A Theoretically Defensible Measure of Risk: Using Financial Market Data from a Middle Income Context

Johannes Fedderke¹ and Neryvia Pillay²

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¹ School of Economics, University of Cape Town
² School of Economics, University of Cape Town
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Johannes Fedderke† and Neryvia Pillay‡

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Abstract

While economic theory assumes that risk is of central importance in financial decision making, it is difficult to measure the uncertainty faced by investors. Commonly used empirical proxies for risk (such as the moving standard deviation of the returns on an asset) are not firmly grounded in economic theory. Risk measures have been developed by other studies, but these are generally based on subjective weights attaching to a range of objective component indicators and are difficult to replicate. The contribution of this paper is to develop a methodology to construct theory-defensible empirical risk measures. It has the advantages of being explicitly consistent with economic theory and easily replicable. We illustrate this methodology by specific application to the South African context. The time-varying risk measure developed in this paper is consistent with the expectations hypothesis and captures the asymmetric nature of shocks. This measure reflects investors’ risk perceptions and accords with the literature on risk in South Africa.

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†University of Cape Town and Economic Research Southern Africa, johannes.fedderke@uct.ac.za.
‡University of Cape Town, neryvia.pillay@uct.ac.za
1 Introduction

While economic theory assumes that risk is important, it is difficult to construct a measure that appropriately captures the risks faced by investors. The contribution of this paper is to develop a methodology to construct a theoretically defensible measure of risk. We illustrate this methodology by specific application to the South African context.¹

The methodology developed here constructs a risk measure that is consistent with the expectations hypothesis of the term structure of interest rates. The framework of the expectations hypothesis enables us to extract the risk implied by the returns on government bonds. Since government bonds are free of default risk, and their prices are determined in the bond market, yields on government bonds can be assumed to reflect the pure country risk perceived by investors.

It should be noted at the outset that the objective is not to develop a formal measure of risk,² nor to operationalize "coherent risk" measures.³ Instead, the objective is to develop an operational measure of risk as manifested in bond prices. Crucially, the measure could be regarded as an aggregate measure of risk, incorporating political risk, and is derived from an explicit theoretical foundation. As such it is distinct from more ad hoc subjective measures of risk (such as the moving standard deviation of the returns on an asset or political instability indexes⁴).

The expectations hypothesis is the simplest and most commonly used hypothesis for explaining the term structure of interest rates.⁵ The expectations hypothesis holds that long-term rates are related to expectations of future short-term rates. Thus, changes in the shape of the term structure reflect a changed outlook for future interest rates relative to current rates.

The expectations hypothesis has been widely studied for US data. One implication of the expectations hypothesis is that long-term rates are related to expectations of future short-term rates. Thus, changes in the shape of the term structure reflect a changed outlook for future interest rates relative to current rates.

¹Fedderke et al. (2001) develop a risk measure based on political instability for South Africa, which is shown to have an impact on the real sector through the investment channel in Kularatne (2002), Mariotti (2002) and Fedderke (2004).
²See for instance the discussion in Gollier (2004) and Jorion (1997).
³See the discussion in Artzner et al (1997, 1999).
⁴For example, those developed by Fedderke, de Kadt and Luiz (2001), Freedom House, Polity and the World Bank.
⁵There are three main alternative theories: the liquidity preference hypothesis developed by Hicks (1946), the market segmentation hypothesis of Culbertson (1957) and arbitrage-free interest rate modelling. The liquidity preference hypothesis also involves the expected values of future spot rates, but emphasises the risk preferences of agents. The risk premium is assumed to increase with the time to maturity of the bond. Under the market segmentation hypothesis, agents have strong preferences for specific maturities. Thus, bond prices are determined in separate markets, and bonds of neighbouring maturities are not substitutes. Heath, Jarrow and Morton (1992) specify a general framework for all arbitrage-free interest rate models. These models can be calibrated to fit the current term structure.
hypothesis is that changes in the yield curve directly reflect changes in expected future rates. In general, the empirical evidence does not support this prediction and thus points to a rejection of the expectations hypothesis (see, for example, Fama (1976), Shiller (1979), Shiller, Campbell and Schoenholtz (1983), Mankiw and Summers (1984), Mankiw (1986), and Campbell and Shiller (1991)).

In their seminal paper, Engle and Granger (1987) employ cointegration techniques to test the expectations hypothesis. The expectations hypothesis posits that yields of different maturities will move systematically together over time, which implies that they may be cointegrated.

Engle and Granger (1987) and Campbell and Shiller (1987) find that the one-month US Treasury bill rate is cointegrated with the yield to maturity on the 20-year US Treasury bond. These studies use a single equation Engle-Granger approach to cointegration. Hall et al. (1992) was the first study to suggest that the term structure might be well modelled as a cointegrating system. They employ twelve yield series and find that while some yields are cointegrated, other yields are not – a finding inconsistent with the expectations hypothesis. In addition, they find that the expectations hypothesis holds for some sub-periods of their sample, but not for others; a further contradiction of the expectations hypothesis.

The evidence on the expectations hypothesis is therefore mixed. One possible explanation for the failure of the expectations hypothesis is the presence of a time-varying risk premium. However, studies that have attempted to incorporate a time-varying risk premium using various proxies continue to generate mixed results. Many of these studies still do not find support for the expectations hypothesis – for example Shiller, Campbell and Schoenholtz (1983), Froot (1989) and Simon (1989); while some of the studies do find support for the expectations hypothesis, such as Tzavalis and Wickens (1997). Harris (2001) uses panel data techniques with the risk premium captured by individual-specific and time-specific fixed effects. This approach has the advantage of allowing a risk premium that varies with maturity and over time without the need for a proxy that may not be based on economic theory, but the study still finds that the expectations hypothesis is rejected.

The development of fully specified general equilibrium models of the term structure, such as affine-yield

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6 The variables used to capture the risk premium include a moving standard deviation of short term rates, a variable representing the relative volumes issued of different maturities (Shiller, Campbell and Schoenholtz, 1983), survey data on interest rate expectations (Froot, 1989), a risk premium that is specified to be proportional to the volatility of excess returns using instrumental variables (Simon, 1989), and the ex post excess holding period return of one maturity as a proxy for the term premium of another maturity (Tzavalis and Wickens, 1997).
models (which have the property that log bond yields are linear in the state variables), may be viewed as another response to the inadequacies of the expectations hypothesis.

In this study we allow for a time-varying risk premium that also captures the asymmetric nature of shocks. Our approach does not use a proxy for the time-varying risk premium, but instead derives a risk premium that is explicitly consistent with the expectations hypothesis. Since the risk premium is derived from the expectations hypothesis, it cannot be incorporated in a test of the expectations hypothesis. However, it does provide a clear indication of the structure the time-varying risk premium would have to exhibit for the expectations hypothesis to hold. This allows us to compare the structure of standard proxies of the time-varying risk premium to the structure that is required for the expectations hypothesis to hold, to gain an indication of how close standard proxies have been to the time-varying risk premium that is consistent with the expectations hypothesis. It also provides an explicit, theoretically defensible measure of risk.

The remainder of the paper is organized as follows. Section 2 outlines the theoretical framework associated with the expectations hypothesis. Section 3 examines the data used in the empirical analysis and establishes the univariate characteristics of the data. Section 4 examines the empirical evidence on the central predictions of the expectations hypothesis. The empirical analysis points to a rejection of the expectations hypothesis, which we show to be attributable to a time-varying risk premium. Section 5 then constructs a time-varying risk measure that is consistent with the expectations hypothesis (modified to allow a time-varying risk premium) and with the literature on risk in South Africa. This risk measure diverges from the standard measures of time-varying risk employed in tests of the expectations hypothesis. Section 6 concludes.

2 The Expectations Hypothesis

Define $R(k, t)$ as the yield to maturity of a $k$ period zero coupon bond at time $t$, and $F(1, t + j)$ as the forward rate contracted at time $t$ to buy a one period zero coupon bond in $t + j$ which matures at the end of that period. Note that $R(1, t + j)$ denotes the actual rate or yield and it is not necessarily equal to the one period forward rate, $F(1, t + j)$ for $j > 0$. It must be the case that $F(1, t) = R(1, t)$.

An efficient market will arbitrage away any expected profit between the alternatives of either (1) buying a $k$ period bond at time $t$, or (2) buying a sequence of one period bonds. This yields the following relationship:
Although the forward rate typically differs from the corresponding actual rate, the forward rate is likely to be related to the expected actual rate. In particular, we assume:

\[ F(1, t + j) = E_t [R(1, t + j)] + \tau(1, t + j) \]  

where \( E_t \) denotes expectations based on the information available at time \( t \), and \( \tau(1, t + j) \) denotes the risk or term premium. This premium could arise, for example, because there is uncertainty about future interest rates, generating risks on forward contracts. In the “pure expectations hypothesis” the term premia are zero, while in the “expectations hypothesis” they are nonzero but constant.

Substituting (2) into (1) yields:

\[ R(k, t) = \frac{1}{k} \left[ \sum_{i=1}^{k} E_t [R(1, t + (i - 1))] \right] + L(k, t) \]

where

\[ L(k, t) = \frac{1}{k} \left[ \sum_{i=1}^{k} \tau(1, t + i) \right] \]

Equation (3) is considered the fundamental relationship of the expectations hypothesis. It states that the yield on a \( k \) period bond is a weighted average, with equal weights summing to one, of the current and expected future yields on a sequence of one period bonds. However, equation (3) cannot be directly estimated since none of the right-hand-side variables are directly observable.

Provided that yields are \( I(1) \), it is straightforward to show that under the expectations hypothesis the yields are cointegrated. Subtracting \( R(1, t) \) from both sides of equation (3) gives:

\[ [(R(k, t) - R(1, t))] = \sum_{i=1}^{k-1} \left( \frac{k - i}{k} \right) E_t [\triangle R(1, t + i)] + L(k, t) \]

where

\[ \triangle R(1, t + i) = R(1, t + i) - R(1, t + (i - 1)) \]
Equation (5) implies the yields on a long- and short-maturity bond differ by the liquidity premium, and by expected changes in the short rate over the horizon of the bond.

If \( R(1,t+i) \) is an \( I(1) \) process, then \( \Delta R(1,t+i) \) is stationary. Provided the premia \( L(k,t) \) are stationary, the right-hand side of equation (5) is stationary.

Recall that \( L(k,t) = \sum_{i=1}^{k} \tau(1,t+i) \), where \( \tau(1,t+i) \) denotes the risk or term premium associated with the forward rate. Under the expectations hypothesis, these risk premia are assumed to be non-zero but constant, rendering the \( L(k,t) \) stationary.

Hence, the left-hand-side of equation (5) must be stationary and, under the expectations hypothesis, each yield \( R(k,t) \) is cointegrated with \( R(1,t) \). We define the spread between the yields \( R(k,t) \) and \( R(1,t) \) as \( S(k,1,t) = R(k,t) - R(1,t) \). It follows that the spread, \( S(k,1,t) \), is the stationary linear combination that results from the cointegration of \( R(k,t) \) and \( R(1,t) \).

Note that the spread vector associated with any two yields (i.e. \( S(k,j,t) \)) is just a linear combination of two of the spread vectors defined using the one period yield (i.e. \( S(k,1,t) \) and \( S(j,1,t) \)). Since linear combinations of stationary variables are also stationary it follows that any spread vector is cointegrated.

The test thus constitutes the null hypothesis that \( S(k,j,t) \sim I(0) \). Under the alternative,\(^7\) consider:

\[
R(k_i,t) - \beta_j R(j,t) = \varepsilon_i(t) = \mu_i(t) + u_i(t)
\]

where \( \mu_i(t) \) represents the time-varying risk premium, associated with the spread of instrument \( i \), approximated by a non-linear trend structure in the error, and \( u_i(t) \sim NID(0,\sigma_u^2) \). Filtering \( \varepsilon_i(t) \) by means of the Hodrick-Prescott filter (Hodrick and Prescott, 1997):

\[
\min_{\{\mu_i(t)\}_{t=1}^{\infty}} \left\{ \sum_{t=1}^{T} u_i(t)^2 + \lambda_i \sum_{t=1}^{T} [(\mu_i(t) - \mu_i(t-1)) - (\mu_i(t-1) - \mu_i(t-2))]^2 \right\}
\]

provides an estimate of the time-varying measure \( L(t) = \sum_{i=1}^{n} \mu_i(t) \), such that, under appropriate specification of \( \lambda \), \( S(k,j,t) - L(t) \sim I(0) \), consistent with the requirement of the expectations hypothesis.

Under a finding rejecting the expectations hypothesis, this paper proceeds by means of the derivation of

\(^7\)This alternative requires that \( \varepsilon(t) \sim I(0) \), satisfied under \( \varepsilon(t) \sim I(d), d \simeq 1. \)
an estimate of the time varying premium in accordance with the approach outlined across equations (7) and (8).

For the sake of completeness, we note the theoretical critique of Cox, Ingersoll and Ross (1981) that most formulations of the pure expectations hypothesis are inconsistent with one another, and with a no-arbitrage equilibrium. However, McCulloch (1993) shows that the expectations hypothesis, under appropriate state assumptions, can be rendered consistent with the no arbitrage equilibrium condition, Fisher and Gilles (1998) generalize the point; and Longstaff (2000) establishes the consistency of the expectations hypothesis under incomplete markets. Since the development of the risk measure in this paper is dependent on an empirical examination of the expectations hypothesis, the most relevant response to the CIR critique is that of Campbell (1986) that the inconsistencies between the forms of the expectations hypothesis are second order, and therefore empirically not critical.

3 Data

The analysis is conducted on nominal yield data obtained from the South African Reserve Bank. We use the yields to maturity on: the three month Treasury bill, denoted $R(3, t)$; zero to three year government bonds, denoted $R(36, t)$; and ten year and over government bonds, denoted $R(120p, t)$.

The sample consists of 304 monthly observations from January 1981 to April 2006. Figure 1 plots the yield, and Figure 2 the differenced yield data.

TABLE 1 reports the relevant ADF tests for stationarity.8 Results confirm that the $R(3, t)$, $R(36, t)$ and $R(120p, t)$ series are $I(1)$ processes, with an absence of trend in all series, and drift present only in the $R(120p, t)$ series.

INSERT FIGURES 1 AND 2 HERE.

Table 1 reports the relevant ADF tests for stationarity.8 Results confirm that the $R(3, t)$, $R(36, t)$ and $R(120p, t)$ series are $I(1)$ processes, with an absence of trend in all series, and drift present only in the $R(120p, t)$ series.

INSERT TABLE 1 HERE.

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8 See Dickey and Fuller (1979, 1981).
4 Evidence of a Time-Varying Risk Premium

The empirical evidence of this section explores the results that emerge from the cointegration tests outlined in the theory section above. Consistent with the literature, the evidence points to a rejection of the expectations hypothesis. In addition, a rolling sample period analysis is conducted to determine whether the expectations hypothesis breaks down over certain sub-periods of the sample. GARCH-M modelling is used to determine whether the failure of the expectations hypothesis is attributable to a time-varying risk premium.

4.1 Empirical Methodology for Testing the Expectations Hypothesis

We employ the standard Johansen technique for multivariate cointegration.\(^9\) Consider the general \(k\)-dimensional VAR specification given by:

\[
Z_t = A_1Z_{t-1} + \ldots + A_mZ_{t-m} + \delta + u_t \tag{9}
\]

where \(m\) is the lag length, \(\delta\) deterministic terms and \(u_t\) a Gaussian error term.

Reparametrization provides the general VECM specification:

\[
\Delta Z_t = \sum_{i=1}^{j-1} \Gamma_i \Delta Z_{t-i} + \Pi Z_{t-j+1} + u_t \tag{10}
\]

The existence of \(r\) cointegrating relationships amounts to the hypothesis that:

\[
H_1(r) : \Pi = \alpha \beta' \tag{11}
\]

where \(\Pi\) is \(p \times p\) and \(\alpha, \beta\) are \(p \times r\) matrices of full rank. Therefore, \(H_1(r)\) is the hypothesis of reduced rank of \(\Pi\). Where \(r > 1\) issues of identification arise,\(^{10}\) resolved by means of restrictions on the loading matrix \((\alpha)\), the matrix representing the short-run dynamics \((\Gamma)\), or on the cointegrating space \((\beta)\).\(^{11}\)

The expectations hypothesis predicts that any spread vector is cointegrated. If we considered a set

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9 See Johansen (1988) and Johansen and Juselius (1990).
11 See the discussion in Greenslade et al. (1999).
of \( n \) yields, then we would expect \((n-1)\) cointegrating relationships since the spread vectors are linearly independent.\(^{12}\) Since for the present study we are using three yields we expect to find two cointegrating vectors, i.e. we expect the \( \Pi \) matrix to have a rank of two.

The previous discussion suggests specific overidentified cointegrating relationships (where the spreads are defined in terms of \( R(3,t) \)):

\[
\Pi Z_{t-j+1} = \begin{pmatrix}
\alpha_{11} & \alpha_{12} \\
\alpha_{21} & \alpha_{22} \\
\alpha_{31} & \alpha_{32}
\end{pmatrix} \begin{pmatrix}
\beta' \\
-1 \\
-1
\end{pmatrix}
\]

\[\begin{pmatrix}
R(3,t-j+1) \\
R(36,t-j+1) \\
R(120p,t-j+1)
\end{pmatrix} \]

(12)

where

\[
\beta' = \begin{pmatrix}
-1 & 1 & 0 \\
-1 & 0 & 1
\end{pmatrix}
\]

(13)

Should we not find the required number of cointegrating vectors or the cointegrating relationships predicted by the theory, this indicates a rejection of the expectations hypothesis. The failure of the expectations hypothesis may suggest that an alternative theory is more appropriate for the term structure (see the discussion in the introduction). Another possibility is the presence of a time-varying risk premium, which is the one explored in this paper.

4.2 Cointegration Analysis: Full Sample Period

Estimation proceeds by means of the VECM technique with a VAR of order two. Based on the results from Table 1, we proceed with unrestricted intercepts and no trends in the VAR. The \( \lambda_{max} \) and \( \lambda_{trace} \) test statistics for the number of cointegrating vectors (\( r \)) are reported in Table 2.

INSERT TABLE 2 HERE.

Both test statistics indicate that we can reject the null hypothesis that \( r = 0 \) at the 5% level. However, we cannot reject the null hypothesis that \( r \leq 1 \) with either of the test statistics. These tests indicate that we

\(^{12}\)While it may be relevant to perform unit root tests on the spreads to test the expectations hypothesis, Hall et al. (1992) note that a unit root test on a spread tests the null hypothesis that the spread vector is not cointegrating instead of the required null that the spread vector is cointegrating. In addition, if we have more than two yields, there will be more than one spread vector. Unit root tests are performed on individual spread vectors and do not test the joint hypothesis that all the spread vectors are cointegrating.
have one cointegrating vector in the data, contradicting the first theoretical prediction of the expectations hypothesis theory of the term structure.

However, given the small sample properties of the cointegration tests, and the strong theoretical prior, we proceed with VECM estimation on the assumption of two cointegrating vectors.

The estimates of the cointegrating relationships under just identifying restrictions are reported in Table 3,\textsuperscript{13} while results for the tests on the overidentifying restrictions of the expectations hypothesis are reported in Table 4.

INSERT TABLES 3 AND 4 HERE.

The first hypothesis test in Table 4 examines whether the homogeneity restrictions hold jointly. The test statistic indicates that we cannot reject the overidentifying restrictions imposed by the expectations hypothesis. The remaining two tests are conducted to determine whether the homogeneity restrictions hold separately for each cointegrating vector. Again the test statistics for the tests that $\beta_{21} = 1$ and $\beta_{32} = 1$ indicate that we cannot reject the null hypotheses that the overidentifying restrictions hold separately.

These results indicate mixed evidence for the expectations hypothesis. The first central prediction of the expectations hypothesis is that there will be two cointegrating vectors present in the data. However, this prediction is not supported by the $\lambda_{max}$ and $\lambda_{trace}$ test statistics. If, in accordance with the theory, we impose two cointegrating vectors then we cannot reject the overidentifying restrictions implied by the expectations hypothesis, providing empirical support for the second central prediction of the expectations hypothesis.

We find empirical support for only one of the two central predictions of the expectations hypothesis. Consequently, we cannot conclude that the expectations hypothesis holds for South Africa over our sample period.

4.3 Cointegration Analysis: Rolling Sample Period

One possibility is that the expectations hypothesis may have broken down over some of the sub-periods covered by our sample. The relatively high political instability faced by South Africa during the 1980s (see

\textsuperscript{13}Impulse response functions confirm stability of the cointegrating vectors. Results available from the authors on request.
Fedderke et al. (2001)) renders this possibility plausible. Consequently, we now investigate whether the expectations hypothesis holds for sub-periods of our full sample period. As before, estimation proceeds by means of the VECM technique with a VAR of order two with unrestricted intercepts and no trends. We roll a ten-year sample period forward by six months at a time.

Figure 3 shows the results from the $\lambda_{\text{max}}$ and $\lambda_{\text{trace}}$ test statistics for the number of cointegrating vectors. As can be seen, in general, we still find only one cointegrating vector, and not the two predicted by the expectations hypothesis, for most sub-periods of the study. Specifically, we find only three sub-periods (1983M1 – 1992M12, 1987M7 – 1997M6 and 1988M7 – 1998M6) with the required two cointegrating vectors.

Figure 4 plots the test statistics of the test for whether the homogeneity restrictions hold jointly (conditional on there being two cointegrating vectors). Figure 5 plots the test statistics to determine whether the homogeneity restrictions hold for the spread between $R(36,t)$ and $R(3,t)$, and Figure 6 plots the test statistics to determine whether the homogeneity restrictions hold for the spread between $R(120p,t)$ and $R(3,t)$.

The figures show that we can reject (both jointly and for each cointegrating vector) the overidentifying restrictions imposed by the expectations hypothesis over most of the eighties while we cannot reject them for most of the nineties.

INSERT FIGURES 3 - 6 HERE.

These results also fail to support the first central prediction of the expectations hypothesis, with the tests generally indicating that fewer than the required number of cointegrating vectors is present in the data. Conditional on there being two cointegrating vectors, we find that the homogeneity restrictions do not hold for the first ten years of the sample period whereas previously full sample tests confirmed the homogeneity restrictions. In effect, support for the second central prediction of the expectations hypothesis is weakened relative to the full sample tests.

### 4.4 Evidence of a Time-Varying Risk Premium

A second possibility for the failure of the expectations hypothesis is that the risk premium is time-varying. The expectations hypothesis assumes that the risk premia are non-zero but constant, which yields the
prediction that the spread between any two yields must be cointegrated (see equation (5)). The theoretical assumption that the risk premium is constant renders it stationary, which is crucial to the cointegration testing procedure carried out above. If the risk premium is time-varying, the testing procedure may become invalid and we cannot draw inferences about the expectations hypothesis from the results.

Since our expectation is that the excess holding yield is a positive function of its own risk, proxied here by its conditional variance, the appropriate model is GARCH-M. Given the presence of asymmetries in the GARCH-M estimation, we proceed by the EGARCH-M formulation of the estimation procedure (Nelson, 1991).

Following Engle, Lilien and Robins (1987) we define the one quarter excess holding yield on the longer bonds relative to the three month T-bill as:

\[ y(i, t) = R(i, t) - R(3, t) - 1 + \frac{R(i, t)}{R(i, t + 1)} \]  

where \( R(i, t) \) is the quarterly yield to maturity on the longer term bond (i.e. the zero to three year and the ten year and over bonds).

We hypothesize that the excess holding yield is a function of its own risk, measured by the conditional variance \( (h_t^2) \). The riskless return is given by \( \mu \). The conditional variance is a function of both the magnitude and sign of shocks, yielding the following model (Nelson, 1991):

\[
\begin{align*}
    y_t & = \mu + \gamma h_t^2 + \varepsilon_t \\
    \log h_t^2 & = \alpha_0 + \sum_{i=1}^{q} \alpha_i \left( \frac{\varepsilon_{t-i}}{h_{t-i}} \right) + \sum_{i=1}^{p} \delta_i \log h_{t-i}^2
\end{align*}
\]

Table 5 gives the results from the EGARCH-M specifications of the excess returns of bonds with zero to three years to maturity \( (y(36, t)) \), and of the excess returns of bonds with ten years and over to maturity \( (y(120p, t)) \).

INSERT TABLE 5 HERE.

Given the significance of the \( h_t^2 \) terms in the mean equation, results confirm the presence of a time-varying
risk premium. While the insignificance of the $|\varepsilon/h_{t-1}| - E(|\varepsilon/h_{t-1}|)$ terms suggests that the magnitude of the shock does not impact the conditional variances; the positive and statistically significant coefficients on $(\varepsilon/h)_{t-1}$ indicate that volatility increases (decreases) when shocks are positive (negative), suggesting an asymmetrical effect of excess return shocks.

To confirm the asymmetrical effect of shocks we run the Engle and Ng (1993) test for asymmetry, with results reported in Table 6. The $S$ variable is a binary variable for negative shocks. The $\varepsilon^+$ and $\varepsilon^-$ variables control for positive and negative shocks respectively.

INSERT TABLE 6 HERE.

The significance of the $\varepsilon^+$ and $\varepsilon^-$ terms indicate that both positive and negative shocks affect the conditional variances of both $y(36,t)$ and $y(120p,t)$. However, the $S$ term is significant only for $y(36,t)$, suggesting that the sign of shocks impacts the conditional variance of $y(36,t)$ only. These results confirm the asymmetrical effect of $y(36,t)$ shocks, while evidence for the asymmetrical effect of $y(120p,t)$ shocks is weaker.

In summary, there is evidence to suggest that both excess returns $y(36,t)$ and $y(120p,t)$ depend on their conditional variances, consistent with a time-varying risk premium. While we find clear evidence of an asymmetric impact of shocks for $y(36,t)$, evidence for asymmetries with respect to $y(120p,t)$ is weaker. This suggests risk premia that are time-varying, asymmetric and distinct between the two spreads examined in this paper. This violates the constant risk premia assumption of the expectations hypothesis, and potentially renders the standard cointegration tests inappropriate.

Moreover, given the differential structure of time varying conditional variances across the term structure, the error structures are potentially complex, differing across the various excess returns. This suggests that if we are to construct a single risk measure that is consistent with all the spread vectors of the expectations hypothesis, the implication is that we may have to extract the risk implied in each of the error structures individually.
5 Constructing a Theoretically Defensible Risk Measure

One of the crucial assumptions of the expectations hypothesis is that the risk premium is constant, which yields the result that any spread vector must be cointegrated (see Section 2). However, the evidence presented in Section 4 rejects this prediction of the expectations hypothesis. Furthermore, this rejection is shown to be the result of a time-varying risk premium.

Since the estimated cointegrating vectors are the spread vectors, they do not allow for a time-varying risk premium. It follows that the time-varying risk premium must be contained in the error processes of the cointegrating vectors. Therefore, we construct a risk measure based on the errors of the estimated single-equation cointegrating vectors in (7). Each error process is filtered by (8), and the resulting series are combined to generate the risk measure that is consistent with the expectations hypothesis.\(^{14}\)

Figures 7 and 8 illustrate the errors from the individual single-equation cointegrating vectors, and the smoothed errors that form the components of the risk measure. These figures suggest that the first error structure contributes the overall trend to the final risk measure, while the second contributes the fluctuations. The final risk measure is an equally weighted combination of the smoothed errors from the individual single-equation cointegrating vectors and is illustrated in Figure 9.

INSERT FIGURES 7 - 9 HERE.

We now show that this risk measure is consistent with the expectations hypothesis (modified to allow a time-varying risk premium). The relevant ADF unit root tests\(^{15}\) reported in Table 7 confirm that the risk measure is an \(I(0)\) process. As such, it will not enter into the long-run equilibrium relationships, but instead into the dynamics of the system. Estimation proceeds, as before, with a VAR of order two (as suggested by the Schwarz Bayesian Criterion) with unrestricted intercepts and no trends.

INSERT TABLES 7 AND 8 HERE.

The \(\lambda_{max}\) and \(\lambda_{trace}\) test statistics now indicate that we can reject the null hypotheses that \(r = 0\) and that \(r \leq 1\) at the 5% level. Thus, we find two cointegrating vectors, consistent with the expectations hypothesis.\(^{16}\)

\(^{14}\)We deviate from the optimal choice of the smoothing coefficient for the Hodrick-Prescott filter suggested by Harvey and Jaeger (1993). We experimented with various smoothing coefficients to generate a risk measure that is consistent with the expectations hypothesis. The object of the exercise is explicitly a risk measure which is consistent with the expectations hypothesis, not an efficient test of the expectations hypothesis.

\(^{15}\)See Dickey and Fuller (1979, 1981).

\(^{16}\)The emergence of two cointegrating vectors on inclusion of an \(I(0)\) variable is explained by the fact that the ADF statistics
Table 9 reports the tests of the overidentifying restrictions implied by the expectations hypothesis.

The first hypothesis test in Table 9 examines whether the homogeneity restrictions hold jointly. The remaining two tests are conducted to determine whether the homogeneity restrictions hold separately for each cointegrating vector. We cannot reject the null hypothesis for any of these tests, indicating that the homogeneity restrictions imposed by the expectations hypothesis hold.

Thus, our evidence shows that our time-varying risk premium is consistent with the expectations hypothesis.

Figure 9 shows that this constructed risk measure is time-varying, and is broadly consistent with the literature on risk in South Africa.

Risk spiked in 1986 and 1994 (though at a lower level than in 1986) as a result of the increased political risk of those years. The year 1986 was a time of peak political instability (Fedderke et al., 2001) characterized by widespread sabotage activities on the part of the African National Congress (ANC) in resistance to apartheid, a second state of emergency with associated extra-judicial forms of repressive state activity, and growing international pressure to dismantle the apartheid system (SAIRR, 1987, 1988). The first democratic elections were held in 1994, and in the lead-up to the elections it was unclear whether the elections would take place peacefully or be derailed by violent resistance. This period saw high racial tensions, sabotage attempts by white right-wing groups, and increased political fatalities (SAIRR, 1995).

The spikes in 2001 and 2004 are largely associated with increased exchange rate risk in those years. Figure 10 illustrates the Rand/$ exchange rate over our sample period. The currency crisis of 2001 was caused by various factors, including a spillover effect of deteriorating economic conditions in Argentina, the political turmoil in neighbouring Zimbabwe, and the tightening of the enforcement of exchange controls (see Bhundia and Ricci, 2006). There was also a global stock market and economic downturn and the beginning of a US technical recession in 2001 (Aron and Muellbauer, 2006), which may have increased the risks associated with emerging market economies. In 2004, the currency strongly appreciated (SARB, 2004) which may have resulted in exchange rate risk if investors perceived the exchange rate to be overvalued.

on the risk measure are borderline stationary. Explicitly controlling for the near non-stationary risk measure is thus likely to generate more stationary linear combinations of the variables in the expectations hypothesis test – which is the result we find.
The peaks of the risk measure have been declining over time, suggesting that the uncertainty surrounding the South African investment climate is improving. In Figure 11 we compare the risk measure constructed here (\textit{RISK}) with some commonly used political risk measures: the political instability measure developed by Fedderke, de Kadt and Luiz (2001), which we denote \textit{FKL}; Moodys’ and Standard and Poors’ ratings on domestic currency long-term government bonds; Freedom House indexes of political rights (\textit{FHPR}) and civil liberties (\textit{FHCL}); the Polity index; the World Bank political stability index (\textit{WBPS}); the moving standard deviation of the short rate (widely used in the international literature as a proxy for time-varying risk in tests of the expectations hypothesis); and the spread between the South African and US 3 month Treasury Bill rate.

The \textit{FKL} measure was only constructed on a yearly basis until 1996. Nevertheless a high correlation emerges between the two risk measures over the 1980s. This suggests that our constructed risk measure is consistent with the political risk faced by investors over this period. The new measure also suggests that the \textit{FKL} measure underestimates risk subsequent to the 1980s. The present measure is theory-defensible, while the \textit{FKL} measure is a subjective measure and was constructed by means of the Delphi technique. Given that the \textit{FKL} measure was constructed in the mid-1990s, it is not surprising that there may have been a framing effect of the reduced levels of political tension of the 1990s, relative to the 1980s.

There is a low correlation of our risk measure with all other risk measures, with our measure giving a more nuanced representation of risk in South Africa. Moreover, our risk measure captures the total risk (including political, economic and exchange rate risk) of investing in the country whereas the other measures capture mainly political risk.

We note specifically that the moving standard deviation of the short rate, which is widely used in the literature as a proxy for time-varying risk, does not match our risk measure well. This suggests that tests using the moving standard deviation as a proxy for the time-varying risk premium are unlikely to generate empirical support for the expectations hypothesis,\footnote{We tested the expectations hypothesis in the presence of the moving standard deviation risk measure. Results continue to reject the expectations hypothesis. Results are available from the authors on request.} given that our risk measure is explicitly derived from...
the error structure of the expectations hypothesis test, to be consistent with the latter.

Further, the alternative risk measure given by the spread between the South African and US 3 month Treasury Bill rates similarly manifests a very different structure from that found in the risk measure consistent with the expectations hypothesis. Most notable is that the cycle found in the theoretically defensible measure is countercyclical in comparison to the SA-US spread.

The inference is that we have constructed a theoretically defensible measure of risk in South Africa that reflects the risk perceived by investors.

6 Conclusion

While economic theory assumes that risk is of central importance in financial decision making, it is difficult to measure the uncertainty faced by investors. Commonly used proxies for risk (such as the moving standard deviation of the returns on an asset) are not firmly grounded in economic theory. Risk measures have been developed by other studies (such as Fedderke et al., 2001), but these are generally based on subjective weights attaching to a range of objective component indicators and are difficult to replicate.

The contribution of this paper is to develop a methodology to construct theory-defensible empirical risk measures. It has the advantages of being explicitly consistent with economic theory and easily replicable. We illustrate this methodology by specific application to the South African context.

The methodology developed here constructs a risk measure that is consistent with the expectations hypothesis of the term structure of interest rates. One of the crucial assumptions of the expectations hypothesis is that the risk premium is constant, which yields the result that any spread vector must be cointegrated. However, the evidence presented in this paper rejects the central predictions of the expectations hypothesis. This rejection is shown to be the result of a time-varying risk premium.

The evidence suggests risk premia that are time-varying, asymmetric and distinct between the two spreads examined in this paper. Given the differential structure of time varying conditional variances across the term structure, the error structures are potentially complex, differing across the various excess returns.

On this basis, we construct a time-varying risk measure that is consistent with the expectations hypothesis and captures the asymmetric nature of shocks. This measure reflects investors’ risk perceptions and accords
with the literature on risk in South Africa.

References


Table 1: Unit Root Tests for Yields

<table>
<thead>
<tr>
<th>Variable</th>
<th>Model</th>
<th>$\tau$</th>
<th>$\Phi_3$</th>
<th>$\tau_\mu$</th>
<th>$\Phi_1$</th>
<th>$\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R(3, t)$</td>
<td>ADF(2)</td>
<td>-3.323</td>
<td>6.078</td>
<td>-2.482</td>
<td>3.081</td>
<td>-0.715</td>
</tr>
<tr>
<td>$R(36, t)$</td>
<td>ADF(1)</td>
<td>-3.338</td>
<td>6.139</td>
<td>-2.551</td>
<td>3.265</td>
<td>-0.675</td>
</tr>
<tr>
<td>$R(120p, t)$</td>
<td>ADF(1)</td>
<td>-2.658</td>
<td>4.483</td>
<td>-1.456</td>
<td>6.085*</td>
<td>-0.655</td>
</tr>
</tbody>
</table>

Test statistics for 2 unit roots

<table>
<thead>
<tr>
<th>Variable</th>
<th>Model</th>
<th>$\tau$</th>
<th>$\Phi_3$</th>
<th>$\tau_\mu$</th>
<th>$\Phi_1$</th>
<th>$\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R(3, t)$</td>
<td>ADF(1)</td>
<td>-9.204*</td>
<td>42.360*</td>
<td>-9.144*</td>
<td>41.810*</td>
<td>-9.160*</td>
</tr>
<tr>
<td>$R(36, t)$</td>
<td>ADF(1)</td>
<td>-10.572*</td>
<td>55.899*</td>
<td>-10.505*</td>
<td>55.178*</td>
<td>-10.521*</td>
</tr>
<tr>
<td>$R(120p, t)$</td>
<td>ADF(1)</td>
<td>-11.232*</td>
<td>63.085*</td>
<td>-11.103*</td>
<td>61.636*</td>
<td>-11.109*</td>
</tr>
</tbody>
</table>

5% Critical values for $T=300$

<table>
<thead>
<tr>
<th>$\tau$</th>
<th>$\Phi_3$</th>
<th>$\tau_\mu$</th>
<th>$\Phi_1$</th>
<th>$\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3.4267</td>
<td>6.340</td>
<td>-2.8716</td>
<td>4.630</td>
<td>-1.945</td>
</tr>
</tbody>
</table>

Note: * significant at 5% level,
$\tau$: no constant or trend, $\tau_\mu$: constant, $\tau_\tau$: constant and trend in regression.

Table 2: Test Statistics for Number of Cointegrating Vectors

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>List of eigenvalues: .095127 .029317 .0042314</td>
<td></td>
</tr>
<tr>
<td>$\lambda_{max}$</td>
<td>$\lambda_{trace}$</td>
</tr>
<tr>
<td>$r = 0$</td>
<td>30.1881*</td>
</tr>
<tr>
<td>$r \leq 1$</td>
<td>8.9860</td>
</tr>
<tr>
<td>$r \leq 2$</td>
<td>1.2806</td>
</tr>
</tbody>
</table>

Note: * significant at 5% level

Table 3: Estimates of Restricted Cointegrating Relations

<table>
<thead>
<tr>
<th>Cointegration with unrestricted intercepts and no trends in the VAR</th>
<th>302 observations from 1981M3 to 2006M4. Order of VAR = 2, $r = 2$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R(3, t)$</td>
<td>$R(36, t)$</td>
</tr>
<tr>
<td>Vector 1</td>
<td>Vector 2</td>
</tr>
<tr>
<td>-1.0000 (<em>none</em>)</td>
<td>-1.0000 (<em>none</em>)</td>
</tr>
<tr>
<td>0.8653 (0.31876)</td>
<td>0.0000 (<em>none</em>)</td>
</tr>
<tr>
<td>0.0000 (<em>none</em>)</td>
<td>0.7012 (0.40581)</td>
</tr>
</tbody>
</table>

Note: standard errors in parentheses
### Table 4: Tests that Homogeneity Restrictions Hold

<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>Restriction</th>
<th>Test statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homogeneity restrictions on $S(36,3,t)$ and $S(120p,3,t)$ hold jointly</td>
<td>$\beta_{21} = \beta_{32} = 1$</td>
<td>2.3149</td>
</tr>
<tr>
<td>Homogeneity restrictions on $S(36,3,t)$ hold</td>
<td>$\beta_{21} = 1$</td>
<td>0.2190</td>
</tr>
<tr>
<td>Homogeneity restrictions on $S(120p,3,t)$ hold</td>
<td>$\beta_{32} = 1$</td>
<td>0.5567</td>
</tr>
</tbody>
</table>

Note: * significant at 5% level

### Table 5: EGARCH-M Models for $y(36,t)$ and $y(120p,t)$

<table>
<thead>
<tr>
<th>Regressor</th>
<th>Dependent variable is $y(36,t)$</th>
<th>Dependent variable is $y(120p,t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EGARCH-M (3,1)</td>
<td>EGARCH-M (3,1)</td>
</tr>
<tr>
<td>constant</td>
<td>-0.53365*** (0.14464)</td>
<td>-0.63568*** (0.20002)</td>
</tr>
<tr>
<td>$h^2$</td>
<td>15.8379*** (5.6972)</td>
<td>10.9492*** (2.8840)</td>
</tr>
</tbody>
</table>

| Conditional variance $h^2$ | Dependent variable is $y(36,t)$ | Dependent variable is $y(120p,t)$ |
|                           | EGARCH-M (3,1)                  | EGARCH-M (3,1)                   |
| constant                 | -2.5541*** (0.44510)            | -1.9790*** (0.26824)             |
| ($e / h$)$_{t-1}$        | 0.23444** (0.10726)             | 0.25980*** (0.072083)            |
| $(e / h)_{t-1} - E[(e / h)_{t-1}]$ | -0.17751 (0.12599)          | -0.074395 (0.049332)            |
| $\log h_{t-3}^2$         | 0.23851* (0.12217)             | 0.25129*** (0.094573)           |

Note: standard errors in parentheses,

*** significant at 1% level, ** significant at 5% level, * significant at 10% level
### Table 6: Engle and Ng Test for $y(36, t)$ and $y(120p, t)$

<table>
<thead>
<tr>
<th>Regressor</th>
<th>Dependent variable is $\varepsilon^2$ from $y(36, t)$</th>
<th>Dependent variable is $\varepsilon^2$ from $y(120p, t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>-0.044287*** (0.010039)</td>
<td>-0.10403*** (0.011423)</td>
</tr>
<tr>
<td>$S$</td>
<td>-0.028491** (0.013621)</td>
<td>0.0074888 (0.018192)</td>
</tr>
<tr>
<td>$\varepsilon^+$</td>
<td>-0.69647*** (0.036871)</td>
<td>0.84123*** (0.028389)</td>
</tr>
<tr>
<td>$\varepsilon^-$</td>
<td>0.52842*** (0.040187)</td>
<td>-0.73764*** (0.037033)</td>
</tr>
</tbody>
</table>

Note: standard errors in parentheses,
*** significant at 1% level, ** significant at 5% level, * significant at 10% level.

### Table 7: Unit Root Tests for Risk Measure

<table>
<thead>
<tr>
<th>Model</th>
<th>$\tau_\tau$</th>
<th>$\tau_\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADF(4)</td>
<td>-3.837*</td>
<td>-3.807*</td>
</tr>
</tbody>
</table>

5% Critical values for $T=300$

<table>
<thead>
<tr>
<th>$\tau_\tau$</th>
<th>$\tau_\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3.4267</td>
<td>-2.8716</td>
</tr>
</tbody>
</table>

Note: * significant at 5% level,
$\tau_\mu$: constant, $\tau_\tau$: constant and trend in regression.

### Table 8: Test Statistics for Number of Cointegrating Vectors

Cointegration with unrestricted intercepts and no trends in the VAR
List of eigenvalues: 0.15252  0.093748  0.0028164

<table>
<thead>
<tr>
<th>Null</th>
<th>$\lambda_{max}$</th>
<th>$\lambda_{trace}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r = 0$</td>
<td>49.9758*</td>
<td>80.5558*</td>
</tr>
<tr>
<td>$r \leq 1$</td>
<td>29.7283*</td>
<td>30.5800*</td>
</tr>
<tr>
<td>$r \leq 2$</td>
<td>0.85175</td>
<td>0.85175</td>
</tr>
</tbody>
</table>

Note: * significant at 5% level
<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>Restriction</th>
<th>Test statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homogeneity restrictions on $S(36,3,t)$ and $S(120p,3,t)$ hold jointly</td>
<td>$\beta_{21} = \beta_{32} = 1$</td>
<td>3.5337</td>
</tr>
<tr>
<td>Homogeneity restrictions on $S(36,3,t)$ hold</td>
<td>$\beta_{21} = 1$</td>
<td>3.5337</td>
</tr>
<tr>
<td>Homogeneity restrictions on $S(120p,3,t)$ hold</td>
<td>$\beta_{32} = 1$</td>
<td>0.84477</td>
</tr>
</tbody>
</table>

Note: * significant at 5% level
Figure 1: Yields to Maturity (Levels)

Figure 2: Yields to Maturity (First Differences)
Figure 3: Number of Cointegrating Vectors

Figure 4: Tests that Homogeneity Restrictions Hold Jointly

Figure 5: Tests that Homogeneity Restrictions Hold for the Spread Between $R(36, t)$ and $R(3, t)$

Figure 6: Tests that Homogeneity Restrictions Hold for the Spread Between $R(120p, t)$ and $R(3, t)$
Figure 7: Risk Measure Component from First CV

Figure 8: Risk Measure Component from Second CV

Figure 9: Final Risk Measure
Figure 10: Rand/$ Exchange Rate
Figure 11: Comparison of Risk Measures