Endogenous market transparency and product differentiation

Witness Simbanegavi ¹

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¹ School of Economics, University of Cape Town
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Witness Simbanegavi*

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Abstract

This paper endogenizes both market transparency and product differentiation in a model of informative advertising à la Grossman and Shapiro (1984). We find, contrary to Schultz (2004), that an increase in market transparency raises firm profits but has no effect on product differentiation. We also find that a move from exogenous to endogenous market transparency is detrimental to welfare.

Compared to the Grossman and Shapiro model, with endogenous product differentiation, firms advertise more, differentiate their products more, charge higher prices and earn higher profits when the advertising cost is "not too low". This is because endogenizing product differentiation relaxes price competition when the advertising cost is not too low.

Keywords: Endogenous market transparency, advertising intensity, exogenous market transparency, product differentiation

JEL classification: L13, L15, M37

1 Introduction

Schultz (2004) studies the effects of increased market transparency on product differentiation and competition in a Hotelling model of product differentiation. He finds that increasing market transparency on the consumers’

*School of Economics, University of Cape Town, Private Bag Rondebosch, 7701 RSA. E-mail: Witness.Simbanegavi@uct.ac.za
side leads to "less product differentiation, lower prices and lower profits" (p. 177). Since transportation costs decrease as firms locate close together, he thus concludes that increasing market transparency on the consumers' side is welfare improving.

In Schultz's model, however, market transparency is exogenous. Firms have no control whatsoever on the degree of market transparency. In practice though, firms have some influence. In particular, for consumer goods, firms generally have considerable influence on the degree of market transparency through their advertising efforts. Indeed, empirical and anecdotal evidence show that firms spend a fortune on advertising. It is important therefore to investigate the implications of relaxing the assumption of exogenous market transparency.

In Schultz (2004), market transparency is exogenous while product differentiation is endogenous. On the other hand, in the Grossman and Shapiro (1984) model and related models of informative advertising, market transparency is endogenous but product differentiation is exogenous. Firms however typically choose both the reach of their advertising campaigns as well as the extent of product diversity vis-à-vis competing brands. This paper endogenizes the choice of market transparency as well as product differentiation. Our approach is to allow firms to have full and sole control over market transparency (as in, for example, Grossman and Shapiro (1984)) and study implications for the degree of market transparency as well as product differentiation. In particular, we seek to address the following questions: How much advertising will the firms choose and will the firms locate closer or

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1 For instance, Bagwell (2005) reports that: "in 2003 in the US, General Motors spent $3.43 billion to advertise its cars and trucks; Procter and Gamble devoted $3.32 billion to the advertisement of its detergents and cosmetics; and Pfizer incurred a $2.84 billion advertising expense for its drugs" (p. 3).


3 Generally, firms do not have sole control over market transparency as consumers can and often do (independently of firms' advertising efforts) engage in information acquisition activities that raise market transparency.
further apart compared to Schultz (2004)? What are the implications for welfare?

We find that, contrary to Schultz (2004), an increase in market transparency raises firm profits but has no effect on product differentiation. We also find that endogenizing market transparency yields lower equilibrium product differentiation, prices and profits – to the benefit of consumers. The lower product differentiation and lower prices are driven by the business stealing effect. The overall welfare effect is however ambiguous as profits are lower.

The base model is then extended by endogenizing the choice of product differentiation. We find that compared to Grossman and Shapiro (1984), firms advertise more, differentiate their products more, charge higher prices and earn higher profits when the advertising cost is "not too low". The implications for consumer welfare however are not obvious: more consumers are informed (good for welfare) but they pay higher prices (bad for welfare). In addition, because firms locate further apart, total transportation costs are higher (bad for welfare). The net effect is thus unclear.

The remainder of the paper is organized as follows. Section 2 sets out the base model (where firms compete on product differentiation and prices but cooperatively determine the level of market transparency). In section 3 we derive the equilibrium prices, market transparency and product differentiation while section 4 synthesizes all the results and contrasts them to the findings of Schultz (2004). Section 5 extends the base model by allowing for strategic advertising and section 6 concludes the paper.

2 Model and Preliminaries

Two firms, firm 1 and firm 2 are located along a linear city (whole real line, \( \mathbb{R} \)) with firm 1 located at \( a \) and firm 2 located at \( 1 - b \), with \( a \leq 1 - b \) (together with Schultz (2004), we allow the firms to locate outside the unit interval). Once locations are chosen the firms proceed to set prices \( p_1 \) and \( p_2 \).
and advertising intensity (market transparency), $\phi \in [0, 1]$ – choosing prices noncooperatively while semicolluding on advertising (Simbanegavi, 2009). We assume that advertising messages (ads) are randomly distributed over consumers, that is, every consumer has an equal chance of receiving any advertising message sent by the firms.\(^4\) The marginal cost of producing a unit of the good by firm $i$, $i = 1, 2$, is constant and equal to $c$. For simplicity, we normalize the marginal cost to zero.

A mass of consumers with density 1 is uniformly distributed on the interval $[0, 1]$. Consumers have unit demands\(^5\) and are completely uninformed about firm locations and prices unless advertised. Each unit of the good generates gross surplus $v$ – which is assumed large enough that there are no gaps in the market, i.e., any informed consumer will purchase from one firm or the other. We assume that consumers face quadratic transportation costs – paying $t$ units per square of the distance traveled. Since ads contain information about the relevant characteristics of both firms, all consumers receiving at least one ad are fully informed. Unlike Schultz (2004), we assume that uninformed consumers do not purchase.\(^6\) This assumption is necessary since the firms jointly decide on the level of market transparency (advertising is a public good). If uninformed consumers would be split equally between the firms, then the free rider problem dictate that neither firm invest in market transparency.

Schultz (2004) defines market transparency as "the fraction of consumers that are informed about [firms'] prices and product characteristics (p. 174). However, the mode by which consumers become informed is not explicitly

\(^4\)We also assume that advertising conveys all the relevant characteristics/information of both firms – hence no firm has a "captive" market.
\(^5\)Each consumer consumes either one unit or zero units of the good.
\(^6\)This assumption can be justified in at least two situations. First, if the product is a new innovation, then consumers who did not receive advertising may not want to search since they do not know the attributes of this product or are unaware of its existence. Second, uninformed consumers may choose not to search if third degree price discrimination is possible (discount coupons). In this case, a consumer with no coupon will have to pay the reservation price, $v$. Since visitation is costly, uninformed consumers will not purchase.
modeled. Consumers learn or become informed about prices and product characteristics either through search, word of mouth or through advertising by firms. In this paper we take the latter view. We assume that search costs are prohibitively high and therefore consumers rely for their information on firm advertising. In this paper, advertising intensity, $\phi$, denotes the fraction/proportion of consumers that are fully informed about the ‘relevant’ characteristics of both firms’ products.\footnote{There is thus an exact equivalence between our measure of advertising intensity, $\phi$, and Schultz’s measure of market transparency, $\phi^s$, where $S$ is a mnemonic for Schultz.} This leaves fraction $1 - \phi$ of consumers uninformed. Let $A(\phi)$ be the cost of informing a fraction $\phi$ of the market. We assume that $\partial A(\phi)/\partial \phi > 0$ and $\partial^2 A(\phi)/\partial \phi^2 > 0$. For what follows, we assume a quadratic functional form for the advertising cost function; that is, $A(\phi) = \lambda \phi^2/2$, where $\lambda$ is the advertising cost parameter.

For brevity, let $\ell_i, \ell_j = \{a, b\}; i, j = 1, 2; j \neq i$. The above model gives rise to the following demands (see Schultz (2004)):

$$D_i(p_1, p_2, a, b, \phi) = \phi \left( \frac{1 + \ell_i - \ell_j}{2} + \frac{p_j - p_i}{2t(1 - \ell_i - \ell_j)} \right).$$  

(1)

3 Prices, market transparency and product differentiation

Given the model above, the firms’ interaction can be modeled as a two stage game in which the firms choose locations (product differentiation) in the first stage (Stage I) and thereafter, after observing each other’s location choices, the firms simultaneously choose prices and market transparency (Stage II). We solve the problem backwards, starting with Stage II.

3.1 Prices and market transparency

Given product differentiation (locations), firms simultaneously decide on prices and market transparency, choosing prices noncooperatively but cooperatively choosing the advertising intensity (market transparency).
The assumption of semicollusion on advertising that we adopt here deserves some explanation—as a more natural assumption would seem to be that advertising levels are chosen noncooperatively. Our assumption here is motivated by the desire to make our model as comparable to Schultz (2004) as possible to allow us to pin down the implications of endogenizing market transparency. In Schultz (2004), market transparency is defined as the proportion of consumers that are informed about the firms’ prices and product characteristics. Of course, since market transparency is exogenous in Schultz (2004), there is no advertising. However, as argued above, his measure of market transparency is exactly the same as our "cooperative" advertising intensity.

In Section 5 we present results of the more general model (firms compete in prices, advertising and locations). As we will see there, the model quickly becomes messy and less tractable.

3.1.1 Market transparency

A novelty of this paper is that we define more precisely the notion of market transparency and in particular, we make it clear that firms are largely responsible for "creating" market transparency through their advertising efforts. Firms choose advertising, $\phi$, to maximize joint profit $\Pi \equiv \pi_1 + \pi_2$, given locations and prices. The firms' objective function for advertising is:

$$\Pi \equiv \phi \left( \frac{p_i + p_j + (\ell_i - \ell_j)(p_i - p_j)}{2} - \frac{(p_j - p_i)^2}{2t(1 - \ell_i - \ell_j)} \right) - \lambda \phi^2. \quad (2)$$

Taking the first order condition with respect to $\phi$ and simplifying gives

$$(p_j - p_i) \left( p_i - p_j + t \left( \ell_i^2 - \ell_j^2 \right) \right) - 4t \lambda \phi (1 - \ell_i - \ell_j) - 2t (\ell_i p_j + \ell_j p_i) + t p_i + t p_j = 0. \quad (3)$$
3.1.2 Prices

In the pricing game, firm’s objective functions are given by

\[ \pi_i = p_i \phi \left( \frac{1 + l_i - l_j}{2} + \frac{p_j - p_i}{2t (1 - l_i - l_j)} \right) - \frac{\lambda \phi^2}{2} ; i; j = 1, 2; j \neq i, \]  

where \( \phi \) is given from (2). Bertrand competition in prices implies the first order conditions

\[ tl_i^2 - 2tl_i - t\ell_j^2 + t + p_i - 2p_j = 0; i, j = 1, 2; j \neq i \]  

(5)

It is immediate from (5) that prices are independent of the advertising level. This is because advertising is non-strategic in the current framework.

Solving the first order conditions (3) and (5) simultaneously for \( \phi, p_i; i = 1, 2 \), gives:

\[ p_i (l_i, l_j) = (1 - l_i - l_j) \left( 1 + \frac{l_i - l_j}{3} \right) t; i, j = 1, 2; j \neq i \]  

(6)

\[ \phi (l_i, l_j) = \frac{(1 - l_i - l_j) ((l_i - l_j)^2 + 9) t}{18 \lambda}. \]  

(7)

We see from (6) that, for given locations, equilibrium prices are increasing in transportation costs, \( t \). The parameter \( t \) is a measure of the degree of exogenous product differentiation and a higher \( t \) implies a softening of price competition. Also, for given locations, the advertising intensity is higher the higher is \( t \). This is because an increase in \( t \), other things being equal, softens price competition and thus raises the profit margin. As profitability increases, firms want to sell more and this can only be realised through increasing the advertising intensity (to expand the size of the market). Also from (7), it is immediate that market transparency decreases as the cost of advertising (\( \lambda \)) increases.

As shown by Schultz (2004), the prices that would emerge under exoge-
nous market transparency are (p. 176)

\[ p_i^S(\ell_i, \ell_j) = (1 - \ell_i - \ell_j) \left( \frac{1}{\phi} + \frac{\ell_i - \ell_j}{3} \right) t \]  

(8)

where superscript \( S \) is a mnemonic for Schultz.

It is immediate from (6) and (8) that, for given locations \( \ell_i \) and \( \ell_j \), prices are higher when market transparency is exogenous relative to when it is endogenous (\( 1/\phi > 1 \) since \( \phi \in (0,1) \)). In other words, price setting in the second stage is more competitive when firms set both price and advertising as opposed to when advertising is exogenously determined. This is intuitive. With endogenous market transparency, firms compete more aggressively as each firm wants to recoup its advertising outlays. Put differently, firms price aggressively in the hope of expanding own demand (business stealing effect). In contrast, when market transparency is exogenous, firms incur no advertising expenses and thus there is relatively less incentive to undercut.

### 3.2 Product differentiation/Locations

Anticipating Bertrand competition in prices and semicollusion on advertising in the second stage, firms choose their locations noncooperatively in Stage I. Firm \( i \)'s optimal location problem is given by

\[ \pi_i = p_i \phi \left( \frac{1 + \ell_i - \ell_j}{2} + \frac{p_j - p_i}{2t (1 - \ell_i - \ell_j)} \right) - \frac{\lambda \phi^2}{2} ; i, j = 1, 2; j \neq i, \]

where \( p_i, p_j \) and \( \phi \) are given by (6) and (7) respectively. Taking the first order condition of firm \( i \)'s objective function with respect to \( \ell_i, i = 1, 2; \) and solving the first order conditions simultaneously for \( \ell_i \) and \( \ell_j \) yields \( \ell_i^* = \ell_j^* = -1/4 \) as the unique (and symmetric) equilibrium in locations. Reverting to familiar notation, this can be written as

\[ a^* = b^* = -1/4 \]  

(9)
Since $a^*$ and $b^*$ are negative, we conclude (as did Schultz (2004)) that firms will locate outside the unit interval. However, since $a^* = b^* < -\infty$, we conclude that firms differentiate their products but not fully. This could be interpreted as firms wanting to be where demand is (Tirole (1988: 286)).

Hotelling (1929) espoused the principle of minimum differentiation – showing that when consumers face linear transportation costs and firms are restricted to locate on the unit interval, the firms will locate at the centre. There are two opposing forces at play here. Locating further apart softens price competition – a strategic effect. Intuitively, consumers located close to firm 1 become more captive as firms locate further apart ($a$ decreases). Consequently each firm has more "monopoly power" over the consumers located on its turf and thus can afford to charge higher prices. On the other hand, other things being equal, if firm 1 is located to the left of the point 1/2, its market share increases as it moves (locates) towards the centre. In effect, firm 1 does strictly better if it can locate to the right of point 1/2 so that it serves more than half of the market. Of course firm 2 behaves symmetrically and thus firms have an incentive to locate close together. This is the market share/business stealing effect (Tirole (1988)). In the case of linear transport costs (Hotelling’s example), the market share effect dominates and firms locate at the center.8

D’Aspremont et al (1979) however showed that when consumers face quadratic transportation costs (as in the present paper) and firms are restricted to locate on the interval $[0,1]$, firms will choose maximal differentiation. That is, firm 1 will locate at point 0 while firm 2 will locate at point 1 (i.e., the strategic effect dominates). In particular, it can be shown (D’Aspremont et al (1979: 1149); Tirole (1988: 281)) that in the case of quadratic transportation costs, the firms’ profits are strictly decreasing in

8D’Aspremont et al (1979) however show that when transport costs are linear, the principle of minimum differentiation is invalid (at least in pure strategies) (p. 1145). This is because, with linear transport costs, the firms’ profit functions are discontinuous and consequently, the price competition problem is not well behaved (p. 1146).
own location so that locating at points 0 and 1 (as in D’Aspremont et al 1979) is only constrained optimal. In other words, the strategic effect is so strong that firms would want to differentiate their products further than is permitted by the restriction [0,1].

4 Analysis

4.1 Equilibrium prices, market transparency and profits

Given the equilibrium locations above (cf. (9)), we can now characterize the firms’ optimal level of market transparency as well as the equilibrium prices and profits. Substituting (9) into (6) and (7) we obtain

\[ p_1^* = p_2^* = \frac{3t}{2} \quad (10) \]
\[ \phi^* = \frac{3t}{4\lambda}. \quad (11) \]

Observe from (9) and (10) that the equilibrium price and locations are unaffected by the advertising cost parameter, \( \lambda \). Although it appears counter-intuitive at first, this is in fact as one would expect: advertising is non-strategic in the current set-up. This is because all the consumers in the market have the same ‘information set’ about the firms’ prices and product characteristics. In this sense, advertising here can be thought of as a common fixed cost.\(^9\) Also from (11), market transparency will be higher the lower is the per unit advertising cost, \( \lambda \). This is intuitive. A higher \( \lambda \) means that it is more costly to inform consumers and as a result, firms will respond by lowering the level of market transparency, other things being equal.\(^10\)

\(^9\)In Section 5 below we present a model where consumers are differentially informed about the firms’ prices and products characteristics – thus giving advertising a strategic dimension.

\(^10\)Observe that from (11), we can pin down the permissible range for \( \lambda \). Since \( \phi \in [0, 1] \), we have that \( \phi^* = \frac{3t}{4\lambda} \leq 1 \) – implying that \( \lambda \geq \frac{3t}{4} \). For \( \lambda > \frac{3t}{4} \), \( \phi \in (0, 1) \) and some consumers are uninformed in equilibrium.
Substituting \( p^*, \phi^*, a^* \) and \( b^* \) into (4) yields the equilibrium profit as;

\[
\pi^* = \frac{9t^2}{32\lambda}. \tag{12}
\]

As one would expect, profits increase with the transportation cost, \( t \), but decrease with the advertising cost, \( \lambda \). Below we summarize the comparative statics effects of changes in the advertising cost on product differentiation, competition and profits:

**Proposition 1** A decrease in the advertising cost parameter \( \lambda \) at equilibrium raises firms’ profits and the advertising intensity but has no effect on product differentiation and competition (prices). Consequently, a decrease in \( \lambda \) unambiguously raises social welfare.

This result closely mirrors Theorem 1 in Schultz (2004). When the cost of advertising decreases, firms respond by raising the advertising intensity thereby growing the market. Since advertising is non-strategic, an increase in the advertising intensity does not directly translate into either more or less price competition or more or less product differentiation. However, because more consumers become informed when the advertising cost decreases, consumers are unambiguously better off.

Given the relationship between our measure of market transparency and the advertising cost parameter, \( \lambda \) (cf. (11)), the above theorem can be restated in a somewhat less parsimonious way, but a way that allows for a direct comparison with Schultz’s (2004) Theorem 1 as follows: *An increase in market transparency at equilibrium (decrease in \( \lambda \)) raise firms’ profits but has no effect on product differentiation and prices. Moreover, an increase in market transparency unambiguously raises welfare.* This result, as stated, contrasts sharply with Schultz’s Theorem 1 (2004: 177) which states that an increase in market transparency leads to less product differentiation and lower prices and profits. Here we show that an increase in market transparency is beneficial (rather than harmful) to firms. The intuition can be
gleaned from the envelope result: a change in $\lambda$ has no first order (direct) effect on the firms’ revenues since both the prices and market transparency are chosen optimally. A first order effect only appears in the advertising cost function, $A(\phi)$, where a decrease in $\lambda$ directly lowers the advertising outlay. This raises firms’ profits. Moreover, a decrease in $\lambda$ does not directly impact prices yet it directly raises consumer welfare (more consumers are informed). Consequently, an increase in market transparency is welfare improving.

4.2 Comparisons with Schultz (2004)

One motivation of this paper is to examine the implications on product differentiation, competition and profitability of relaxing the assumption of exogenous market transparency. We compare our results for these variables (9) – (12)) to those of Schultz (2004). See the appendix for details.

We compare the firms’ locations ($a^S$ and $a^*$) given the optimal level of market transparency, $\phi^*$. In the choice of location, two opposing considerations are at play: On the one hand, for given prices and advertising, each firm would like to locate close to the competitor as this enhances business stealing. On the other hand, for given advertising intensity, $\phi$, firms realize that locating close together squeezes margins and therefore firms would want to increase product differentiation in order to soften price competition. We find that, although firms locate outside the unit interval, they locate close together than is implied by Schultz (2004). Put differently, the business stealing effect is stronger under endogenous market transparency.

We next compare prices $p^S$ and $p^*$. The question we seek to answer is whether endogenizing market transparency carries with it procompetitive effects or otherwise. We find that prices are lower when market transparency is endogenous. In other words, price competition is tougher under endogenous market transparency. The explanation is as follows: because firms incur

\footnote{Variables with a superscript $S$ are the equilibrium quantities as computed by Schultz (2004), see appendix for details.}
advertising expenses, for given advertising intensity, $\phi$, there is greater incentive to undercut the competitor in order to "steal" the competitor's market share and hence increase own profits (the business stealing effect). Lastly, we compare profits under endogenous and those under exogenous market transparency ($\pi^S$ and $\pi^*$). We find that firms earn lower profits if they advertise to inform consumers. The low profits can be explained by the following: First, firms incur advertising costs which tend to lower profits, other things being equal and second, products are less differentiated and therefore price competition is much tougher than under exogenous market transparency and third, the fact that uninformed consumers do not purchase makes the market thinner (compared to Schultz (2004)).

The following proposition summarizes the implications of endogenizing market transparency, in particular, how the equilibrium locations, prices and profits compare with those when market transparency is exogenous.

**Proposition 2** Let market transparency be given by (11). When firms advertise to inform the market (endogenous market transparency) product differentiation, prices and profits are lower than under exogenous market transparency. However, consumers are better off since for given advertising intensity (market transparency), they pay lower prices.

Observe that in Schultz (2004), firms incur no advertising expenses. The fact that firm profits are lower, coupled with the fact that uninformed consumers do not purchase points to lower social welfare under endogenous market transparency. However, tough price competition mitigates the tendency to lower welfare as tough price competition implies greater consumer surplus. In addition to paying lower prices, on average each consumer travels a shorter distance than under exogenous market transparency. This lowers aggregate transportation costs (good for welfare). It follows therefore that the welfare effect of moving from exogenous to endogenous market transparency is ambiguous.
5 Strategic Advertising

We extend the model to allow for strategic advertising. In particular, in addition to competing on prices and product differentiation, firms also compete on advertising.\textsuperscript{12} This case is interesting as it allows for differentially informed consumers and thus gives a strategic dimension to advertising.

Let $\phi_1$ and $\phi_2$ be, respectively, firm 1 and firm 2’s advertising intensities. Given these advertising intensities, the market is delineated as follows; fraction $\phi_1 \phi_2$ of consumers receive advertising messages from both firms and is thus fully informed; fraction $\phi_i (1 - \phi_j) ; i,j = 1,2; j \neq i$ receive ads from firm $i$ but not firm $j$ and is thus partially informed (consumers captive to firm $i$) and fraction $(1 - \phi_1) (1 - \phi_2)$ receive no ads from either firm (uninformed). We assume that $\phi_1 \phi_2$ is large enough so that firms find it worthwhile to compete for the fully informed consumers.

Fully informed consumers purchase from whichever firm guarantees them the greatest surplus while partially informed consumers purchase whenever it is individually rational to do so. Given prices $p_i$ and $p_j$; and locations $\ell_i$ and $\ell_j$, $\ell_i, \ell_j = \{a,b\} ; i,j = 1,2; j \neq i$, firm $i$ thus faces the demands

$$D^i_{\text{full}} = \phi_i \phi_j \left( \frac{1 + \ell_i - \ell_j}{2} + \frac{p_j - p_i}{2t(1 - \ell_i - \ell_j)} \right)$$

and

$$D^i_{\text{partial}} = \phi_i (1 - \phi_j) \frac{v - p_i}{t(1 - \ell_i)^2}$$

from the fully and respectively partially informed consumers. However, if $v - p_i - t (1 - \ell_i)^2 \geq 0$, all consumers who receive at least one ad from firm $i$ will make a purchase, that is, $D^i_{\text{partial}} = \phi_i (1 - \phi_j)$.\textsuperscript{13} Thus the total demand

\begin{itemize}
  \item \textsuperscript{12}In this sense this section amounts to an extension of the model of Grossman and Shapiro (1984) to allow for endogenous product differentiation.
  \item \textsuperscript{13}Observe that the full market coverage condition $(v - p_i - t (1 - \ell_i)^2 \geq 0)$ implies that $\frac{v - p_i}{t(1 - \ell_i)^2} \geq 1$. Since consumers are uniformly distributed with density 1, we have that $\frac{v - p_i}{t(1 - \ell_i)^2} = 1$: that is, all the consumers who are captive to firm $i$ buy from firm $i$ with
\end{itemize}
that firm $i$ faces is:

$$D_i (p_1, p_2, \phi_1, \phi_2; a, b) = \phi_i (1 - \phi_j) + \phi_i \phi_j \left( \frac{1 + \ell_i - \ell_j}{2} + \frac{p_j - p_i}{2t (1 - \ell_i - \ell_j)} \right)$$

(13)

and its objective function is given by

$$\pi_i = p_i D_i (p_1, p_2, \phi_1, \phi_2; a, b) - \lambda \phi_i^2 / 2; i = 1, 2. \quad (14)$$

Taking the first order conditions of (14) with respect to $p_i$ and $\phi_i; i = 1, 2$ for given locations ($\ell_i$ and $\ell_j$) and setting them equal to zero we obtain

$$\partial \pi_i / \partial p_i = 0 \text{ and } \partial \pi_i / \partial \phi_i = 0; i = 1, 2. \quad (15)$$

Solving the equations $\partial \pi_i / \partial p_i = 0$ simultaneously for the prices $p_i; i = 1, 2$, given locations and the advertising intensities yields

$$p_i = t (1 - \ell_i - \ell_j) \left[ \frac{4}{3\phi_j} + \frac{2}{3\phi_i} - \left( 1 + \frac{\ell_j - \ell_i}{3} \right) \right]; i \neq j. \quad (16)$$

We saw earlier (p. 7) that when advertising is non-strategic, equilibrium prices are independent of the advertising level. It is immediate from (16) that the advertising intensity does affect prices. The question therefore is one of the nature of the relationship between the advertising intensity and prices. Since $\phi_i$ and $\phi_j; j \neq i$ are in the denominator, visual inspection of (16) shows that higher advertising is associated with lower prices ($\partial p_i / \partial \phi_i < 0$ and $\partial p_i / \partial \phi_j < 0$). An inspection of the demand function helps to build the intuition:

$$D_i (.) = \phi_i (1 - \phi_j) + \phi_i \phi_j \left( \frac{1 + \ell_i - \ell_j}{2} + \frac{p_j - p_i}{2t (1 - \ell_i - \ell_j)} \right).$$

probability one. Hence $D_{\text{partial}}^i = \phi_i (1 - \phi_j)$. 

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All else equal, an increase in $i$ raises both the proportion of consumers captive to firm $i$ ($\phi_i (1 - \phi_j)$) and the proportion of consumers who are fully informed and thus selective ($\phi_i \phi_j$). If $\phi_j$ is high, then the increase in the share of captive consumers resulting from an increase in $\phi_i$ is small (because $(1 - \phi_j)$ is small) while the share of selective consumers increases significantly. Thus, given the larger selective segment facing both firms, the firms will compete aggressively for these consumers thereby causing the negative relationship between advertising and prices. Intuitively higher advertising, by increasing the proportion of selective consumers in the market, raises the relative importance of these consumers (relative to the captive consumers).\footnote{In a symmetric advertising equilibrium ($\phi_1 = \phi_2 = \phi$), the proportion of captive consumers is given by $\phi (1 - \phi)$ while the fraction of fully informed consumers is $\phi^2$. Now, $\frac{\partial}{\partial \phi} [\phi (1 - \phi)] = 1 - 2\phi \leq 0$ for $\phi \geq \frac{1}{2}$ while $\frac{\partial^2}{\partial \phi^2} [\phi^2] = 2\phi > 0 \forall \phi$.}

As a result, firms compete more aggressively for these consumers.

Due to complexity, it is not possible to solve the equations (15) simultaneously for $p_i, p_j, \phi_i$, and $\phi_j$. Instead, we solve for a symmetric equilibrium, $p_i = p_j = p, \phi_i = \phi_j = \phi$. This yields (reverting back to the familiar notation),

$$p = \sqrt{2t\lambda(1-a-b)}$$

$$\phi = \frac{2(-t-a^2t+b^2t+2at+\sqrt{2t\lambda(1-a-b)})}{-t+2\lambda-3at+b^3t+3at+3bt-ab^2t-a^2bt+2abt}.$$ (18)

It is immediate from (17) that as firms locate close together ($a$ and $b$ increase) price competition intensifies thereby squeezing margins. When firms move close together, (endogenous) product differentiation is reduced and as a result consumers become less captive to their nearest store (products are more substitutable). With less monopoly power over the consumers on one’s turf, firms compete vigorously for the consumers on each other’s turf.

Substituting (17) and (18) into (14) and taking the first order condition with respect to firm 1’s location choice $a$, and solving for a symmetric equi-
librium in location we obtain\(^{15}\)

\[
\hat{a} = \hat{b} = \left(1 - \sqrt[3]{\lambda / 2t}\right) / 2. \tag{19}
\]

Unlike Schultz (2004) and/or our earlier model where advertising is non-strategic, in the present model, the optimal location is a function of the advertising cost parameter \(\lambda\). Observe that \(\lambda = 2t\) implies \(\hat{a} = \hat{b} = 0\). That is, firms locate at the extreme points of the linear city (as in D’Aspremont et al (1979)). Firm 1 locates at point 0 and firm 2 locates at point 1. If \(\lambda > 2t\), then \(\hat{a} = \hat{b} < 0\). That is, firms locate outside the unit interval (as in Schultz (2004)). For \(\lambda < 2t\), \(\hat{a} = \hat{b} > 0\). That is, firms locate inside the unit interval. However, firms will never find it optimal to locate at the center of the market, that is, \(\hat{a} = \hat{b} \neq 1/2\).\(^{16}\) Again we see that firms differentiate their products but not fully (Tirole (1988: 286-287) discusses possible explanations for why firms may choose not to differentiate their products fully).

Substituting (19) into (17) and (18) we obtain

\[
\hat{p} = (2t)^{1/3} \lambda^{2/3} \tag{20}
\]
\[
\hat{\phi} = 2^{3/2} \sqrt{2t \lambda^2} / \left(2 \lambda + 3^{1/2} \sqrt{2t \lambda^2}\right). \tag{21}
\]

The firms’ profits at equilibrium are given by

\[
\hat{\pi} = \frac{\hat{p} \hat{\phi}}{2} \left(2 - \hat{\phi}\right) - \frac{\lambda \hat{\phi}^2}{2}
\]

and substituting for \(\hat{p}\) and \(\hat{\phi}\) from (20) and (21) yields

\[
\hat{\pi} = 2\lambda \left(\sqrt[3]{2t \lambda^2}\right)^2 / \left(2 \lambda + \sqrt[3]{2t \lambda^2}\right)^2. \tag{22}
\]

\(^{15}\)The expressions are quite complex so much that it is impossible to check for the second order conditions.

\(^{16}\)Prescott and Visscher (1977) show that firms have incentives to locate "far apart" but not necessarily at the extreme points.
Equations (19) – (22) can be summarized in the following proposition:

**Proposition 3** An increase in the advertising cost parameter \( \lambda \) reduces market transparency and induces firms to differentiate their products more. This softens price competition and increases firms’ profits.

**Proof.** \( \partial \phi / \partial \lambda = -4\sqrt{2t\lambda^2}/3 \left( 2\lambda + \sqrt{2t\lambda^2} \right)^2 < 0 \), \( \partial a / \partial \lambda = -\frac{1}{6\lambda} \sqrt{2t\lambda} < 0 \), \( \partial p / \partial \lambda = 2 \left( 2t \right)^{3/2} \lambda^{-1/3} > 0 \) and \( \partial \pi / \partial \lambda = 2 \left( 2\lambda + \sqrt{2t\lambda^2} \right) \sqrt{2t\lambda^2}/3 \left( 2\lambda + \sqrt{2t\lambda^2} \right)^3 > 0 \).

Intuitively, when advertising becomes more costly, firms respond by reducing advertising. This reduction in advertising gives rise to the "strategic effect" (Tirole, 1988: 293). A reduction in the advertising intensity increases informational product differentiation and thus allows for softening of price competition.\(^{17}\) Observe that consumers who receive advertising from both firms can make across firm price comparisons and thus buy from the firm quoting the lowest ‘delivered’ price. On the other hand, captive consumers are "totally" price insensitive. When the advertising cost is low (advertising intensity is high), a greater proportion of the market is fully informed and thus firms compete aggressively for this segment of the market and competition to sell to these consumers drives the price down. However, when the advertising cost is high (advertising intensity low), the tables are turned as a lesser proportion of the market is now fully informed. In this case, the captive consumers are relatively more profitable as the optimal price applicable to this group is higher compared to that applicable to the fully informed group. Thus, high advertising costs soften price competition by constraining the proportion of fully informed consumers and hence limiting comparison shopping (Simbanegavi, 2009). Also, when advertising becomes more expensive, firms, in addition to reducing their advertising intensity, also raises

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\(^{17}\)Observe that this dynamic was non-existent in the model of Section 2 where advertising was nonstrategic.
product diversity. This adds to the softening of price competition thereby allowing firms to charge higher prices.

That profits increase with the advertising cost simply means that the strategic effects (resulting from increased product differentiation and reduced advertising) outweigh the direct effect of higher advertising cost on the advertising outlays. The above result has important implications for consumer welfare: High advertising costs, by constraining informative advertising, exacerbates product differentiation and thus softens price competition to the detriment of consumers. This points to the importance of policies that promote and /or lower the costs of advertising.

In the Grossman and Shapiro model, firms are (exogenously) located at the end points of a unit interval and they simultaneously decide on price and advertising. As we pointed out in the introduction, our model extends the Grossman and Shapiro (1984) analysis to the case where product differentiation is endogenous. The question of interest here is the following: What are the additional implications, if any, that flow from our relaxing of the assumption of exogenous product differentiation?

The equilibrium price, advertising and profit in the Grossman and Shapiro (1984) model (as simplified by Tirole (1988: 292-293)) are given by

\[ p^{GS} = \sqrt{2t\lambda} \]  
\[ \phi^{GS} = 2 / \left( 1 + \sqrt{2\lambda/t} \right) \]  
\[ \pi^{GS} = 2t\lambda / \left( \sqrt{t} + \sqrt{2\lambda} \right)^2. \]

Comparing \( \hat{p} \) to \( p^{GS} \), \( \hat{\phi} \) to \( \phi^{GS} \) and \( \hat{\pi} \) to \( \pi^{GS} \) we obtain the following:

\[
\begin{align*}
\hat{p} &> p^{GS} \\
\hat{\phi} &> \phi^{GS} \\
\hat{\pi} &> \pi^{GS}
\end{align*}
\]

if \( \lambda > 2t \) and
\[
\begin{align*}
\hat{p} &\leq p^{GS} \\
\hat{\phi} &\leq \phi^{GS} \\
\hat{\pi} &\leq \pi^{GS}
\end{align*}
\]

for \( \lambda \leq 2t \).  

(26)
Thus for $\lambda > 2t$, equilibrium prices and the equilibrium advertising intensities are higher when product differentiation is endogenous. As we showed earlier, if $\lambda > 2t$, $\bar{a} = \bar{b} < 0$. That is, firms locate outside the unit interval thereby according more differentiation (and thus greater softening of price competition) than in the standard Grossman and Shapiro model. On the other hand, for $\lambda < 2t$, $\bar{a} = \bar{b} > 0$. That is, firms differentiate their products less than the exogenous product differentiation accorded in the standard Grossman and Shapiro model. The result is more vigorous price competition than in the model of Grossman and Shapiro, other things being equal. This explains the lower price ($\bar{p} < p^{GS}$).

As equation (26) shows, $\bar{\phi} \geq \phi^{GS}$ if $\lambda \geq 2t$. This is quite intuitive. When $\lambda < 2t$, $\bar{a} = \bar{b} > 0$ implying that the firms’ products are less differentiated than in the standard Grossman and Shapiro model (where firms are located at the extremes of unit interval). In this case, firms advertise less as a way to soften price competition. By advertising less, firms increase informational product differentiation which accords them more "monopoly power". With respect to profits, we find that $\bar{\pi} > \pi^{GS}$ if $\lambda > 2t$. That is, profits are higher with endogenous product differentiation for $\lambda \geq 2t$. As we pointed out earlier, for $\lambda > 2t$, firms locate outside the unit interval and they also advertise more than they would were differentiation exogenous. Consequently, profits are higher than in the standard Grossman and Shapiro model. However, when $\lambda < 2t$, product differentiation is weak (products more similar) and the resulting tough price competition erodes potential profits. Consequently, firms earn lower profits than in the Grossman and Shapiro (1984) model.

The implications for consumer welfare however are not obvious though. When the advertising cost is high, more consumers are informed (which is good for welfare) but they pay higher prices (bad for welfare). In addition, locating outside $[0,1]$ raises the consumers’ shopping costs and thus makes them more captive to their nearest store. In this way, endogenous product differentiation reinforces exogenous product differentiation. This gives the firms more market power and thus softens price competition.

In terms of overall welfare however, higher prices are not necessarily welfare reducing.
because firms locate further apart, total transportation costs are higher – implying lower consumer surplus. Thus, a proper examination is required if we are to make definitive statements about the welfare implications of endogenizing product differentiation. This we leave for future research.

6 Conclusion

This paper is an attempt to "kill two birds with one stone". We extend Schultz’s (2004) model by endogenizing market transparency and Grossman and Shapiro’s (1984) seminal model by endogenizing product differentiation. We find that Schultz’s result is reversed. In particular, an increase in market transparency raises firm profits but has no effect on product differentiation. Furthermore, endogenizing market transparency leads to less product differentiation, more price competition and lower profits. Because consumers pay lower prices, endogenizing market transparency raises consumer welfare. The intuition is that profitability requires firms to recoup the advertising outlays. This creates a strong incentive for firms to compete aggressively by undercutting each other.

In the extended model firms, in addition to competing on locations and prices, also compete on advertising. We find that a reduction in the advertising cost parameter (analogous to an increase in market transparency) leads to more advertising, less product differentiation and lower prices and profits. This is explained by the business stealing effect. Because firms advertise more when the cost of advertising is low, the resulting "bigger" market creates incentives for business stealing. Consequently, product diversity is reduced. Also, the strategic effect (reduced informational product differentiation) works to increase competitiveness as a higher advertising intensity implies greater scope for comparison shopping. Thus, a decrease in the advertising cost favours consumers and hurts firms.

as they constitute only a transfer from consumers to firms. By definition, welfare equals profits plus consumer surplus.
Appendix A.


In Schultz (2004), the equilibrium locations, prices and profits are given by: $a^S = b^S = g \frac{\phi - g}{\phi^2} < 0$; $p^S = \frac{3}{4} \frac{(3-\phi)t}{\phi^2}$ and $\pi^S = \frac{3}{8} \frac{(3-\phi)t}{\phi^2}$ (p. 176), where $S$ is a mnemonic for Schultz. The ‘optimal’ quantities (in the sense that market transparency is endogenously determined) are denoted by a star ($\ast$). Thus $a^\ast$, $p^\ast$, $\phi^\ast$ and $\pi^\ast$ are given by equations (9), (10), (11) and (12) respectively.

**Locations** Let $\phi^\ast = \frac{3t}{4}$. Then, $a^S - a^\ast = g \frac{\phi - g}{\phi^2} \mid_{\phi^\ast} - \frac{1}{4} = -\frac{3}{8} (4\lambda - 3t) < 0$ for all $\lambda > \frac{3t}{4}$. Thus firms locate close together than is implied by Schultz (2004).

**Prices** $p^S = \frac{3}{4} \frac{(3-\phi)t}{\phi^2}$ and $p^\ast = \frac{3t}{2}$.

Let $\phi^\ast = \frac{3t}{4}$. Then, $p^S - p^\ast = \frac{3}{4} \frac{(3-\phi)t}{\phi^2} - \frac{3t}{2} = -\frac{1}{2t} \left( 3t^2 + 2t\lambda - 8\lambda^2 \right).$ Now, $(p^S - p^\ast) \bigg|_{\lambda = \frac{3t}{4}} = 0.$ Observe that $p^S \mid_{\phi^\ast} = -\frac{1}{t} \lambda (t - 4\lambda)$ and $\frac{\partial (p^S \mid_{\phi^\ast})}{\partial \lambda} = -\frac{1}{t} (t - 8\lambda) > 0$ for all $\lambda \geq \frac{t}{8}$ while $\frac{\partial (p^\ast)}{\partial \lambda} = 0.$ Thus $p^S$ and $p^\ast$ diverge as $\lambda$ increases. It follows therefore that $p^S - p^\ast > 0$ for all $\lambda > \frac{3t}{4}$. That is, equilibrium prices are lower under endogenous market transparency.

**Profits** $\pi^S - \pi^\ast = \frac{3}{8} \frac{(3-\phi)t}{\phi^2} - \frac{9t^2}{32\lambda} = -\frac{1}{32t\lambda} \left( 9t^3 + 16t\lambda^2 - 64\lambda^3 \right).$ Observe that $(\pi^S - \pi^\ast) \bigg|_{\lambda = \frac{3t}{4}} = \frac{3t}{8} > 0.$ Moreover, $\pi^S \mid_{\phi^\ast} = -\frac{1}{2t} \lambda (t - 4\lambda)$, and $\frac{\partial (\pi^S \mid_{\phi^\ast})}{\partial \lambda} = -\frac{1}{2t} (t - 8\lambda) > 0$ for all $\lambda \geq \frac{t}{8}.$ On the other hand, $\frac{\partial \pi^\ast}{\partial \lambda} < 0$ for all $\lambda$. Since
for our purposes, we conclude that $\pi^S - \pi^* > 0$. That is, profits are lower under endogenous market transparency.


A.2.1 Prices

$$\hat{p} = \sqrt[3]{\frac{2t}{2\lambda}}$$
$$\hat{\phi} = \frac{2\sqrt{2t\lambda^2}}{2\lambda + \frac{3}{2}\sqrt{2t\lambda^2}}$$

$$p^{GS} = \sqrt{2t\lambda}$$
$$\phi^{GS} = \frac{2}{1 + \sqrt{\frac{2\lambda}{t}}} = \frac{2\sqrt{t}}{\sqrt{t} + \sqrt{2\lambda}}$$

$$\hat{p} = (2t\lambda^2)^\frac{1}{3} \geq (2t\lambda)^\frac{1}{2} = p^{GS} \iff 2t\lambda^2 \geq (2t\lambda)^\frac{2}{3}$$. Dividing both sides by $\lambda^{\frac{3}{2}}$ and then by $2t$ and then squaring both sides we obtain: $\hat{p} = (2t\lambda^2)^\frac{1}{3} \geq (2t\lambda)^\frac{1}{2} = p^{GS} \iff \lambda \geq 2t$ as required.

A.2.2 Advertising

$$\widehat{\phi} - \phi^{GS} = \frac{2\sqrt{2t\lambda^2}}{2\lambda + \frac{3}{2}\sqrt{2t\lambda^2}} - \frac{2\sqrt{t}}{\sqrt{2\lambda} + \sqrt{2\lambda} + \frac{3}{2}\sqrt{2t\lambda^2}}$$

$$= \frac{(\sqrt{7+2\lambda})^2(2\sqrt{2t\lambda^2}) - 2\sqrt{t}(2\lambda + \frac{3}{2}\sqrt{2t\lambda^2})}{(2\lambda + \frac{3}{2}\sqrt{2t\lambda^2})(\sqrt{7+2\lambda} + \sqrt{2\lambda})} = \frac{2\sqrt{2\lambda^2} \sqrt{2t\lambda^2} - 4\sqrt{t}\lambda}{(2\lambda + \frac{3}{2}\sqrt{2t\lambda^2})(\sqrt{7+2\lambda} + \sqrt{2\lambda})}.$$  

Since the denominator is positive, $\widehat{\phi} - \phi^{GS} \geq 0 \iff 2\sqrt{2\lambda^2} \sqrt{2t\lambda^2} - 4\sqrt{t}\lambda \geq 0$. Simplifying and solving for $\lambda$ we have that $\widehat{\phi} \geq \phi^{GS} \iff \lambda \geq 2t$. Thus firms advertise more when their products are more differentiated.
A.2.3 Profits

\[ \pi^{GS} = \frac{2\lambda}{\left(1 + \sqrt{\frac{2\lambda}{t}}\right)^2} = \frac{2\lambda}{\left(\sqrt{t} + \sqrt{2\lambda}\right)^2} \]

\[ \hat{\pi} = \frac{2\lambda \left(\sqrt{2\lambda t^2}\right)^2}{\left(2\lambda + \sqrt[3]{2\lambda t^2}\right)^2} \]

\[ \hat{\pi} - \pi^{GS} = \frac{2\lambda \left(\sqrt[3]{2\lambda t^2}\right)^2}{\left(2\lambda + \sqrt[3]{2\lambda t^2}\right)^2} - \frac{2t\lambda}{\left(\sqrt{t} + \sqrt{2\lambda}\right)^2} = \frac{\left(\sqrt{t} + \sqrt{2\lambda}\right)^2 2\lambda \left(\sqrt{2\lambda t^2}\right)^2 - 2t\lambda \left(2\lambda + \sqrt[3]{2\lambda t^2}\right)^2}{\left(2\lambda + \sqrt[3]{2\lambda t^2}\right)^2 \left(\sqrt{t} + \sqrt{2\lambda}\right)^2} \]

Since the denominator is positive, \( \hat{\pi} - \pi^{GS} \geq 0 \iff \left(\sqrt{t} + \sqrt{2\lambda}\right)^2 2\lambda \left(\sqrt[3]{2\lambda t^2}\right)^2 - 2t\lambda \left(2\lambda + \sqrt[3]{2\lambda t^2}\right)^2 \leq 0 \). Thus, it suffices find the condition under which \( \left(\frac{1}{2}\sqrt{t}\sqrt{\lambda} - \frac{1}{4}\sqrt[3]{2\lambda t^2}\right) \leq 0 \). But this is straightforward. Simplifying and solving for \( \lambda \) yields \( \lambda \geq 2t \).

Thus, profits are higher with endogenous product differentiation than with exogenous product differentiation for \( \lambda \geq 2t \).

References


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