Optimal Exchange Rate Policy

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2nd ERSA/CEPR Workshop on Macroeconomic Policy in Emerging Markets

January 26, 2024

Motivation

- What is the optimal exchange rate policy?
  1. exchange rate as a target
     — trilemma vs. fear of floating
  2. exchange rate is not a policy instrument
     — what mix of monetary policy, FX interventions, capital controls?

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- Build on a realistic GE model of exchange rates consistent with
  — PPP, UIP, Backus-Smith, Meese-Rogoff puzzles $\Rightarrow$ UIP shock ψ_t
  — Mussa puzzle ▸ show $\Rightarrow$ $\psi_t = \psi_t(\sigma_e^2)$

- Dual role of exchange rates:
  a) expenditure switching in goods markets
  b) risk sharing in financial markets

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- Dual role of exchange rates:
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- Develop a rich framework for policy analysis
  — intuitive linear-quadratic Ramsey problem (cf. CGG'99, GM'05)
  — optimal targets, pecking order of instruments, divine coincidence, time consistency, forward guidance, gains from cooperation

Main Results

1. First best:
   - — one-to-one mapping between instruments, targets, shocks
   - — exchange rate targeting is suboptimal
2. Divine coincidence in an open economy
   - — requires that the frictionless real exchange rate is stable
   - — peg can implement the first-best
3. More generally, optimal MP partially stabilizes exchange rate
4. Capital controls are required when foreign traders
5. Gains from international cooperation under second-best policies

Relation to the Literature

- Portfolio models:
  - Segmented markets: Kouri (1976), Blanchard, Giavazzi & Sa (2005), Alvarez, Atkeson & Kehoe (2009), Camanho, Hau & Rey (2021), Greenwod, Hanson, Stein & Sunderam (2020), Jiang, Krishnamurthy & Lustig (2021), Gourinchas, Ray & Vayanos (2021), Kollmann (2005), Jeanne & Rose (2002), Gabaix & Maggiori (2015), Itskhoki & Mukhin (2021a,b)
  - Financial channel of MP: Obstfeld & Rogoff (2002), Rey (2013), Kekre & Lenel (2021), Fanelli (2017), Hassan, Mertens & Zhang (2021), Akinci, Kalemli-Ozcan & Queralto (2022), Fornaro (2021)
- Optimal policy in open economy:
  - Monetary policy: Obstfeld & Rogoff (1995), Clarida, Gali & Gertler (1999, 2001, 2002), Devereux & Engel (2003), Benigno & Benigno (2003), Gali & Monacelli (2005), Engel (2011), Goldberg & Tille (2009), Corsetti, Dedola & Leduc (2010, 2018), Egorov & Mukhin (2021)
  - Capital controls: Jeanne & Korinek (2010), Bianchi (2011), Farhi & Werning (2012, 2013, 2016, 2017), Costinot, Lorenzoni & Werning (2014), Schmitt-Grohe & Uribe (2016), Basu, Boz, Gopinath, Roch & Unsal (2020)
  - FX interventions: Jeanne (2013), Cavallino (2019), Amador, Bianchi, Bocola & Perri (2016, 2020), Fanelli & Straub (2021)

SETUP

Setup

- SOE with T and NT, segmented asset markets
- **Households**:

$$\max \ \mathbb{E} \sum_{t=0}^{\infty} \beta^t \Big[\gamma \log C_{Tt} + (1-\gamma)(\log C_{Nt} - L_t) \Big]$$

$$\text{s.t.} \quad \frac{B_t}{R_t} + P_{Tt} C_{Tt} + P_{Nt} C_{Nt} = B_{t-1} + W_t L_t + \Pi_t + T_t$$

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- **Firms**:
  1. tradables: exogenous endowment Y_{Tt}, law of one price $P_{Tt} = \mathcal{E}_t P^*_{Tt} = \mathcal{E}_t$
  2. non-tradables: technology $Y_{Nt} = A_t L_t$, fully sticky prices $P_{Nt} = 1$

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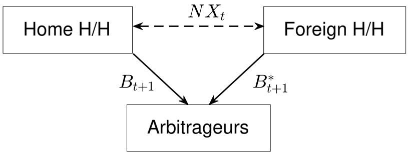

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  - arbitrageurs choose zero-capital portfolio (D_t, D^*_t): $\frac{D_t}{R_t} + \frac{\mathcal{E}_t D^*_t}{R^*_t} = 0$
  - earn carry trade returns $\tilde{R}_{t+1} \equiv R^*_t - R_t \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}$, transfer to home h/h

$$\max_{D^*_t} \; \mathbb{E}_t[\Theta_{t+1} \mathcal{W}_{t+1}] - \frac{\omega}{2} \mathrm{var}_t[\mathcal{W}_{t+1}], \qquad \mathcal{W}_{t+1} = \tilde{R}_{t+1} \frac{D^*_t}{R^*_t}$$

  - market clearing for bonds:

$$B^*_t = D^*_t + N^*_t + F^*_t$$

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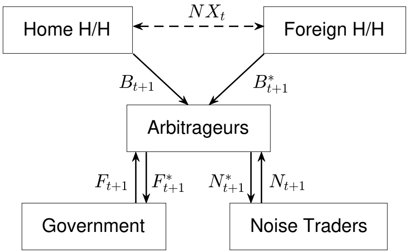

First Best

- **Social planner's problem**:

$$\max_{\{C_{Tt}, C_{Nt}, L_t, B_t^*\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \Big[\gamma \log C_{Tt} + (1-\gamma)(\log C_{Nt} - L_t) \Big]$$

$$\text{s.t.} \quad \frac{B_t^*}{R_t^*} - B_{t-1}^* = Y_{Tt} - C_{Tt} \qquad \text{and} \qquad C_{Nt} = A_t L_t$$

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- **Quadratic loss function**: proof

$$\frac{1}{2}\,\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\big[\gamma z_t^2+(1-\gamma)x_t^2\big]$$

$$\text{s.t.}\quad \beta b_t^*-b_{t-1}^*=-z_t$$

Equilibrium Conditions

- **Goods market**:

$$\frac{\gamma}{1-\gamma}\frac{C_{Nt}}{C_{Tt}} = \frac{\mathcal{E}_t P^*_{Tt}}{P_{Nt}} = \mathcal{E}_t$$

Equilibrium Conditions

- **Goods market**:

$$\boxed{e_t = \tilde{q}_t + x_t - z_t}$$

  - $\tilde{q}_t = \log \tilde{C}_{Nt} - \log \tilde{C}_{Tt}$ is natural RER
  - $x_t \equiv \log(C_{Nt}/\tilde{C}_{Nt}), \quad z_t \equiv \log(C_{Tt}/\tilde{C}_{Tt})$
  - EE + sticky prices $\Rightarrow$ R_t determines x_t ▸ PT ▸ NKPC

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- **Financial market:**

$$\beta R_t^* \mathbb{E}_t \frac{C_{Tt}}{C_{Tt+1}} = 1 + \omega\sigma_t^2 \frac{B_t^* - N_t^* - F_t^*}{R_t^*}$$

  - ω is arbitrageurs' risk aversion
  - σ_t^2 is the volatility of carry-trade returns
  - $B_t^* - N_t^* - F_t^*$ is net demand of h/h, n/t, gov't = arbitrageurs' gross position

$\Rightarrow$ e.g. $N_t^* \uparrow \;\Rightarrow\; D_t^* \downarrow \;\Rightarrow\; \mathbb{E}_t\big[R_t \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} - R_t^*\big] > 0 \;\Rightarrow\; \mathcal{E}_t \uparrow \;\Rightarrow\; C_{Tt} \downarrow$

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$$\boxed{\mathbb{E}_t \Delta z_{t+1} = -\bar{\omega}\sigma_t^2(b_t^* - n_t^* - f_t^*)} \qquad \boxed{\sigma_t^2 = \mathrm{var}_t(\Delta e_{t+1})}$$

  — $\mathbb{E}_t \Delta z_{t+1} = i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1}$ (UIP deviations $\leftrightarrow$ RS wedge)

  — first-order risk premium $(X_t = \bar{X}(1 + \nu x_t),\ \omega = \bar{\omega}/\nu^2$ and $\nu \to 0)$ ▸ details

Ramsey Problem

- **Lemma**: To the first-order approximation, the optimal policy solves

$$\min_{\{z_t, x_t, e_t, b_t^*, f_t^*, \sigma_t^2\}} \quad \frac{1}{2}\,\mathbb{E}\sum_{t=0}^{\infty}\beta^t\Big[\gamma z_t^2 + (1-\gamma)x_t^2\Big]$$

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  - **Shocks**:
    1. macro/fundamental: $(A_t, Y_{Tt}, R_t^*) \longrightarrow \tilde{q}_t$
    2. financial/liquidity: $(N_t^*, \tilde{B}_t^*) \longrightarrow n_t^*$
  - **Instruments**:
    1. monetary policy (MP): $R_t \longrightarrow x_t$
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1. **Relaxed Trilemma**: it is possible to simultaneously have (i) no capital controls, (ii) inward-looking MP, (iii) independent ER policy (cf. Wallace'81)
   — subject to country's budget constraint and $\sigma_t^2 > 0$

TWO POLICY INSTRUMENTS

Optimal Policy

- Planner's problem:

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2. **Optimal targets**: MP $\rightarrow$ inflation/output, FX policy $\rightarrow$ UIP deviations
   - implements efficient allocation ▸ Friedman
   - closes UIP rather than CIP deviations
   - targeting ER is suboptimal

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3. **Responses to shocks**: FX policy offsets n_t^* and accommodates $\tilde{q}_t$ ▸ BiU
   - unobservable $\tilde{q}_t$, n_t^*, $\mathbb{E}_t\Delta z_{t+1}$ (cf. potential output, NAIRU, natural rate)

MONETARY POLICY

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6. **Optimal currency area**: countries with stable RER $\tilde{q}_t$, large spreads n_t^*, high openness γ benefit more from a common currency (Mundell'61)
   — yet, may be subject to fickle capital flows

show

Monetary Peg

- More generally, the optimal monetary rule is

$$(1-\gamma)\underbrace{x_{t+1}}_{\text{output gap}} = -\gamma\bar{\omega}\underbrace{\mu_t(b_t^* - n_t^* - f_t^*)}_{\geq 0}\left(\underbrace{e_{t+1} - \mathbb{E}_t e_{t+1}}_{\text{ER volatility}}\right)$$

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7. **Crawling peg**: if FXI are unconstrained at $t-1$, $t+1$, but not at t:

$$x_{t+1} = -\frac{2\gamma\bar{\omega}}{1-\gamma}\frac{\bar{\omega}\sigma_t^2}{1+\beta+\bar{\omega}\sigma_t^2}(b_t^* - n_t^* - f_t^*)^2(e_{t+1} - \mathbb{E}_t e_{t+1})$$

   - leans against the wind: $e_{t+1} > \mathbb{E}_t e_{t+1} \Rightarrow i_{t+1}\uparrow \Rightarrow e_{t+1}\downarrow, x_{t+1}\downarrow$
   - closes average output gap $\mathbb{E}_t x_{t+1} = 0$, no constraint on $\mathbb{E}_t \Delta e_{t+1}$
   - puts more weight on ER stability when $\gamma\bar{\omega}\sigma_t^2(b_t^* - n_t^* - f_t^*)$ is large
   - non-linear dynamics with time-varying volatility ▸ show

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— leans against the wind: $e_{t+1} > \mathbb{E}_t e_{t+1} \;\Rightarrow\; i_{t+1}\uparrow \Rightarrow\; e_{t+1}\downarrow,\, x_{t+1}\downarrow$

— closes average output gap $\mathbb{E}_t x_{t+1} = 0$, no constraint on $\mathbb{E}_t \Delta e_{t+1}$

— puts more weight on ER stability when $\gamma\bar{\omega}\sigma_t^2(b_t^*-n_t^*-f_t^*)$ is large

— non-linear dynamics with time-varying volatility ▸ show

8. **Forward guidance**:

$$z_t = \mathbb{E}_t z_{t+1} - \bar{\omega}\sigma_t^2\big(n_t^* + f_t^* - b_t^*\big)$$

— FX forward guidance: via future $\mathbb{E}_t z_{t+1}$

— ER forward guidance: via $\sigma_t^2 = \mathrm{var}_t(\tilde{q}_{t+1} + x_{t+1} - z_{t+1})$

Monetary Peg

- More generally, the optimal monetary rule is

$$(1-\gamma)\underbrace{x_{t+1}}_{\text{output gap}} = -\gamma\bar{\omega}\underbrace{\mu_t(b_t^* - n_t^* - f_t^*)}_{\geq 0}\big(\underbrace{e_{t+1} - \mathbb{E}_t e_{t+1}}_{\text{ER volatility}}\big)$$

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9. **Time consistency**: optimal *discretionary* policy closes output gap $x_t = 0$

▸ Markov problem

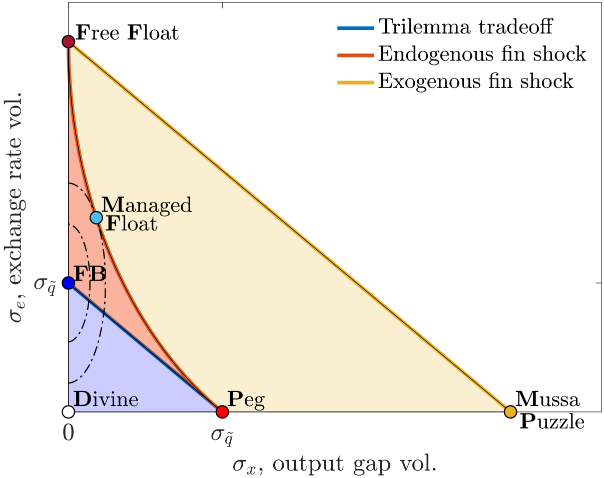
Illustration

$$e_t = \tilde{q}_t + x_t - z_t$$

$$\mathbb{E}_t \Delta z_{t+1} = -\Gamma \left(b_t^* - n_t^* - f_t^* \right)$$

$\Rightarrow$

a) trilemma $\Gamma \approx 0$



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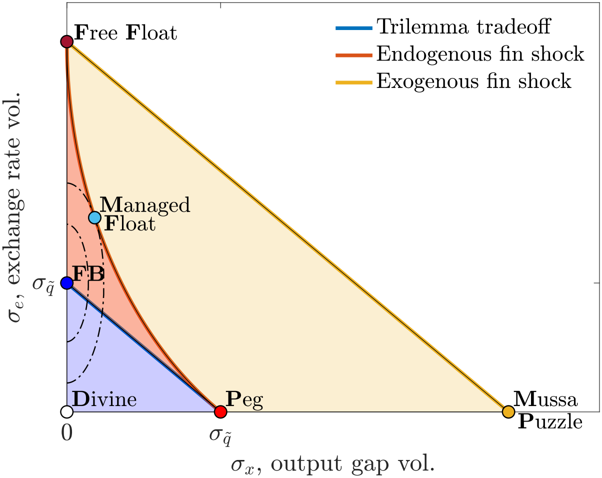

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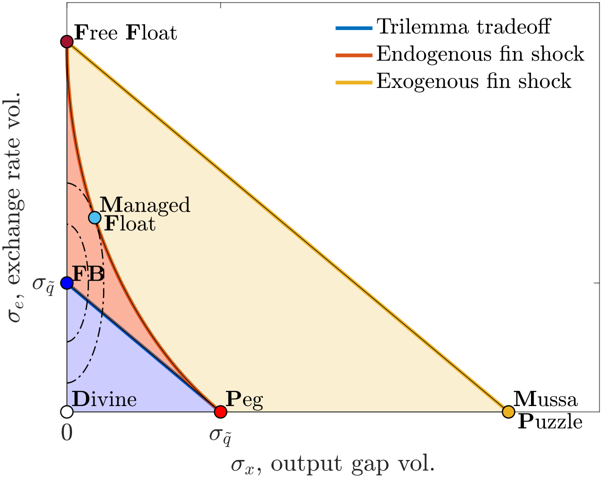
c) exogenous Γ



FX POLICY

9. **FX policy** cannot close output gap and should focus on UIP deviations
   — ZLB $\Rightarrow$ $0 = \mathbb{E}_t \Delta c_{Nt+1} = \mathbb{E}_t \big[\Delta x_{t+1} + \Delta \tilde{c}_{Nt+1} \big] \Rightarrow x_t \perp f_t^*$ ▸ Markov
   — does not require commitment

10. **Gains from commitment**: forward guidance relaxes FX constraints

$$z_t = \mathbb{E}_t z_{t+1} + \bar{\omega} \sigma_t^2 \big(b_t^* - n_t^* - f_t^* \big)$$

   a) FX forward guidance (cf. Werning'2011)
      — increase future imports $\mathbb{E}_t z_{t+1}$ to stimulate z_t
   b) ER forward guidance
      — stabilize future ER $e_{t+1} = \tilde{q}_{t+1} - z_{t+1}$ to mitigate risk-sharing wedge

CAPITAL CONTROLS

Capital Controls

- Add to the model
  - **foreign** arbitrageurs and noise traders
  - **tax** τ_t on *international* positions of all traders

details

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  - $\psi_t \equiv i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} - \tau_t$ is carry-trade return for foreign agents

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- Loss function includes international transfers:

$$\frac{1}{2}\,\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\gamma z_t^2 + (1-\gamma) x_t^2 + 2\beta\gamma \left(\frac{1}{\bar{\omega}\sigma_t^2} \psi_t - n_t^* \right) \psi_t \right]$$

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12. **Transfers**: while $x_t = z_t = 0$ can be implemented with MP and FXI at zero costs, the optimal policy with capital controls can also extract rents
    - optimal targets: $x_t = 0$, $\quad f_t^* = -n_t^*/2$, $\quad \tau_t = -\bar{\omega}\sigma_t^2 n_t^*/2 \;\Rightarrow\; \mathbb{E}_t \Delta z_{t+1} = 0$

INTERNATIONAL SPILLOVERS

International Spillovers

- Global equilibrium:
  - continuum of SOEs trading dollar bonds
  - global interest rate

$$r_t^* = \mathbb{E}_t \Delta y_{Tt+1} + \int \bar{\omega} \sigma_{it}^2 (b_{it}^* - n_{it}^* - f_{it}^*) \mathrm{d}i$$

  - deviations from globally optimal risk sharing

$$\mathbb{E}_t \Delta z_{t+1} = \psi_{it} - \bar{\psi}_t, \quad \psi_{it} \equiv -\bar{\omega} \sigma_{it}^2 (b_{it}^* - n_{it}^* - f_{it}^*)$$

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i) first-best policies $\Rightarrow$ NE is cooperatively optimal

ii) second-best policies $\Rightarrow$ negative spillovers, cooperative solution $\psi_{it} = \bar{\psi}_t$

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14 **Anchor currency**: countries import U.S. MP under second-best policies

$$e_{it} = \hat{q}_{it} + x_{it} - z_{it} - p_{Tt}^*$$

- funding currency $\Rightarrow$ anchor/reserve currency ▸ IRR'2019
- cf. *gold standard* with $i_t^* = 0$ and p_{Tt}^* determined by market clearing

Extensions

- **ToT + PCP/DCP** show
- **NKPC and costs of inflation** show
- **Incomplete pass-through** show
- **Risk-premium and default shocks** show
- **H/h demand for foreign currency** show

Conclusion

- New **policy framework** to think about exchange rate policies
  - i) **realistic**: consistent with exchange rate puzzles
  - ii) **tractable**: attains linear-quadratic representation
  - iii) **practical**: revisits classical policy questions

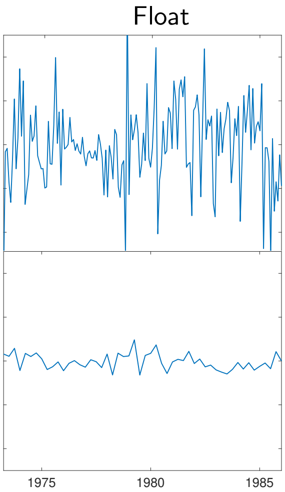
- Motivates future research:
  - — What is the elasticity of currency demand?
    (Koijen-Yogo'21, Camanho-Hau-Rey'21...)
  - — How to measure UIP deviations?
    (Kalemli-Özcan-Varela'21, Engel'16, Kollmann'05, Bekaert'95...)
  - — Financial channel in closed economy?
    (Caballero-Simsek'22, Kekre-Lenel'22...)

APPENDIX

Mussa Puzzle Redux

Δq_t:

Δc_t:



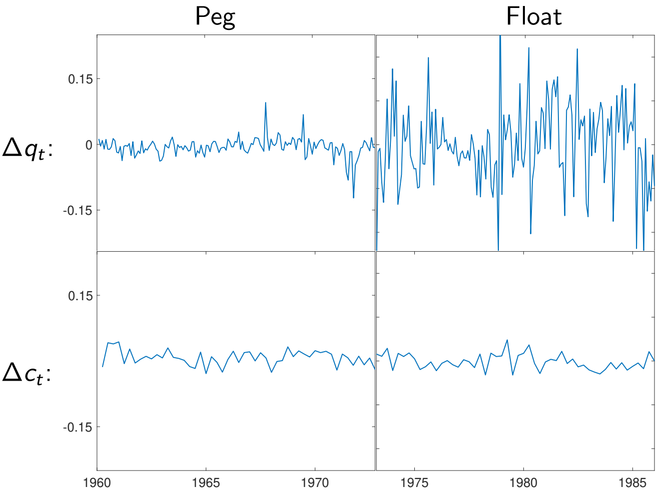
$q_t = e_t + p_t^* - p_t$

$\Downarrow$

✓ ER Disconnect

$i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} = \psi_t$

Mussa Puzzle Redux



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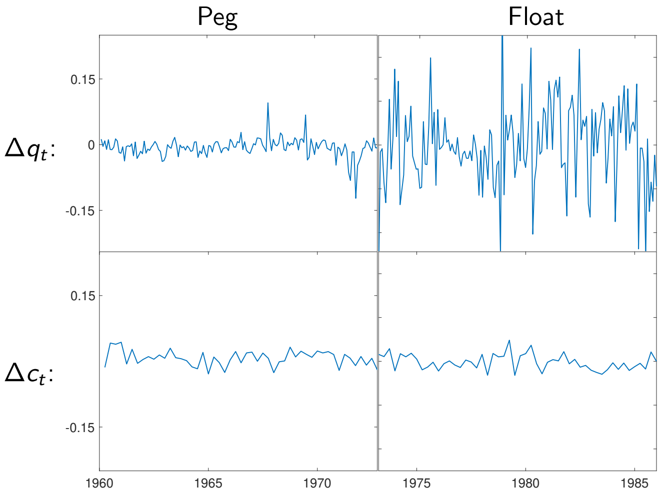

$q_t = e_t + p_t^* - p_t$

$\Downarrow$ ✓ Mussa Redux

$\Downarrow$ ✓ ER Disconnect

back

$$i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} = \psi_t(\sigma_e^2)$$

$$\max_{\{R_t, F_t^*, C_{Tt}, C_{Nt}, \mathcal{E}_t, B_t^*, \sigma_t^2\}_{t\geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\gamma \log C_{Tt} + (1-\gamma)\left(\log C_{Nt} - \frac{C_{Nt}}{A_t} \right) \right]$$

subject to

$$\frac{B_t^*}{R_t^*} - B_{t-1}^* = Y_{Tt} - C_{Tt},$$

$$\beta R_t^* \mathbb{E}_t \frac{C_{Tt}}{C_{Tt+1}} = 1 + \omega \sigma_t^2 \frac{B_t^* - N_t^* - F_t^*}{R_t^*},$$

$$\beta R_t \mathbb{E}_t \frac{C_{Nt}}{C_{Nt+1}} = 1,$$

$$\mathcal{E}_t = \frac{\gamma}{1-\gamma} \frac{C_{Nt}}{C_{Tt}},$$

$$\sigma_t^2 = R_t^2 \cdot \mathrm{var}_t \left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \right),$$

back

Quadratic Loss Function

- **Lemma**: Let $\tilde{x}$ solve $\max_x \; F(x) \;$ s.t. $g(x) = 0$. Then the second-order approximation to the problem is given by

$$\mathcal{L}(dx) \propto \frac{1}{2} dx' \left[\nabla^2 F(\tilde{x}) + \bar{\lambda} \nabla^2 g(\tilde{x})\right] dx,$$

where $\bar{\lambda}$ is the steady-state values of the Lagrange multipliers.

- Non-tradable sector (NK block):

$$\mathcal{L}_N = \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[\log C_{Nt} + \lambda_t \left(A_t L_t - C_{Nt}\right)\right] \propto -\frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \big(\underbrace{c_{Nt} - \tilde{c}_{Nt}}_{x_t}\big)^2$$

- Tradable sector (portfolio choice):

$$\mathcal{L}_T = \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[\log C_{Tt} + \lambda_t \left(B^*_{t-1} + Y_t - C_{Tt} - \frac{B^*_t}{R^*}\right)\right] \propto -\frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \big(\underbrace{c_{Tt} - \tilde{c}_{Tt}}_{z_t}\big)^2$$

- Total welfare:

$$\mathcal{L} = \gamma \mathcal{L}_T + (1-\gamma) \mathcal{L}_N \propto -\frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[\gamma z_t^2 + (1-\gamma) x_t^2\right]$$

back

Back to Friedman (1953)

1. **Flexible exchange rates** "combine interdependence among countries through trade with a maximum of internal monetary independence"
2. **Nominal peg**: "if internal prices were as flexible as exchange rates, it would make little economic difference whether adjustments were brought about by changes in exchange rates or by equivalent changes in internal prices. But this condition is clearly **not** fulfilled"
3. **Trade tariffs and capital controls** are the most realistic way to support a fixed exchange rate and is the least desirable one because of distortions, loopholes, and political economy issues
4. **FXI**

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3. **Trade tariffs and capital controls** are the most realistic way to support a fixed exchange rate and is the least desirable one because of distortions, loopholes, and political economy issues

4. **FXI**: "it may be that private speculation is at times destabilizing"
   - "this device is **feasible and not undesirable**, though it is largely **unnecessary** since private speculative transactions will provide currency demand with only **minor movements** in exchange rates
   - "the objective of smoothing out **temporary fluctuations** and not interfering with fundamental adjustments
   - "there should be a **simple criterion of success – whether the agency makes or loses money**"

back

Approximation $\mathcal{O}(\nu)$

- Non-linear system: $F(\hat{X}_t, \omega\sigma^2(\hat{X}_t)) = 0$, ▸ back

where $\hat{X}_t = \bar{X}(1 + \nu\hat{x}_t)$ for $\nu = 1$, and $\bar{X} = 1$: $F(1, 0) = 0$.

- Conventional approximation:

$$F(X_t, \omega\sigma^2(X_t)) = F(1, 0) + \overbrace{F'_X(1, 0) \cdot x_t}^{B \cdot x_t = 0} \cdot \nu + \mathcal{O}(\nu^2),$$

$X_t = \bar{X}(1 + \nu x_t)$ such that $x_t - \hat{x}_t = \mathcal{O}(\nu)$ and $\omega\sigma^2(X_t) = \mathcal{O}(\nu^2)$.

▸ back

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  $X_t = \bar{X}(1 + \nu x_t)$ such that $x_t - \hat{x}_t = \mathcal{O}(\nu)$ and $\omega\sigma^2(X_t) = \mathcal{O}(\nu^2)$.

- Our approximation: $\omega = \bar{\omega}/\nu^2$ such that $\omega\sigma^2(X_t) = \bar{\omega}\sigma^2(x_t) = \mathcal{O}(1)$
$$F(X_t, \omega\sigma^2(X_t)) = F(1, \bar{\omega}\sigma^2(x_t)) + F'_X(1, \bar{\omega}\sigma^2(x_t)) \cdot x_t \cdot \nu + \mathcal{O}(\nu^2).$$

- **Lemma**: $F(1, \bar{\omega}\sigma^2(x_t)) = 0$, and the non-linear system
$$F'_X(1, \bar{\omega}\sigma^2(x_t)) \cdot x_t = 0$$
  has solution $x_t = \hat{x}_t + \mathcal{O}(\nu)$ with $\bar{\omega}\sigma(x_t) - \omega\sigma(\hat{X}_t) = \mathcal{O}(\nu)$.

Approximation $\mathcal{O}(\nu)$

- Parametrize shocks and $\bar{\omega}$ by ν:

back

$$
\begin{aligned}
n_t^* &= \rho n_{t-1}^* + \nu \sigma_n \varepsilon_t^n, \qquad && \varepsilon_t^n \sim \mathcal{N}(0,1) \\
\tilde{q}_t^* &= \rho \tilde{q}_{t-1}^* + \nu \sigma_q \varepsilon_t^q, \qquad && \varepsilon_t^q \sim \mathcal{N}(0,1) \\
\bar{\omega} &= \tilde{\omega}/\nu^2
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(1-\gamma)x_{t+1} = -\gamma\bar{\omega}\mu_t(b_t^* - n_t^* - f_t^*)\big(e_{t+1} - \mathbb{E}_t e_{t+1}\big)
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$$

- Substitute in $e_t = \tilde{q}_t - z_t + x_t$:

$$
x_{t+1} = -\frac{\gamma\bar{\omega}\mu_t(b_t^* - n_t^* - f_t^*)}{(1-\gamma) + \gamma\bar{\omega}\mu_t(b_t^* - n_t^* - f_t^*)}\left[\tilde{q}_{t+1} - z_{t+1} - \mathbb{E}_t(\tilde{q}_{t+1} - z_{t+1})\right]
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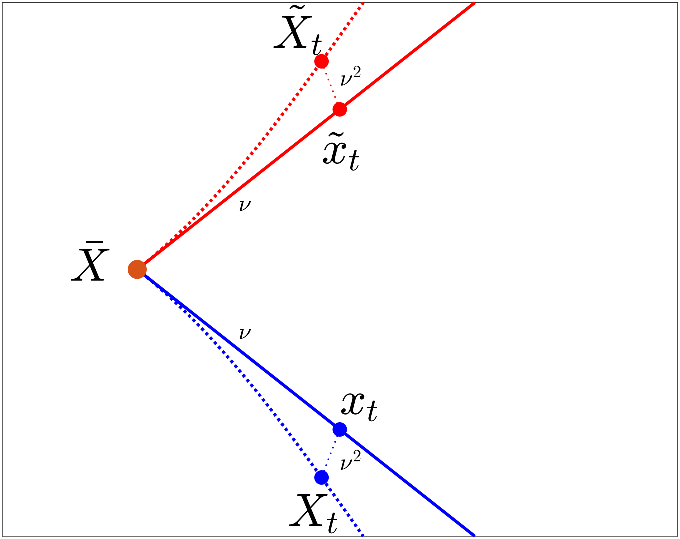
- **Lemma**:
  - i) this system is first-order approximation to the exact solution as $\nu \to 0$,
  - ii) $(n_t^*, \tilde{q}_t^*, b_t, x_t, z_t) = \mathcal{O}(\nu)$ and $(\delta_t, \bar{\omega}\sigma_t^2) = \mathcal{O}(1)$,
  - iii) (δ_t, ς_t) are time-varying with $\{\varepsilon_{t-j}^n, \varepsilon_{t-j}^q\}_{j\geq 0}$ and thus the solution is generally non-linear in $(\varepsilon_t^n, \varepsilon_t^q)$

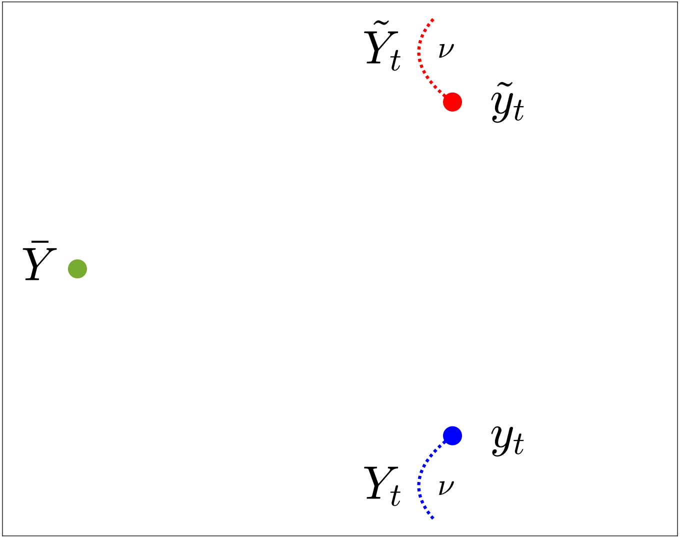
$\Rightarrow$ non-linear dynamics with stochastic time-varying volatility back

Approximation $\mathcal{O}(\nu)$



$$X_t = \left\{ C_{Tt}, \ldots \sigma_t \right\}$$

back



$$Y_t = \left\{ \frac{\log(C_{Tt}/\bar{C}_T)}{\nu}, \ldots \omega\sigma_t^2 \right\}$$

back

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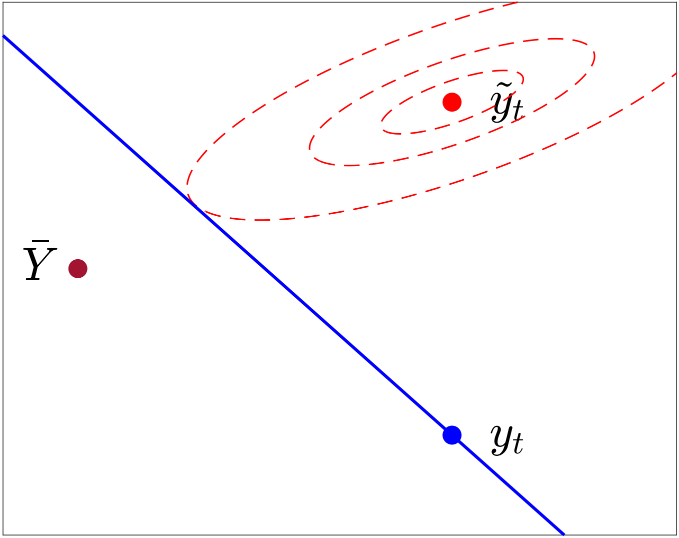

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back

Example: Two Periods, $\beta = 1$

- Planner's problem:

$$\begin{aligned}
\min_{z_0,\sigma^2,\{z_1,x_1\}} \quad & \frac{1}{2}\,\mathbb{E}\Big\{(1-\gamma)x_1^2 + \gamma(z_0^2 + z_1^2)\Big\} \\
\text{s.t.} \quad & z_0 + z_1 = 0 \\
& \mathbb{E}\Delta z_1 = \bar{\omega}\sigma^2 n_0^* \\
& \sigma^2 = \mathrm{var}\big(\tilde{q}_1 - z_1 + x_1\big)
\end{aligned}$$

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- Optimal policy:

$$x_1 + 2\bar{\gamma}\left(\frac{\bar{\omega}n_0^*}{2}\right)^2 \sigma^2 \underbrace{\left(\tilde{q}_1 + x_1\right)}_{e_1 - \mathbb{E}e_1} = 0$$

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$$\sigma^2 = \mathbb{E}(\tilde{q}_1 + x_1)^2 = (1-\delta)^2\mathbb{E}\tilde{q}_1^2 = \left(\frac{1}{1 + \frac{\bar{\gamma}}{2}\bar{\omega}^2 n_0^{*2}\sigma^2}\right)^2 \mathbb{E}\tilde{q}_1^2$$

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$\Rightarrow$ $\sigma^2 = \mathcal{O}(\nu^2)$, $\bar{\omega}\sigma^2 = \mathcal{O}(1)$, $\delta = \mathcal{O}(1)$, $z_0, \{x_1\} = \mathcal{O}(\nu)$

▸ back

Numerical Algorithm

- Assume: i.i.d. symmetric n_t^* shocks, no h/h or gov't FXI ▸ back

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$$\min_{\{\delta_t, \sigma_t^2\}} \quad \mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\beta^2(1-\gamma)\left(\frac{\delta_t}{1-\delta_t}\right)^2\sigma_t^2 + \gamma\bar{\omega}^2\left(\sigma_t^2 n_t^*\right)^2\right]$$
$$\text{s.t.} \quad \frac{\sigma_t^2}{(1-\delta_t)^2} = \sigma_q^2 + \bar{\omega}^2\ \mathbb{E}_t\left(\sigma_{t+1}^2 n_{t+1}^*\right)^2 \tag{1}$$

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- Optimal policy:
$$\frac{\delta_t}{1-\delta_t} = \frac{\bar{\omega}^2}{\beta}\left[\frac{\gamma}{1-\gamma}\frac{1}{\beta} + \delta_{t-1}(2-\delta_{t-1})\right]\sigma_t^2\left(n_t^*\right)^2 \tag{2}$$

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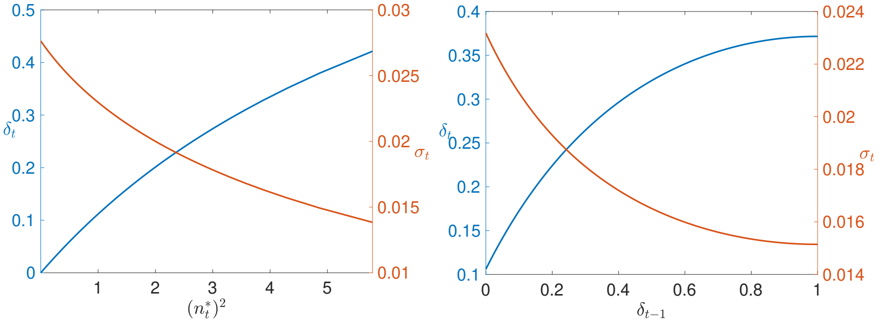
  — conjecture $\sigma_t^2 = \sigma^2(\delta_{t-1}, n_t^{*2})$
  — solve for $\sigma_t^2 = \sigma^2(\delta_t, n_t^{*2})$ from eq. (1)
  — solve for $\delta_{t-1} = \delta_{-1}(\delta_t, n_t^{*2})$ from eq. (2)
  — invert $\delta_t = \delta(\delta_{t-1}, n_t^{*2})$ and update $\sigma_t^2 = \sigma^2(\delta(\delta_{t-1}, n_t^{*2}), n_t^{*2})$

Policy Functions



- Calibration: $\beta = 0.96^{\frac{1}{12}}$, $\gamma = 0.2$, $\sigma_q^2 = \frac{0.02^2}{12}$, $\bar{\omega}^2\sigma_n^2$ to $\times 5$ ER volatility
- More aggressive peg δ_t in response to large shocks $\{n_{t-j}^{*2}\}$
- ER volatility is $< 3\%$ per annum even when $\delta_{t-1} = n_t^* = 0$ because *future* policy offsets large n_t^*

back

Discretionary Policy

- Markov problem:

$$V(b^*, s) = \min_{z,x,b^{*\prime}} \quad \gamma z^2 + (1-\gamma)x^2 + \beta \mathbb{E} V(b^{*\prime}, s')$$

$$\text{s.t.} \quad \mathbb{E} z(b^{*\prime}, s') = z - \omega\sigma^2 \left(b^{*\prime} - n^*\right),$$

$$\beta b^{*\prime} = b^* - z,$$

$$\sigma^2 = \mathrm{var}\Big(\tilde{q}' + x(b^{*\prime}, s') - z(b^{*\prime}, s')\Big),$$

$\Rightarrow$ path of $\{z_t, b_t^*\}$ is independent of x_t

$\Rightarrow$ optimal policy focuses on closing the output gap

back

Optimal FX Policy

- FX policy problem:

$$\min_{\{z_t, b_t^*\}} \quad \frac{1}{2}\,\mathbb{E}\sum_{t=0}^{\infty} \beta^t z_t^2$$

$$\text{s.t.} \quad \beta b_t^* = b_{t-1}^* - z_t$$

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- Has standard recursive formulation:

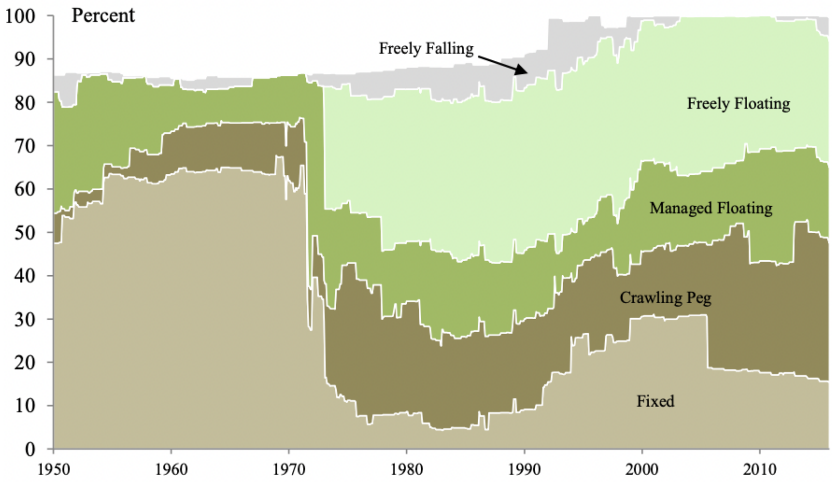
$$V(b^*) = \min_{b^{*\prime}} \; \frac{1}{2}\left(b^* - \beta b^{*\prime}\right)^2 + \beta V(b^{*\prime})$$

Proposition

Optimal FX policy is time consistent and implements efficient risk sharing $z_t = 0$.

back

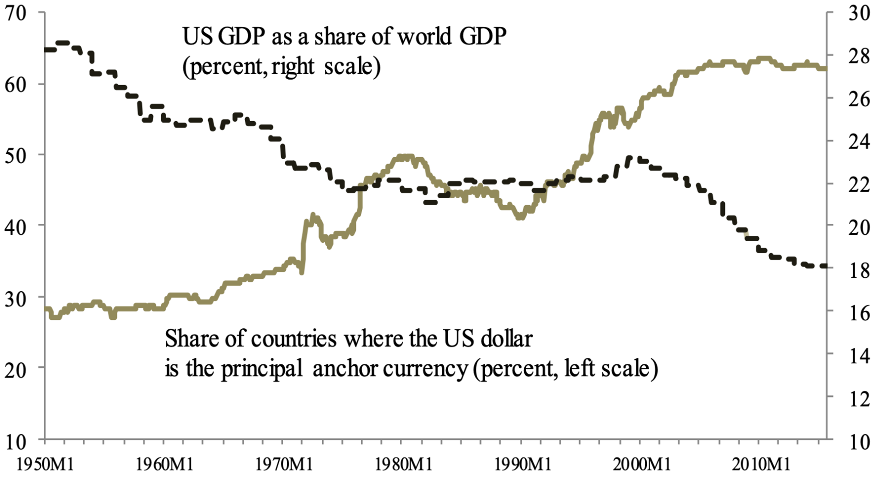
Exchange Rate Regime



Source: Ilzetzki, Reinhart, and Rogoff (2019)

back

Anchor Currencies



Source: Ilzetzki, Reinhart, and Rogoff (2019) back

Capital Controls

- Assume fraction κ_a of arbitrageurs are foreigners and $N_t^* = N_{Ht}^* + N_{Ft}^*$
- Capital controls:
  - tax on h/h deposits/loans: $\beta \frac{R_t}{1+\tau_t^h} \mathbb{E}_t \frac{C_{Nt}}{C_{Nt+1}} = 1,$
  - tax on bond holdings of domestic traders: $\tilde{R}_{Ht+1}^* \equiv \frac{R_t}{1+\tau_{Ht}} \frac{1+\tau_{Ht}^*}{R_t^*} \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} - 1,$
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- Efficient **risk sharing** requires offsetting low demand for H bonds $N_t^* > 0$:

$$\beta R_t^* \mathbb{E}_t \frac{C_{Tt}}{C_{Tt+1}} = (1+\tau_t^h)\left[(1-\kappa_a)\frac{1+\tau_{Ht}^*}{1+\tau_{Ht}} + \kappa_a \frac{1}{1+\tau_{Ft}}\right] + \omega\sigma_t^2 \frac{B_t^* - N_t^* - F_t^*}{R_t^*}$$

  i) FXI increase supply of dollars
  ii) $R_t \uparrow$ offsets depreciation, while $\tau_t^h > 0$ keeps x_t undistorted
  iii) subsidize H bonds for all traders $\tau_{Ht} = \tau_{Ft} < 0$
  iv) tax F bonds $\tau_{Ht}^* > 0$ and subsidize H bonds $\tau_{Ft} < 0$ for int'l flows

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- Collecting **rents** requires manipulating $\tilde{R}_{Ft}^*$:

$$\frac{B_t^*}{R_t^*} = B_{t-1}^* + Y_{Tt} - C_{Tt} - \tilde{R}_{Ft}^* \left(\kappa_a \frac{\mathbb{E}_{t-1} \tilde{R}_{Ft}^*}{\omega \sigma_{t-1}^2} - N_{Ft-1}^* \right)$$

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i) FXI are effective

ii) financial repression of h/h τ_t^h changes returns $\tilde{r}_{Ft}^*$ only via b_t^*

iii) uniform taxes on H bonds $\tau_{Ht} = \tau_{Ft}$ or int'l flows $\tau_{Ht}^* = -\tau_{Ft}$ do not work

iv) taxes on inflows or outflows work if $0 < \kappa_a < 1$

back

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- Baseline model assumes T and NT:
  - might be a good approximation for commodity exporters
  - contrasts with OR'95, DE'03, GM'05, etc.

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- Currency of invoicing:
  1. producer (PCP) = sticky wages
  2. dominant (DCP)

Optimal Policy: PCP

- Planner's problem under **PCP**:

$$\min_{\{z_t, x_t, b_t^*, f_t^*, \sigma_t^2\}} \quad \frac{1}{2}\,\mathbb{E}\sum_{t=0}^{\infty}\beta^t\Big[\kappa \underbrace{z_t^2}_{c_{Ft}-\tilde{c}_{Ft}} + \underbrace{x_t^2}_{y_t-\tilde{y}_t}\Big]$$

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$$\sigma_t^2 = \mathrm{var}_t\big(\tilde{q}_{t+1} - (1-\bar{\gamma})z_{t+1} + x_{t+1}\big), \quad \tilde{q}_t \equiv a_t - \tilde{c}_{Ft}$$

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- **First-best policy**: same as in the baseline model

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- **Markup shocks**: the optimal policy does not result in long-term price targeting $p_{Nt}\nrightarrow 0$

▸ equilibrium ▸ extension

Incomplete Pass-Through

- Extend model to allow for:
  1. elasticity of substitution θ: $U = \gamma C_{Tt}^{\frac{\theta-1}{\theta}} + (1-\gamma)(C_{Nt}^{\frac{\theta-1}{\theta}} - L_t)$
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- More volatile exchange rate if $\theta, \alpha < 1$:

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  $\Rightarrow$ no divine coincidence

  $\Rightarrow$ same optimal policy

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Fickle Flows

- Allow for exogenous default shocks δ_t
  - e.g. home bonds are issued by government

- Risk-sharing condition:

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- A **fixed exchange rate** amplifies

  a) *capital reversals*:
  - boom: $\delta_{t+1} \approx 0 \;\Rightarrow\; \mathbb{E}_t \Delta z_{t+1} = 0 \;\Rightarrow\; b_t^* \downarrow$
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- **Policy side-effects**: the capital flows and UIP spreads are more fickle under a fixed exchange rate regime

MP extensions

Preference Shocks

- H/h can hold and enjoy utility from FC bonds:

$$\max \ \mathbb{E} \sum_{t=0}^{\infty} \beta^t \Big[\gamma \log C_{Tt} + (1-\gamma)(\log C_{Nt} - L_t) - \frac{\kappa}{2}(N_t^* - \Psi_t^*)^2 \Big]$$

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⇒ Optimal policy is the same when n_t^* is driven by preference shocks