The term structure of the natural rate of interest

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[1] Views expressed here do not imply the ECB or the FRB

Motivation

- The natural rate of interest (NR) is the real short-term interest rate consistent with stable inflation and absence of inflationary pressures
- A useful benchmark to measure the monetary policy stance: expansionary if the short-term real interest rate is below the natural rate and vv
- At any point in time, there is a full term structure of current and expected future natural rates
    - path that real short rates should follow to ensure that inflation is stabilized in the face of inflationary/deflationary pressures
- Our objective: provide an empirical estimate of the term structure of natural rates of interest using a structural model

Our approach, I

- Combine macroeconomic data and information from bond yields in a unified, structural framework
  - Bond market information is valuable, as shown in Christensen and Rudebusch (2019)
  - After accounting for term premia, bond yields provide direct evidence on expected future real interest rates
  - At far ahead future horizons, real interest rates are tied to, and thus provide information on, the NR

Our approach, II

- We need a structural framework to operationalize the notion of natural rate, i.e. to ensure that it does represent a useful indicator of the monetary policy stance
- We define the natural rate as the real interest rate that would be observed in a counterfactual world without nominal rigidities and inefficient, cost-push shocks (Woodford, 2003)
  - The real NR is independent of monetary policy
  - If tracked by monetary policy, it ensures absence of inflationary pressures in the face of all but cost-push shocks.

Preview of preliminary results, I

- Imposing model-consistent, no-arbitrage restrictions on both actual and natural yields delivers NR estimates with plausible level of high-frequency volatility.
    - This is in contrast to many existing estimates in the literature, that are purely based on macroeconomic data and are very highly volatile
    - While volatility plays no role in linearized macro models,in our model, excessively high volatility would have repercussions on the solution, and specifically on the level of actual interest rates.
    - Matching the data on bond yields, therefore, requires a reasonable volatility of the natural rate
    - This explains the low high-frequency volatility of our estimates.

Preview of preliminary results, II

- Regarding policy, our results real rates have been quite consistent with the natural rate over most of the 1987-2019 period.
- With persistent discrepancy at the very end of the sample.
  - A sign that Federal Reserves stabilized appropriately all shocks which did not produce a trade-off between inflation and the output gap.
- Regarding long-run NR, our estimates are consistent with existing results : gentle downward trend in the expected natural rate.

Related literature, I

- Long tradition of estimating the natural rate of interest in the macroeconomic literature
  - Edge, Kiley, and Laforte (2008), Justiniano and Primiceri (2010), Barsky, Justiniano, and Melosi (2014), Cúrdia et. al (2015)
  - By and large, this literature has ignored financial market information.
- Del Negro et al. (2017): natural rate in the context of a medium-scale, DSGE model, using uses some financial market information in estimation, but it relies on the expectations hypothesis.
- In contrast, we model term premia endogenously and impose no-arbitrage restrictions.
- These restrictions are useful to take on board he whole term structure in estimation and use it to inform NR estimates

Related literature, II

- Alternative strand of research: reduced-form, or semi-reduced form measures of the natural rate, focusing on its long-run equilibrium level, often referred to as "r-star"
  - Laubach and Williams (2003): r-star as "the real short-term interest rate consistent with output equaling its natural rate and constant inflation".
  - r-star defined as as long-horizon forecast of the observed real rate (Lubik and Matthes, 2015, Hamilton et al., 2016, Del Negro et al., 2017, Fiorentini et al., 2018).
  - Christensen and Rudebusch: r-star defined as investors' expectations of the real short rate for the five-year period starting five years ahead (5Y5Y)
  - Gonzalez-Astudillo and Laforte (2024): semi-structural model to jointly estimate r-star, the natural rate of interest, and the output gap.

The model

- Starting from Amisano and Tristani (2023)
- two distinguishing features: nonexpected utility preferences and heteroskedasticity, in the form of regime switching, in the conditional variance of structural shocks.
- These two features produce time-variation in bond risk premia following switches in variance regimes.
- use macro and yield data to estimate model over different subsamples. Here we focus on 1966Q1-2019Q4
- Model fits the the yield data well and our estimated parameters are consistent with previous studies.
- As an external validation, we compute model-based real yields and compare them to index-linked yields, available since the early 2000s.

Regime switches, I

- Regime switching on conditional variances of productivity and monetary policy shocks.

$$\sigma_{z,s_{z,t}} = \sigma_{z,H} s_{z,t} + \sigma_{z,L} \left(1 - s_{z,t}\right)$$
$$\sigma_{\eta,s_{\eta,t}} = \sigma_{\eta,H} s_{\eta,t} + \sigma_{\eta,L} \left(1 - s_{\eta,t}\right)$$

  where $s_{z,t}$ and $s_{\eta,t}$ index the H (high) and L (low) states for each of the variances.
- For each $s_{j,t}$ ($j = z, \eta$), the probabilities of remaining in states H and L are constant and equal to $p_{j,H}$ and $p_{j,L}$, while the probabilities of switching to the other state are $1 - p_{j,H}$ and $1 - p_{j,L}$, respectively.

Regime switches, II

- When estimating the model over longer samples (eg 1966-2019), we allow the inflation target to have 2 regimes:

$$\pi_t^* = \pi_H^* s_{\pi^*,t} + \pi_L^* (1 - s_{\pi^*,t})$$

- where $s_{\pi^*,t}$ is another independent MS variable

Non-expected utility, I

- As in Amisano and Tristani (2023) a non-expected utility that also accounts for habit persistence in consumption and labor-leisure choice.
- The utility of household *i* defined recursively through an aggregator of current and future utility

$$V_t = \left\{ (1-\beta)\, u_t^{1-\psi} + \beta \left(\mathrm{E}_t V_{t+1}^{1-\gamma} \right)^{\frac{1-\psi}{1-\gamma}} \right\}^{\frac{1}{1-\psi}}, \qquad \psi, \gamma \neq 1, \tag{1}$$

where ψ is the inverse of the elasticity of intertemporal substitution, γ is a parameter which contributes to affect risk aversion

Non-expected utility, II

- current utility is a function of consumption and leisure

$$u_t = (C_t - h\Xi_t C_{t-1}) \left(1 - \eta (1 - \psi) N_t^{1+\frac{1}{\phi}}\right)^{\frac{\psi}{1-\psi}}.$$

where the h is external habits, Ξ_t is the rate of growth of technology, N_t are hours worked, ϕ is the Frisch elasticity of labour supply,

Other shocks

- A mark-up shock $\mu_{w,t}$
- A demand shock G_t , such that output $Y_t = C_t + G_t$, where C_t is aggregate consumption,
- A technology shock A_t including both a permanent and a transitory component.
- All these shocks are assumed to follow AR(1) processes.

Taylor rule

- Taylor rule with interest rate smoothing

$$I_t = \left(\frac{\Pi_t^* \Xi_t^{\psi}}{\beta} \right)^{1-\rho_I} \left(\frac{\Pi_t}{\Pi_t^*} \right)^{\psi_\Pi} \left(\frac{\widetilde{Y}_t}{\widetilde{Y}} \right)^{\psi_Y} I_{t-1}^{\rho_I} e^{\eta_{t+1}},$$

where I_t denotes the nominal interest rate, Π_t denotes the inflation rate, Π_t^* is the inflation target, $\widetilde{Y}_t$ is output detrended by the level of technology, and η_t is a monetary policy shock with mean zero and time-varying standard deviation.

The natural rate of interest

- Given the relevant stochastic discount factor, the natural rate R_t^n can be defined as usual as

$$\frac{1}{R_t^n} = \mathrm{E}_t q_{t,t+1}^n \tag{2}$$

- where $q_{t,t+1}^n$ is the real stochastic discount factor produced by the model.

Solution

- Based on Amisano and Tristani (2011, 2023), the reduced form of the model can be approximated to second order as follows:

$$g\left(\mathbf{x}_t, \widetilde{\sigma}, \mathbf{s}_t\right) = F\widehat{\mathbf{x}}_t + \frac{1}{2}\left(I_{n_y} \otimes \widehat{\mathbf{x}}_t'\right) E\widehat{\mathbf{x}}_t + k_{y,s_t}\widetilde{\sigma}^2$$

(Control variables)

and

$$h\left(\mathbf{x}_t, \widetilde{\sigma}, \mathbf{s}_t\right) = P\widehat{\mathbf{x}}_t + \frac{1}{2}\left(I_{n_x} \otimes \widehat{\mathbf{x}}_t'\right) G\widehat{\mathbf{x}}_t + k_{x,s_t}\widetilde{\sigma}^2$$

(State variables)

where F, E, P and G are constant vectors and matrices and only the vectors k_{y,s_t} and k_{x,s_t} are regime dependent. As a result, regime-switching affect the conditional means of vectors $\mathbf{x}_t$ and y_t.

Estimation

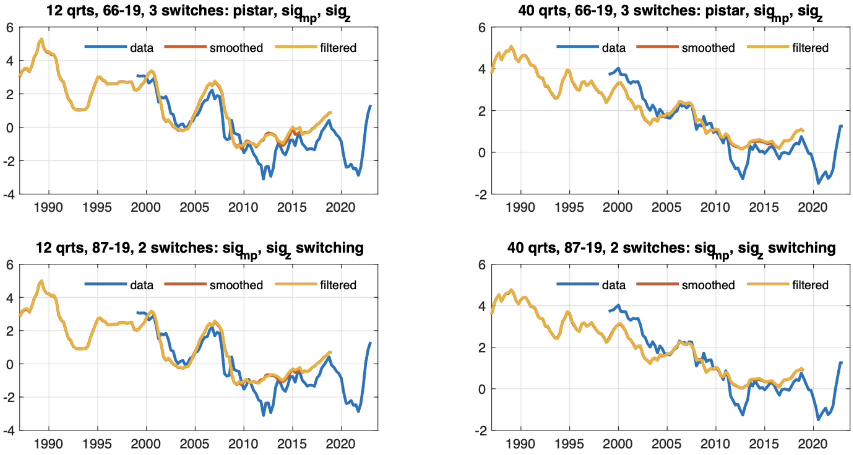
- Model is nonlinear and non-Gaussian
- Likelihood not available in closed form
- we use approximation described in Amisano and Tristani (2023): fast and reliable
- combine likelihood and prior and simulate posterior distribution of latent variables and parameters using MCMC.

TIPS and model's real rates, I

- We compare the model-based measures of real rates with data on TIPS zero coupon yields (not used in model estimation).
- Given that the specification of our model does not consider the role of liquidity premia, we compare model measures of real rates with TIPS yields net of liquidity premia.
- We find that model-based and actual TIPS yields are close (but no cigar), in this way providing some support to the specification of our model.
- It might be a good idea to use TIPS data in estimation.

TIPS and model's real rates, II

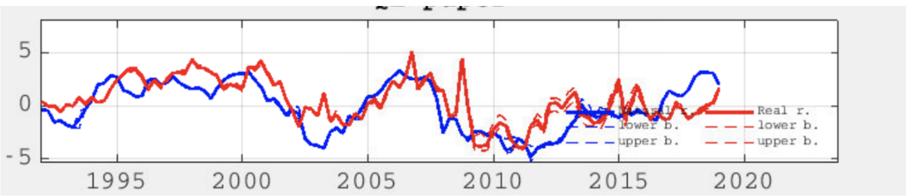
ZC-LP TIPS data and model



The natural rate in the short run, I

- Figure in next slide, with NR and real rate
- nominal 3-month rate is estimated as a shadow rate over the period in which actual short-term rates were constrained by the zero lower bound (ZLB).
- this means that the 3 month rate is treated as missing during the ZLB period.

The natural rate in the short run, II



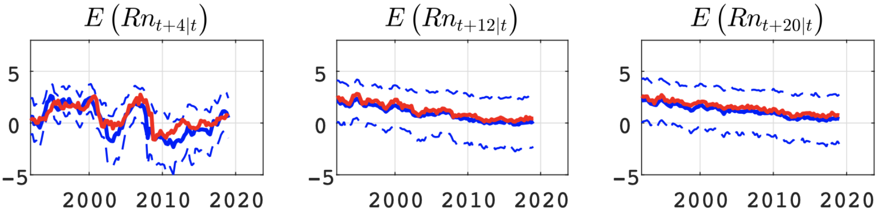
The natural rate in the short run, III

1. The natural rate is highly cyclical. It increases during expansions and then starts falling at the beginning of each recessions in our sample
2. NR and RR remarkably close
   - persistently different at the end of the sample, after 2017.
   - there, the natural rate keeps increasing towards a 2% level, while real rates tend hover around 0
   - Rise in the natural rate is a signal that the model sees positive aggregate inflation pressures
   - All in all, the model interprets monetary policy as consistent with the aim of offsetting all efficient shocks, i.e those shocks that do not produce a trade-off between inflation and the output gap
3. The natural rate is not particularly volatile at high frequency.
   - Unlike in linear DSGE models

The natural rate at longer horizons, I

- In our model, we can also compute a notion of long-run equilibrium for the natural rate, or r-star.
- This will be defined as the expected natural rate over a distant future horizon.
- Compare with Laubach and Williams (2003), Holston, Laubach and Williams (2017), and Lubik and Matthes (2015).
- Consistent with the idea of a progressive fall in r-star over the estimation sample.
- The decline in r-star is more abrupt at the 5-year horizon, while it is smoother at the 10-year horizon.

The natural rate at longer horizons, II



Recap and ways forward, I

- We present empirical estimates of the term structure of natural rates of interests in the U.S. economy based on a unified, structural framework that combines macroeconomic data and information from bond yields
- We define the natural rate as the real interest rate which would prevail in the absence of nominal rigidities
- Our estimates show contained high-frequency volatility. We argue that this is due to our imposing model-consistent, no-arbitrage restrictions on the data
- Our results suggest that real rates in the U.S. have been quite consistent with the natural rate over most of the 1987-2019 period

Recap and ways forward, II

- We obtain these results in a relatively simple structural model augmented with some finance-like features. It would be interesting to explore whether our results are robust to some model extensions.
- One important feature missing from our analysis is an explicit account of the ZLB. We have accounted for the ZLB in estimation through the maintained assumption that large-scale asset purchases were perfect substitutes for interest rate policy, when this was constrained. In future research, it would be important to explore ways to account for the ZLB through a less dogmatic approach