



# **Welfare analysis of bank capital requirements with endogenous default**

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# Welfare analysis of bank capital requirements with endogenous default

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## Abstract

This paper presents a tractable framework with endogenous default and evaluates the welfare implication of bank capital requirements. We analyze the response of social welfare to a negative technology shock under different capital requirement regimes with and without default. We show that including default as an additional indicator of capital requirements is welfare improving. When implementing capital requirements, a more aggressive reaction to the default rate is more effective for weakening the negative effect of the shock on welfare. Compared with output gap, the credit-to-output gap is a better indicator for implementing the countercyclical capital buffer.

*JEL Classification:* E44, E47, E58, G28

*Keywords:* Bank capital requirement, Default, Welfare, DSGE

## 1 Introduction

This paper develops a real business cycle model with endogenous default, and investigates the welfare implication of bank capital requirements when the default rate is considered as an additional indicator. [Van den Heuvel \(2008\)](#) argues that it is critical to understand the welfare implication of bank capital requirements as one would simply raise capital adequacy ratio to 100% if there were no costs of implementing capital requirements. In this paper we argue that changes in the loan default rate can have a significant impact not only on financial intermediaries' bank capital, but also on borrowers' balance sheet and future borrowing capability. It is, therefore, critical for regulatory authorities to consider the effect of default on capital requirements and the implications for social welfare.

Our welfare analysis of capital requirements is related to [Van den Heuvel \(2008\)](#) and [Angeloni and Faia \(2013\)](#). Using a general equilibrium growth model with liquidity-creating banks, [Van den Heuvel \(2008\)](#) investigates the welfare cost of Basel I and II, and shows that it is equivalent to a 0.1% to 1%

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permanent loss in consumption. [Angeloni and Faia \(2013\)](#) report similar findings on Basel II: risk-weighted capital requirements amplify the cycle and are welfare deteriorating. Basel III, on the other hand, is welfare improving. The above mentioned two articles, however, do not take into the consideration of default when investigating the welfare cost of capital requirements.

The Bank for International Settlements (BIS hereafter) has been consistently emphasizing the critical role of default played in the bank capital requirement decision makings (e.g., [BCBC, 2009, 2010](#); [BCBS, 2011](#)). On the academic front, among others, [Catarineu-Rabell et al. \(2005\)](#) evaluate three possible scenarios where risk weights assigned to bank assets are constant, or depend positively or negatively on the probability of default. The authors find that setting risk weight positively to default is desirable from the regulation point of view. The countercyclical capital buffer of Basel III requires banks to increase their holdings of capital during economic booms. This precautionary regulation aims to curtail credit booms that might end in financial crises. However, not all credit booms lead to crises (see, [Bakker et al., 2012](#)). The probability of default can be a good candidate for correcting this potential error: non-beneficial reductions in bank loans. In this paper, we therefore augment the default rate in capital requirement regimes and study the implications for welfare.

Which indicator should be used when implementing the countercyclical capital buffer of Basel III is an empirical question. The BIS suggests that the difference between the aggregate credit-to-output ratio and its long term trend can be a good candidate ([BCBC, 2009](#)). There is, however, no general consensus on this. Some studies criticize this indicator (credit-to-output gap) proposed by the BIS. For instance, [Drehmann and Tsatsaronis \(2014\)](#) argue that the credit-to-output gap is not useful as a warning indicator of banking crises, especially for emerging market economies. [Repullo and Saurina \(2011\)](#) suggest that regulatory authorities should use output growth as the indicator when implementing the countercyclical capital buffer, instead of the credit-to-output gap. Some studies suggest that excessive credit growth is a valid indicator for potential banking crises (e.g., [Lowe and Borio, 2002](#)); some suggest the aggregate credit contains information of the likelihood of future financial distresses (e.g., [Schularick and Taylor, 2012](#)); and some suggest both deviations of aggregate output and credit from their steady states should be considered (e.g., [Resende et al., 2013](#)). In this paper, we consider output gap and credit-to-output gap as potential candidates.

The contribution of this paper is three-fold. First, to the best of our knowledge, this is the first attempt to study the welfare implication of capital requirements with endogenous default. Second, we investigate whether the proposed countercyclical capital buffer of Basel III does a better job than Basel II in terms of welfare. Third, using social welfare as the criterion, we evaluate different potential indicators for the implementation of the countercyclical capital buffer.

To study the welfare implication of different capital requirement regimes with endogenous default, we develop a real business cycle model (RBC) with banking, in which borrowers may default on their financial obligations. We introduce endogenous default along the lines of [Shubik and Wilson \(1977\)](#) and [de Walque et al. \(2010\)](#), where borrowers may default on the loans borrowed from the previous period upon paying a penalty cost. We then examine the social welfare response to a negative technology shock under different capital requirement regimes with and without default, that is, a capital requirement regime responding to

the default rate, or otherwise.

By augmenting endogenous default in a capital requirement regime we introduce a stabilizer not only in the financial sector but also in the real sector. First, the imposed penalty costs provide firms incentives not to, or default less on bank loans. Second, banks benefit from the augmented capital requirement rule as banks are more profitable and better capitalized with a lower default rate. Banks are, therefore, able to accumulate more funds and supply more credit to firms. This is, in turn, beneficial for production. Last, households can consume and invest (in the form of deposits) more with a higher production. Ex ante, a capital requirement regime responding to the default rate is welfare improving.

The capital requirement regimes studied in this paper are as follows. Following [Angeloni and Faia \(2013\)](#), we assume a fixed rate of bank capital requirement for Basel I. Both Basel II and III evolve a Taylor-type rule. In the case of Basel II, the capital requirement reacts negatively with respect to output gap. There are two specifications for Basel III. For the first specification, namely Basel III, the capital requirement reacts positively to output gap; and for the second specification, namely Basel III credit-to-output, the capital requirement reacts positively to the credit-to-output gap. We then augment the default rate gap (deviation from its steady state) into Basel II, Basel III, and Basel III credit-to-output.<sup>1</sup>

The welfare analysis results are as follows. First, introducing default in Basel II, Basel III, and Basel III credit-to-output is welfare improving in all cases. It is through the bank funding channel that introducing default in capital requirement regimes attenuates the negative effect of the shock on welfare. Moreover, a more aggressive reaction to default is more effective in mitigating the negative effect of the shock. Second, compared with Basel II the countercyclical capital buffer (both Basel III and Basel III credit-to-output regimes) is slightly welfare deteriorating. Last, between Basel III and Basel III credit-to-output, the later gives a better performance and, hence, credit-to-output gap can serve as a better indicator for implementing the countercyclical capital buffer, as opposed to output gap. This conclusion is obtained based on the analysis of both the first and second moments of social welfare, complemented by the analysis on the transmission mechanisms through which introducing default in the capital requirement regimes attenuates the negative effect of the shock on welfare.

The rest of the paper is structured as follows. [Section 2](#) describes the model. [Section 3](#) presents the functional forms and parameters values in the model. [Section 4](#) discusses the welfare analysis results and [Section 5](#) concludes.

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<sup>1</sup>We only consider the case where the capital requirement reacts positively to the default gap since the otherwise makes no intuitive sense.

## 2 The model

This section describes the structure of the model and the interaction between agents in the model economy. The model economy is inhabited by households, firms, banks, and a government. Banks intermediate credit between borrowers (firms) and savers (households), facing capital requirement regulation imposed by the government.

Following [de Walque et al. \(2010\)](#), we introduce endogenous probability of default for firms, whereby deposits are safe assets for households. Firms accumulate physical capital with own profits and bank loans. In each period, firms may default on a fraction of loans borrowed from previous period upon paying a penalty cost. We assume households demand liquidity (deposits) and deposits yield utility in the spirit of [Sidrauski \(1967\)](#). We introduce deposits as households' assets and one kind of banks' liabilities, and model the preferences in a less restrictive way, not depending on modeling choices ([Van den Heuvel, 2008](#)).<sup>2</sup> For simplicity, we assume banks do not default on deposits. Banks supply loans to firms and finance these loans with deposits and own funds (capital). We further assume banks can recover a fraction of defaulted loans upon paying an insurance premium to the government. In our welfare analysis, we consider various types of bank capital requirements that banks face, which are explained in the introduction.

### 2.1 Households

There is a continuum of identical households with mass one. In each period households consume consumption goods,  $C_t$ , and hold bank deposits,  $D_t$ . Households supply labor,  $H_t$ , inelastically to firms and receive a wage of  $W_t$ .<sup>3</sup> Households are subject to lump-sum taxes,  $T_t$ . Households maximize their expected discounted utility as:

$$\max_{\{C_t, D_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t, D_t), \quad (1)$$

subject to the budget constraint:

$$C_t + D_t + T_t = R_{t-1}^d D_{t-1} + W_t H_t + (1 - \nu_b) \pi_{t-1}^b + (1 - \nu_f) (1 - \xi_f) \pi_{t-1}^f, \quad (2)$$

where  $\beta \in (0, 1)$  is the discount factor and  $R_t^d$  is the gross rate of return on deposits. Households receive dividend payments from banks,  $(1 - \nu_b) \pi_{t-1}^b$ , and firms,  $(1 - \nu_f) (1 - \xi_f) \pi_{t-1}^f$ , where  $\nu_b$  represents the fraction of profits banks keep as retained earnings and  $\nu_f$  represents the fraction of profits that firms reinvest in physical capital.  $\xi_f$  captures firms' administrative expenses.

Considering  $\lambda_t^h$  as the Lagrange multiplier of the budget constraint, the first order conditions (FOCs) for households are as follows:

$$C_t : u_{C_t}(C_t, D_t) + \beta \mathbb{E}_t u_{C_t}(C_{t+1}, D_{t+1}) = \lambda_t^h, \quad (3)$$

$$D_t : u_{D_t}(C_t, D_t) = \lambda_t^h - \beta \mathbb{E}_t \lambda_{t+1}^h R_{t+1}^d, \quad (4)$$

<sup>2</sup>We acknowledge that choices of utility function affect welfare analysis results. This is, however, beyond the scope of the current study.

<sup>3</sup>For simplicity we normalize labor supply to 1.

where  $u_C$  and  $u_D$  are the partial derivatives of the utility function with respect to consumption and deposits, respectively. [Equation 3](#) is the consumption Euler equation and [Equation 4](#) is the asset pricing equation for deposits.

## 2.2 Firms

Firms produce output,  $Y_t$ , using a standard Cobb-Douglas function,  $Y_t = A_t F(K_t, H_t)$ , where function  $F$  is homogenous of degree one. The technology shock,  $A_t$ , follows a logarithmic AR(1) process. The size of firms in the economy is normalized to one. In each period firms demand an amount of loans,  $L_t$ , from banks and invest in physical capital,  $K_t$ . The law of motion for physical capital is determined as follows:

$$K_t = (1 - \delta) K_{t-1} + L_t + \nu_f (1 - \xi_f) \pi_{t-1}^f, \quad (5)$$

where  $\delta \in (0, 1)$  is the depreciation rate of physical capital. We assume a fraction  $\xi_f$  of profits are used as administrative expenses, and only a fraction  $\nu_f$  of net profits are reinvested in physical capital. We introduce these frictions to pin down the fraction of profits that firms pay out as dividends and, more importantly, to ensure that in equilibrium firms cannot retain enough earnings so that they are completely self-financed. In equilibrium, firms borrow a positive amount of loans to finance the gap between capital depreciation and the amount of net profits that are reinvested in capital:<sup>4</sup>

$$L = \delta K - \nu_f (1 - \xi_f) \pi^f. \quad (6)$$

In each period firms may choose to default a fraction  $\chi_t$  of loans borrowed from the previous period. Following [de Walque et al. \(2010\)](#), we assume firms have to pay a penalty cost (or reputation losses) in the following period based on the amount of loans defaulted, i.e.,  $z(\chi_{t-1}, R_{t-2}^l, L_{t-2})$ , where  $R_t^l$  is the gross rate of return on loans. The function  $z$  is assumed to be non-negative, and at least twice differentiable on  $\chi_t$ ,  $R_t^l$ , and  $L_t$ . Moreover, the partial derivatives of function  $z$  with respect to its arguments,  $z_\chi$ ,  $z_{R^l}$ , and  $z_L$ , are non-negative.

The representative firm's profits are defined as follows:

$$\pi_t^f = A_t F(K_t, H_t) - W_t H_t - (1 - \chi_t) R_{t-1}^l L_{t-1} - z(\chi_{t-1}, R_{t-2}^l, L_{t-2}). \quad (7)$$

The representative firm maximizes the discounted sum of profits as:

$$\max_{\{\chi_t, H_t, K_t, L_t, \pi_t^f\}_{t=0}^\infty} \mathbb{E}_0 \sum_{t=0}^\infty \beta^t \log(\pi_t^f), \quad (8)$$

subject to the law of motion of capital [\(5\)](#) and the firm's profits [\(7\)](#).

Denoting  $\lambda_t^a$  and  $\lambda_t^b$  as the Lagrange multipliers of [Equation 5](#) and [Equation 7](#), the first order conditions

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<sup>4</sup>A letter without time subscript represents its corresponding steady state.

are:

$$\chi_t : \lambda_t^b R_{t-1}^l L_{t-1} = \beta \mathbb{E}_t \lambda_{t+1}^b z_\chi (\chi_t, R_{t-1}^l, L_{t-1}), \quad (9)$$

$$H_t : A_t F_H (K_t, H_t) = W_t, \quad (10)$$

$$K_t : \lambda_t^b A_t F_K (K_t, H_t) = \lambda_t^a - \beta (1 - \delta) \mathbb{E}_t \lambda_{t+1}^a, \quad (11)$$

$$L_t : \lambda_t^a = \beta \mathbb{E}_t \lambda_{t+1}^b (1 - \chi_{t+1}) R_t^l + \beta^2 \mathbb{E}_t \lambda_{t+2}^b z_L (\chi_{t+1}, R_t^l, L_t), \quad (12)$$

$$\pi_t^f : \frac{1}{\pi_t^f} + \nu_f \beta (1 - \xi_f) \mathbb{E}_t \lambda_{t+1}^a = \lambda_t^b, \quad (13)$$

where  $F_K$  and  $F_H$  are the partial derivatives of the production function with respect to capital and labor. Equation 9 equalizes the marginal cost of defaulting with its possible benefit. Equation 10 gives the optimal wage. Equation 11 shows the marginal product of capital as the difference between its shadow value today and its discounted shadow value tomorrow. Equation 12 equalizes the shadow value of loans to its costs, which is its discounted rate of return when there is no default ( $\chi_{t+1} = 0$ ). When default occurs the shadow value of loans today equals to the sum of its discounted rate of return and the marginal penalty cost of loans defaulted.

In steady state, Equation 9 implies that default probability depends negatively on the penalty costs of default:

$$\chi = \frac{1}{\beta \omega_z R^l L}, \quad (14)$$

where  $\omega_z$  is the parameter that measures the size of the penalty costs.<sup>5</sup> By equalizing Equation 11 and Equation 12, we have, in steady state:

$$\chi = \frac{\{F_K(K, H) - \beta R^l [1 - \beta(1 - \delta)]\} \omega_z L + [1 - \beta(1 - \delta)]}{\beta \omega_z R^l L [1 - \beta(1 - \delta)]}. \quad (15)$$

Given that the second term in the numerator is positive, a sufficient condition for the existence of a positive default rate in equilibrium is:

$$F_K(K, H) > \beta R^l [1 - \beta(1 - \delta)]. \quad (16)$$

That is, in equilibrium, the marginal product of capital must be greater than the right hand side of (16).

## 2.3 Banks

As in the case of firms, there is a continuum of banks with mass one in each period. Table 1 shows the representative bank's balance sheet, where loans are on the asset side, and deposits and own funds are on the liability side.

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<sup>5</sup>See Section 3 for the details of the functional forms.

Table 1: Balance sheet.

Assets		Liabilities	
Loans	$L_t$	Deposits	$D_t$
		Funds	$F_t^b$

The representative bank's profits are defined as follows:

$$\pi_t^b = [(1 - \chi_t) R_{t-1}^l L_{t-1} - R_{t-1}^d D_{t-1} + q(\chi_{t-1}, R_{t-2}^l, L_{t-2})]^+, \quad (17)$$

where the notation  $[x]^+$  is  $\max\{x, 0\}$ , which guarantees that banks' profits are not negative. Following [Van den Heuvel \(2008\)](#) we assume that deposits are fully insured by the government. Banks are able to recover  $q(\chi_{t-1}, R_{t-2}^l, L_{t-2})$  fraction of defaulted loans through insurance. The function  $q$  is non-negative and once differentiable, and the partial derivatives with respect to its arguments,  $q_\chi$ ,  $q_{R^l}$ , and  $q_L$ , are non-negative.

The representative bank's maximization problem is defined as follows:

$$\max_{\{D_t, F_t^b, L_t, \pi_t^b\}_{t=0}^\infty} \mathbb{E}_0 \sum_{t=0}^\infty \beta^t \log(\pi_t^b), \quad (18)$$

subject to the balance sheet ([Table 1](#)), bank's profits ([17](#)), and the following restrictions:

$$F_t^b = (1 - \xi_b - \varsigma) F_{t-1}^b + \nu_b \pi_{t-1}^b, \quad (19)$$

$$F_t^b \geq \tau_t L_t, \quad (20)$$

where [Equation 19](#) is the law of motion for bank own funds and [Equation 20](#) is the minimum bank capital requirement constraint. The representative bank needs to pay a fraction  $\varsigma$  of own funds to the insurance authority (the government). A fraction  $\xi_b$  of profits are being used to cover the administrative expenses, whereby a fraction  $\nu_b$  of net profits are kept as retained earnings. We introduce the administrative expenses to pin down the fraction of profits that banks pay out as dividends and insurance premium. More importantly, we calibrate these parameters ( $\xi_b$ ,  $\varsigma$ , and  $\nu_b$ ) such that a positive Lagrange multiplier of the bank capital requirement constraint exists, implying a binding capital requirement constraint in equilibrium. The minimum bank capital requirement rule  $\tau_t$  will be discussed in details in the following section.

Denoting  $\lambda_t^d$ ,  $\lambda_t^e$ ,  $\lambda_t^f$  and  $\lambda_t^g$  as the Lagrange multipliers of the balance sheet, bank's profits, flow of bank funds, and the minimum bank capital requirement constraint, the first order conditions are as follows:

$$D_t : \lambda_t^d = \beta \mathbb{E}_t \lambda_{t+1}^e R_t^d, \quad (21)$$

$$F_t^b : \lambda_t^f - \lambda_t^d - \lambda_t^g = \beta (1 - \xi_b - \varsigma) \mathbb{E}_t \lambda_{t+1}^f, \quad (22)$$

$$L_t : \beta \mathbb{E}_t \lambda_{t+1}^e (1 - \chi_{t+1}) R_t^l + \beta^2 \mathbb{E}_t \lambda_{t+2}^e q_L(\chi_{t+1}, R_t^l, L_t) = \lambda_t^d + \lambda_t^g \tau_t, \quad (23)$$

$$\pi_t^b : \frac{1}{\pi_t^b} + \nu_b \beta \mathbb{E}_t \lambda_{t+1}^f = \lambda_t^e, \quad (24)$$



where  $q_L$  is the partial derivative of function  $q$  with respect to loans. Equation 21 equalizes the shadow value of deposits to its returns. Equation 22 shows that the shadow value of funds equals to its discounted future value after paying insurance premium and administrative costs. Equation 23 gives the shadow value of loans, which equals to the return of loans plus insurance premium.

The bank capital requirement (20) is always binding in steady state and bank own funds equals a fraction  $\tau$  of loans,  $F^b = \tau L$ . In steady state, Equation 23 becomes:

$$\lambda^g = \frac{\beta\lambda^e [(1-\chi)R^l + \beta\omega_q\chi R^l - R^d]}{\tau}, \quad (25)$$

where  $\omega_q \in (0, 1)$  is the insurance coverage ratio.<sup>6</sup> We see that  $\lambda^g$  is positive so long as  $\lambda^e$  is positive and the following condition holds:

$$\frac{R^l}{R^d} > \frac{1}{1-\chi(1-\beta\omega_q)}. \quad (26)$$

With a positive default probability in equilibrium, we have  $\frac{R^l}{R^d} > 1$ , in other words, there is always a positive spread between the rates of loans and deposits.

From the steady state condition of Equation 24, we have:

$$\lambda^e = 1/\pi^b + \nu_b\beta\lambda^f, \quad (27)$$

which indicates that  $\lambda^e$  is positive as long as bank profits are positive and  $\lambda^f \geq 0$ . By equalizing Equation 22 and Equation 23, we can solve for  $\lambda^f$ :

$$\lambda^f = \frac{[\beta\lambda^e(1-\chi)R^l + \beta^2\lambda^e\omega_q\chi R^l + \lambda^g(1-\tau)]}{[1-\beta(1-\xi_b-\varsigma)]}, \quad (28)$$

which is positive so long as  $\tau \neq 1$ , and  $\chi \neq 1$ .

In steady state, bank profits (17) becomes:

$$\pi^b = (1-\chi)R^lL - R^dD + \omega_q\chi R^lL. \quad (29)$$

Making use of the bank's balance sheet and the binding bank capital requirement constraint, the necessary condition for bank profits to be positive is:

$$\frac{R^l}{R^d} > \frac{1-\tau}{1-\chi+\omega_q}. \quad (30)$$

Based on our calibration the right-hand-side of (30) is 1, which implies there is a positive spread between the rates of loans and deposits  $\frac{R^l}{R^d} > 1$ .

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<sup>6</sup>See Section 3 for the details of the function forms.

## 2.4 Government

The government plays three roles in the economy: (i) sets lump-sum taxes, (ii) collects and manages insurance fund, and (iii) acts as bank capital regulation authority.

As for the first two tasks, the government sets the lump-sum taxes to meet the need for the insurance repayment:

$$T_t = q(\chi_{t-1}, R_{t-2}^l, L_{t-2}) - z(\chi_{t-1}, R_{t-2}^l, L_{t-2}) - \varsigma F_{t-1}^b. \quad (31)$$

That is, the government finances insurance repayment with the insurance premium paid by banks and the penalty costs paid by firms, whereby if there is a shortfall a lump-sum tax,  $T_t$  is levied on households.

As for the third task, we consider the following types of capital requirement regimes:

$$\tau_t = \tau \left( \frac{Y_t}{Y} \right)^{\phi_y} \left( \frac{\chi_t}{\chi} \right)^{\phi_\chi}, \quad (32)$$

$$\tau_t = \tau \left( \frac{L_t/Y_t}{L/Y} \right)^{\phi_l} \left( \frac{\chi_t}{\chi} \right)^{\phi_\chi}. \quad (33)$$

As pointed out in the introduction, we consider the government follows a Taylor-type of rule, adjusting the capital requirement ratio in response to output gap  $\left( \frac{Y_t}{Y} \right)$  or the credit-to-output gap  $\left( \frac{L_t/Y_t}{L/Y} \right)$ . In addition, we consider the default rate gap  $\left( \frac{\chi_t}{\chi} \right)$  as an additional indicator in different capital requirement regimes. Variables without a time subscript denote steady-state values of the respective variables. The coefficients  $\phi_y$ ,  $\phi_l$ , and  $\phi_\chi$  are related to  $\frac{Y_t}{Y}$ ,  $\frac{L_t/Y_t}{L/Y}$ , and  $\frac{\chi_t}{\chi}$ , respectively.

## 3 Functional forms and parameter values

In this section we show the functional forms in the model and parameter values being used for the welfare analysis.

The functional forms of households' preferences, penalty costs, insurance coverage, production technology, and technology shock are as follows:

$$u(C_t, D_t) = \log(C_t - jC_{t-1}) + \phi_d \log(D_t), \quad (34)$$

$$z(\chi_{t-1}, R_{t-2}^l, L_{t-2}) = \frac{\omega_z}{2} (\chi_{t-1} R_{t-2}^l L_{t-2})^2, \quad (35)$$

$$q(\chi_{t-1}, R_{t-2}^l, L_{t-2}) = \omega_q \chi_{t-1} R_{t-2}^l L_{t-2}, \quad (36)$$

$$Y_t = A_t F(K_t, H_t) = A_t (K_t)^\alpha (H_t)^{1-\alpha}, \quad (37)$$

$$\log(A_t) = \rho_A \log(A_{t-1}) + \xi_t^A. \quad (38)$$

The instant utility function,  $u(C_t, D_t)$ , considers satisfaction from consumption with habit persistence ( $j \in (0, 1)$ ) and deposits, where  $\phi_d > 0$  is a parameter.  $\omega_z > 0$  is the coefficient related to the penalty charged to firms in case firms choose to default.  $\omega_q \in (0, 1)$  is the fraction of the amount of defaulted loans that banks are able to recover from the insurance.  $\alpha \in (0, 1)$  is the capital-output share in the production function.  $\rho_A$

governs the persistence of the productivity shock.  $\xi_t^A$  is the productivity shock, which is normally distributed with zero mean and variance  $\sigma_A^2$ .

In general, we use standard values found in the literature to conduct our welfare analysis. We consider the period length to be one quarter. [Table 2](#) lists the values of the parameters and [Table 3](#) shows the main steady state ratios. The discount factor  $\beta = 0.99$ . The habit persistence parameter  $j$  is fixed at 0.7. This value is close to the estimated value of 0.73 in [Boldrin et al. \(2001\)](#) and the one (0.65) reported in [Christiano et al. \(2005\)](#). The discount factor together with the deposits utility coefficient ( $\phi_d = 0.00137454$ ) imply an annualized return on loans of 5.5%, such that  $R_t^l = 1.01375$ , and an annualized return on deposits of 2%, such that  $R^d = 1.005$ . Both rates assigned here are in line with the literature (e.g., [Iacoviello, 2015](#)). In steady state, the ratio of consumption to output is 76%, whereas the ratio of total taxes to output is 0.24%.

On firms' side, following [de Walque et al. \(2010\)](#) we assume that the steady state default rate of non-financial corporate loans  $\chi = 5\%$ . We calibrate the coefficient of penalty cost for default  $\omega_z = 30.5854$ . The reinvestment rate of firms  $\nu_f = 1/4$ . The administrative expenses count 7% of firms' profits,  $\xi_f = 0.07$ . The value of depreciation rate,  $\delta$ , is set at 3%. In steady state the capital-output ratio is approximately 8.4.

For the banking sector, we set the steady state bank capital requirement at 10%. This implies that, in steady state, the ratio of banks funds over loans  $F^b/L = 10\%$ . We assume that banks are able to recover 85% the loans that firms defaulted from the insurance. We assume banks need to spend 7% of own funds as administrative expenses to pin down insurance premium to 7% of own funds and their re-investment rate to  $\nu_b = 0.137732$ .

Table 2: Parametrization.

Description	Parameter	Value
Capital share in production	$\alpha$	1/3
Discount factor	$\beta$	0.99
Depreciation	$\delta$	0.03
Habit persistence	$j$	0.7
Utility coefficient of deposits	$\phi_d$	0.00137454
Penalty cost coefficient ( $z$ -function)	$\omega_z$	30.5854
Insurance coverage ( $q$ -function)	$\omega_q$	0.85
Technology Persistence	$\rho_A$	0.9
Bank capital requirement	$\tau$	0.10
Insurance payment of banks	$\varsigma$	0.07
Banks administrative costs	$\xi_b$	0.07
Firms administrative costs	$\xi_f$	0.07
Banks reinvestment rate	$\nu_b$	0.137732
Firms reinvestment rate	$\nu_f$	1/4

## 4 Welfare analysis

In this section we first study the welfare implication of different capital requirement regimes with and without endogenous default. We then extend our welfare analysis with different parameterizations in different

Table 3: Main steady state ratios.

Description	Value
Consumption to output	$C/Y = 0.762223$
Taxes to output	$T/Y = 0.00236179$
Capital to output	$K/Y = 8.36608$
Deposits to Loans	$D/L = 0.9$

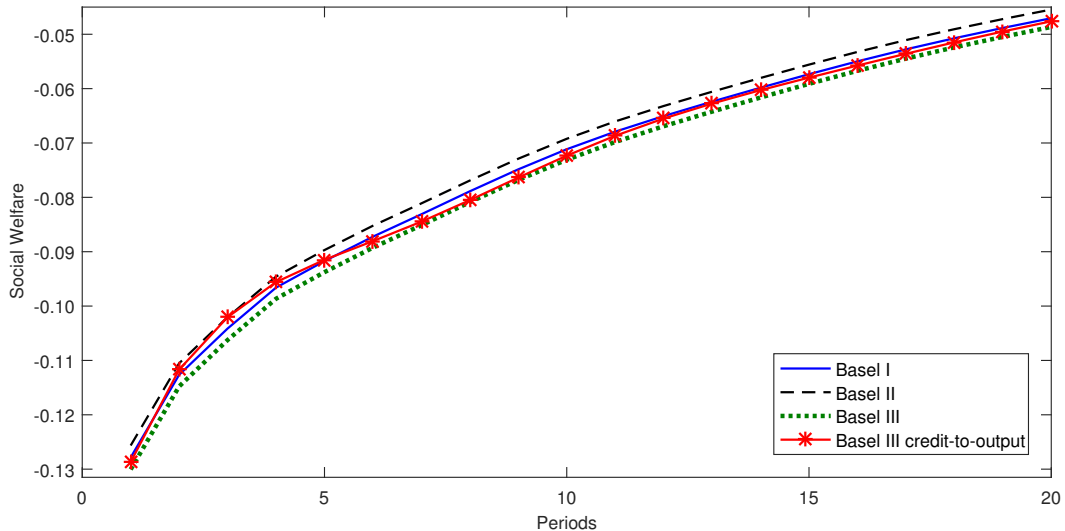


Figure 1: Social welfare IRFs: Basel I, II, III and III credit-to-output, simple Basel rule.

capital requirement regimes. To complement our welfare analysis, in the end, we investigate the transmission mechanisms through which bank capital requirement regimes with endogenous default attenuate the negative effect of the shock on welfare.

Following [Angeloni and Faia \(2013\)](#), we define three Basel capital requirement regimes as follows. By setting  $\phi_y = 0$  and  $\phi_l = 0$  in equations [32](#) and [33](#), we obtain Basel I. As for Basel II, we consider a negative value of  $\phi_y$ , which mimics the pro-cyclicality of Basel II. We reverse the sign of  $\phi_y$ , to obtain Basel III. This captures the counter-cyclical capital buffer regulation proposed in Basel III. For Basel III credit-to-output we set  $\phi_l > 0$ . Finally, by setting  $\phi_\chi = 0$ , we have the scenario of no endogenous default in the capital requirement regime, or the *simple Basel rule*, whereas a positive value of  $\phi_\chi$  implies otherwise, or the *extended Basel rule*. To quantify and numerically evaluate the welfare implication of each case we define the social welfare as follows:  $SW_0 \equiv \sum_{t=0}^{\infty} \beta^t u(C_t, D_t)$ .

#### 4.1 Simple and extended rules: Basel II vs III

We start our analysis by first looking at the social welfare dynamics when implementing the *simple Basel rule*. [Figure 1](#) reports the dynamics of the social welfare in response to a negative technology shock of size 1%. Compared with Basel I (solid line), the social welfare declined lesser under Basel II (dashed line).<sup>7</sup> As

<sup>7</sup>We set  $\phi_y = -0.5$  for Basel II,  $\phi_y = 0.5$  for Basel III, and  $\phi_l = 0.5$  for Basel III credit-to-output, respectively.

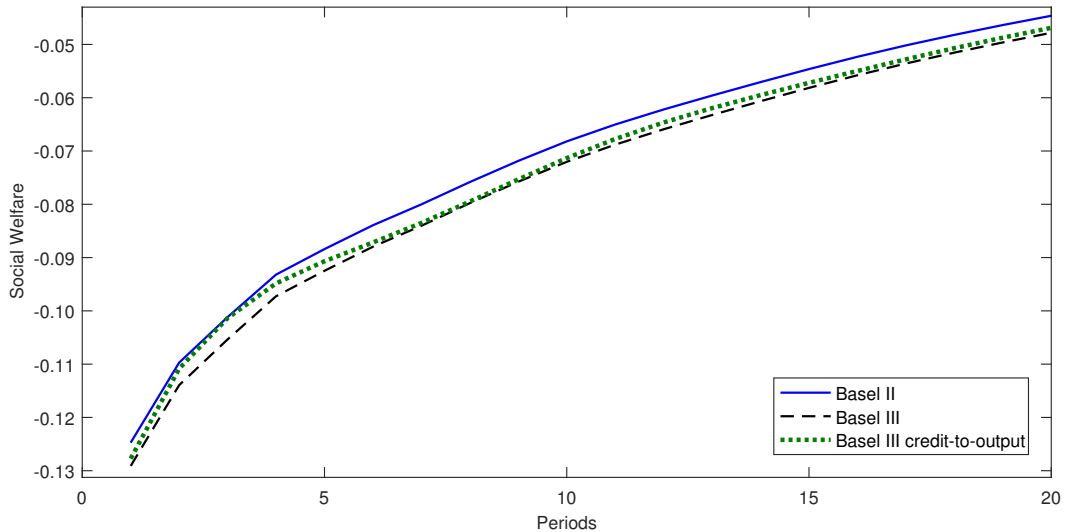


Figure 2: Social welfare IRFs: Basel II, III and III credit-to-output, extended Basel rule.

for Basel III (dotted line), and Basel III credit-to-output (starred line), under both cases the social welfare declined slightly more than that under Basel II. We, therefore, conclude that given our model the welfare cost of Basel III is higher than that of Basel I and II under a *simple Basel rule*.

We now investigate whether the proposed Basel III is welfare improving compared to Basel II, if we introduce endogenous default in the capital requirement regimes. [Figure 2](#) plots the social welfare response to the same shock under the different capital requirement regimes with default, or *extended Basel rule*.<sup>8</sup> As the figure indicates, the same conclusion emerges. That is, the welfare cost under both regimes of Basel III and Basel III credit-to-output is slightly higher than that of Basel II. It is, however, worth noting that, for the first four periods after the shock occurs, Basel III credit-to-output performs almost as good as Basel II, and better than Basel III.

We show that, for both simple and extended Basel rules, there is no improvement for welfare under Basel III compared to its predecessor. This result contradicts the findings of [Angeloni and Faia \(2013\)](#), in which the authors show that the social welfare deteriorates more severe under Basel II. On the other hand, [Repullo and Saurina \(2011\)](#); [Repullo and Suarez \(2013\)](#) argue that Basel III has a potential to exacerbate the procyclicality of its predecessor. This is especially case for using credit-to-output gap as the indicator when implementing the counter-cyclical buffer, because it is negatively correlated with the business cycle. If this is the case, it is unlikely that the implementation of Basel III will improve the social welfare, regardless it is a simple or an extended Basel rule.

## 4.2 Different parametrization of the extended Basel rules

In this section we take our investigation one step further and compare the social welfare response under different capital requirement regimes with vary parametrization under the *extended Basel rule*. We first

<sup>8</sup>We set  $\phi_\chi = 0.15$  and the values of the parameters of  $\phi_y$  and  $\phi_l$  are the same as in the previous exercise. For the rest of our analysis we only focus on Basel II and the two Basel III policy regimes and exclude Basel I.

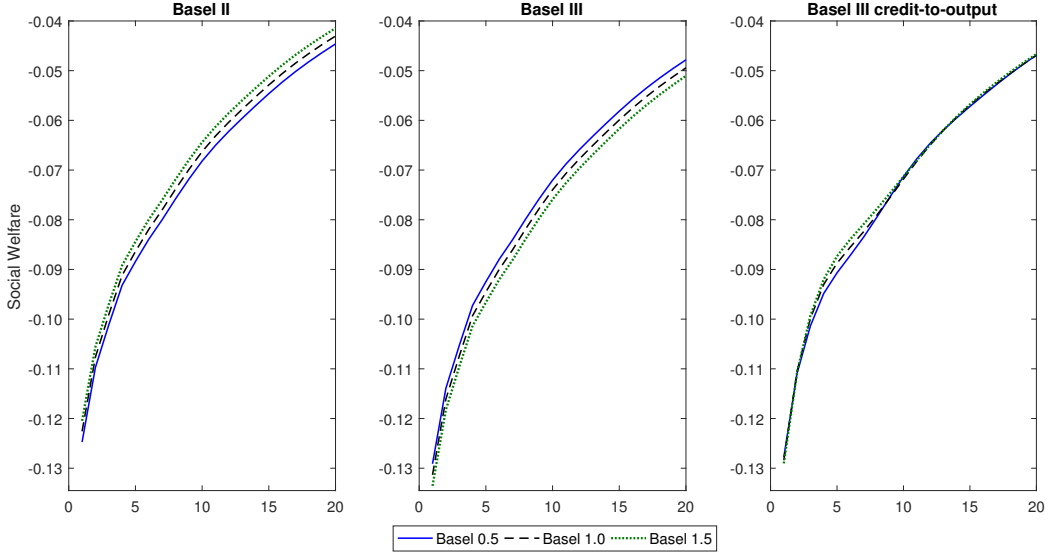


Figure 3: Social welfare IRFs: Basel II, III and III credit-to-output with different values of  $\phi_y$  and  $\phi_l$ .

consider different values for  $\phi_y$  or  $\phi_l$ , while keeping  $\phi_\chi$  constant ( $\phi_\chi = 0.15$ ). The results are reported in Figure 3, where the numbers in the description represent the different values for  $\phi_y$  or  $\phi_l$ . We find that a more aggressive capital requirement regime of Basel II and Basel III credit-to-output mitigates the negative effect of the shock on welfare, while a more aggressive Basel III deteriorates the social welfare. These findings are consistent with those of Resende et al. (2013) and Clerc et al. (2015). Resende et al. (2013) conclude that the counter-cyclical capital requirement is optimal when either it is relatively aggressive or it is fixed (a-cyclical). Clerc et al. (2015) show that business cycle fluctuations are mitigated with a more aggressive Basel III.

We now repeat this experiment but considering different values of the parameter of default, while keeping  $\phi_y$  and  $\phi_l$  constant.<sup>9</sup> Figure 4 reports the response of social welfare with different values of parameter  $\phi_\chi$ : 0 (solid line), 0.15 (dashed line), and 0.3 (dotted line). The results show that the higher the parameter  $\phi_\chi$  is the lesser the social welfare would decline in response to a negative productivity shock. This implies that it is welfare improving if a capital requirement rule reacts to default more aggressively. This is true for all three capital requirement regimes.

To have a better understanding of the welfare response with different parameter values, we report the second moments of social welfare in Figure 5.<sup>10</sup> The left panel displays the variance of social welfare under Basel II and Basel III and the right panel shows the same under Basel III credit-to-output. It is clear that for Basel II a lower variance is obtained with a more aggressive policy. The opposite is true for Basel III and Basel III credit-to-output: a more aggressive policy leads to a higher social welfare variance. In contrast to this mixed results, a higher value of the parameter governing the default ( $\phi_\chi$ ) helps in reducing the variance of social welfare under all regimes. This effect is more significant under Basel III credit-to-output. Considering the most aggressive Basel III credit-to-output ( $\phi_l = 4$ ) in this experiment, the variance of social

<sup>9</sup>We set both  $\phi_y$  and  $\phi_l$  to 0.5 for the two Basel III regimes and  $\phi_y = -0.5$  for Basel II.

<sup>10</sup>We use Dynare to approximate the model around its steady state up to the second order and calculate the second moments of the social welfare by solving the model at each point on a grid of 429 points for parameters  $\phi_y \in [-4, 0)$  and  $\phi_\chi \in [0, 0.3]$  for Basel II,  $\phi_y \in (0, 4]$  and  $\phi_\chi \in [0, 0.3]$  for Basel III, and 221 points for  $\phi_l \in (0, 4]$  and  $\phi_\chi \in [0, 0.3]$  for Basel III credit-to-output.

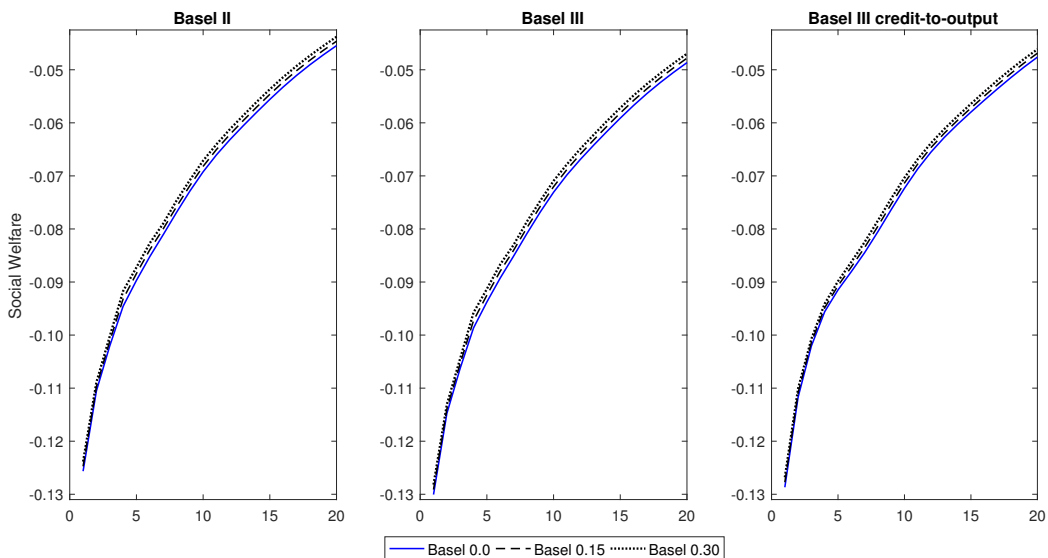


Figure 4: Social welfare IRFs: Basel II, III and III credit-to-output with different values of  $\phi_\chi$ .

welfare will reduce 9.11% if  $\phi_\chi$  increases from 0 to 0.3.

### 4.3 The transmission mechanisms: Simple vs extended rule

In order to have a better understanding of why augmenting the default rate in a capital requirement regime is welfare improving, we investigate the transmission mechanisms through which it mitigates the negative effects of the shock. A negative productivity shock results in an increase in the default rate (see the last row in Figure 7). Since capital requirement is increasing in default, the regulatory authority will increase the capital requirement accordingly based on the extended Basel rule. Figure 6 reports the IRFs of capital requirement ratio in response to the shock under different capital requirement regimes. Under Basel II, the use of the *extended Basel rule* reinforces the regulation. This, however, can potentially exacerbates the pro-cyclicality of Basel II.<sup>11</sup> Under Basel III, in the first four periods after the shock occurs the counter-cyclical capital buffer is offset considerably by augmenting endogenous default in the extended Basel rule. This is especially the case for a more aggressive capital requirement in response to default ( $\phi_\chi = 0.3$ ). The same holds for Basel III credit-to-output, despite the IRF of capital requirement ratio is less volatile.

Figures 7 and 8 report the IRFs of the key variables to a negative technology shock. The solid-line represents the IRFs under the Basel requirement regime specified at the top of each column in its *simple Basel rule* form, and the dashed-line represents the IRFs when implementing the *extended Basel rule* ( $\phi_\chi = 0.3$ ). The results show that it is through the bank funding channel that introducing endogenous default in a capital requirement regime improves social welfare. As discussed earlier, banks benefit from the *extended Basel rule* as banks will be more profitable and better capitalized with a lower default rate (especially in the Basel III credit-to-output). As evidenced in figure Figure 7, this results in a significant recovery in bank funds after the shock, compared with the case under the *simple Basel rule*. Banks can, therefore, supply more credit to

<sup>11</sup>For studies on the pro-cyclicality of Basel II see, e.g. Repullo and Suarez (2013); Liu and Seeiso (2012). This is, however, not the focus of the current study.

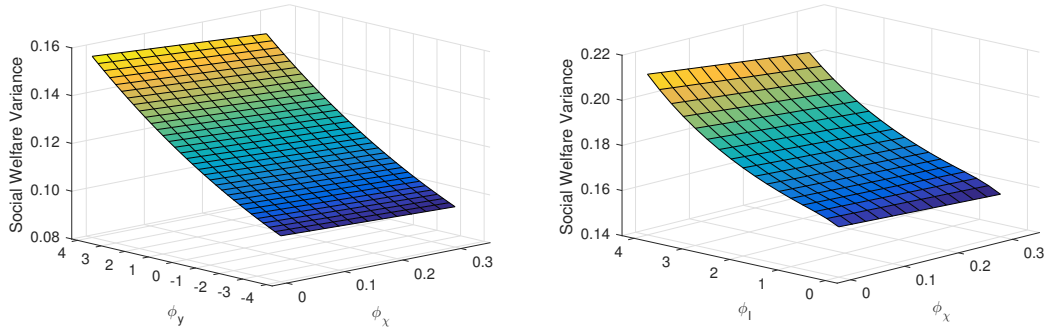


Figure 5: Social welfare variance with different values of  $\phi_y$ ,  $\phi_l$ , and  $\phi_x$ .

firms. This, in turn, mitigates the negative effects of the shock and helps the economy to recovery quicker and, hence, welfare improving.

Figure 8 suggests that the *extended Basel rule* attenuates the negative effects of the shock directly through financial aggregates not through the interest-rate channel. This is evidenced, when implementing the *extended Basel rule*, the decline in loans and deposits is less pronounced and they return to their steady state quicker, despite almost identical IRFs of their corresponding rates emerge. With a binding bank capital requirement, any reduction in loans is transferred into a  $\tau$  fraction of reduction in bank funds and a  $1 - \tau$  fraction of reduction in deposits, resulting in a shrinkage in banks' balance sheet. In this regard, the *extended Basel rule* can mitigate the shrinkage of banks' balance sheet.



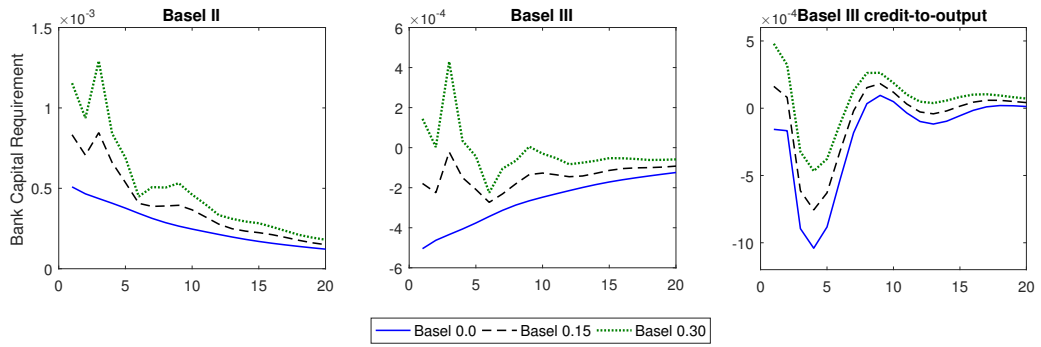


Figure 6: Bank Capital Requirement IRFs with different values of  $\phi_\chi$ .

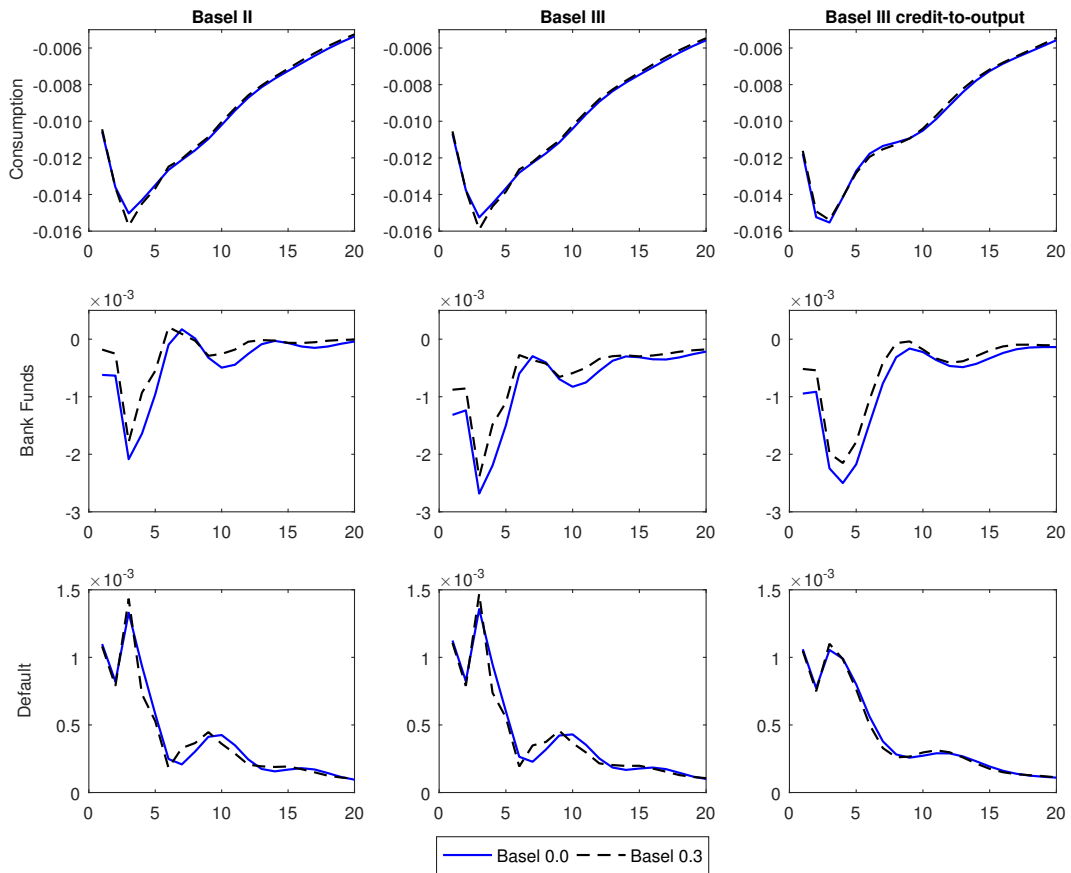


Figure 7: Main variables IRFs: Basel II, III and III credit-to-output.

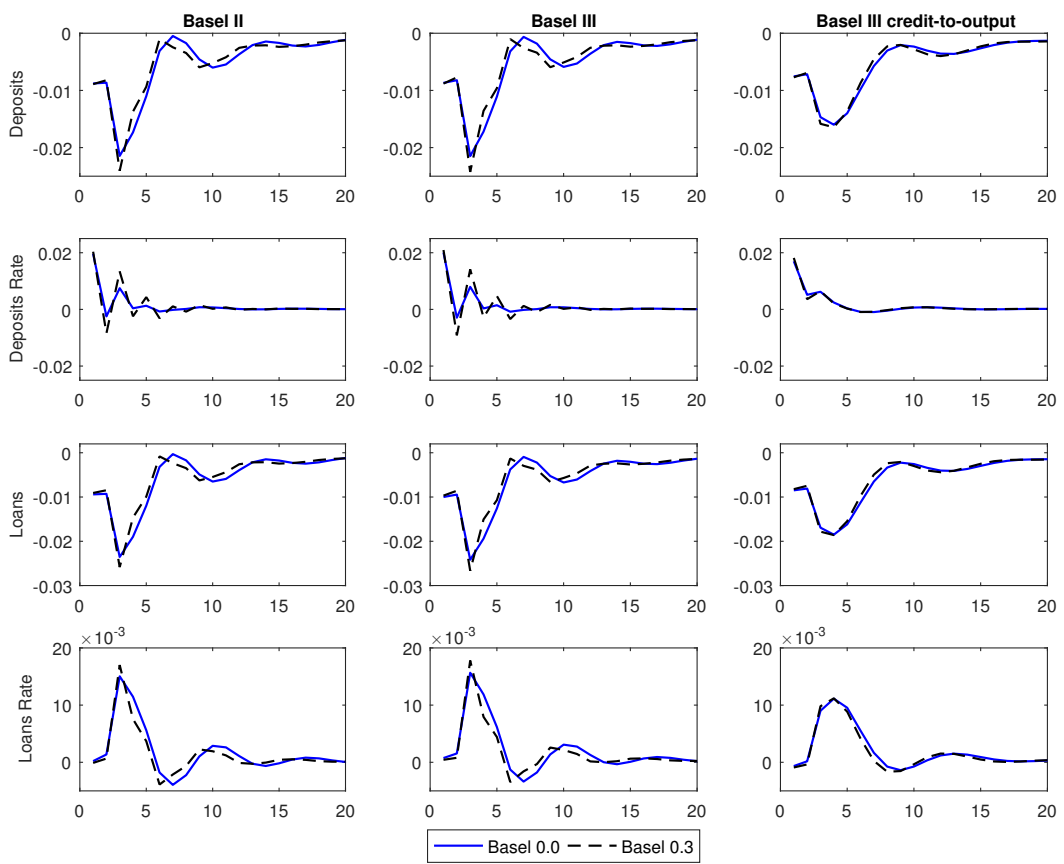


Figure 8: Main variables IRFs (2): Basel II, III and III credit-to-output.

## 5 Conclusions

To study the welfare implication of capital requirements augmented with default rate, we develop an RBC model with banking, in which borrowers may default on their financial obligations upon paying a penalty cost. We then examine the response of welfare to a negative technology shock under different capital requirement regimes with and without default. We show that including default in the capital requirement rule is welfare improving. A more aggressive reaction to the default rate mitigates the negative effect of the shock on welfare. The credit-to-output gap is a better indicator than output gap for the implementation of countercyclical capital buffer.

## References

- Angeloni, I., Faia, E., 2013. Capital regulation and monetary policy with fragile banks. *Journal of Monetary Economics* 60 (3), 311–324.
- Bakker, B. B., Dell’Ariccia, G., Laeven, L., Vandenbussche, J., Igan, D., Tong, H., Jun. 2012. Policies for macrofinancial stability: How to deal with credit booms. IMF Staff Discussion Notes 12/06, International Monetary Fund.
- BCBC, 2009. Strengthening the resilience of the banking sector. Consultative Document, Bank for International Settlements, Basel.
- BCBC, 2010. Countercyclical capital buffer proposal. Consultative Document, Bank for International Settlements, Basel.
- BCBS, 2011. The transmission channels between the financial and real sectors: A critical survey of the literature. Basel Committee on Banking Supervision, Working Paper, No. 18. Bank for International Settlements. Basel.
- Boldrin, M., Christiano, L. J., Fisher, J. D. M., 2001. Habit persistence, asset returns, and the business cycle. *American Economic Review* 91 (1), 149–166.
- Catarineu-Rabell, E., Jackson, P., Tsomocos, D., October 2005. Procyclicality and the new Basel Accord - banks choice of loan rating system. *Economic Theory* 26 (3), 537–557.
- Christiano, L. J., Eichenbaum, M., Evans, C. L., February 2005. Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy* 113 (1), 1–45.
- Clerc, L., Derviz, A., Mendicino, C., Moyen, S., Nikolov, K., Stracca, L., Suarez, J., Vardoulakis, A. P., June 2015. Capital regulation in a macroeconomic model with three layers of default. *International Journal of Central Banking* 11 (3), 9–63.
- de Walque, G., Pierrard, O., Rouabah, A., December 2010. Financial (in)stability, supervision and liquidity injections: A dynamic general equilibrium approach. *Economic Journal* 120 (549), 1234–1261.
- Drehmann, M., Tsatsaronis, K., March 2014. The credit-to-GDP gap and countercyclical capital buffers: Questions and answers. *BIS Quarterly Review*.
- Iacoviello, M., January 2015. Financial business cycles. *Review of Economic Dynamics* 18 (1), 140–164.
- Liu, G., Seeiso, N. E., 2012. Basel ii procyclicality: The case of south africa. *Economic Modelling* 29, 848–857.
- Lowe, P., Borio, C., Jul. 2002. Asset prices, financial and monetary stability: Exploring the nexus. *BIS Working Papers* 114, Bank for International Settlements.
- Repullo, R., Saurina, J., Mar. 2011. The countercyclical capital buffer of Basel III: A critical assessment. *Working Papers* wp2011-1102, CEMFI.

- Repullo, R., Suarez, J., 2013. The procyclical effects of bank capital regulation. *Review of Financial Studies* 26 (2), 452–490.
- Resende, C. D., Dib, A., Lalonde, R., Perevalov, N., April 2013. Countercyclical bank capital requirement and optimized monetary policy rules. Working Papers 13-8, Bank of Canada.
- Schularick, M., Taylor, A. M., April 2012. Credit booms gone bust: Monetary policy, leverage cycles, and financial crises, 1870-2008. *American Economic Review* 102 (2), 1029–61.
- Shubik, M., Wilson, C., 1977. The optimal bankruptcy rule in a trading economy using fiat money. *Zeitschrift für Nationalökonomie / Journal of Economics* 37 (3/4), pp. 337–354.
- Sidrauski, M., 1967. Rational choice and patterns of growth in a monetary economy. *American Economic Review* 57 (2), 534–544.
- Van den Heuvel, S. J., March 2008. The welfare cost of bank capital requirements. *Journal of Monetary Economics* 55 (2), 298–320.

## A Appendix: Maximization problems

### A.1 Households

The Lagrangian of the representative household reads as follows:

$$\mathcal{L}^h = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log(C_t - jC_{t-1}) + \phi_d \log(D_t) \right. \quad (39)$$

$$\left. + \lambda_t^h \left[ R_{t-1}^d D_{t-1} + W_t H_t + (1 - \nu_b) \pi_{t-1}^b + (1 - \nu_f) (1 - \xi_f) \pi_{t-1}^f - T_t - C_t - D_t \right] \right\}. \quad (40)$$

Defining  $\lambda_t^h$  as the Lagrange multiplier, the first order conditions with respect to  $\{C_t, D_t\}_{t=0}^{\infty}$  are:

$$C_t : \frac{1}{(C_t - jC_{t-1})} - \beta j \mathbb{E}_t \frac{1}{(C_{t+1} - jC_t)} = \lambda_t^h, \quad (41)$$

$$D_t : \phi_d \frac{1}{D_t} = \lambda_t^h - \beta \mathbb{E}_t \lambda_{t+1}^h R_t^d. \quad (42)$$

### A.2 Firms

The Lagrangian of the representative firm reads as follows:

$$\mathcal{L}^f = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log(\pi_t^f) + \lambda_t^a \left[ (1 - \delta) K_{t-1} + L_t + \nu_f (1 - \xi_f) \pi_{t-1}^f - K_t \right] \right. \quad (43)$$

$$\left. + \lambda_t^b \left[ A_t K_t^\alpha H_t^{1-\alpha} - W_t H_t - (1 - \chi_t) R_{t-1}^l L_{t-1} - \frac{\omega_z}{2} (\chi_{t-1} R_{t-2}^l L_{t-2})^2 - \pi_t^f \right] \right\}.$$

The first order conditions with respect to  $\{\chi_t, H_t, K_t, L_t, \pi_t^f\}_{t=0}^{\infty}$  are:

$$\chi_t : \lambda_t^b R_{t-1}^l L_{t-1} = \beta \mathbb{E}_t \lambda_{t+1}^b \omega_z \chi_t (R_{t-1}^l L_{t-1})^2, \quad (44)$$

$$H_t : (1 - \alpha) A_t K_t^\alpha H_t^{-\alpha} = W_t, \quad (45)$$

$$K_t : \alpha \lambda_t^b A_t K_t^{\alpha-1} H_t^{1-\alpha} - \lambda_t^a + (1 - \delta) \beta \mathbb{E}_t \lambda_{t+1}^a = 0, \quad (46)$$

$$L_t : \lambda_t^a - \beta \mathbb{E}_t \lambda_{t+1}^b (1 - \chi_{t+1}) R_t^l - \beta^2 \mathbb{E}_t \lambda_{t+2}^b \omega_z (\chi_{t+1} R_t^l)^2 L_t = 0, \quad (47)$$

$$\pi_t^f : 1/\pi_t^f - \lambda_t^b + \nu_f \beta (1 - \xi_f) \mathbb{E}_t \lambda_{t+1}^a = 0. \quad (48)$$

### A.3 Banks

The Lagrangian of the representative bank reads as follows:

$$\mathcal{L}^b = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log(\pi_t^b) + \lambda_t^d (D_t + F_t^b - L_t) \right. \quad (49)$$

$$+ \lambda_t^e \left[ (1 - \chi_t) R_{t-1}^l L_{t-1} - R_{t-1}^d D_{t-1} + \omega_q \chi_{t-1} R_{t-2}^l L_{t-2} - \pi_t^b \right]$$

$$+ \lambda_t^f \left[ (1 - \xi_b - \zeta) F_{t-1}^b + \nu_b \pi_{t-1}^b - F_t^b \right]$$

$$+ \lambda_t^g \left[ F_t^b - \tau_t L_t \right] \left. \right\}.$$

The first order conditions with respect to  $\{D_t, F_t^b, L_t, \pi_t^b\}_{t=0}^\infty$  are:

$$D_t : \lambda_t^d = \beta \mathbb{E}_t \lambda_{t+1}^e R_t^d, \quad (50)$$

$$F_t^b : \lambda_t^d + \lambda_t^g - \lambda_t^f + \beta (1 - \xi_b - \varsigma) \mathbb{E}_t \lambda_{t+1}^f = 0, \quad (51)$$

$$L_t : \beta \mathbb{E}_t \lambda_{t+1}^e (1 - \chi_{t+1}) R_t^l + \beta^2 \mathbb{E}_t \lambda_{t+2}^e \omega_q \chi_{t+1} R_t^l = \lambda_t^d + \lambda_t^g \tau_t, \quad (52)$$

$$\pi_t^b : 1/\pi_t^b - \lambda_t^e + \nu_b \beta \mathbb{E}_t \lambda_{t+1}^f = 0. \quad (53)$$

Taxes:

$$T_t = q(\chi_{t-1}, R_{t-2}^l, L_{t-2}) - z(\chi_{t-1}, R_{t-2}^l, L_{t-2}) - \varsigma F_{t-1}^b. \quad (54)$$

Capital requirement rule:

$$\tau_t = \tau \left( \frac{Y_t}{Y} \right)^{\phi_y} \left( \frac{\chi_t}{\chi} \right)^{\phi_x}, \quad \text{or} \quad \tau_t = \tau \left( \frac{L_t/Y_t}{L/Y} \right)^{\phi_l} \left( \frac{\chi_t}{\chi} \right)^{\phi_x}. \quad (55)$$

Technology shock:

$$\log(A_t) = \rho_A \log(A_{t-1}) + \xi_t^A. \quad (56)$$