implicitly assumes that technologies are common across sectors. For a South African assessment of industry-variation in technology, see Rankin et al. (2015) and Kreuser and Newman (2018).

[5]Given the 5 series being used, the quarterly sample provides 365 data points for the seven parameters in the least restricted model specification.

When assuming constant growth instead (Column 2), rapid capital augmenting growth of 6.8 percent is offset by rapid regress in labour-augmenting growth ($\hat{\gamma}_L$) of about 6.4 percent. However, the time-varying specification is preferred over the constant growth model for its superior determinant and model fit (based on root mean squared errors (RMSE)). When Hicks neutrality is assumed (ie. where technical progress is assumed to be symmetric for labour and capital) in Columns (4 and 5), factor-augmenting technical progress is estimated to be 0.2 and 0.3 percent in the time-varying and constant growth specifications, respectively. The most general model specification in Column 1 is preferred to both Hicks neutral specifications since it has a lower determinant and a likelihood ratio tests reject the more restricted specifications. Likewise, the most general specification is favoured over the Harrod- and Solow-neutral specifications. ADF unit root tests also do not reject the null of non-stationarity of the residuals of the constant growth Hicks, Harrod neutral and Solow neutral specifications.

The results from this paper show that the elasticity of substitution is estimated to be well below unity in most specifications. An aggregate elasticity below one implies, in a standard neo-classical growth model, that an increase in the capital-to-labour ratio would, ceteris paribus, be associated with an increase in the labour share. Although there has been capital deepening in South Africa, the estimates suggest that productivity growth of capital has not been fast enough (recall also the fall in the output-capital ratio in Figure 2) for changes in the effective capital-labour ratio[6] to meaningfully raise the labour share.

[6] Expressed as $\frac{\gamma_t^K K}{\gamma_t^L L}$ with definitions in Appendix section A.

Table 1: Estimates (1999Q1-2017Q1)

	1	2	3	4	5	6	7	8
	Time-Varying Factor-Augmenting Technical Growth	Constant Factor-Augmenting Technical Growth	Calibrated α Factor augmenting	Time-Varying Hicks Neutral Technical Growth	Constant Hicks Neutral Technical Growth	Calibrated α Hicks Neutral	Constant Harrod Neutral Technical Growth	Constant Solow Neutral Technical Growth
ξ	1.022***	1.026***	1.020***	1.021***	1.020***	1.019***	1.022***	1.021***
	(0.002)	(0.010)	(0.022)	(0.002)	(0.026)	(0.022)	(0.026)	(0.026)
α	0.507***	0.502***		0.503***	0.510***		0.513***	0.511***
	(0.001)	(0.010)		(0.001)	(0.001)		(0.013)	(0.011)
σ	0.442***	0.975***	0.648***	0.569**	0.777***	0.778***	1.21***	0.914***
	(0.000)	(0.019)	(0.001)	(0.001)	(0.001)	(0.001)	(0.034)	(0.018)
γ_K	0.001***	0.068***	0.001***	0.002***	0.003***	0.003***		0.006***
	(0.000)	(0.004)	(0.000)	(0.000)	(0.000)	(0.000)		(0.000)
γ_L	0.004***	-0.064***	0.004***	0.002***	0.003***	0.003***	0.006***	
	(0.000)	(0.004)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	
λ_K	0.077**			0.060				
	(0.029)			(0.046)				
λ_L	-0.205***			0.060				
	(0.102)			(0.046)				
	Tests and restrictions							
$\sigma = 1$	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
LR test	-	-	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
RMSE: α	0.0343446	0.0452987	0.0209977	0.0396116	0.0308848	0.0306612	0.0566419	0.0410099
RMSE: $1-\alpha$	0.0240716	0.0463319	0.0582387	0.0425094	0.0512401	0.0506069	0.0415339	0.0470975
RMSE: y	0.0193493	0.017929	0.0417174	0.0183719	0.041191	0.0408139	0.0430947	0.0415204
ADF: α	-1.695*	-1.988**	-3.211***	-2.204**	-3.35***	-3.356***	-1.79*	-0.794
ADF: $1-\alpha$	-4.247***	-2.050***	-2.762***	-3.500***	-2.912***	-2.914***	-1.963**	-0.715
ADF: y	-2.604**	-2.997***	-0.515	-2.415**	-1.333	-0.629	-0.549	-0.44
TFP growth (annualised)	0.017	0.062	0.023	0.016	0.017	0.017	0.016	0.013
Determinant	4.65E-14	3.58E-13	5.28E-13	4.42E-13	5.73E-13	5.69E-13	6.43E-13	5.85E-13

*, ** and *** indicate 10, 5 and 1 percent level of significance. ( ) denotes standard errors based on Stata's nlsur command using Gauss-Newton regression and [ ] denotes p-values. Wald tests used to test coefficient restrictions and 'LR test' denotes likelihood ratio test. TFP growth calculated using the Kmenta approximation. Lag selections for unit root tests based on the Ng-Perron criteron using Dickey-Fuller GLS tests in Stata.

5 Comparison of results to other estimates

To estimate TFP, statistical agencies typically use index number approaches that do not separately identify the different parameters of the production function. This dramatically simplifies estimation: TFP calculations can be done in a spreadsheet. A Cobb-Douglas form is most commonly used, and this usually involves weighting each factor by its income share under the assumption of constant returns to scale, unitary elasticity of substitution and perfectly competitive markets. In a South African context, Productivity South Africa calculates TFP residually from an equation that captures changes in real output, labour inputs and capital inputs. The alternative estimates presented in this paper are based on those of Botha et al. (2018) who use TFP a semistructural framework that includes a Cobb-Douglas production function, estimated using a filtering approach. The disadvantage of the estimation approach used in this paper is that it assumes constant growth in TFP since 1999, and is therefore not able to provide insights into whether TFP has slowed in recent years.

Table 2 compares the estimates from this paper to other TFP estimates for South Africa. The table shows that the estimates produced in this paper are higher than those from Botha et al. (2018) which are based on the same data, and have averaged about 1.1 percent since 2000. Productivity South Africa's estimates for the period 1999-2015 are 1.5 percent. Estimates from The Conference Board (2018) have averaged only 0.1 percent between 1999 and 2016, while those from the Penn World Tables (Feenstra et al. (2015)) have averaged about -0.3 percent per annum between 1999 and 2014.

Table 2: Comparison of TFP estimates for South Africa

	Sample	TFP estimate (average, percent)
Factor Augmenting	1999Q1-2017Q1	1.7
Hicks neutral	1999Q1-2017Q1	1.6
Harrod neutral	1999Q1-2017Q1	1.6
Solow neutral	1999Q1-2017Q1	1.3
Botha et al.	2000Q1-2017Q1	1.1
Productivity SA	1999-2015	1.5
Conference Board	1999-2016	0.1
Penn World Table	1999-2014	-0.3

Table 3 presents various estimates of the elasticity of substitution for South Africa. The finding that the elasticity of substitution is generally below one at aggregate level is in line with Bonga-bonga (2009) (with an estimate of 0.13 covering 1970-2006) and industry estimates for South Africa by Rankin et al. (2015) (covering 1994-2012).

Table 3: Comparison of σ estimates for South Africa

	Steenkamp	Rankin et al
Factor Augmenting	0.44-0.97	1.32
Hicks neutral	0.57-0.78	0.99
Harrod neutral	1.21	0.84
Solow neutral	0.91	1.52

6 Implied levels of potential and the output gap

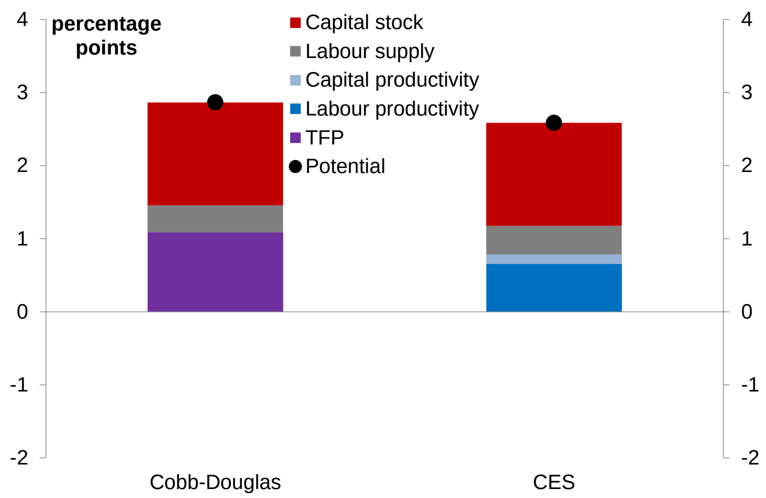
This paper also backs-out estimates of potential output (usually defined as the level of output consistent with stable inflation) and the output gap (defined as the difference between actual output and its potential level) based on various CES production functions.[7] Performing growth decompositions based on CES production functions is more challenging than when Cobb-Douglas production functions are estimated since CES functions are non-linear as there can be many possible combinations of parameter values that can fit the function. This paper constructs estimates of potential using the fitted values of the estimated CES production functions by decomposing estimated real output using a first order Taylor expansion following Felix and Almeida (2006) (see Steenkamp 2018 for details). The definition of the potential (and the output gap) here is therefore very different from those on which estimates produced by filters or production functions where TFP is calculated residually are based. The estimate of potential implied by the estimates in this paper is consistent with the estimated structural parameters, observed variation in factor shares and factor accumulation.

Figure 3 presents growth decompositions based on the preferred CES specification when assuming constant TFP growth to simplify the growth accounting.[8] In the factor augmenting specification, potential output is estimated to have averaged 2.6 percent since 2000Q1. Capital stock growth added approximately 1.4 percent to potential output, while an increase in labour supply contributed 0.4 percent. Capital- and labour-augmenting technical change contributed and estimated 0.1 and 0.7 percent, respectively. The estimates therefore suggest that productivity played a smaller role than factor accumulation in the growth of potential.

[7] The estimation approach is described in more detail in Steenkamp (2018).

[8] The model used in this section is from column 3 in Table 1 in the Appendix. Though not the global optimum, alternative model specifications produced (even more) implausible estimates of the output gap.

Figure 3: Growth decomposition (2001Q1 to 2017Q1)



The South African Reserve Bank uses a suite of models to estimate South Africa's potential growth, which includes statistical filters, semistructural multivariate filter models, a production function model, and a SVAR model (see Anvari et al. 2014 and Botha et al. 2018 for an overview). Figure 3 compares the CES decomposition to the filter-based decomposition of Botha et al. (2018) and shows that on, average, their estimate has been slightly higher.[9]

Figures 4 and 5 decompose the drivers of potential under the CES and Botha et al. (2018) approaches. Under a CES specification, labour supply and the capital stock have dominated shifts in underlying potential. Extending the approach of Botha et al. (2018), produces a decomposition that suggests that factor accumulation played a similarly important role in raising potential pre-GFC, while TFP has recently been the most important factor responsible for the decline in estimated potential.

Since the preferred CES specification produces an higher estimate of potential in 2017Q1, its output gap estimate is lower than the estimate from Botha et al. (2018) (Figure 6).[10] All else equal, the CES model therefore suggests that there have recently been weaker underlying inflationary pressures than implied by estimates of Botha et al. (2018).

[9]Section B in the Appendix presents alternative estimates of potential over time.

[10]These estimates are lower than filter-based estimates by Fedderke and Mengisteab (2017), which are estimated to have been positive in 2014 and 2015.

Figure 4: CES growth decomposition

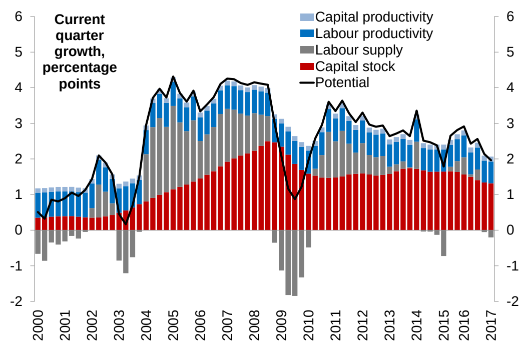


Figure 5: Extension of Botha et al. approach

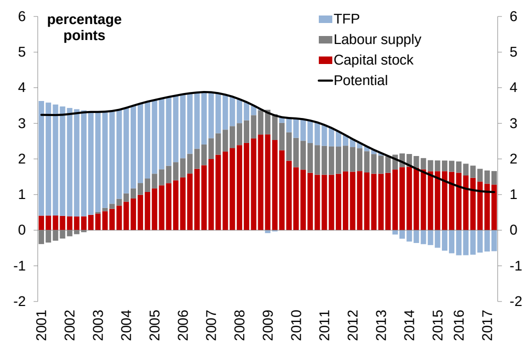
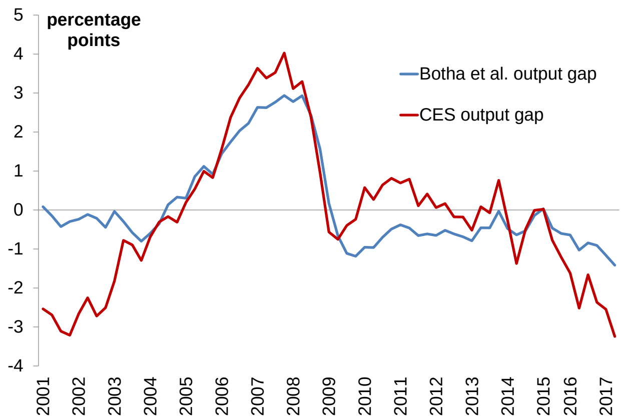


Figure 6: Output gap estimates



7 Conclusion

TFP estimates are typically quite sensitive to the production function, estimation approach and data definitions used. This paper highlights how production function assumptions can affect estimates of TFP for South Africa. The estimates produced are slightly higher than those produced using a Cobb-Douglas approximation or alternative estimates. The finding that the estimate of the elasticity of substitution is below one (i.e. factors are gross complements) is in line with other estimates for South Africa. This suggests that labour-augmenting technical change will tend to increase the labour share, all else equal. The results from this paper also imply that the output gap has recently been lower than the alternative estimates presented and that accumulation of capital and labour played a larger role than productivity in the growth of potential.

References

Anvari, V., N. Ehlers, and R. Steinbach (2014, November). A semi-structural approach to estimate South Africa's potential output. Working Paper WP/14/08, South African Reserve Bank.

Bonga-bonga, L. (2009). The south african aggregate production function: Estimation of the constant elasticity of substitution function. *South African Journal of Economics 77*(2), 332–349.

Botha, B., F. Ruch, and R. Steinbach (2018). Short-lived supply shocks to potential growth. South African Reserve Bank Working Paper 2.

du Toit, C., A. du Toit, N. Ehlers, K. Kuhn, and M. Mashiane (2006). South Africa's growth potential: Prospects and challenges. Discussion Paper DP/06/04, South African Reserve Bank.

Fedderke, J. W. and D. K. Mengisteab (2017). Estimating south africa's output gap and potential growth rate. *South African Journal of Economics 85*(2), 161–177.

Feenstra, R. C., R. Inklaar, and M. Timmer (2015). The Next Generation of the Penn World Table. *American Economic Review 105*(10), 3150–3182.

Felix, R. and V. Almeida (2006). Computing Potential Output and the Output Gap for the Portuguese Economy. *Economic Bulletin and Financial Stability Report Articles*.

Kemp, J. and B. Smit (2015). Estimating and explaining changes in potential growth in South Africa. Technical report.

Klump, R., P. McAdam, and A. Willman (2007, February). Factor Substitution and Factor-Augmenting Technical Progress in the United States: A Normalized Supply-Side System Approach. *Review of Economics and Statistics 89*(1), 183–192.

Klump, R., P. McAdam, and A. Willman (2012). The normalized ces production function: Theory and empirics. *Journal of Economic Surveys 26*(5), 769–799.

Kreuser, C. F. and C. Newman (2018). Total factor productivity in south african manufacturing firms. *South African Journal of Economics 86*, 40–78.

Rankin, N., R. Burger, and F. Kreuser (2015). The elasticity of substitution and labour-displacing technical change in post-apartheid South Africa.

Steenkamp, D. (2018). Factor Substitution and Productivity in New Zealand. *Economic Record 94*(304), 64–79.

The Conference Board (2018). The Conference Board Total Economy Database. Retrieved January 2018: http://www.conference-board.org/data/economydatabase/.

Appendices

A CES production function estimation

The full system to be estimated is:

$$\ln(\frac{Y_t}{L_t}) = \ln(\frac{A\tilde{Y}_t}{L_t}) + \gamma^L(t-\bar{t}) - \frac{\sigma}{\sigma-1}\ln[\bar{\alpha}e^{\frac{1-\sigma}{\sigma}(\gamma^L(t-\bar{t})-\gamma^K(t-\bar{t}))}(\frac{K_t/\tilde{K}}{L_t/\tilde{L}})^{\frac{\sigma-1}{\sigma}} + (1-\bar{\alpha})] \quad (1)$$

$$\ln(\frac{R_tK_t}{P_tY_t}) = \ln(\frac{\bar{\alpha}}{1+\mu}) + \frac{1-\sigma}{\sigma}[\ln(\frac{Y_t/\tilde{Y}}{K_t/\tilde{K}}) - \ln(A) - \gamma^K(t-\bar{t})] \quad (2)$$

$$\ln(\frac{W_tL_t}{P_tY_t}) = \ln(\frac{1-\bar{\alpha}}{1+\mu}) + \frac{1-\sigma}{\sigma}[\ln(\frac{Y_t/\tilde{Y}}{L_t/\tilde{L}}) - \ln(A) - \gamma^L(t-\bar{t})] \quad (3)$$

where ˜ denotes geometric mean and ¯ denotes arithmetic mean. The scaling parameter A is a 'normalisation constant' which has an expected value of close to one (see Klump et al. 2007). μ is a mark-up, which under the assumption of perfect competition equals zero (and ignored in this paper). Following Klump et al. (2007), additional curvature parameters can be estimated to allow for time-varying TFP growth. To calculate γ_j for $j = K, L$, a Box-Cox transformation can be applied $\gamma^j(t-\bar{t}) = \frac{\gamma_j t_0}{\lambda^j}[(\frac{t}{t_0})^{\lambda^j} - 1]$, where $t > 0$, $t_0 = \bar{t}$ and λ is the curvature parameter.[11] This implies that when $\lambda = 1$ technical progress is linear (i.e. constant), $\lambda = 0$ it is log-linear and when $\lambda < 0$ it is hyperbolic. An important advantage of using a CES specification is that σ is estimated jointly with TFP and different restrictions on the parameters in the model can be tested empirically. Regarding the latter, this paper explicitly tests three common neutrality conditions: (1) Solow neutral technology ($\gamma^K > 0$ and $\gamma^L = 0$); (2) Harrod neutral technical change ($\gamma^K = 0$ and $\gamma^L > 0$); and (3) Hicks neutral technological change ($\gamma^K = \gamma^L > 0$). TFP growth can be backed out using what is known as the Kmenta approximation.[12] Estimation uses seemingly unrelated regression using Stata's iterative feasible generalized non-linear least squares estimator (see Steenkamp 2018 for details).

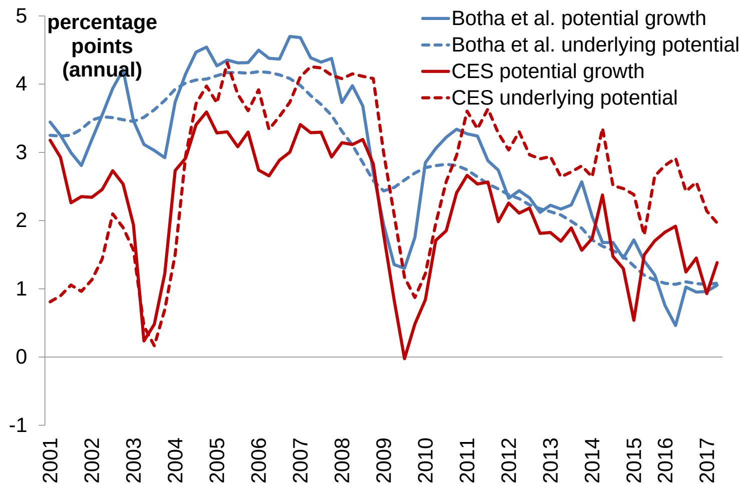
[11] Assuming $e^{\gamma^K} = e^{\gamma^L} = 1$ at the point of normalisation (t_0) ensures that factor shares are exactly equal to $\bar{\alpha}$ and $1-\bar{\alpha}$, respectively, at time t_0 (Klump et al. 2012).

[12] That is, $\frac{dTFP}{dt} = (\bar{\alpha})\gamma_K + (1-\bar{\alpha})\gamma_L - \frac{1-\sigma}{\sigma}(\bar{\alpha})(1-\bar{\alpha})(\gamma_K - \gamma_L)(\frac{d\gamma_L}{dt} - \frac{d\gamma_K}{dt})$.

B Additional potential estimates

While the estimates of potential of Botha et al. (2018) have been higher on average since 2000, they have recently been lower than those from the CES model. Figure 7 compares CES-based estimates of potential from the estimated production function (labelled 'potential growth') and 'underlying potential' based on a growth decomposition to the estimates of Botha et al. (2018) from estimated production function and growth decompositions, respectively. While the CES model is estimated using a systems approach as in Steenkamp (2018) and growth decompositions are based on the linearised production function, the Cobb-Douglas production function estimates of Botha et al. (2018) is estimated using a filter approach, and its growth decomposition similarly uses a filtering methodology. These estimates are slightly lower than filter-based estimates by Fedderke and Mengisteab (2017).[13]

Figure 7: Potential estimates



[13]The underlying potential estimates from a growth decomposition of Botha et al. (2018) are also similar to those from Kemp and Smit (2015). See du Toit et al. (2006) for earlier estimates of potential.