Inflation Targets as Focal Points

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Abstract
In a world characterised by noisy information and conflicting signals, no Central Bank is always able to affect private sector expectations. Based on Morris and Shin’s model, monetary policy then becomes an information game, in which individuals form their expectations based on all the information that is available to them (public and private). However individual agents also know that ultimately inflation is affected by both the objectives of the Central Bank (and hence the policies it pursues) as well as the average expectation formed by the all agents. They thus need to evaluate both actions. Central to our argument is the way that individuals interpret these actions to form their expectations. We apply Bacharach’s methodology to provide a framework for assessing everyone’s interpretations. Our contribution is to merge these two models to show that a monetary policy regime that has explicit quantitative objectives may provide individuals with better anchors for expectations to coordinate at. However, that is only true first, if no great shocks are anticipated to hit the economy and second, when all other public information is very unclear thus rendering the inflation target the only clear piece of information. We derive in detail the conditions under which this is true.

JEL codes: C71, C78, E52
Keywords: inflation targeting, information games, matching games, focal points

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1 Introduction

Modern monetary policy theory emphasizes the central role of private sector expectations in determining policy outcomes. Recent empirical evidence by Paloviita and Virén (2005) demonstrate this for inflation in the euro area. It is thus widely acknowledged, that the success of maintaining a stable monetary environment depends crucially on the ability of the policy regime to control inflation expectations (Blinder et al, 2001). Evidence of that is shown by Orphanides and Williams (2004) in their analysis of US monetary policy history, where they argue that monetary policy failures are connected with changes in public sentiment about the future state of the economy. In other words, policy mistakes alone are not enough to produce long term negative effects on monetary stability.

The practice of monetary policy in the past ten to fifteen years has thus concentrated on providing institutional set-ups that provide an explicit information platform for expectations to be formed. The main features of such institutional set-ups are:

- credible institutions, mainly through independence and the pursuit of the principal objective of price stability;
- clear policy frameworks, captured by well defined intermediate policy objectives and procedures, and finally;
- transparent policy making, implemented through publication and distribution of the information set used in the decision making process (inflation forecasts, modelling strategies, well defined assumptions) and a clear demonstration of accountability (publication of minutes, regular appearance in front of parliamentary committees and regular press conferences).

Practically every monetary policy authority nowadays defines its policies according to these criteria, emphasising one or another aspect, depending on preferences. The set-up of the twelve-country Euro area for example, has emphasised the importance of building and sustaining credibility and independence from governments, as an instrument towards low expected inflation. In the US experience instead, credibility, independence but also flexibility in following multiple objectives has helped achieve a stable monetary environment. Alternatively, inflation targeting as implemented first, by the Reserve Bank of New Zealand and then the Bank of England, and increasingly more and more banks around the world, is understood to provide clear and immediate objectives for monetary policy. Inflation targeting practitioners argue that the main advantage of an explicit numerical inflation target is its ability to provide a focal point for private sector expectations. As Mervyn King (2002, p.4) has claimed for the UK case, inflation expectations have indeed been anchored to the pre-announced target. The ability of explicit quantitative targets to tie down expectations, is also confirmed by the empirical analysis of Levin et al (2004), Mishkin and
Schmidt-Hebbel (2001) and more recently by Fatás et al (2004). However, conventional monetary policy models (Svensson, 1999, 2003) allow for no difference in the way inflation targeting is modelled by comparison to other regimes. There is thus no explicit analysis of the way the provision of a specific numerical target may constitute a better anchor for private sector expectations. What is the mechanism that makes the inflation target a better anchor and which conditions are required for this to be true? In our view, to be able to answer that, we need a mechanism of expectations formation more complex than the standard full information rational expectations paradigm.

The recent model put forward by Morris and Shin (2002a, 2002b) (and used in Amato et al, 2003 and Amato and Shin, 2003) renders itself to identifying first, how private agents form expectations based on private and public information available to them and second, to showing how policy makers affect these expectations by providing greater or lesser information. It is shown in this set-up, that in forming these expectations, private agents care not only about their own views but also about other people’s expectations, as a means to confirming their own beliefs. In fact Phelps (1983) noted that “...in order to reduce the price level (in relation to the accustomed trend), it is not sufficient that the central bank persuade each agent to reduce his private expectation of the money supply (in relation to the past trend) by the warranted amount. The prevalence of this expectation must be public knowledge - an accepted fact” (p.35). And as the ‘beauty contest’ element (based on Keynes, 1936) plays a greater role in expectations forming, signals provided by public institutions can conceivably become tantamount to coordination devices. This therefore, implies that monetary policy can be viewed as a coordination game between the Central Bank and the private sector but also as a matching game between the private sector themselves. Due to the latter, public information then acquires a dual role - “...of conveying fundamentals information as well as serving as a focal point for beliefs” (Morris and Shin, 2002a, henceforth, MS 2002). The question that arises following this argument is then, what monetary policy regimes provide better signals and in which way these signals constitute focal points? The aim of this paper is to formalise the widely believed but little analysed benefits of inflation targeting in coordinating private individuals’ expectations and the conditions necessary for this to be achieved.

The theory on coordination games provides valuable insight into the way that such games are resolved. For example, it is often observed that in matching games players coordinate much more frequently than by randomising (Casajus, 2000). Indeed, according to Wilson and Rhodes (1997), it is to the benefit of all

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1 See also Leiderman and Svensson (1995) and Bernanke et al (1999) for earlier accounts of experiences with inflation targeting.

2 Kuttner 2004, also alludes to this fact. The benefits of inflation targeting as a coordination device have been discussed by Hughes Hallett and Viegi, 2002, but then in the context of two policy authorities, the policies of which might have strong "spillovers".

3 See Sugden (1999) for a theory on focal points. Also Bryan and Palmqvist (2005) apply the term "focal points" to monetary policy, in a similar fashion to ours.
actors to avoid the conflict that escalates as solutions are delayed. To achieve that, players rely heavily on salient features when deciding on their actions. And salience in this context, can be a “...social custom or convention, namely, a mode of behaviour that finds automatic acceptance” (Dixit and Skeath, 1999) and a salient item is “...one that stands out from the rest by its uniqueness in some conspicuous respect.” (Bacharach 1993). Furthermore, Wilson and Rhodes (1997) argue that all that is required for such salience to be achieved is a signal from somebody that can be recognised as the ‘leader’ in the game, to send a signal. A commonly accepted leader, in a clearly defined leader-follower(s) game, can thus provide such a focal point. In our set-up, the Central Bank acquires this leadership role by the sheer extent of its contribution to the final inflation outcome. Providing then a numerical target, we argue, grants private agents the choice between two actions, by comparison to a regime in which a numerical target is absent. These are, either to internalise the announced target and treat it as an additional piece of public information or, driven by their incentive to coordinate, ignore all other information and fix their expectations at that level. Given the intentions of the Central Bank, the latter is naturally the preferred option, but then only if everybody else pursues this option as well. In deciding between these two options therefore, the agent remains uncertain as to how others view the target. Trusting the target for oneself is neither necessary nor is it sufficient for pursuing it; what is actually pivotal to one’s choice is how every individual understands others’ interpretation of the target and the Central Bank’s ability to achieve it. What the individual needs therefore, is a framework that will help first, identify what her options are and second, evaluate how these options are understood by all others. To this end, we employ the Variable Universe Games approach put forward by Bacharach (1993) which describes how players evaluate their options in the context of what everyone else might believe about them.

Our contribution then is to merge the MS (2002) framework to that of Bacharach (1993), in order to provide the individual with a framework to identify her best action, allowing for her personal interpretation of the options available to her. We find that numerical targets are effective coordinators of expectations when no great shocks are anticipated, or when all other public information available to individuals is prohibitively imprecise.

The paper is organised as follows. With the aid of a standard monetary policy model, section 2 describes how monetary policy can be seen as an information game based on the work by MS (2002). We then explain how the provision of numerical targets increases the options available to individuals. Section 3 then describes Bacharach’s (1993) approach to interpreting the options available to all players and section 4 merges the two models. Section 5 concludes.
2 Monetary Policy as an Information Game

The Central Bank has a standard loss function in which it chooses the rate of inflation $x$ to minimise the distance of inflation from its target $(x - x^T)$ and close the output gap $y$,

$$L_{CB|x} = E \frac{1}{2} \left[ (x - x^T)^2 + y^2 \right]$$  \hspace{1cm} (1)

subject to a standard Lucas supply function, $y = x - x^e + \xi$ where $\xi$ is a supply shock with zero mean and constant variance, $\sigma_{\xi}^2$. Note that any Central Bank will have an objective $x^T$ irrespective of whether it has communicated it to the public clearly, or even at all. We assume for simplification that the CB’s instrument is $x$. Optimisation of (1) implies that

$$x|\xi = \frac{x^T}{2} + \frac{x^e}{2} - \frac{\xi}{2}$$  \hspace{1cm} (2)

where $x$ is now the ex post inflation outcome conditional on the shock $\xi$ and $x^e$ is private sector expectations about the relevant rate of inflation. Representation (2) is of a structural form\(^4\) in the sense that expectations are not replaced (Leiteno, 2005). Svensson (2003) argues in favour of such a representation in order to indicate that factors like judgement that contribute to the way expectations are formed but cannot always be modelled, are an important contributor to monetary policy. In a typical commitment game, where the Central Bank communicates its target $x^T$ and commits to it, expectations formed by all individuals collectively are equal to the CB’s objectives, $x^e = x^T$ and the ex post outcome is

$$x|\xi = x^T - \frac{\xi}{2}$$  \hspace{1cm} (3)

$$E(x) = x^T$$  \hspace{1cm} (4)

The objective of this paper however, is to depart from the assumption that expectations are always equal to the objective of the Central Bank and analyse

\(^4\)Note that (2) is specific to the underlying Lucas supply function assumed but demonstrates that the outcome will be a function of both the policy the Central Bank pursues as well as what the private sector anticipates. Similarly, had the model been of the standard Neokeynesian type,

$$x_t = \beta E_t x_{t+1} + k y_t + \varepsilon_t$$

$$y_t = E_t y_{t+1} - \gamma (i_t - E_t x_{t+1}) + \eta_t$$

then the structural representation of the ex post inflation outcome would be

$$x_t = \frac{k^2}{1 + k^2} x^{e,T} + \frac{1}{1 + k^2} E_t x_{t+1} + \frac{\varepsilon_t}{1 + k^2}.$$  

Our point is to show that the ex post outcome is a function of both the CB objective as well as the expectations of the private sector.
how individuals go about interpreting the information that is available to them when forming expectations. Every individual $i$ will be forming an expectation of inflation $x_i$, such that the collective outcome (for a continuum of agents) is $x^e = \int_0^1 x_j d_j$, which is the expectation that is relevant to the inflation outcome. The time of the game assumed has the Central Bank deciding what its objectives are first, shocks occur next, then private agents form expectations based on information available about these shocks and policy objectives and finally the CB reacts to the supply shock $\xi$.

### 2.1 The Formation of Expectations

We thus start by arguing that while the CB may be clear itself about what its objectives are, it is not always possible to assume that private individuals form expectations that are consistent with these objectives. It becomes important then to examine, the information that is available to the private sector and how they use it to form expectations. Typically, every individual forms expectations based on two information sets, namely what is publicly available and therefore common to everyone, and what is available to them privately. Furthermore, every individual is aware of the fact that the ex post outcome of inflation $x$ will be determined by (2), in other words will be affected equally (given the model assumed) by the policy the Central Bank pursues to attain its objectives, as well as the average of expectations formed by the public. However, as the individual is interested in predicting the ex post level of inflation correctly, she needs to interpret both components of (2) based on the information she has. Her objectives are captured by a standard expected dis-utility\(^5\),

$$u_i (x_e, x_T) \equiv \frac{1}{2} E_i (x_i - x)^2$$

(5)

Note that subscript $i$ in the expectations operator, indicates that the individual will be seeking to minimise her expected dis-utility, given her own perceptions. $x_i$ is individual $i$’s expectation of what inflation will be and $x$ is again the ex post inflation outcome. We use $x^e$ to refer to the expectations profile over all agents. The objective of each individual $i$ is thus to form expectations $x_i$, as accurately as possible, which she will then use, for example, in wage negotiations. The individual decides her action $x_i$ based on the first-order condition of (5). This is:

$$\arg \min_{x_i} u_i (x^e, x^T) = E_i (x)$$

and from (2),

\(^5\) We assume that the individual consumer sets a price variable (individual wage) and supply elastically to the amount of labour demanded. This is just a narrative trick: the argument would work equally well in a set up as in Lucas’ island model in which individuals set the price of a good in an imperfect knowledge set-up. See appendix A for details.
The optimal action for individual $i$ is thus a function of three things: the objectives of the central Bank and hence the policy it will pursue, the shock that will occur and finally the average expectation formed by all individuals. Moreover, in forming expectations $x_i$, individual $i$ needs to evaluate these three things, captured here by the expectations operator, subscript $i^6$. It follows that if $x_i = x_j \forall j$, then $x_i = x^e$ and individuals’ expectations are matched. However, although desirable, coordination between agents at any level of inflation is not sufficient; the optimal outcome occurs when agents coordinate at the objective pursued by the Central Bank. Coordination at any other expectation rate still leaves agents away from the level of inflation that the CB aims to achieve. We will argue further down, that knowledge of the CB objective is necessary but not sufficient for coordination at it. Following MS (2002), we argue that information used by the agents is available in the form of a public signal common to all, and a private signal which is specific to each agent in the economy. These take the following form:

Public signal: $y = (x^T - \xi) + \eta$  \hspace{1cm} (7)  
Private signal: $z_i = (x^T - \xi) + \varepsilon_i$  \hspace{1cm} (8)  

Both $\eta$ and $\varepsilon_i$ are normally distributed with a zero mean and variance $\sigma^2_\eta$ and $\sigma^2_{\varepsilon_i}$ respectively. Furthermore, the two error terms are independent of $x$ and of each other, such that $E(\varepsilon_i \varepsilon_j) = 0$ for $i \neq j$. Contrary to MS then, the clarity of public information is not under the full control of the CB but it is affected by a combination of the CB’s information strategy, general market information available and noise. Based on these two types of signals, MS show that action for agent $i$ then is

$$ x_i = \frac{2\alpha \eta + \beta \varepsilon_i}{2\alpha + \beta} = x^T - \xi + \frac{2\alpha \eta + \beta \varepsilon_i}{2\alpha + \beta} \hspace{1cm} (9) $$

$^6$Equation (6) is not dissimilar to equation (2) of MS (2002) in which the individual forms a view about the state $\theta$ and the average action, $\bar{a}$. The strength with which she pursues that is given by the “beauty term” parameter $r$, equal here to the value $\frac{1}{2}$, provided by the model. An important difference to the MS (2002) approach however, is that the state $x$ is now endogenous, in the sense of being affected by the average action, whereas in the MS approach $\theta$ is independent of $\bar{a}$. 

7
where $\alpha = \frac{1}{\sigma_x^2}$ and $\beta = \frac{1}{\sigma_z^2}$, precision for the two information sets respectively. We call this the MS action. It follows that expectations across all agents are then equal, to

$$x^e = \int_0^1 x_j dj = x^T - \xi + \frac{2\alpha\eta}{2\alpha + \beta}$$

(10)

Equation (10) shows that the average expectation across all agents will be distorted by the (lack of) precision of the two signals as well as the preference for the ‘beauty term’ $r$, here equal to $\frac{r}{2}$ (in 6).

2.2 The Role of Inflation Targets

As argued earlier, the CB aims to set inflation equal to $x^T$. However this objective is not necessarily common knowledge to all private agents; it is instead subject to interpretation, affected by the way the CB communicates its objectives to the public, or indeed by its ability to achieve it, given its track record (credibility). The objective of the central bank is therefore, not uniquely ‘conspicuous’, in the sense of being seen and understood in the same way by everyone. As indicated in (9), private individuals weigh their own private information against what is publicly known and decide accordingly. This is true for any monetary policy regime. We argue next, that a Central Bank that provides a quantitative target differs to a Central Bank that does not in the following sense. The individual is now effectively presented with two options: either pursue the action indicated in (9) in which the inflation target is internalised and judged just like any other piece of public information, or alternatively, driven by her desire to coordinate, adopt the inflation target and fix her expectations at it. In modelling terms, the provision of a quantitative objective has the following implication. Every individual is faced with the choice between two alternatives for her action $a_i$: either to weigh all information available to her and thus follow the strategy suggested by Morris and Shin $x_i$, or simply form an expectation equal to the quantitative level of inflation announced by the Central Bank, $x^T$. By analogy, the same applies for the collective action $\bar{a}$, such that respectively for the two alternatives, the MS action leads to an average inflation expectation of $x^e$, whereas following the CB target leads to $x^Ts$. Based on (5), we turn next to the general form of individual $i$’s dis-utility which is affected by both her own action $a_i$, as well as the average action $\bar{a}$.

$$u_i(a_i, \bar{a}) \equiv E_i(a_i - x)^2$$

(11)

7 We depart from the MS (2002) set up here in so far that we do not assume that the level of clarity of the signal is completely within the CB’s control. The clarity of the signal is partly determined by the central bank, partly parametric and partly dependent on the shocks hitting the economy. This allows us to introduce meaningfully the provision of a numerical inflation target as a communication instrument.

8 For simplicity we assume that the collective action is either $x^T$ or $x^e$ and nothing inbetween.
Following the announcement of an inflation target, then individual $i$ is faced with two options. Either she weighs the public information available against her own private information and thus follows $a_i = x_i$ as suggested by MS, where

$$x_i = x^T - \xi + \frac{2\alpha \eta + \beta \varepsilon_i}{2\alpha + \beta}$$  \hfill (12)

or, simply ignores all information and follows the target, leading to her action being

$$a_i = x^T$$  \hfill (13)

Similarly, we can infer what the collective action will be. If all individuals follow the action suggested by Morris and Shin, then $a = x^e$, where,

$$x^e = x^T - \xi + \frac{2\alpha \eta}{2\alpha + \beta}\)  \hfill (14)

By contrast when all individuals follow the target, then inflation expectation is

$$\bar{a} = x^T$$

For a continuum of agents, the inflation outcome is affected by the collective action $\bar{a}$, and not the action of the individual $a_i$. For the former case, the $ex \ post$ inflation outcome (from (2)) which is relevant to the individual’s dis-utility is thus:

$$x|_{\xi,x^e} = \frac{x^T}{2} + \frac{x^e}{2} - \frac{\xi}{2}$$  \hfill (15)

$$= x^T - \xi + \frac{\alpha \eta}{2\alpha + \beta}$$

and for the latter,

$$x|_{\xi,x^T} = \frac{x^T}{2} + \frac{x^T}{2} - \frac{\xi}{2}$$  \hfill (16)

$$= x^T - \frac{\xi}{2}$$

Following these four possible actions, we calculate next the payouts for individual $i$ based on (11).

**Choice 1:** $a_i = x_i, \bar{a} = x^e$

$$u_1(x_i, x^e) = \frac{\alpha + \beta}{(2\alpha + \beta)^2}$$  \hfill (17)

**Choice 2:** $a_i = x^T, \bar{a} = x^e$

\footnote{All derivations that follow in this section are presented in detail in Appendix B.}
\[ u_1 (x^T, x^e) = \sigma_i^2 + \frac{\alpha}{(2\alpha + \beta)^2} \]  

Choice 3: \( a_i = x_i, \bar{a} = x^T \)

\[ u_1 (x_i, x^T) = \frac{1}{4} \sigma_i^2 + \frac{4\alpha + \beta}{(2\alpha + \beta)^2} \]  

Choice 4: \( a_i = x^T, \bar{a} = x^T \)

\[ u_1 (x^T, x^T) = \frac{1}{4} \sigma_i^2 \]  

We summarise the payout matrix for individual \( i \), given the two alternative collective actions, as follows:

\begin{table}[h]
\centering
\begin{tabular}{c|cc}
\hline
\( a_i \) & \( x^e \) & \( x^T \) \\
\hline
\( x_i \) & \( \frac{\alpha + \beta}{(2\alpha + \beta)^2} \) & \( \frac{1}{4} \sigma_i^2 + \frac{4\alpha + \beta}{(2\alpha + \beta)^2} \) \\
\( x^T \) & \( \sigma_i^2 + \frac{\alpha}{(2\alpha + \beta)^2} \) & \( \frac{1}{4} \sigma_i^2 \) \\
\hline
\end{tabular}
\end{table}

Table 1 summarises the pure form strategies available to individual \( i \), and the possible dis-utility outcomes associated with them. However, Table 1 fails to capture an important element in the game, and that is the uncertainty that individual \( i \) faces, with respect to how all other individuals perceive this target and therefore, what the collective action will be. From her own perspective therefore, before deciding on her own action, individual \( i \) needs to evaluate how likely the two alternative collective actions, \( x^e \) and \( x^T \), are. It is not sufficient therefore, to evaluate for herself, the clarity or credibility of the signal that the Central Bank provides; more important to her decision is how she interprets what others believe about that signal. What the individual is concerned with therefore, is the following: given the inherent incentive to coordinate, are there conditions under which it is always to her benefit to follow the signal? If on the other hand, the collective action applied is of paramount importance to which action the individual should choose, how does she go about deciding? We provide next a framework for her to do that, the last step required before she decides on an action.

### 3 A framework for Interpreting Expectations

While Table 1 identifies the options that are available to the individual, we provide next a framework for interpreting these options, given the action of the counter-player (i.e. the collective action). This is based on Bacharach’s *Variable Universe Games* (1993) framework, which helps describe how players evaluate their strategies to identify salient points when forming expectations in matching games\(^{10}\). The novelty of this approach is that it allows explicitly for differences

\(^{10}\) Used and extended by Janssen (2001).
in perceptions which then helps players choose rationally between alternative outcomes. The framework provided shows that in matching games, the players’ incentive to coordinate induces them to look for salient points. However, as salience is subject to personal interpretation, the existence of such features is not necessarily uniquely defined. We describe the approach first, and apply it explicitly to our question then in section 4.

3.1 An Expected Utility Approach

The game of blockmarking is played in the following way. Two players are shown a number of wooden blocks and each has to secretly pick one. If both players pick the same block, they receive an identical pecuniary prize; otherwise they receive nothing. The author then describes three variants of the game, summarised in figure 1.

In Blockmarking 1, (B1), the players are given five identical blocks (in size, colour, shape and material). In Blockmarking 2, (B2), the same game is repeated, except now one of the five blocks is of a different colour, (white). In Blockmarking 3, (B3), players are now given 20 blocks, eighteen of which are grey and two are white. Furthermore, closer inspection of the blocks, allows players to see that the grain of the wood in just one of the grey blocks is wavy. B3 can thus be described either in terms of colour, \( C \) or in terms of the grain of the wood, \( G \). As the game is of a matching nature, it is to the players’ interest to look for salient features that help achieve tacit coordination. In example B1 above, there is no clear way of differentiating between the blocks, so one is inclined to simply pick at random. At example B2 however, the difference in colour allows players to distinguish between the blocks in such a way, that it is always wise to go for the one that is white. The unique instantiation of the white block thus provides the two players with a focal point. Similarly in

\[ \text{Figure 1: The Game of Blockmarking} \]
B3, if colour is the distinguishing feature that occurs to the players, they are then inclined to pick one of the white blocks, even though such action does not automatically lead to coordination. However, if a player has managed to see that not only colour but also the grain of the wood differentiates the blocks, uniqueness is again guaranteed. The difficulty now however, is that the grain pattern of the wood is not necessarily identifiable (conspicuous) by (to) all players. In forming her choice therefore, having seen the difference in grain herself, player 1, needs to assess how likely her partner is to distinguish the blocks in terms of the grain as well. Bacharach’s analysis shows, that if this likelihood is big, then it is to her interest to pick the grey block with the wavy pattern; otherwise she is better off picking one of the white blocks and face an at most, 50% chance of matching the choice of her partner.

Bacharach provides a thorough proof to B3 in the appendix to his paper, but the essence of the game faced by the two players individually can be summarised as follows. In solving B3, player 1 is effectively faced with two alternative actions: \( M_h \) mark a white block at random, or \( M_w \), mark the grey block with the wavy grain. Furthermore, as explained above, the crucial point in this analysis is the likelihood with which player 1 believes player 2 has noticed the grain. She is thus left with the following two choices when forming her views about player 2. Either she believes that her opponent has seen the grain (and assigns probability \( v \) to that event), or she does not believe that he has seen the grain (and assigns probability \( 1 - v \) to that event). It is reasonable to assume that if player 2 has indeed noticed the grain, then he will pick it with some non-zero probability. However, if he has not noticed the grain then he can never mark a block accordingly. From player 1’s perspective therefore, her expected utility from choosing one of her two actions is the following.

**Definition 1:** Both players have an identical set of feasible strategies, \( R^+ = \{C, G\} \) and possible actions, \( A = \{M_h, M_w\} \). Define \( U_1(x_{1,a(\cdot)}, x_{2,a(\cdot)}) \), player 1’s utility from following action \( x_{1,a(\cdot)} \) and player 2 following action \( x_{2,a(\cdot)} \), for \( a \in A \) where \( a(C) = M_h \) and \( a(G) = M_w \).

We need to deal with two cases:

**Case 1:** Player 2 always marks a block according to colour, either because he has not seen the grain himself, or because he believes his partner has not. Then player 1’ expected utility is

\[
E_1 U(M_h, M_h) = (1 - v)U_1(M_h, M_h) + vU_1(M_h, M_h)
\]

\[
E_1 U(M_w, M_h) = (1 - v)U_1(M_w, M_h) + vU_1(M_w, M_h)
\]

We normalise next \( U(x_1 = x_2) = 1 \), and calculate the expected utilities:
\[ E_1U(\tilde{M}_h, \tilde{M}_h) = (1 - v)\frac{1}{2}U(x_1 = x_2) + v\frac{1}{2}U(x_1 = x_2) = \frac{1}{2} \]
\[ E_1U(Mw, \tilde{M}_h) = (1 - v) \cdot 0 + v \cdot 0 = 0 \]

This implies that \( E_1U(\tilde{M}_h, \tilde{M}_h) > E_1U(Mw, \tilde{M}_h) \) and therefore player 1 has an incentive to match her partner’s action by also picking a white block at random.

**Case 2:** Player 2 now, marks a block based on the grain when he has noticed it. Otherwise, he marks a block according to colour. Then expected utility for player 1 is now

\[ E_1U \left[ M\tilde{h}, \left( M\tilde{h} \text{ or } Mw \right) \right] = (1 - v)U_1 \left( M\tilde{h}, M\tilde{h} \right) + vU_1 \left( M\tilde{h}, Mw \right) \]
\[ E_1U \left[ Mw, \left( M\tilde{h} \text{ or } Mw \right) \right] = (1 - v)U_1 \left( Mw, M\tilde{h} \right) + vU_1 \left( Mw, Mw \right) \]

and therefore

\[ E_1U \left[ M\tilde{h}, \left( M\tilde{h} \text{ or } Mw \right) \right] = (1 - v)\frac{1}{2}U(x_1 = x_2) + v \cdot 0 = \frac{1 - v}{2} \]
\[ E_1U \left[ Mw, \left( M\tilde{h} \text{ or } Mw \right) \right] = (1 - v) \cdot 0 + vU(x_1 = x_2) = v \]

It follows that,

\[ E_1U \left[ Mw, \left( M\tilde{h} \text{ or } Mw \right) \right] > E_1U \left[ M\tilde{h}, \left( M\tilde{h} \text{ or } Mw \right) \right], \text{ iff } v > \frac{1}{3}. \]

But between the two cases, the necessary and sufficient condition for player 1 to decide to mark a block according to the grain, is

\[ E_1U \left[ Mw, \left( M\tilde{h} \text{ or } Mw \right) \right] > E_1U(\tilde{M}_h, \tilde{M}_h) \iff v > \frac{1}{2} \]

(21)

In other words, the balance of reasons favours marking the block with the wavy grain, only if \( v \) is a large enough number by comparison to \( \frac{1}{m} \) where \( m \) is the number of white blocks. Bacharach argues therefore, that the relative rarity of the white blocks, captured here by \( \frac{1}{m} \), is pulling against the conspicuousness \( v \) of the grain pattern, as the less rare the white blocks (bigger \( m \)), the more likely the player is to pick the wavy grey block.

The point that is crucial to Bacharach’s analysis is the fact that players have particular ways of perceiving the game, such that the framing of the game
(universe) available to them individually is not necessarily available to other players as well (variable). Before deciding on a possible action therefore, for example picking the block with the wavy grain (provided they have seen it), the player has to form a view as to how likely her counterpart is to have noticed the grain (as well as the colour) as a possible distinguishing feature. Evaluating that is necessary before picking a strategy, implying that having noticed the grain for oneself is not sufficient to pick it. Any player therefore, needs to assess whether their own beliefs as to what is conspicuous to them is also conspicuous to others. Picking the wavy block is the desirable strategy only if, given her assessment of this likelihood, the expected value of doing so is greater than the expected value of picking a white block at random.

4 Variable Universes and Inflation Targets

How does the analogy carry over to monetary policy? As argued above, a Central Bank that provides an explicit quantitative target presents the individual with two options: to either treat the target like any other piece of public information and simply apply the MS rule in deciding on her actions or, alternatively, adopt the inflation target and fix her expectation on it. The latter is naturally an attractive alternative because of her desire to coordinate, reflected in the second term in equation (6). But for this latter option to be attractive enough, the individual needs to have sufficient confidence that others will pick the target as well. Just as in the blockmarking game, the fact that a target (wavy block) is provided (is seen for oneself) is not sufficient for the individual to coordinate it; what is required further is for the individual to have enough confidence that others will think the same way. In that respect, her interpretation of the way the target is perceived by others is key to her decision. So while there is no uncertainty as to what strategies are available to people by comparison to the blockmarking game, there is uncertainty as to how these strategies are perceived. It is in this sense that Bacharach’s approach is useful to our analysis.

Following Bacharach’s approach, we then need to deal with two cases regarding the actions taken:

Case 1: First we assume that when the Central Bank announces its quantitative inflation objective, the collective action applies it occasionally. Player \(i\) therefore, forms a belief about the likelihood \(v\) with which the collective action will be equal to that target, in which case, with likelihood \((1 - v)\), the collective action will simply treat the inflation target as just an extra piece of public information. Expected dis-utility for player \(i\) of pursuing either of her two options, is therefore,

\[
E \{ u_i \left( x_i, (x^e \text{ or } x^T) \right) \} = (1 - v)u_i \left( x_i, x^e \right) + vu_i \left( x_i, x^T \right)
\]

\[
E \{ u_i \left( x^T, (x^e \text{ or } x^T) \right) \} = (1 - v)u_i \left( x^T, x^e \right) + vu_i \left( x^T, x^T \right)
\]
and from Table 1,

\[ E \left\{ u_i \left[ x_i, (x^e \text{ or } x^T) \right] \right\} = (1 - v) \frac{\alpha + \beta}{(2\alpha + \beta)^2} + v \left[ \frac{1}{4} \sigma^2 + \frac{4\alpha + \beta}{(2\alpha + \beta)^2} \right] \]

\[ E \left\{ u_i \left[ x^T, (x^e \text{ or } x^T) \right] \right\} = (1 - v) \left[ \sigma^2 + \frac{\alpha}{(2\alpha + \beta)^2} \right] + v * \frac{1}{4} \sigma^2 \]

It follows that,

\[ E \left\{ u_i \left[ x^T, (x^e \text{ or } x^T) \right] \right\} < E \left\{ u_i \left[ x_i, (x^e \text{ or } x^T) \right] \right\}, \text{ iff} \]

\[ \sigma^2 < \frac{\beta + v\alpha}{(1 - v)(2\alpha + \beta)^2} \] (22)

or in other words, for any given value of \( v \) (even zero), for the inflation target to be the optimal strategy for player \( i \), it is sufficient that the supply shock is small enough.

**Case 2:** However, Player \( i \) may have to do with the fact that others do not interpret the target in the way intended by the Central Bank and therefore, prefer to simply weigh public against private information and thus follow the action derived by Morris and Shin. This may be either because they do not understand or believe the intentions of the Central Bank, or because they do not trust that others understand or believe the intentions of the Central Bank. This achieves an average expectation for inflation equal to \( x^e \). Then player \( i \)'s expected dis-utility of following either of her two options is:

\[ E \left[ u_i(x_i, x^e) \right] = (1 - v)u_i(x_i, x^e) + vu_i(x_i, x^e) \]

\[ E \left[ u_i(x^T, x^e) \right] = (1 - v)u_i(x^T, x^e) + vu_i(x^T, x^e) \]

Based on Table 1, then these are

\[ E \left[ u_i(x_i, x^e) \right] = (1 - v) \left[ \frac{\alpha + \beta}{(2\alpha + \beta)^2} + v \frac{\alpha + \beta}{(2\alpha + \beta)^2} \right] = \frac{\alpha + \beta}{(2\alpha + \beta)^2} \]

\[ E \left[ u_i(x^T, x^e) \right] = (1 - v) \left[ \sigma^2 + \frac{\alpha}{(2\alpha + \beta)^2} \right] + v * \left[ \sigma^2 + \frac{\alpha}{(2\alpha + \beta)^2} \right] = \sigma^2 + \frac{\alpha}{(2\alpha + \beta)^2} \]

It follows that

\[ E \left[ u_i(x^T, x^e) \right] < E \left[ u_i(x_i, x^e) \right], \text{ iff} \]

\[ \sigma^2 < \frac{\beta}{(2\alpha + \beta)^2} \] (23)

Note that if (23) holds, then (22) is also satisfied, so that the former is the necessary and sufficient condition for individual \( i \) to pick the target.
4.1 Inflation Targeting as a dominant strategy

Note that if (23) holds, then based on Table 1, following the target becomes the dominant pure form strategy, in that the individual will always choose to form inflation expectations according to the inflation target. At the same time, and allowing for interpretation uncertainty to characterise the game, satisfying this condition guarantees that the expectation value of adopting the target is improved. In other words, for this to be true, it is important that the supply shock is smaller than a given ratio. Figure 2 shows that this condition (condition (23) drawn against values of $\alpha$ and $\beta$) is very stringent, in the sense that inflation targeting is dominant only if shocks are very small in size. Indeed, if the economy is hit by very big shocks then the condition is not satisfied, and the provision of a target does not help agents coordinate at the level intended by the Central Bank. This is intuitively appealing because it evaluates the effectiveness of the target within the context of economic conditions in which it is applied.

Figure 2: Inflation Targeting as a dominant Strategy

Moreover, figure 2 also shows that if public information is very imprecise ($\alpha$ is low) then the provision of an inflation target becomes helpful, in the sense that the condition becomes easier to satisfy. This implies that numerical targets become substitutes for imprecise public information; in the absence of concrete information, the provision of one clear inflation target becomes the only unequivocal piece of public information. And that is true irrespective of the interpretation parameter, $v$. To carry the analogy to the blockmarking game, if white blocks become more and more numerous, then having seen the grain, one is more likely to pick it.

4.2 Expectations formation as a matching game

However if (23) is not satisfied, i.e., $\sigma_k^2 > \frac{\beta}{(2\alpha + \beta)}$ then, from Table 1, individual’s optimal action in pure strategies requires "matching" the average action. When allowing for the uncertainty of interpretations, then for inflation targeting to
produce higher expected dis-utility, condition (22) must be satisfied and by consequence the value of the interpretation parameter $v$ is relevant. Condition (22) can be re-written as

$$\frac{(2\alpha + \beta)^2 \sigma^2_s - \beta}{4\alpha + (2\alpha + \beta)^2 \sigma^2_s} < v$$

(24)

Figure 3 plots condition (24) in the $\alpha$ and $\beta$ space for four different values of the supply shock ($0 < \sigma_s < 1$)$^{11}$. 

Figure 3: The Role of Interpretations

There are two interesting features that arise from figure 3. First, it is the case that as the variance of the shock increases, then (24) becomes more difficult to satisfy. In other words, if larger shocks are expected, then individual $i$ needs an ever great degree of confidence $v$ that others will follow the target, before she picks it herself. This is consistent with what is mentioned above when inflation targeting is a dominant strategy for individual $i$, namely that in the presence of large shocks, inflation targeting is less convincing in its role as a coordinator of expectations. Second, as public information suffers from lack of clarity (i.e. $\alpha$ small), the provision of a clear and unique quantitative signal helps relax the stringency of the condition.

In both cases from above, the role of private information is de-emphasised in that it does not impose a constraint on either (23) or (24) to hold. This is demonstrated in figure 4 for the latter condition for different values of $\beta$ (overlapping plates on the graph).

$^{11}$As $v$ is constrained to take values between 0 and 1, so is condition (24).
5 Conclusions

Any private individual forms expectations of inflation based on information that is available to her. Our paper concentrates on the way the Central Bank’s communication strategy might affect these expectations. We begin our analysis by arguing that it is not always possible for a monetary policy authority to assume that it can affect private expectations in the way that they will match its own intentions. Private individuals rely on information that is available to them publicly (and thus common to everyone) and information that might be unique to them individually. Monetary policy then, becomes an information game, in which private individuals base their decision on a combination of all information available, corrected for their respective degree of precision, (or lack of). As the level of expectations affects the final outcome of inflation, the private sector needs to deduce both what the objective of the Central Bank is and its ability to achieve it, as well as what everybody else’s beliefs are. We apply the Morris and Shin model to demonstrate that the latter point implies that coordinated expectations are preferable although not necessarily the guarantee of optimal outcomes. Further to that, we then use Bacharach’s Variable Universe approach to demonstrate exactly how people interpret the options available to them given the actions of all other players in the game. Our contribution has therefore, been to merge the two models and provide a comprehensive framework for individuals to enumerate their options and thus form expectations. Based on this, we find that a Central Bank that announces a very precise quantitative target may, ceteris paribus, benefit from helping private sector expectations coordinate at the level of its objectives. We describe the conditions for which this happens and discover that inflation targeting does indeed achieve coordination, first when the supply shocks expected are small - in other words the
economy is stable and second, when public information fails in all other respects to provide the private sector with clear signals as to what the level of inflation relevant to them is going to be. It is in this sense that we argue that inflation targets are substitutes for poor, otherwise, public information. Naturally, as we show above, it is not sufficient for any individual to view this quantitative signal just for herself, as a satisfactory substitute. She must have a high enough degree of confidence that all other agents do too. If this holds, then following the signal constitutes her preferred strategy.
APPENDICES

A Individual i’s Dis-Utility

Individual i dis-utility function is

$$u_i(x^e, x^T) = \frac{1}{2} E_i(x_i - x)^2$$  \hfill (25)

The most direct way to rationalise the loss function (25) is to consider it an approximation of a model of a monopolistically competitive labour market in which firms set prices and individuals set wages, as in Erceg at al (2000). and Woodford (2003). Individual i then supplies a type of labour i, such that total demand for labour is the aggregate of all the individual types, according to a standard CES specification with elasticity of substitution equal to $\theta$, i.e.,

$$L_t = \left[ \int_0^1 l_t(i)^{\theta-1}/\theta \; di \right]^{\theta/(\theta-1)}$$

It follows that demand for labour type i on the part of the wage taking firm is given by

$$l_t(i) = L_t \left( \frac{w_t(i)}{W_t} \right)^{-\theta}$$

where $w_t(i)$ is the wage of individual i and $W_t$ is the aggregate wage index. Thus individual i takes the demand for labour as given and fixes the nominal wage to maximise a standard utility function in consumption and leisure, under a standard budget constraint. One of the first order conditions for the maximisation problem is:

$$\frac{w_t(i)}{P_t} = -\frac{w_t'(i)}{v'_t(i)} = MRS_{cl}$$  \hfill (26)

where $P$ is the consumer price index. Individual i determines her nominal wage, her consumption and her labour supply at the beginning of period t, given her expected level of prices and the form of her utility function. For a marginal rate of substitution between consumption and leisure equal to $-1$, condition (26) reduces to:

$$w_t(i) = E(P_t)$$  \hfill (27)

which is equivalent to the first order condition of (25). The actual form used is a simplification for reason of tractability.
The Role of Inflation Targets

In terms of the model adopted, the provision of a quantitative objective has the following implication. Every individual is faced with the choice between two actions, $a_i$: either to weigh all information available to her and thus follow the strategy suggested by Morris and Shin, $x_i$, or simply form an expectation equal to the quantitative level of inflation announced by the Central Bank, $x^T$. By analogy, the same applies for the collective action $\bar{a}$, such that respectively for the two alternatives, the MS action leads to an average inflation expectation of $x^e$, whereas following the CB target leads to $x^T$. Based on (5), we turn next to the general form of individual $i$’s dis-utility which is affected by her own action $a_i$, as well as the average action $\bar{a}$. The latter in turn affects the inflation outcome, $x$.

$$u_i(a_i, \bar{a}) \equiv E_i(a_i - x)^2$$  (28)

Following the announcement of an inflation target, then individual $i$ is faced with two options. Either she weighs the public information available against her own private information and thus follows $a_i = x_i$ as suggested by MS, where

$$x_i = \frac{2\alpha y + \beta x_i}{2\alpha + \beta}$$
$$= x^T - \xi + \frac{2\alpha y + \beta \varepsilon_i}{2\alpha + \beta}$$
$$= x^T - \xi + \frac{2\alpha y + \beta \varepsilon_i}{2\alpha + \beta}$$  (29)

or, simply ignores all information and follows the target, leading to her action being

$$a_i = x^T$$  (30)

If individuals follow collectively the action suggested by Morris and Shin, $\bar{a} = x^e$ where

$$x^e = x^T - \xi + \frac{2\alpha y}{2\alpha + \beta}$$  (31)

By contrast when all individuals follow the target, then inflation expectation is

$$\bar{a} = x^T$$

For a continuum of agents the inflation outcome is affected by the average action (31), such that for the former case it is
and for the latter

\[ x|_\xi = \frac{x^T}{2} + \frac{\xi}{2} \]

Following these four possible actions, we calculate next the payouts for individual \( i \) based on (28).

**Choice 1:** \( a_i = x_i, \bar{a} = x^e \)

\[ u_1(x_i, x^e) \equiv E_i(a_i - x)^2 \]

\[ = E_i \left[ x^T - \xi + \frac{2\alpha \eta + \beta \xi}{2\alpha + \beta} - \left( x^T - \xi + \frac{\alpha \eta}{2\alpha + \beta} \right)^2 \right] \]

\[ = E_i \left[ \frac{2\alpha \eta + \beta \xi}{2\alpha + \beta} - \frac{\alpha \eta}{2\alpha + \beta} \right]^2 \]

\[ = E_i \left[ \frac{\alpha \eta + \beta \xi}{2\alpha + \beta} \right]^2 \]

\[ = \frac{\alpha^2 \sigma^2}{(2\alpha + \beta)^2} + \frac{\beta^2 \sigma^2}{(2\alpha + \beta)^2} \]

\[ = \frac{\alpha}{(2\alpha + \beta)^2} + \frac{\beta}{(2\alpha + \beta)^2} = \frac{\alpha + \beta}{(2\alpha + \beta)^2} \]  

(34)

**Choice 2:** \( a_i = x^T, \bar{a} = x^e \)
\[ u_1(x^T, x^e) \equiv E_i(a_i - x)^2 \]
\[ = E_i \left[ x^T - \left( x^T - \xi + \frac{\alpha \eta}{2\alpha + \beta} \right) \right]^2 \]
\[ = E_i \left( \xi - \frac{\alpha \eta}{2\alpha + \beta} \right)^2 \]
\[ = \sigma_x^2 + \frac{\alpha^2 \sigma_\eta^2}{(2\alpha + \beta)^2} \quad (35) \]
\[ = \sigma_\xi^2 + \frac{\alpha}{(2\alpha + \beta)^2} \quad (36) \]

**Choice 3:** \( a_i = x_i, \bar{a} = x^T \)

\[ u_1(x_i, x_i^T) \equiv E_i(a_i - x)^2 \]
\[ = E_i \left[ x^T - \xi + \frac{2\alpha \eta + \beta \varepsilon_i}{2\alpha + \beta} - \left( x^T - \frac{\xi}{2} \right) \right]^2 \]
\[ = E_i \left( \frac{\xi}{2} + \frac{2\alpha \eta + \beta \varepsilon_i}{2\alpha + \beta} \right)^2 \quad (37) \]
\[ = \frac{1}{4} \sigma_\xi^2 + \frac{4\alpha}{(2\alpha + \beta)^2} + \frac{\beta}{(2\alpha + \beta)^2} \quad (38) \]
\[ = \frac{1}{4} \sigma_\xi^2 + \frac{4\alpha + \beta}{(2\alpha + \beta)^2} \quad (39) \]

**Choice 4:** \( a_i = x^T, \bar{a} = x^T \)

\[ u_1(x^T, x^T) \equiv E_i(a_i - x)^2 \]
\[ = E_i \left[ x^T - \left( x^T - \frac{\xi}{2} \right) \right]^2 \]
\[ = E_i \left( \frac{\xi}{2} \right)^2 \quad (40) \]
\[ = \frac{1}{4} \sigma_\xi^2 \quad (41) \]

We summarise the payout matrix in normal form for player \( i \), given the two alternative collective actions, as follows:

<table>
<thead>
<tr>
<th>( a_i )</th>
<th>( \bar{a} )</th>
<th>( x^T )</th>
<th>( x^T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_i )</td>
<td>( \frac{x_i + \delta}{(2\alpha + \beta)^2} )</td>
<td>( \frac{1}{4} \sigma_\xi^2 + \frac{\delta}{(2\alpha + \beta)^2} )</td>
<td></td>
</tr>
<tr>
<td>( x^T )</td>
<td>( \sigma_\xi^2 + \frac{\alpha}{(2\alpha + \beta)^2} )</td>
<td>( \frac{1}{4} \sigma_\xi^2 + \frac{1}{4} \sigma_\xi^2 )</td>
<td></td>
</tr>
</tbody>
</table>
References


