The reliability of South African real-time output gaps estimates

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Abstract

Estimates of the output gap are an important component of policy-makers’ toolkits. Both the theory underlying monetary policy analysis and the empirical models employed by central banks suggest that the output gap is a key variable explaining inflation. In this view, the estimate of the output gap provides not only an indication of how well the economy is operating relative to its potential, it also signals whether inflation is likely to increase or decrease in the future. The reliability of estimates of the output gap is therefore extremely important for policy making. However, a large literature has highlighted both conceptual and practical problems in measuring the gap, including the difficulty of using real-time data that will be revised in the future. The contribution of this paper is to assess the reliability of real-time estimates of the South African output gap, by estimating output gaps using a range of univariate methods applied to real-time gross domestic product (GDP) data. Consistent with the results of similar studies conducted in other countries, it is found that the real-time South African output gap estimates are in fact quite unreliable and are significantly revised over time. Furthermore, the source of these revisions is largely attributed to new data points becoming available, indicating the unreliability of end-of-sample estimates, rather than data or parameter revisions.

JEL Classification: C32, E32.
Keywords: Output gap; real-time data; monetary policy; South Africa.

∗The views expressed are those of the authors and do not necessarily represent those of the University of the Witwatersrand or the South African Reserve Bank. Comments from an anonymous referee that materially improved the paper are gratefully acknowledged.
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1. Introduction

Knowing the size and sign of the output gap at a particular point in time is important for policy-makers, so it is pertinent to ask whether estimates of output gaps are reliable. Defined as the log difference between potential and actual real gross domestic product (GDP), where potential output is only realised if all resources and available technology are fully employed in the economy, the output gap not only gives an indication of the extent to which the economy is fully employed, but it also provides insight regarding future inflationary pressures. The output gap is particularly relevant for policy-makers in countries where an inflation-targeting monetary policy framework has been adopted, both because it is viewed as having information content for inflation and also because flexible inflation targeters aim not only at stabilising inflation around the inflation target but also allow some weight to be placed on the real economy in the form of the output gap.1

To effectively inform policy decisions, the output gap is estimated using the 'most up-to-date' or real-time macroeconomic data. However, the use of real-time data raises a number of important issues.3 First, real-time data points used to estimate the output gap can be subsequently revised a number of times as more relevant information becomes available. According to van Walbeek (2006), in the case of South African gross domestic expenditure (GDE), the ‘Final’ growth rate measures can deviate by up to 5 percentage points in size from when they were first released. Revisions can also occur as a result of changes in the definitions of

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1For example, Svensson (1999) stated "...there is considerable agreement among academics and central bankers that the appropriate loss function both involves stabilizing inflation around an inflation target and stabilizing the real economy, represented by the output gap". In the policy environment, Bank of England MPC member Andrew Sentance (2011) acknowledged that in the United Kingdom (UK), economic stabilisation and the trade-off between growth and inflation "leans heavily on the idea of the path of inflation being driven by an 'output gap'". In South Africa, the South African Reserve Bank (SARB) MPC Statements invariably refer to recent developments in the output gap (e.g the Bank recently supported the decision to leave the repurchase rate unchanged by noting that the "output gap remains negative and gross domestic product (GDP) growth is expected to remain below potential over the next two years" (Marcus, 2011)).

2It should be noted, however, that even the most up-to-date data are generally released with a lag. For instance, South African GDP estimates for each quarter are only released by Statistics South Africa approximately 50-60 days after the quarter has ended.

3Croushore (2011) provides a recent review of work that uses real-time data.
macroeconomic variables and the methods used to compile them. Second, end-of-sample bias can occur where end data points can vary considerably with the addition of a few observations.\footnote{Watson (2007, 144) distinguishes between these first two points as follows, ”there are two distinct problems using real-time data to estimate trends and gaps. First, data published in real time are often subsequently revised, and these revisions can be large. Second, for the purpose of estimating trends and gaps, future values of the series are needed, so that estimates of a trend at time $t$ will change as data becomes available for time $t + 1, t + 2$, etc., even if the data at time $t$ is not revised”.} Third, it may become apparent as data are revised, that the economy has in fact undergone a structural change. This would not only inform expectations about the output gap going forward but may also mean that historical real-time output gaps were over- or under-estimated (depending on the nature of the structural change).

As a result, real-time output gap estimates may not provide a sufficiently accurate measure of economic activity to properly inform economic policy-making.\footnote{This has implications for both current, forward-looking policy decisions and the policy decisions of the past. The analysis of past decisions requires that we know the data that were available to policymakers at the time.} The contribution of this research is to assess the reliability of South African real-time output gap estimates, the first study of this issue for South Africa that we are aware of. The paper follows Orphanides and van Norden’s (2002) approach, where a number of output gaps are estimated from real-time GDP data using standard detrending techniques, both at each point in time and over time.

The paper is structured in the following manner: Section 2 provides a brief review of the relevant literature on real-time output gap estimates and their reliability. Section 3 provides a description of the data used, including a detailed explanation of how the real-time South African dataset was constructed for the purpose of this analysis. Section 4 sets out the approach and methodology adopted, including an overview of the models employed to estimate the output gaps. Sections 5 provides and discusses the results and Section 6 concludes.
2. The literature on real-time output gaps

The seminal work on the reliability of real-time output gap estimates was carried out by Orphanides and van Norden (2002). Using the Croushore and Stark (2001) real-time dataset for the US, they tested the reliability of output gap estimates by using a number of univariate and multivariate detrending methods. Three key findings emanated from their results. First, and foremost, it was concluded that the reliability of the real-time estimates was, in fact, quite low and, as a result, had serious implications for monetary policy. Second, rather than unreliability stemming from data revisions, it was found to stem from end-of-sample bias and thirdly, unreliability varied according to the method used to estimate the gap (although there was no evidence of multivariate methods being superior to their univariate counterparts). Also for the US, Orphanides (2003) found that Federal Reserve Board staff estimates of the US output gap, obtained from the Greenbook document, were similarly unreliable.

Orphanides and van Norden’s (2002) research spurred on similar studies in a number of countries including (but not limited to) Canada, New Zealand, Australia, Norway, Germany, the euro area, Brazil, Japan and the Organisation for Economic Co-operation and Development economies. Applying a similar approach to Canadian data, for example, Cayen and van Norden (2005) not only confirmed Orphanides and van Norden’s (2002) findings but found that the real-time estimates were in fact even more unreliable. They found significant measurement error with real-time estimates having less than 50 per cent correlation with the estimates that were based on revised data. More recently, Cusinato et al (2010) and Marcellino and Musso (2010) came to comparable conclusions in relation to the reliability of real-time estimates for Brazil and the euro area, respectively.

It should be noted that while these studies have come to similar overall conclusions, the underlying data issues that drive the unreliability of the real-time estimates are not the same in all cases (reflecting perhaps characteristics unique to each of the economies). For example, data revision played a greater role in

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6Discussions on related issues in the wider literature on monetary policy and macroeconomic stabilisation, however, date at least to Friedman (1947, 1953).
the Canadian data than it did for the US, although it was important for both. However, for the euro area, data revisions "played a minor role" (Marcellino and Musso, 2010).

Not all studies have determined that real-time estimates of the output gap are unreliable. In contrast to the studies discussed above, Gruen et al. (2005) found that in the case of Australia, real-time estimates were reasonable but only if "a sufficiently flexible and robust approach was used to obtain them". More recently, Garratt et al. (2008) argued that the unreliability of the US real-time output gap estimates is potentially over-stated by Orphanides and van Norden (2002). They argue that it is likely that policy-makers take future revisions into account when estimating the real-time output gap, and that Orphanides and van Norden, in assuming that future revisions are not taken into account, may overestimate the effect of subsequent data revisions. However, this assumes that the magnitude and direction of future revisions are predictable by policy-makers in real time, which would need to be established on a case-by-case basis. Furthermore, in the case of South Africa "there is no evidence for a systematic upward or downward bias in the GDE revisions" (van Walbeek, 2006: 753). Edge and Rudd (2012) also argue that the findings of Orphanides and van Norden (2002) and Orphanides (2003) for earlier periods in the US are too pessimistic in that more recent Federal Reserve Board estimates perform better than those documented in these studies.

In South Africa, there has been no research testing the reliability of real-time output gap estimates, although several studies have estimated potential output and the output gap. Generally the studies fall loosely into three categories: those that solely focus on obtaining output gap estimates from a number of different methods, those that generate estimates to gain a better understanding the South African economy and finally those that test whether certain (standard) methods are appropriate to measure the South African output gap. The first set of studies typically estimate and compare the output gap generated from an univariate statistical time series approach (such as the Hodrick-Prescott Filter) and the production function approach. One of the first of these types of studies was conducted by Smit and Burrows (2002). A similar study was completed more recently by Akinboade (2005). Examples of the second set of studies are Arora and Bhundia
du Toit et al. (2006) and du Plessis et al. (2008), which estimate the output gap to understand post-apartheid productivity growth, determine South Africa’s growth potential and identify supply and demand shocks, respectively. The final set of studies considers whether standard methods should be used to estimate the output gap. For example, Boshoff (2010) considers the merits of using band-pass filters to measure high-frequency and medium-term deviation cycles for South Africa, while Du Toit (2008) considers what the optimal Hodrick-Prescott filter is for South Africa.

3. The data

3.1. Data sources, revisions and definitions

Quarterly expenditure on GDP at constant prices (seasonally adjusted and annualised) was used as the measure for real GDP. Raw data for series KBP6006D were extracted from the SARB Quarterly Bulletins (as well as Quarterly Bulletin Supplements). In the Quarterly Bulletins, the most recent observation for GDP is published, as well as historical quarterly observations (usually the last 16 quarters). As noted earlier, the most recent or ‘real-time’ observation is published with a lag. For example, for data for the quarter ending 30 September 2010, Statistics South Africa released its first estimates on 23 November 2010 (a lag of 53 days), and the relevant Quarterly Bulletin was released 15 days later on 8 December 2010.

As is the case in other countries, the South African national accounts are revised on a regular basis. More specifically, variables are subject to regular minor revisions and also to more comprehensive revisions that occur every few years. Minor revisions occur on a quarterly basis where the most recent observations are revised as more relevant and accurate information becomes available. It is not uncommon for an estimate to be revised up to 4 or 5 times after its initial release, with most of the revision occurring in the first few releases. In addition, there have been 8 comprehensive revisions of the national accounts in the pe-

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7These are published to document major revisions to South Africa’s national accounts.
8Up to 16 historical quarters are revised in each Quarterly Bulletin.
roid considered here. A comprehensive revision is undertaken to accommodate a number of improvements such as changes in definitions and classifications of variables, updated methodologies and statistical techniques and rebasing exercises (South African Reserve Bank, 2010). When a comprehensive revision occurs, quarterly variables are revised from 1960Q1 to the most recent observation. Examples of minor and major (benchmarking and rebasing) revisions to the GDP series are presented in Figure 1.

For the purpose of this analysis, each Quarterly Bulletin represents a particular ‘vintage’ of the data where the first vintage is the December 1981 Quarterly Bulletin (or 1981Q4), which publishes data from 1960Q1 to 1981Q3. Although national accounts data have been published on a quarterly basis since March 1971 (1971Q1), vintages prior to 1981Q4 are not used in this analysis. Reasons for this are twofold. First, prior to March 1979, data were only published in current prices. Second, despite comprehensive revisions occurring in 1975Q2 and 1980Q3, available vintages for these periods are incomplete. The final vintage of data is 2010Q4, which was the most recent vintage of GDP at the time of the study. Altogether, there are 117 vintages over the period 1981Q4 to 2010Q4.

3.2. Construction of the real-time dataset

To be able to analyse the reliability of the real-time output gap estimates it is necessary, at a minimum, to have access to a real-time dataset for the GDP series. This dataset comprises the most recent (or real-time) estimates of GDP for a particular point in time, with each point in time representing a vintage as defined above. Such a dataset was constructed for the purpose of this study. This involved extracting and collating the 117 vintages of the South African GDP data series, with each series starting in 1960Q1.

It should be noted that while some vintages documented the GDP series from

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9 These revisions were released in 1981Q4, 1986Q1, 1988Q2, 1991Q2, 1994Q2, 1999Q2, 2005Q2 and 2010Q1.
10 The rebasing results in the change in the base year evident in the graph showing major revisions, and does not reflect revisions due to changes in definitions and classifications of variables, updated methodologies and statistical techniques.
11 For multivariate approaches to estimating the output gap real-time datasets for other variables are also required.
Figure 1: Examples of minor and major revisions to South African Real GDP

Source: SARB Quarterly Bulletins
1960Q1 (typically when a comprehensive revision occurred), other vintages contained only a limited sample of the more recent observations. To complete these vintages, earlier observations from the most recent comprehensive revision were used to fill in the missing data points. For example, for the most recent 2010Q4 vintage the GDP estimates were published in the Quarterly Bulletin for the period 2006Q4 to 2010Q3 (the most recent observation and the previous 15 quarters). In this case, the data from the most recent comprehensive revision, which were given in the 2009Q4 vintage, were used to obtain the data points between 1960Q1 and 2006Q3.

4. Measuring the reliability of South African real-time output gap estimates

4.1. Methods for estimating the output gaps

Following Orphanides et al. (1999), and because our real-time dataset is currently limited to GDP data, we used univariate methods to estimate output gaps. These include:

1. Deterministic Trends
2. Time-series Filters
3. The Beveridge-Nelson Decomposition
4. Unobserved Components Models

Each of these methods is briefly described below.\(^{12}\)

4.1.1. Deterministic trends

By far the simplest way to estimate the output gap, this detrending method is based on the hypothesis that a GDP series follows a deterministic trend. More

\(^{12}\)The trend-cycle decompositions using deterministic trends, mechanical filters and the Beveridge-Nelson decomposition were undertaken using RATS and Eviews. The RATS code made available by Cayen and van Norden, which was adapted for use here, is gratefully acknowledged. The unobserved components models were estimated using STAMP (Koopman et al., 2000).
specifically (the log of) GDP can be decomposed into two parts: a deterministic trend component and a cyclical component (which is assumed to be any deviation from the trend). The latter component represents the output gap. Several deterministic trend functions exist, two of which are considered here: linear and quadratic trends.

When the linear trend model is applied, it is assumed that GDP is a linear function of time and can be decomposed into a linear deterministic trend and a cyclical component. This is illustrated in equation (1) below

$$y_t = \alpha + \beta.t + c_t$$  (1)

where $y_t$ is logged GDP, $\alpha + \beta.t$ describes the deterministic trend and $c_t$, the output gap. An alternative, slightly more sophisticated method is the quadratic trend

$$y_t = \alpha + \beta.t + \gamma.t^2 + c_t$$  (2)

where $\gamma.t^2$ represents the additional nonlinear component. Once again this function can be decomposed into a deterministic trend, $\alpha + \beta.t + \gamma.t^2$, and cyclical component, $c_t$.

It should be noted that while deterministic trends are a simple and easy way to estimate the output gap, they are not always useful. For example, not only do they not take into account supply shocks, they assume that trend GDP growth is constant. Furthermore, the output gap estimates may not be stationary as it is possible that any stochastic trend in the function may not be completely removed (Cotis, Elmeskov, and Mourougane, 2005).

### 4.1.2. Time-series filters

One of the most popular methods used to estimate the output gap is the Hodrick-Prescott (HP) filter (Hodrick and Prescott, 1997). Similar to the deterministic trend methods, the HP filter assumes that the GDP series can be decomposed into a trend or growth component, $g_t$, and a cyclical component, $c_t$:

$$y_t = g_t + c_t$$  (3)
In order to obtain the cyclical component, $g_t$ is selected to minimise:

$$\{g_t\}_{t=0}^{T+1} = \text{argmin} \sum_{t=1}^{T} \{ (y_t - g_t)^2 + \lambda [ (g_{t+1} - g_t) - (g_t - g_{t-1}) ]^2 \}$$

(4)

In equation (4), $\lambda$ is the ‘smoothness parameter’ or ‘smoothing constant’ and determines how smooth the trend will be by penalising variation in its growth rate. Hodrick and Prescott (1997) recommended a value of $\lambda = 1600$ for quarterly data. In this paper, an HP filter with an alternative value for $\lambda$ is also considered. According to Du Toit (2008), business cycles in the South African economy typically have a frequency of six years or less, suggesting an optimal value of $\lambda$ of 352.

While the Hodrick-Prescott filter is widely used as a tool to measure business cycles and is popular in the South African literature, there are several disadvantages to using this technique. These range from (1) estimates tending to become less accurate towards the end of the sample as a result of the symmetry of the filter (St-Amant and van Norden, 1997; Baxter and King, 1995), (2) estimates found to be imprecise (Laxton and Tetlow, 1992), (3) the most optimal data generating process for the filter not being suitable for macroeconomic time series (Harvey and Jaeger, 1997) (Guay and St-Amant, 1996), (4) impractical conditions for the HP filter to perform optimally (Guay and St-Amant, 1996), (King and Rebelo, 1993) and (5) potential spurious cycles being generated as a result of the filter as opposed to the properties of the time series (Cogley and Nason, 1995).

The suboptimality of the Hodrick-Prescott filter at time series endpoints has been explored using Monte Carlo simulations by Mise et al (2005), who support Kaiser and Maravall (1999) in recommending the application of the Hodrick-Prescott filter to forecast-augmented data in an attempt to reduce the revision errors of the most recent trend values. In line with this recommendation, each vintage of the GDP data is augmented here with a forecast of 25 observations using an AR(4) process, and the Hodrick-Prescott filter is applied to the augmented sample to obtain an estimate of the real-time gap.

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13Mise et al (2005, 58) note that the theoretical analysis of forecast-augmentation requires forecasts infinitely far ahead, although the weights assigned to far-ahead forecasts become very small. Their experiments suggest augmenting by 28 quarterly observations.
The second filter employed in this study is the Band-Pass filter, which defines the cycle as having a particular frequency. A linear filter isolates the cycle by taking a two-sided weighted moving average of the data and, for a specified range (band), removes components that fall outside the frequency. In the case of output gaps, the range consists of an upper and lower band that indicates the typical minimum and maximum length of a business cycle. According to the National Bureau of Economic Research (NBER) definition, this entails retaining components of the GDP series with periodic fluctuations between six and 32 quarters, and removing components at higher and lower frequencies.

It should be noted that several Band-Pass filters exist, which differ in the way that the weighted moving average is computed. In this study, the Baxter-King Band-Pass filter is employed (Baxter and King, 1999). This is a fixed-length symmetric filter, so the filter is again applied to forecast-augmented GDP series created using an AR(4) process to augment each vintage by 25 observations.\textsuperscript{14}

### 4.1.3. The Beveridge-Nelson Decomposition

Beveridge and Nelson (1981) view any GDP trend as stochastic and define it as the level to which a series is expected to converge in the long run. To estimate the output gap, an ARIMA process is used to decompose a non-stationary series into a stochastic trend and a stationary component. More specifically, any ARIMA\((p,1,q)\) model can be decomposed into a sum of a random walk with drift and a stationary component, where the latter is the cyclical component.\textsuperscript{15}

Consider the following example where the first difference of the series is stationary and can be represented in the following infinite-order ARIMA\((0,1,\infty)\)\textsuperscript{16}

\[
y_t - y_{t-1} = \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2} + \ldots
\]  

(5)

and \(e_t\) is defined as \(e_t = \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2} + \ldots\). If we assume the initial condition to be \(y_0\) for \(y_t\), after \(s\) periods we have:

\textsuperscript{14}Christiano and Fitzgerald (2003) propose an optimal band-pass filter that avoids the need to augment the series.

\textsuperscript{15}The algorithm used by the RATS code is from Newbold (1990).

\textsuperscript{16}Assuming that the deterministic component is part of the trend component and has already been removed.
The stochastic trend is the level to which a series is expected to converge in the long run and thus is defined as the limiting value of the forecast

\[
\lim_{s \to \infty} E_t (y_{t+s}) = y_t + \lim_{s \to \infty} E_t \left( \sum_{i=1}^{s} e_{t+i} \right)
\]

(7)

Since \( E_t \varepsilon_{t+i} = 0 \), the changes in trend are not forecastable and the time series has been successfully decomposed into a random walk and cyclical component:

\[
y_t = \mu_t + c_t
\]

(8)

Where the trend and cyclical component are defined as \( \mu_t = \mu_{t-1} + e_t \) and \( c_t = y_t - \mu_t \), respectively.

In general, a three-step process can be followed when using the Beveridge-Nelson decomposition:

1. Estimate an appropriate ARIMA(p,q) model for the time series as well as identify all \( \beta_j \)
2. Use the model to generate \( E_t \varepsilon_{t+i} \) for all \( t \) and \( i = 1, \ldots, s \)
3. Using information from 1 and 2 calculate the trend and output gap.

An advantage of the Beveridge-Nelson decomposition is that it does not have an end-of-sample problem in the way that the deterministic trend and time-series filters have, although the decomposition can generate noisy cycles.

4.1.4. Unobserved components models

A basic structural time series model (Harvey, 1989) with unobserved components specified as a trend plus cycle plus irregular is fitted to the log of the relevant GDP series \( y_t \):

\[
y_t = \mu_t + \nu_t + \varepsilon_t
\]

(9)

where \( \mu_t \) is the trend, \( \nu_t \) is a second-order autoregressive cyclical component
and \( \varepsilon_t \) is the irregular \((\varepsilon_t \sim NID(0, \sigma^2_\varepsilon))\), and \( t = 1, \ldots, T \).

The trend component is specified as the local linear trend model:

\[
\begin{align*}
\mu_t &= \mu_{t-1} + \beta_{t-1} + \eta_t, \quad \eta_t \sim NID(0, \sigma^2_\eta) \\
\beta_t &= \beta_{t-1} + \xi_t, \quad \xi_t \sim NID(0, \sigma^2_\xi)
\end{align*}
\]

(10) (11)

where \( \beta_t \) is the slope. The disturbances of the irregular \( \varepsilon_t \), the level \( \eta_t \) and the slope \( \xi_t \) are mutually uncorrelated. The \( \eta_t \) (\( \xi_t \)) disturbance allows the level (slope) of the trend to change. Maximum likelihood estimation using the Kalman filter is undertaken in Stamp 8.2 (Koopman et al., 2000).

### 4.2. Revision analysis

To determine the reliability of real-time output gap estimates for South Africa, this paper follows, for the most part, the approach used by Orphanides and van Norden (2002) and Cayen and van Norden (2005). From the real-time dataset, the output gap is estimated in a number of different ways, using the approaches discussed in the previous section.

To begin with, the "Final" output gap estimates are generated by detrending the final, or most recent, vintage of the logged GDP data (i.e., 2010Q4).\(^{17}\) The "Final" estimate provides the benchmark against which other output gap estimates are compared, consistent with the view that measurement of the past improves with time. When the unobserved components model is used, the smoothed estimates of the output gap obtained from the Kalman filter are used to generate the Final series.

Next, the "Real-time" output gap estimates are constructed. In this case, each vintage in the real-time GDP dataset is detrended to obtain an output gap series. Then from each vintage, the most recent estimates of the output gap are collated to create the series of "Real-time" estimates. For example, the output gap series estimated from the vintage 2008Q2 would provide the real-time estimate for 2008Q1. A comparison between the "Real-time" and "Final" estimates reveals the extent to which output gap estimates have been revised at each point in time.

\(^{17}\)So 100 times these estimates are approximate percentage gaps.
In order to illustrate the importance of data revisions, the “Quasi-real” output gap estimate is also constructed. To construct this “Quasi-real” series, the final vintage of the GDP data is employed to estimate output gaps recursively. For example, to obtain the Quasi-real estimate for 1988Q1, observations of the 2010Q4 vintage up to, and including, 1988Q1 are detrended. The difference between the Real-time and the Quasi-real output gap for a particular period is due to data revisions alone, since the estimates use the same data sample periods.

The last type of output gap considered is the “Quasi-final” gap obtained from the filtered estimates of the unobserved components model applied to the full sample of the final vintage of the GDP series. Comparing these gaps to the Quasi-real gaps provides insight into parameter instability in the unobserved components model used to estimate the output gap (parameters are estimated using the full-sample for the Quasi-final gaps, while they are estimated using partial samples for the Quasi-real estimates).

5. Results

5.1. Output gap estimates

Table 1 provides summary statistics for the Real-Time, Quasi-Real, Quasi-Final and Final output gap estimates generated by the different methods for the period 1981Q3-2010Q3. For example, the Real-Time Hodrick-Prescott ($\lambda = 1600$) estimates of the output gap have a mean value of 0.002, a standard deviation of 0.011, and reach a minimum value of -0.030 and a maximum value of 0.019. The remaining statistics compare the different output gap measures obtained from each of the methods. The “Cor (Final)” statistic provides the correlation with the Final estimate (ideally, this should be close to 1) whereas the “Cor (Real time)” statistic provides the correlation with the Real-Time estimate. “Opp. Sign” indicates the frequency at which the sign of the estimate differs from the sign of the respective Final estimate (ideally, close to 0). It is evident that the Beveridge-Nelson Real-Time estimates were relatively highly correlated with the Final estimates with a correlation coefficient of 0.678. However, the remaining Real-Time
estimates tend to be poorly correlated, with correlations of around 0.55 or less. In terms of opposite signs, the Real-Time estimates generally have the opposite sign to their Final counterparts 35-40 per cent of the time, with only the Beveridge-Nelson estimates having opposite signs less than 20 per cent of the time. It is important to note that the Cor (Real time) statistic shows that the correlations between Real-time (RT) and Quasi-RT (QR) estimates are in general very high and hence data revision, calculated as $QR - RT$, is small. Therefore, the correlations between RT and Final (F) estimates and between QR and F estimates are also similar (the Cor (Final) statistics). This indicates that the impact of data revisions tend to be minor and that what explains the bulk of the total revision is the impact of new data points becoming available (as discussed in Footnote 4).

Figure 2 graphically shows the evolution of the Real-Time and Final output gap estimates over the period for the seven different methods. The comparison between the output gap estimates reveals a number of interesting characteristics. First, consider the evidence for short-term co-movements among the various Real-Time and Final estimates (disregarding the sign of the gap for the moment). The plots reveal that there are relatively few periods where the output gap estimates of different methods appear to move together. To illustrate, consider a minimum period of four quarters or one year. For the Real-Time estimates, there are only four cases when all 7 output gap estimates move in the same direction over a 4-quarter period (1989Q4 - 1990Q3, 1992Q2 - 1993Q1, 1998Q1 - 1998Q4 and 2008Q3 - 2009Q2). However, for the Final estimates, not only are co-movements more common, but they also occur over longer periods of time. There are 6 periods of co-movement, ranging from 4 to 8 quarters (1983Q3 - 1985Q2, 1992Q1 - 1993Q3, 1997Q3 - 1998Q4, 1999Q2 - 2000Q1, 2006Q1 - 2007Q1 and 2008Q3 - 2009Q2). Interestingly, it is only for the periods starting in 1992Q2 and 2008Q03 that co-movements are apparent for both the Real-Time and Final estimates. In this instance all estimates move downwards reflecting the recessions experienced in South Africa around those periods.

Second, even when co-movement exists, this does not necessarily translate into the different methods producing output gap estimates of a similar size. As illustrated in Figure 2, the magnitude of estimates can vary significantly depending on the method employed. In the case of the Real-Time estimates, the linear
### Table 1: Output Gap Summary Statistics: 1981Q3 – 2010Q3

<table>
<thead>
<tr>
<th>Method</th>
<th>Mean</th>
<th>Std. Dev</th>
<th>Min</th>
<th>Max</th>
<th>Cor (Final)</th>
<th>Cor (Real-time)</th>
<th>Opp. Sign</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Linear Trend</strong></td>
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</tr>
<tr>
<td>Real-Time</td>
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<td>0.064</td>
<td>-0.207</td>
<td>0.047</td>
<td>0.339</td>
<td>1.000</td>
<td>0.333</td>
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<td>Quasi-Real</td>
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<td>0.065</td>
<td>-0.206</td>
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<td>0.322</td>
<td>0.986</td>
<td>0.333</td>
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<tr>
<td>Final</td>
<td>-0.008</td>
<td>0.066</td>
<td>-0.095</td>
<td>0.163</td>
<td>1.000</td>
<td>0.339</td>
<td>0.000</td>
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<td></td>
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<td></td>
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</tr>
<tr>
<td>Real-Time</td>
<td>0.061</td>
<td>0.061</td>
<td>-0.053</td>
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<td>0.421</td>
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<td>0.675</td>
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<td>0.135</td>
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</tr>
<tr>
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<td>0.011</td>
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<td>0.561</td>
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<td>0.350</td>
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<td>0.606</td>
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<td>1.000</td>
<td>0.561</td>
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<td>0.008</td>
<td>-0.030</td>
<td>0.012</td>
<td>0.525</td>
<td>1.000</td>
<td>0.368</td>
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<td>0.008</td>
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<td>0.010</td>
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<td>0.012</td>
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<td>0.014</td>
<td>0.678</td>
<td>1.000</td>
<td>0.171</td>
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<td>0.013</td>
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<td>0.020</td>
<td>1.000</td>
<td>0.678</td>
<td>0.000</td>
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<tr>
<td>Real-Time</td>
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<td>0.008</td>
<td>-0.026</td>
<td>0.013</td>
<td>0.527</td>
<td>1.000</td>
<td>0.359</td>
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<tr>
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<td>0.009</td>
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<td>0.019</td>
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<td>0.045</td>
<td>1.000</td>
<td>0.527</td>
<td>0.000</td>
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Figure 2: Real-time (RT) and final (FL) estimates of the output gap

and quadratic methods produce noticeably dissimilar results, with the difference sometimes as much as 0.205 or 20.5 per cent.

Third, it is not uncommon for methods to produce Real-Time estimates that have different signs for the same period. Again this is especially apparent for the linear and quadratic Real-Time estimates where more times than not they produce estimates that have opposite signs for the same period. Despite variation
in the magnitude and signs of estimates, however, there are periods where estimates tend to be quite similar. If one excludes the linear and quadratic trend estimates, the remaining Real-Time estimates are particularly clustered around 1982 and 2004, while the remaining Final estimates are clustered around a larger number of periods (including 1984, 1986, 1988, 1992, 1996 and 1998).

Since the linear and quadratic trend approaches appear to provide estimates of the output gap that are relatively uninformative for our study (relative to the other approaches), they are excluded for the remainder of this analysis.

5.2. Analysis of revisions

This section provides an analysis of the total revision between the Real-Time and Final output gap estimates for the different methods (plotted in Figure 3). The magnitudes of the revisions over the sample period generally range between $-0.03$ and $0.05$, which is largely in line with Orphanides and van Norden (2002) for the US and Cayen and van Norden (2005) for Canada as well as Cusinato et al (2010) and Marcellino and Musso (2010) for Brazil and the euro area, respectively.

Figure 3: Total revision in output gap estimates
Table 2 provides descriptive statistics for the total revisions, as well as three additional statistics: "\( \rho \)"", "N/S" and "Xsize". The first order autocorrelation coefficient, \( \rho \), indicates how persistent the revisions are or how likely the estimates are to stay in the same state over an extended period. The closer \( \rho \) is to 1, the greater the persistence. With the exception of the Beveridge-Nelson estimates \( (\rho = 0.652) \), revisions to estimates of the output gap are highly persistent with \( \rho \) values above 0.86.

Following Cayen and van Norden (2005), the noise-to-signal ratio (N/S) is calculated by dividing the root-mean-square of the revision by the standard deviation of the Real-Time estimate. This statistic indicates the relative importance of the revision with the ratio equalling zero if there was no revision at all. For all the models except the Beveridge-Nelson, "N/S" exceeds 1 indicating that output gap revisions are substantial in South Africa. The Unobserved Components Model estimates perform the worst in this regard with an N/S ratio of around 1.867, followed by the Band-Pass filter estimates at 1.561.

The final statistic in Table 2, Xsize, measures the proportion of time that the absolute value of the revision exceeds that of the absolute value of the Final estimate. The Xsize statistic for the Beveridge Nelson estimates is just below 30 per cent, while the statistics for the UC Model, Band-Pass and HP filters range between 41 and 46 per cent.

<table>
<thead>
<tr>
<th>Method</th>
<th>Mean</th>
<th>Std. Dev</th>
<th>Min</th>
<th>Max</th>
<th>( \rho )</th>
<th>N/S</th>
<th>Xsize</th>
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</thead>
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<td>Hodrick-Prescott ((\lambda = 1600))</td>
<td>-0.002</td>
<td>0.014</td>
<td>-0.025</td>
<td>0.035</td>
<td>0.895</td>
<td>1.306</td>
<td>0.462</td>
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<td>0.011</td>
<td>-0.017</td>
<td>0.031</td>
<td>0.860</td>
<td>1.288</td>
<td>0.462</td>
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<tr>
<td>Band-Pass</td>
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<td>0.014</td>
<td>-0.017</td>
<td>0.048</td>
<td>0.883</td>
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<td>0.444</td>
</tr>
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<td>-0.032</td>
<td>0.040</td>
<td>0.652</td>
<td>0.819</td>
<td>0.299</td>
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<tr>
<td>UC model</td>
<td>0.001</td>
<td>0.016</td>
<td>-0.026</td>
<td>0.040</td>
<td>0.913</td>
<td>1.867</td>
<td>0.410</td>
</tr>
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</table>
5.3. Decomposition of revisions

This section looks in more detail at the different sources of revisions. Figure 4 plots the Real-Time estimate, the total revision and finally, the data revision for each method. As defined in section 4.2, the last of these series is calculated by subtracting the Quasi-Real estimate from the Real-Time estimate, and measures the extent to which revision of the real-time estimate can be attributed to revised data published in subsequent periods - and not new data points. For the unobserved components model, the parameter revision (the difference between the Quasi-Final and Quasi-Real estimate) is also given, providing an indication of how much revision can be attributed to parameter instability. Table A1 in Appendix 1 provides the summary statistics for each of the different methods and respective revisions.

Figure 4 illustrates that for both the standard and alternative Hodrick-Prescott estimates, data revision appears to play a more significant role, sometimes being of the same order of magnitude as the Real-Time estimate. However, in comparison to Total Revision, its contribution is still relatively minor. The Real-Time estimate of the Band-Pass filter is subject to a large degree of revision over the sample period, but little of it stems from data revision. In general, the Band-Pass Filter produces ‘pessimistic’ Real-time estimates, with the output gap almost always being negative or close to zero - which is not always a true reflection of the economy. The total revision and contribution of data revision are not clear for the Beveridge-Nelson decomposition due to significant outliers in the Quasi-Real estimates (which have subsequently been transferred into the data revision series). Nevertheless, when taking a closer look at the data, it was found that at least 60 per cent of the time, data revision was of the same order of magnitude as the Real-Time estimate and total revision (although not necessarily at the same time). The latter indicates that for many points in time, total revision can be exclusively explained by data revision. Similar results were found by Orphanides and Van Norden (2002) for the US economy. Finally, Figure 4 illustrates that data revisions explain little of the revision of the Real-Time estimates of the Unobserved Components model. Interestingly, however, the role of parameter revision is just as small, if not smaller.

By decomposing the revisions of the different methods used in this analysis
Figure 4: Decomposition of revisions of the output gap

To estimate South African output gaps, it is clear that both data and parameter revisions generally play a relatively small role in total revisions. This indicates that revision can be largely attributed to new data points becoming available.
5.4. Alignment of Real-Time estimates with business cycle phases of the South African economy

An additional perspective on the reliability of real-time output gap estimates is provided by an analysis of the alignment of Real-Time estimates with the business cycle phases of the South African economy. In this sense, despite the pessimistic nature of revisions documented in this paper, many of the Real-Time estimates provide a relatively realistic view of the economy. Figure 5 compares the Real-Time output gap estimates with the business cycle phases of the South African economy dated by the South African Reserve Bank (2010). The shaded areas reflect the downward phases of the business cycles while the unshaded areas reflect the upward phases. The Real-Time output gaps identified by the methods in this study appear to be aligned with the SARB’s view of South African business cycles. Given lags in the transmission to inflation, though, what this alignment means for the real-time forecasting ability of these output gaps remains an open question.

Figure 5: Real-time output gap estimates and South African business cycle down swings
6. Conclusion

This study constructed a new real-time dataset for South African GDP, and generated and analysed output gap estimates using a number of different univariate methods. A number of results were documented. First, the analysis demonstrated that significant revisions to real-time output gap estimates occur in the South African case, regardless of the method used to estimate the gaps. The proportion of time that the absolute value of the revision exceeded that of the absolute value of the Final estimate was always above 30 per cent (and generally ranged between 41 and 46 per cent, depending on the method used).

Second, the Real-Time gaps were found to be quite unreliable in that most were poorly correlated with Final estimates (only one real-time gap estimate had a correlation coefficient above 60 per cent). Furthermore, it was not uncommon for the Real-Time estimates to have opposite signs from the Final estimates. For most methods this occurred at least 35 per cent of the time.

Third, several differences among the methods used to estimate the output gaps were apparent, although no method clearly outperformed all others. For example, the simpler linear and quadratic trend Real-Time estimates were the worst correlated with Final estimates. The mechanical filters tended to perform better in this regard, but the Real-Time estimates had the opposite sign to the Final estimates more often than the Beveridge-Nelson approach. The South African adaptation of the Hodrick-Prescott Filter improved only marginally on the standard Hodrick-Prescott filter in terms of the metrics considered. The Beveridge-Nelson Decomposition had many good features, although outliers in the gap estimates raise questions regarding real-time reliability.

Fourth, regarding the sources of revisions, it was found that both data and parameter revision tended to play a relatively small role. Instead, revision stemmed from new data points becoming available, despite forecast-augmenting adjustments being made in certain cases in an attempt to improve the end-of-sample estimates. The end-of-sample problem is the largest source of unreliability in these real-time output gap estimates for South Africa.

Finally, on a more positive note, trends in the Real-Time output gap estimates were generally aligned with the business cycle phases of the South African econ-
omy in that negative output gaps were generally associated with the downward phases of the business cycle.

Overall, these results suggest that real-time estimates of the output gap for South Africa may be misleading and could potentially negatively influence analysis of the state of the economy and monetary policy decision-making. This is of particular concern for South Africa as the real-time output gap remains a key tool for policy-makers and is regularly used as a justification for policy decisions.

Further research is essential in order to better understand the nature of the uncertainties associated with real-time output gap estimates in South Africa. The merits of alternative, more complex, measures of output gaps need to be investigated in a real-time context, and it is also important to know how useful real-time estimates of output gaps (rather than final estimates) are for predicting the future path of inflation. Importantly, the questions that are raised regarding the weight that should be given to real-time output gap estimates, and how best to factor in the uncertainty arising from unreliable real-time estimates, need to be considered carefully when taking policy decisions.
References


South African Reserve Bank. (2010, March). South Africa’s national accounts


Table A.1: Detailed Breakdown of Revision Statistics: 1981Q3 – 2010Q3

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<thead>
<tr>
<th>Method</th>
<th>Mean</th>
<th>Std. Dev</th>
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<th>Max</th>
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</tr>
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Note: (FL: final; RT: real time; QR: quasi-real; QF: quasi-final).