Flexible Priors and Restrictions for Structural Vector Autoregressions

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Motivation

• What is a Structural VAR?

Structural VAR

$$\mathbf{A}_{\mathbf{0}}\mathbf{y}_{t} = \mathbf{c} + \mathbf{A}_{\mathbf{1}}\mathbf{y}_{t-1} + \dots + \mathbf{A}_{\mathbf{p}}\mathbf{y}_{t-p} + \boldsymbol{\Sigma}\boldsymbol{\epsilon}_{t}, \quad \boldsymbol{\epsilon}_{t} \sim \mathcal{N}(\mathbf{0}, \mathcal{I})$$

Reduced-Form VAR

$$\mathbf{y}_t = \mathbf{d} + \mathbf{B}_1 \mathbf{y}_{t-1} + \dots + \mathbf{B}_p \mathbf{y}_{t-p} + \mathbf{u}_t, \quad \mathbf{u}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{\Omega})$$

- On the importance of SVARs:
 - Essential tool for understanding economic dynamics and policy impacts.
 - Useful to inform and validate DSGE models.

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Problem Statement

Conventional approaches to estimate SVARs:

- Rely on reduced-form representation of the VAR.
- Separate estimation and identification steps.
- 8 Rely on algorithms that restrict the flexibility and scope of priors and identifying information.
- Onditional on an estimated parameter vector, multiple theories (SVARs) derived from the same RFVAR share equal likelihood.

Our Solution and This Paper

New framework to apply a broad range of priors and restrictions on (functions of) structural parameters **drawing on a wealth of economic knowledge**.

1 Leverages structural-form representation.

- 2 Integrates estimation and identification in a single step.
- Ombines identifying information, eliminating separate algorithms.
 - Acommodates rich set of priors and restrictions:
 - Priors on structural parameters
 - Priors on (non-)linear functions of structural parameters \rightarrow sign, shape, narrative sign and relative magnitude priors
 - (In-)equality restrictions
 - \rightarrow zero and bound restrictions
 - Combinations of the above
 - Can be used to identify one, a subset or all shocks.

() Allows for Bayesian model comparison to evaluate different economic theories.

More Benefits of Our Approach

- Elicits transparent and informative priors on structural parameters
 avoids imposition of unintended beliefs on structural objects of interest.
- Flexibly tailors model dynamics to prior beliefs \implies no rigid imposition.
- Does not rely on terative procedures \implies scalable to complex models.

Roadmap

- 1. Methodology
- 2. Building Intuition: A Simple Simulation Study
- 3. Empirical Illustration: Replicating and Extending Belongia and Ireland (2021)
- 4. Summing Up

Methodology

Structural VAR

Expanded Notation

$$\mathbf{A}_{\mathbf{0}}\mathbf{y}_{t} = \mathbf{c} + \mathbf{A}_{\mathbf{1}}\mathbf{y}_{t-1} + \dots + \mathbf{A}_{\mathbf{p}}\mathbf{y}_{t-p} + \boldsymbol{\Sigma}\boldsymbol{\epsilon}_{t}, \qquad (1)$$

- \mathbf{y}_t is the $n \times 1$ vector of endogenous variables
- $\boldsymbol{\epsilon}_t \sim \mathcal{N}(\boldsymbol{0}, \mathcal{I})$ is a vector of mutually uncorrelated structural innovations
- WLOG diagonal elements of A₀ normalized to 1
 ⇒ diagonal elements of Σ = standard deviations of structural shocks

Compact Notation

$$\mathbf{A}_{\mathbf{0}}\mathbf{y}_{t} = \mathbf{A}^{+}\mathbf{x}_{t} + \boldsymbol{\Sigma}\boldsymbol{\epsilon}_{t}, \qquad (2$$

where $\mathbf{x}_t = \begin{bmatrix} \mathbf{l}, \ \mathbf{y}_{t-1}', \ \dots, \ \mathbf{y}_{t-p}' \end{bmatrix}'$, and $\mathbf{A}^+ = [\mathbf{c}, \ \mathbf{A}_{\mathbf{l}}, \ \dots, \ \mathbf{A}_{p}]$.

An Example

Aggregate Demand

$$y_t = \alpha_{yr}r_t + \rho_y y_{t-1} + \rho_{yr}r_{t-1} + \sigma^{ad}\epsilon^{ad}$$
(3)

Monetary Policy

$$r_t = \alpha_{ry} y_t + \rho_{ry} y_{t-1} + \rho_r r_{t-1} + \sigma^{mp} \epsilon^{mp} \tag{4}$$

Structural Matrices

$$\mathbf{A_0} = \begin{bmatrix} 1 & -\alpha_{yr} \\ -\alpha_{ry} & 1 \end{bmatrix}, \ \mathbf{A_l} = \begin{bmatrix} \rho_y & \rho_{yr} \\ \rho_{ry} & \rho_r \end{bmatrix}, \ \boldsymbol{\Sigma} = \begin{bmatrix} \sigma^{ad} & \mathbf{0} \\ \mathbf{0} & \sigma^{mp} \end{bmatrix}$$

Likelihood

Let $\mathbf{Y}_T = [\mathbf{y}_1, \dots, \mathbf{y}_T]'$ and $\theta = [\mathbf{A}_0, \mathbf{A}^+, \mathbf{\Sigma}]$, then the likelihood can be written as

$$p(\mathbf{Y}_{T}|\theta) = \frac{|\det\left(\mathbf{A}_{\mathbf{0}}\right)|^{T}}{(2\pi)^{\frac{nT}{2}}} |\mathbf{\Sigma}|^{-T} e^{-\frac{1}{2}\left(\mathbf{A}_{\mathbf{0}}\mathbf{y}_{t}-\mathbf{A}^{+}\mathbf{x}_{t}\right)'\mathbf{\Sigma}^{-2}\left(\mathbf{A}_{\mathbf{0}}\mathbf{y}_{t}-\mathbf{A}^{+}\mathbf{x}_{t}\right)}$$
(5)

Priors on Structural VAR Parameters

- Overall dynamic behavior $f_{sz}(\theta)$
 - Sims and Zha (1998) prior
- Structural parameters $f_{sparam}(\theta)$
 - · Priors in the spirit of Baumeister and Hamilton (2019)
- Priors on structural objects $f_{sobj}(\theta)$
 - "Endogenous priors" as in Del Negro and Schorfheide (2008) and Christiano, Trabandt and Walentin (2011), also known as "system priors" in Andrle and Plasil (2018).
 - Can be applied to any (linear and nonlinear) function of the structural parameters, e.g., IRFs, HD, VD, shocks.
- Overall prior density is $f(\theta) = f_{sz}(\theta) f_{sparam}(\theta) f_{sobj}(\theta)$

An Aside: Behavioral Priors vs. Entropic Tilting

• Objective:

- Incorporate additional information.
- Maintain closeness to original beliefs or distributions.
- Method:
 - Behavioral/Property/Endogenous/System Priors: Bayesian updating.
 - Priors on complex functions, e.g., $f_{sobj}(\theta) \propto \exp(-\lambda \cdot \text{Penalty}(\theta))$, λ = belief strength.
 - · Penalty term reflects the deviation from the desired properties.
 - Entropic Tilting: e.g. Robertson, Tallman and Whiteman (2005)
 - Adjust distributions to satisfy moment conditions: $\mathbb{E}_Q[h(X)] = c$.
 - New set of weights to minimize Kullback-Leibler divergence: $\min_{Q} \mathbb{E}_{Q} \left[\log \frac{Q}{P} \right]$.

(In-)Equality Restrictions

- Can be thought of as dogmatic priors: Researcher has strong belief that specific (combinations of) structural parameters have certain values or lie in a given range.
- Our methodology allows us to impose:
 - (Non-)linear range and inequality restrictions on the structural VAR parameters, which exclude certain values of the parameter space.
 - Linear equality restrictions, handled via the techniques by Binning and Maih (2015).

Deriving the Posterior

• Posterior combines the likelihood and the prior:

$$g(\theta|\mathbf{Y}_T) = \frac{p(\mathbf{Y}_T|\theta)f(\theta)}{p(\mathbf{Y}_T)} \propto p(\mathbf{Y}_T|\theta)f(\theta) \equiv \tilde{g}(\theta|\mathbf{Y}_T)$$

- Mode found by maximizing the posterior kernel $\tilde{g}(\theta | \mathbf{Y}_T)$ as with DSGE models.
- The posterior kernel can be initialized using a candidate parameter vector and standard maximization routines can be used to find its maximum.
- Practitioners can first inspect the mode before moving to posterior sampling.

Modeling Environment: RISE Toolbox - Maih (2015)

- Regime Switching DSGE + Optimal policy + 5th-order perturbation
- Regime Switching VAR + SVAR + Panel VAR + Proxy SVAR
- MLE, Bayesian Estimation, Indirect Inference
- Conditional forecasting for nonlinear models + Relative Entropy
- Uncertainty Quantification: GSA, HDMR, etc.
- PDF and HTML reporting systems
- Support for various time series frequencies: D-ly, W-ly, M-ly, Q-ly, H-ly, Y-ly, and undated

Building Intuition: A Simple Simulation Study

Our DGP: A Toy Model

Aggregate Demand

$$y_t = \alpha_{yr}r_t + \rho_y y_{t-1} + \rho_{yr}r_{t-1} + \sigma^{ad} \epsilon^{ad}$$
(6)

Monetary Policy

$$r_t = \alpha_{ry} y_t + \rho_{ry} y_{t-1} + \rho_r r_{t-1} + \sigma^{mp} \epsilon^{mp} \tag{7}$$

Implementation

Structural VAR Model

$$\mathbf{A_0}\mathbf{y}_t = \mathbf{A_1}\mathbf{x}_t + \boldsymbol{\Sigma}\boldsymbol{\epsilon}_t, \qquad (8)$$
with one lag and where $\mathbf{y}'_t = [y_t, r_t]'$ and $\boldsymbol{\epsilon}'_t = \left[\boldsymbol{\epsilon}^{ad}_t, \boldsymbol{\epsilon}^{mp}_t\right]'$.

Structural Matrices

$$\mathbf{A}_{\mathbf{0}} = \begin{bmatrix} 1 & -\alpha_{yr} \\ -\alpha_{ry} & 1 \end{bmatrix}, \ \mathbf{A}_{\mathbf{i}} = \begin{bmatrix} \rho_{y} & \rho_{yr} \\ \rho_{ry} & \rho_{r} \end{bmatrix}, \ \mathbf{\Sigma} = \begin{bmatrix} \sigma^{ad} & \mathbf{0} \\ \mathbf{0} & \sigma^{mp} \end{bmatrix}$$
(9)

Parameterization

$$\mathbf{A_0} = \begin{bmatrix} 1 & 0.1 \\ -0.2 & 1 \end{bmatrix}, \ \mathbf{A_l} = \begin{bmatrix} 0.8 & -0.3 \\ 0.2 & 0.9 \end{bmatrix}, \ \boldsymbol{\Sigma} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
(10)

- Simulate data for t = 200 periods.
- Estimate a suite of SVARs, using flat Sims and Zha (1998) priors.

True IRFs



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Experiment I : Honoring Traditions

• Conventional VAR with sign restrictions

• In the tradition of Faust (1998), Canova and De Nicolò (2002), Uhlig (2005).

Sign restrictions

- Output responds negatively, on impact, to MP shock.
- Interest rate responds positively, on impact, to MP shock.

Implementation details

• Rubio-Ramírez, Waggoner and Zha (2010) algorithm to impose sign restrictions.

IRFs



Histograms of Impact IRFs



- True impact IRF of interest rate assigned high density but different from median.
- True impact IRF of output not assigned high density and different from median.
- These marginal posterior distributions of the IRFs are not uniform as intended.

Histograms of Impact IRFs



- True impact IRF of interest rate assigned high density but different from median.
- True impact IRF of output not assigned high density and different from median.
- These marginal posterior distributions of the IRFs are not uniform as intended.
- Implications for structural parameters: $\alpha_{yr} \in (-\infty, 0]$ and $\alpha_{ry} \in [0.01, 114.83]$ \rightarrow we do not learn anything about the structural parameters.

Experiment II: Innovating on the Approach

Structural VAR with endogenous prior on signs of impact IRFs.

$$-I\!RF(y_t, \epsilon_t^{mp}) \sim folded \ \mathcal{N}(0.2, 0.2)$$

 $I\!R\!F(r_t, \epsilon_t^{mp}) \sim folded \ \mathcal{N}(0.75, 0.2)$



IRFs



Histograms of Impact IRFs



- Sign priors considerably reduce posterior range of IRFs.
- True impact IRF of interest rate assigned high density and closer to median.
- True impact response of output is still in the tail of the posterior distribution.

Histograms of Structural Parameters



• Sign priors considerably reduce posterior range of the structural parameters.

• True values of structural parameters within posterior distribution.

Experiment III: Advancing Frontiers

Structural VAR with parameter priors à la Baumeister and Hamilton (2018).

$$I\!RF(y_t, \epsilon_t^{mp}) = \frac{\alpha_{yr}}{1 - \alpha_{yr}\alpha_{ry}} \quad \text{and} \quad I\!RF(r_t, \epsilon_t^{mp}) = \frac{1}{1 - \alpha_{yr}\alpha_{ry}}$$

• Case A: Prior on sensitivity of aggregate demand to interest rate

$$a_{0,12} = -\alpha_{yr}$$

 $-\alpha_{yr} \sim folded \ \mathcal{N}(1,1)$ (11)

• Case B: Prior on monetary policy response to output

$$a_{0,21} = -\alpha_{ry}$$

$$\alpha_{ry} \sim folded \ \mathcal{N}(1,1) \tag{12}$$

Illustrating the Priors



IRFs

Case A: $-\alpha_{vr} \sim folded \ \mathcal{N}(1,1)$



Histograms of Impact IRFs

Case A: $-\alpha_{yr} \sim folded \ \mathcal{N}(1,1)$



• Prior on α_{yr} considerably reduces posterior range of IRFs.

• True impact response of output is outside the posterior distribution, but close.

Historgrams of Structural Parameters

Case A: $-\alpha_{yr} \sim folded \ \mathcal{N}(1,1)$



• Prior on α_{yr} considerably reduces posterior range of structural parameters.

- $-\alpha_{yr}$: from [0, 1.4] to [0.025, 0.075]
- $-\alpha_{ry}$: from [-1.6, 0.2] to [-0.3, 0.2]
- True sensitivity of AD to interest rate α_{yr} outside of distribution, but close.

IRFs

Case B: $\alpha_{ry} \sim folded \ \mathcal{N}(1,1)$



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Histograms of Impact IRFs

Case B: $\alpha_{ry} \sim folded \ \mathcal{N}(1,1)$



• Prior on α_{ry} considerably reduces posterior range of IRFs.

• Truth still outside the posterior distribution.

Histograms of Structural Parameters

Case B: $\alpha_{ry} \sim folded \ \mathcal{N}(1,1)$



• Prior on α_{rv} considerably reduces posterior range of structural parameters:

- $-\alpha_{yr}$: from [0, 1.4] to [-0.1, 0.075]
- $-\alpha_{ry}$: from [-1.6, 0.2] to [-0.15, 0]
- True response of MP to output α_{ry} outside of distribution, but close.

Experiment IV: Harmonizing Echoes

Put all prior information on the table!

• Prior on the sensitivity of aggregate demand to interest rate

$$-\alpha_{yr} \sim folded \ \mathcal{N}(1,1)$$
 (13)

• Prior on the monetary policy response to output

$$\alpha_{ry} \sim folded \ \mathcal{N}(1,1)$$
 (14)

• Signs of impulse responses

$$-IRF(y_t, \epsilon_t^{mp}) \sim folded \ \mathcal{N}(0.2, 0.2) \tag{15}$$

$$I\!RF(r_t, \epsilon_t^{mp}) \sim folded \ \mathcal{N}(0.75, 0.2)$$
 (16)

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IRFs



Histograms of Impact IRFs



• True values of impact IRFs within posterior distribution and with high density.

Histograms of Structural Parameters



• True values of structural parameters within posterior distribution and close to median.

Key Takeaways

- Sign priors and priors on structural parameters can give extremely helpful identifying information.
- Even information about parameters that do not directly enter the equation of the shock of interest can be very powerful in terms of identification.
- Combining prior information can be very helpful and is, in fact, advisable.
- Priors on (functions of) structural parameters are an improvement over sign restrictions used in conventional VAR approaches.

Bayesian Model Comparison

Table: (Log) Marginal Data Densities

| | $\mathbf{p}(\alpha_{\mathbf{yr}})$ | $\mathbf{p}(\alpha_{\mathbf{ry}})$ | endo | combo | $\alpha_{\rm yr} = 0$ | $\alpha_{\mathbf{ry}} = 0$ |
|------------------------|------------------------------------|------------------------------------|-------|-------|-----------------------|----------------------------|
| Reciprocal IS | 570.9 | 601.2 | 577.5 | 587.4 | 565.8 | 565.7 |
| Meng and Wong's Bridge | 571.1 | 601.4 | 577.6 | 587.5 | 565.9 | 565.9 |
| Ulrich Mueller | 576.5 | 607 | 582.7 | 590.7 | 570.8 | 571 |
| Laplace | 583.7 | 617.5 | 580.7 | 600 | 573 | 574.2 |
| Sims, Waggoner and Zha | 572.5 | 602.8 | 578.2 | 588.1 | 567.4 | 567.4 |
| Laplace MCMC | 578.3 | 612.4 | 579.9 | 591.7 | 572.9 | 575.2 |
| Chib and Jeliazkov | 582.5 | 616.8 | 584.3 | 595.7 | 576.7 | 578.8 |
| Modified Harmonic Mean | 570.8 | 601.1 | 577.4 | 587.3 | 565.7 | 565.6 |

Empirical Illustration: Replicating and Extending Belongia and Ireland (2021)

Belongia and Ireland (2021)

Structural VAR Model à la Baumeister and Hamilton (2018)

$$\mathbf{A}_{\mathbf{0}}\mathbf{y}_{t} = \mathbf{A}^{+}\mathbf{x}_{t} + \boldsymbol{\Sigma}\boldsymbol{\epsilon}_{t}, \qquad (17)$$

with four lags and where
$$\mathbf{y}_t' = \left[p_t, y_t, r_t\right]'$$
 and $\epsilon_t' = \left[\epsilon_t^{as}, \epsilon_t^{ad}, \epsilon_t^{mp}\right]'$.

- p_t inflation: year-over-year percentage changes in PCE
- y_t output gap: p.p. difference between real GDP and CBO's potential GDP.
- rt: mix of effective FFR and Wu and Xia (2016) shadow FFR

Contemporaneous Structural Parameters Matrix A₀

$$\mathbf{A_0} = \begin{bmatrix} 1 & -\alpha_{py} & 0\\ -\alpha_{yr} & 1 & \alpha_{yr}\\ -\alpha_{rp} & -\alpha_{ry} & 1 \end{bmatrix}$$
(18)

Replication in RISE

• Sample 1968:Q1 - 2017:Q4. Sims and Zha (1998) prior with $\lambda_1 = 1$ and $\lambda_3 = 1$.

• Restrictions

- $a_{0,21} = -a_{0,23}$ (related to α_{yr})
- $a_{0,13} = 0$ (no contemporaneous relationship between π_t and r_t in Phillips curve)

• Priors on elements in A₀:

similar to original Student-t priors (sc = 0.3, df = 2), with larger prior variance

| Parameter | Structural Equation | Mode | Scale |
|---------------------------|---------------------------|-----------------------|-------|
| | | Logistic Distribution | |
| $-\alpha_{py} = a_{0,12}$ | Phillips Curve | -0.5 | 0.6 |
| $-\alpha_{yr} = a_{0,21}$ | Aggregate Demand | -1 | 0.6 |
| $\alpha_{yr} = a_{0,23}$ | Aggregate Demand | 1 | 0.6 |
| $-\alpha_{rp} = a_{0,31}$ | MP Response to Inflation | -0.375 | 0.6 |
| $-\alpha_{ry} = a_{0,32}$ | MP Response to Output Gap | -0.125 | 0.6 |

Estimated IRFs after Posterior Sampling



Posterior of Coefficients

Flexible Priors and Restrictions for SVARs

Laboratory: An Augmented VAR

• Extend VAR of Belongia and Ireland (2021) with

- commodity price index
- total reserves
- non-borrowed reserves

from Uhlig (2005).

• Sample: 1968:Q1 - 2007:Q4.

IRFs: Belongia and Ireland (2021) Variables



IRFs: Belongia and Ireland (2021) Variables



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Flexible Priors and Restrictions for SVARs

Combining Prior Information

- Problem: Extending the dataset affects the identification of all shocks.
- Solution: Add information on top of Belongia and Ireland (2021) priors.
- Uhlig (2005) inspired sign priors
 - A contractionary monetary policy shock reduces the commodity price index, total reserves, non-borrowed reserves from 0 to 2 quarters after shock.
 - Our prior for these IRFs is $\sim \mathcal{N}(-0.5, 0.25)$.
- Add "DSGE-model-inspired" shape priors for responses to MP shock
 - inflation negative in h = 1 and $h = 3 \sim truncated \mathcal{N}(-0.2, 0.25, -20, 0)$
 - inflation back to zero at $h = 4 \sim \mathcal{N}(0, 0.25)$
 - output gap at $h=2\sim\mathcal{N}(-0.1,0.25)$
 - output gap trough at $h=4\sim\mathcal{N}(-0.2,0.25)$
 - output gap at $h=6\sim\mathcal{N}(-0.1,0.25)$

What We Are Up Against

- Priors on VAR dynamics.
- Linear restrictions on structural VAR parameters.
- Priors on individual parameters of the structural form.
- Priors on signs and shapes of the IRFs.

What We Are Up Against

- Priors on VAR dynamics.
- Linear restrictions on structural VAR parameters.
- Priors on individual parameters of the structural form.
- Priors on signs and shapes of the IRFs.

Not a problem for our procedure!

IRFs with Added Priors: Belongia and Ireland (2021) Variables



IRFs with Added Priors: Belongia and Ireland (2021) Variables



Uhlig (2005) Variable

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Flexible Priors and Restrictions for SVARs

Summing Up

Summing Up

- We offer a comprehensive and user-friendly framework for estimating and identifying SVARs in one step and with one coherent and efficient algorithm.
- Framework is flexible enough to <u>accommodate all available sources of information</u> that have been proposed in the literature jointly.

• Added benefit of:

- Openly acknowledging uncertainty about prior information.
- ② Conducting proper posterior inference.
- 3 Tailoring model dynamics to prior beliefs without rigid imposition.
- The proposed method will be available in the RISE toolbox (see Maih, 2015), simplifying the estimation of structural VARs for practitioners.

Thank You!

References I

Andrle, M. and Plasil, M. (2018). Econometrics with System Priors. Economics Letters, 172, 134-137.

- Baumeister, C. and Hamilton, J. D. (2018). Inference in Structural Vector Autoregressions When the Identifying Assumptions are Not Fully Believed: Re-evaluating the Role of Monetary Policy in Economic Fluctuations. *Journal of Monetary Economics*, **100** (C), 48–65.
- and (2019). Structural Interpretation of Vector Autoregressions with Incomplete Identification: Revisiting the Role of Oil Supply and Demand Shocks. American Economic Review, 109 (5), 1873–1910.
- Belongia, M. T. and Ireland, P. N. (2021). A Classical View of the Business Cycle. Journal of Money, Credit and Banking, 53 (2-3), 333-366.
- Binning, A. and Maih, J. (2015). Applying Flexible Parameter Restrictions in Markov-Switching Vector Autoregression Models. Working Papers No 12/2015, Centre for Applied Macro- and Petroleum Economics (CAMP), BI Norwegian Business School.
- Canova, F. and De Nicolò, G. (2002). Monetary Disturbances Matter for Business Fluctuations in the G-7. Journal of Monetary Economics, 49 (6), 1131-1159.
- Christiano, L. J., Trabandt, M. and Walentin, K. (2011). Introducing Financial Frictions and Unemployment into a Small Open Economy Model. *Journal of Economic Dynamics and Control*, 35 (12), 1999–2041.
- Del Negro, M. and Schorfheide, F. (2008). Forming Priors for DSGE Models (and How it Affects the Assessment of Nominal Rigidities). *Journal of Monetary Economics*, **55** (7), 1191–1208.
- Faust, J. (1998). The Robustness of Identified VAR Conclusions About Money. Carnegie-Rochester Conference Series on Public Policy, 49 (1), 207–244.
- Maih, J. (2015). Efficient Perturbation Methods for Solving Regime-Switching DSGE Models. Working Paper 2015/01, Norges Bank.

References II

- Robertson, J. C., Tallman, E. W. and Whiteman, C. H. (2005). Forecasting Using Relative Entropy. Journal of Money, Credit and Banking, 37 (3), 383-401.
- Rubio-Ramírez, J. F., Waggoner, D. F. and Zha, T. (2010). Structural Vector Autoregressions: Theory of Identification and Algorithms for Inference. *The Review of Economic Studies*, **77** (2), 665-696.
- Sims, C. A. and Zha, T. (1998). Bayesian Methods for Dynamic Multivariate Models. International Economic Review, 39 (4), 949–968.
- Uhlig, H. (2005). What are the Effects of Monetary Policy on Output?: Results from an Agnostic Identification Procedure. Journal of Monetary Economics, 52 (2), 381–419.
- Wu, J. C. and Xia, F. D. (2016). Measuring the Macroeconomic Impact of Monetary Policy at the Zero Lower Bound. Journal of Money, Credit and Banking, 48 (2-3), 253-291.

Estimated Contemporaneous Coefficients

Phillips Curve and Aggregate Demand



Estimated Contemporaneous Coefficients

Monetary Policy Rule - Response to Inflation and Output Gap



IRFs

Uhlig (2005) Variables



IRFs with Added Priors

Uhlig (2005) Variables

