Phases of Higher Education, Tuition Grants, and Equity-Efficiency Tradeoff

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ERSA working paper 848

January 2021

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Abstract

We study how taxes and alternative higher education subsidies affect equity–efficiency trade-off for countries at different phases of higher education development. We find a scholarship program is the most efficient higher-education-subsidy program at all stages of higher education development due to its highly regressive nature. Laissez-faire (no-government subsidy) Lorenz dominates universal grant in the early stages of development; vice versa, in the later stages of development. Higher education subsidy could thus be regressive in developing countries but progressive in advanced economies. We also find, at the later stages of higher education development, enrollment rate increases in universal subsidy but decreases in other policies, implying the recent shift away from universal grant scheme in the UK could lead to a decline in the enrollment rate.

Key words:
Education subsidy; equity; externality; phases of higher education
JEL Classification: H2; I2; O1; O4

[*]I am grateful to Zhang Jie for extensive comments on a previous draft of the paper and to participants of various conferences (2020 World Congress: The Econometric Society; The 2019 African Meeting of the Econometric Society; 2019 Biennial Conference of the Economic Society of South Africa; Brown Bag Seminar, Economics Department, University of Pretoria) for fruitful discussions and suggestions. The usual disclaimers apply.

"*For me, education has never been simply a policy issue – it's personal. Neither of my parents and hardly anyone in the neighborhood where I grew up went to college. But thanks to a lot of hard work and plenty of financial aid, I had the opportunity to attend some of the finest universities ... .*"
Michelle Obama

1. Introduction

College funding is personal. On September 19, 2016, the South African higher education minister Blade Nzimande announced that higher institutions in the country could hike next year's fees by a maximum of 8%. This has sparked a national student protest that has led to at least seventeen (out of twenty-six) major universities closure in the country that longed for weeks, causing major disruptions in academic activities. In 2010, students across Britain protested tuition hikes that turned violent after the government's plan to lower the government subsidy to higher education significantly. Subsidy to higher education was lowered by about eighty percent in 2012, leading sophomore students to pay triple the tuition fee that their seniors paid. The government has considered alternative funding such as subsidizing students from poor backgrounds while the impacts of the policy shift on efficiency and inequality are still debatable.

In the extant literature, the debate over higher education financing revolves around regressivity and externality effects. On one hand, higher education subsidies and grants become a concern of transferring resources away from unskilled workers towards skilled ones (e.g., Hanson and Weisbrod, 1969; Fernandez and Rogerson, 1995; Garcia-Penalosa and Walde, 2000; De Fraja, 2002). On the other, they are justified on the basis of externality effects of human capital[2] and the pervasiveness

[2]There is some support from the empirical literature with respect to human capital externality but not without dispute. Moretti (2004) estimated human capital externality (the effects of one more year of average education on income) up to 25% for the US. In contrast, Krueger and Lindahl (2001) and Acemoglu and Angrist (2000) argued that the difference between the social and private returns of education is not significantly different from zero for the US. Benhabib and Spiegel (1994) found no relationship between human capital and growth but a positive relationship between human

of borrowing constraints that prevent individuals from investing optimally by borrowing against future human capital (e.g., Barham et al. 1995; Fender and Wang, 2003). There is also a third case for education subsidies, alleviating the distortions in human capital caused by redistributive policies such as progressive taxation (see, e.g., Benabou, 2002, Bovenbreg and Jacobs, 2005, Krueger and Ludwig, 2016).

A common feature of this literature is its failure to account for the different forms of the higher education system. The structure of a country's higher education system, particularly the stage of development it exists, however, largely determines the equity and efficiency impact of any higher education financing policy that it adopts. When two countries are at different stages of higher education development, not only they will have different enrollment rates but also different class compositions which, in turn, creates disparities in the effectiveness of higher education policies. For instance, a universal tuition fee grant in Uganda may not have the same regressive effect in Spain because they are at different levels of higher education development. In Uganda, only less than 5 percent of the age group has access to tertiary education whereas the enrollment rate in Spain exceeds 87 percent. Many countries in the developing world are at stages where higher education is a luxury consumption good enjoyed by few elites (Table 1).[3] In contrast, for economies in the developed world, the "massification" of higher education is at an advanced stage where the majority of their population has access to it (Table 2). This leads to the important question that we address in this paper: how do alternative higher education financing schemes impact efficiency and equity when accounting for the *transition* and *different phases of higher education*, in line with Trow's (1973, 2007) work?

capital and total factor productivity.

[3]Source: https://ourworldindata.org/tertiary-education#enrollment-in-tertiary-education

Table 1: Higher education enrollment of age group, sample of low income countries

Country	**2013**
Benin	15.3628
Burkina Faso	4.77591
Burundi	4.40817
Congo, Dem. Rep.	6.64076
Guinea	10.3789
Madagascar	4.24579
Mozambique	5.04323
Rwanda	7.52925
Tanzania	3.64732
Togo	10.0422
Zimbabwe	5.87175

Table 2: Higher education enrollment of age group, high income countries

Country	**1971**	**2013**
Argentina	15.3701	79.9867
Australia	17.0328	86.5546
Austria	12.2113	80.3868
Belgium	16.8641	72.3096
Chile	11.1577	83.8164
Czech Republic	8.92373	65.3774
Denmark	18.8583	81.237
Finland	13.1341	91.0658
France	18.5413	62.1469
Hong Kong SAR, China	6.83597	67.2759
Hungary	10.0217	57.0167
Ireland	10.5903	73.1685
Italy	16.8803	63.4551
Japan	17.6406	62.4116
Korea, Rep.	7.24645	95.3454
Malta	6.51885	45.6805
New Zealand	16.9108	79.7143
Norway	15.7949	76.1179
Panama	10.3144	38.7393
Poland	13.3588	71.1587
Portugal	7.27266	66.2216
Spain	8.66966	87.0658
Sweden	21.7328	63.3929
Switzerland	10.0385	56.2682
United Kingdom	14.5679	56.8701
United States	47.3235	88.8086

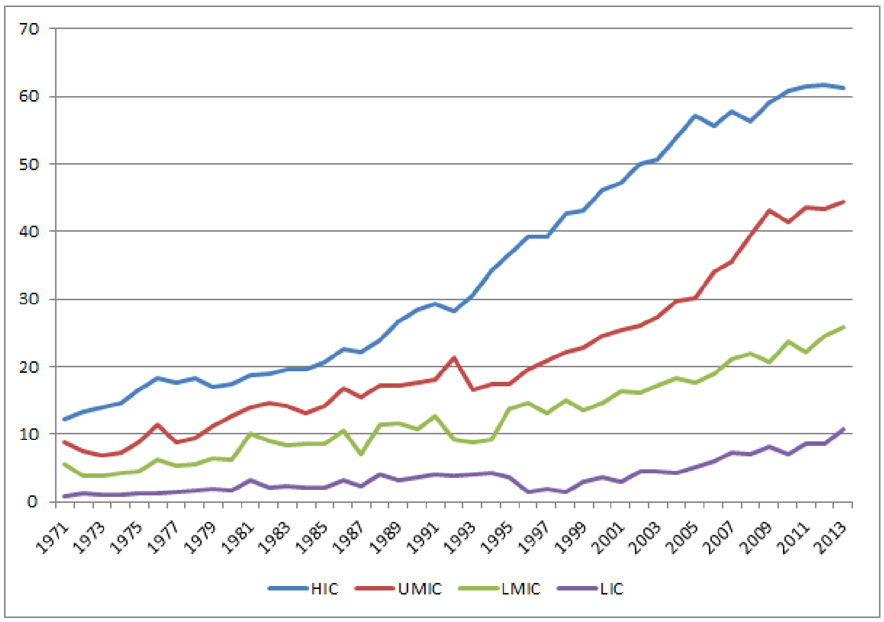
Almost half a century ago, Trow (1973), in a seminal work, predicted the transformation of the higher education system of today's advanced economies from an elite, to a mass, and a universal phase.[4] In a later work (Trow, 2007), he argued that the industrialized society is further moving towards a more advanced phase, what he called "a learning society," where the large parts of the population engaged in the formal education of one form or another. Trow's prediction was made at the time when most of today's industrialized economies have less than 20% of the enrollment rate in their higher education system. It is now believed that the "massification" of higher education is real and many of today's industrialized economies have, more or less, passed through Trow's phases of development since the Second World War (see Table 2 and Figure 1).

The paper develops a simple model that captures the endogenous transformation of higher education development. It then provides a comprehensive analysis of the effects of alternative higher education financing policies on efficiency, equity, and enrollment rates with closed-form solutions. In the model, agents are heterogeneous in terms of their initial human capital and their ability to learn. Individuals are differentiated as college-educated and non-college-educated based on their family background, and as high and regular ability based on their learning ability. Children of parents who afford to pay the minimum college tuition fee up-front will join college. But, those whose parents do not afford to pay the college tuition fee will join the unskilled labor force and earn a lower wage income. Individuals with college education receive an additional skill premium. Skilled labor (human capital) and raw labor are the only factors of production at the aggregate level.

Parents' ability to pay for their children's college education depends on their income, which also depends on whether they are college-educated (rich) or not, and their children's ability. But, individuals' productivity or income depends on the level of aggregate human capital at the time. At the early stage of development, the ag-

[4] Trow (1973) identifies the elite phase where less than 15% of the high school cohorts move beyond the secondary level; the mass phase where 16% to 50% of high school graduates continue their educations; and, the universal phrase where over 50% of graduates continue their higher education.

Figure 1: Evolution and stages of higher education of countries at different stages of development



Note: UIC – Upper income countries; HMIC – Upper middle income countries; LMIC – Lower middle income countries; LIC – Lower income countries.

gregate human capital is too low and thence individuals' income is low, only the elite (rich parents with high-ability children) afford to pay the college tuition fee while the rest of the population do not. We refer to this as Stage I of the higher education development process. As the economy continues to grow and individuals' productivity and income increase due to positive human capital externality effects, more and more individuals start to afford to pay the college tuition fee. In Stage II, the middle-class (rich families with regular ability children) will afford to pay the tuition fee and in Stage III, the lower-middle-class (rich families with regular ability children) will. In Stage IV, aggregate productivity is large enough for everyone (including poor families with regular ability children) to afford college tuition. The different phases of higher education can be associated with the stages of economic development that today's higher-, upper- and lower-middle, and lower-income countries exist, as reflected in the data (Figure 1).

In any of the developmental stages, the government could adopt one of the three commonly practiced tuition subsidy programs and finance it with flat-rate taxes. It can apply a universal tuition grant scheme that targets any individual that joins college, a scholarship grant scheme that targets high-ability individuals, or a means-tested grant scheme that targets high-ability individuals from poor background. We examine these policies affect individuals' ability and decision to invest in higher education, and their implications to equity–efficiency trade-off, at the different phases of higher education development. We compare each policy with a laissez-faire education system. Our analysis does not include student loans, which is extensively studied in the literature.[5] While a student loan is widely practiced in many advanced countries, it is not popular in developing countries due to a low recovery rate.[6]

Among the findings, a scholarship program is the most *efficient* higher-education-

[5]See, for instance, Garcia-Penalosa and Walde (2000), De Fraja, 2002), Del Rey and Racionero (2010), Abbott et al. (2019), Gary-Bobo and Trannoy (2015), Heijdra et al. (2017) among others.

[6]Even in South Africa, a country with a much-developed institution and economy in the continent, the student loan recovery rate is quite low. According to the National Student Fund Aid Scheme, recovery rates have fallen substantially since 2009 to under 4% in 2014. (https://www.gtac.gov.za/perdetail/11.1%20Summary.pdf).

subsidy program at all phases of higher education development due to its regressive nature. Means-tested is the least efficient policy in the early Stages I & II, as few are eligible for this program during these stages. However, it is the most efficient one (along with the scholarship scheme) in Stages III and IV through mobilizing resources to the ablest individuals in the economy. Laissez-faire is preferable at the initial or last stage but not in the middle Stages II & III; particularly, it is the least efficient one in Stage III when high-ability individuals of poor background have access to higher education. At this stage, government intervention in any form is preferable to ensure resource-poor but high-ability individuals would not be left behind. A universal subsidy scheme performs as the second-best in most of the developmental phases.

However, the *equity* effects of higher education subsidy are rather ambiguous. In general, the distribution under means-tested grant schemes Lorenz dominates the one in scholarship in all of the stages. In Stages I & II, laissez-faire is the second-best followed by the universal grant scheme. In the early stages, means-tested leaves everyone worse off, as non of the groups who invest in education at these stages qualify for the program. But, taxation seems to hurt the high-ability individuals more. In the later stages (particularly in Stage IV), we have established that the universal subsidy grant scheme Lorenz dominates laissez-faire. More interestingly, the universal funding Lorenz dominates the rest of the grant schemes when it comes to the poorest section of the society at this late stage of development. However, if the purpose is to narrow the gap between the top earners and the rest of society, the scholarship program is the second-best, next to means-tested.

In regards to college *enrollment rate*, in Stage I, universal and scholarship grant schemes (*vis-à-vis* laissez-faire) have similar positive effects. Apparently, means-tested has a negative effect. In Stage II, enrollment increases in universal grant but decreases in other policies. Means-tested is the first best in increasing enrollment, in Stage III, whereas scholarship and universal are the second and the third best, respectively. We find enrollment rate increases in universal subsidy but decreases in other policies in Stage IV. This result, particularly, confirms with other studies that find the policy shift in 2012 has led to a decline in enrollment rate in the UK (Geven, 2015).

The paper is related to strands of literature. Particularly, it is closely related to the literature that compares the efficiency and equity effects of different financing systems. A non-comprehensive list includes Garcia-Penalosa and Walde (2000), Caucutt and Kumar (2003), Cigno and Luporini (2009), Del Rey and Racionero (2010), and Abbott et al. (2019). Garcia-Penalosa and Walde (2000), for instance, examine the equity and efficiency effects of a general tax-subsidy, pure and income-contingent loan schemes and graduate tax.[7] They argue efficiency targets could be achieved with the general tax-subsidies scheme but not equity and efficiency targets at the same time, as the scheme is regressive. Loan schemes and graduate tax fare better than the traditional tax-subsidy system in achieving efficiency–equity where the latter is preferable when education outcome is uncertain as it could provide partial insurance.[8] However, there are no externality effects from human capital investment to the general population which is the deriving force of higher education transformation in the current work.

A more comprehensive and unifying work has been done by Abbott et al. (2019) who consider individuals' decision through different stages of their life cycle – from high school to retirement: whether to attend high school and college and whether to complete or dropout high school and college. They also consider uncertain return to investment in education, endogenous life span and parental transfer of resources; furthermore, there are different types of human capital that correspond to different levels of education such as high school and college. They then calibrate their model for the US economy and conclude that the current financial system in the US is welfare improving. In particular, they find the partial and general equilibrium effects of different financing programs such as means-tested, ability-tested and general ex-

[7] Cigno and Luporini (2009) argue that all student loans basically are income-contingent loans because anyway if unsuccessful it would be difficult to enforce repayment.

[8] Del Rey and Racionero (2010) build on this and rather divide the income-contingent loans into two types: those with risk-sharing and risk-pooling, the difference being unpaid costs from unsuccessful students to be covered by the general population and successful cohorts, respectively. However, they do not model externality and only analyze the efficiency and participation effects of alternative financing schemes with a focus on the role of insurance.

pansion of future grant to be welfare improving.[9] They don't address equity issues, though. Beside, their solely focus on advanced economies is in contrast to ours that examines different stages of higher education development analytically.

The paper is also related to the unified growth theory and the literature that focuses on altruistic parents that face a warm glow utility and human capital investment threshold (e.g., Galor and Zier, 1993, Moav, 2002, Galor and Moav, 2004, Galor and Mountford, 2008), which defines individual investment and consumption decision. However, this literature abstracts from education policies but inequality and growth issues.

The paper is structured as follows. Section 2 develops the model. Section 3 characterizes the different phases of higher education development under laissez-faire while Section 4 and 5 study the problem under government intervention. Section 6, 7, and 8 examine the efficiency, equity, and enrollment impacts of the policies (*vis-à-vis* laissez-faire) at each stage of higher education development respectively. Section 9 concludes. Proofs to the Propositions are relegated to the Appendix.

2. The Model

Suppose heterogeneous agents in overlapping generations model. The size of the population is one. In the beginning, λ number of individuals are college-educated; therefore, the remaining $1-\lambda$ number of individuals are non-college educated. Individuals also differ in their innate learning ability: high and regular ability individuals. The probability to be born as talented is p. Individuals live for two periods as young and as an adult. Each individual is born with a unit of time. Conditioned on parental investment (covering a fixed college tuition fee plus other variable costs such as books, laptops, etc.), they could build on it by joining a college. A college education is a possibility only if the minimum tuition fee is paid up-front. Therefore, only households that can afford the tuition fee (and, finds optimal to do so) will send

[9] This finding is supported by Akyol and Athreya (2005) who argue that not only existing higher education subsidies in the US are welfare improving but even more higher subsidies could be beneficial since it encourages students to invest in higher education, which is risky and lumpy, by reducing college failure risk.

their children to college. Otherwise, the child joins the unskilled labor force when she becomes an adult.

2.1. Human Capital and Preferences

The human capital of individual i with ability j who is born at date t is given by:

$$h_{it+1}^{j} = \epsilon^{j} e_{it}^{j} + 1 \tag{1}$$

e_{it}^{j} represents additional parental investment (other than a fixed tuition cost) in education. Implicit in condition (1) is human capital will be fully depreciated at the end of each period. Such specification is not only not as restrictive as may at first appear but it might also be quite appropriate given that human capital is embedded in individuals that have a finite life.[10] Parents who send their children to college need to pay a fixed tuition fee cost up-front. If a parent chooses $e_{it}^{j} = 0$, then she does not need to pay the tuition fee. But her child grows as an unskilled worker while her human capital is given by $h_{it+1}^{j} = 1$. Therefore, even without a formal college education, individuals will have basic knowledge when joining the labor force.[11] ϵ^{j} represents the learning ability of a child, $j \equiv \{g, r\}$. If j takes g that implies the child i is gifted (or born with high-ability); otherwise, she is a regular ability individual. $i \equiv \{c, n\}$ denotes the family background, where c and n stand for college-educated and non-college-educated parents, respectively.

Suppose the following warm-glow utility function, with logarithmic preferences:[12]

[10] Besides, it enables us to obtain closed-form solutions, without loss of generality. Incorporating parental human capital in the production function, to capture intergenerational externality, for instance, will not change the main results.

[11] An alternative interpretation of this is that since all children go through a compulsory primary and secondary school education, they have at least a minimum level of skill when joining the labor force.

[12] The use of such utility function is ubiquitous in the literature (see for instance, Glomm and Ravikumar, 1992, Galor and Zier, 1993, Banerjee and Newman, 1993, Galor and Weil, 2000, Benabou, 2000, Galor and Maov, 2004 among many other). Its main advantage (*vis á vis* other dynastic altruistic models that assume parents derive utility from the utility of their children) is its greater analytical tractability while the qualitative results of the model remain unaffected.

$$u_{it}^{j} \equiv \ln\left(c_{it}^{j}\left(h_{it+1}^{j}\right)^{\beta}\right) \tag{2}$$

The utility of the ith agent is subject to the budget constraints:

$$c_{it}^{j} + \widetilde{s}.\mathbf{1}_{\{e_{it}^{j}>0\}} + e_{it}^{j} \equiv I_{it}^{j} = \begin{cases} (1-\tau_w)\,\omega_t \text{ if } h_{it}^{j} = 1 \\ (1-\tau_w)\,\omega_t + (1-\tau_y)\,\phi h_{it}^{j} \text{ if } h_{it}^{j} > 1 \end{cases} \tag{3a}$$

where

$$\widetilde{s} \equiv \begin{cases} s - x_t \text{ if eligible for subsidy} \\ s \text{ otherwise} \end{cases} \tag{3b}$$

and

$$c_{it}^{j} \geq 0,\ e_{it}^{j} \geq 0 \tag{3c}$$

$$s > 1,\ \epsilon^{j} > 1 \tag{3d}$$

x_t represents a per capita tuition grant, provided by the government to eligible individuals.[13] τ_w and τ_h denote the fixed tax rates imposed in wage and capital incomes, respectively. c_{it}^{j} is the household consumption. I_{it}^{j} is the disposable income of the adult; its value will be determined based on the individual's educational background, i.e., whether or not she has received a college education when a child at date $t-1$ (or, equivalently, whether or not she is skilled or unskilled at date t). ω_t and $\phi_t h_{it}^{j}$ are the wage rate per unit of labor and the skill premium per se, respectively.[14] For a skilled individual, her disposable income constitutes labor income and skill premium, minus the respective labor and capital taxes; for an unskilled person, it is after-tax labor income. The fixed tuition cost that an eligible household has to pay up-front, if it chooses to send the child to college ($e_{it}^{j} > 0$), is $\widetilde{s} \equiv s - x_t$. There-

[13]We defer the definition and discussion of x_t to Section 4, where we study the equilibrium conditions under government interventions.

[14]We see later, $\phi_t = \phi$.

fore, with government intervention, the tuition fee for those eligible individuals who invest in education is reduced by x_t. However, individual incomes that are available for investment are also reduced due to tax duties. Ineligible households who send their children to college, however, incur the full tuition cost ($\widetilde{s} = s$) and still pay their taxes accordingly. Families who do not invest in higher education ($e_{it}^{j} = 0$) will not pay the tuition fee ($\widetilde{s} = s = 0$) and consume the full amount of their after-tax income. Finally, eq. (3c) represents a no-borrowing condition, as individuals are restricted to have a non-negative consumption and they are not allowed to carry over a negative asset in the future.

Note that the specification in (1) and (2) acknowledges the parent's good knowledge of her child's ability. According to Caucutt and Kumar (2003), such an assumption is reasonable given that the parent lives together with her child for an extended period of time. Our setting is in contrast to the literature that emphasizes that parents care only about a bequest they leave for their children (see, for e.g., Galor and Moav, 2004). In the current setting, the parent rather cares for the human capital of her child, but not only just for the bequest she leaves. This does not necessarily make parents more altruistic but make them consider additional factors in their investment in the current model, which, we believe, is a better reflection of the reality. In particular, parents are aware of their children's possession of a unit of human capital (in addition to their knowledge of their ability) regardless of their investment, which affects their marginal benefit of investing in their children's education.

2.2. The Firm

There is a representative firm that operates in a perfectly competitive market. The firm uses both skilled and unskilled labors to produce the final product where the later is augmented by the aggregate capital stock in the economy (in the spirit of Romer, 1986). Prices per unit of unskilled and skilled labor are thus given by, respectively:

$$\omega_t = (1-\alpha) A h_t \tag{4a}$$

$$\phi = \alpha A \tag{4b}$$

where A is a constant total factor productivity (TFP); α is a factor share and h_t is the aggregate human capital at time t. Implicit in condition (4b) is perfect substitutability (or homogeneity) among skilled workers. Both high and regular ability individuals receive similar rate per unit of human capital holdings. The only difference between these individuals is thus on the quantity but quality of human capital that they possess.

2.3. Government Budget

Given that there are λ college-educated and $1-\lambda$ non-college-educated individuals at time t, the total number of tax-payer individuals is unity. This implies that in a balanced budget the total government revenue, which is the sum of taxes collected from labor income of skilled ($\lambda\omega_t$) and unskilled individuals ($(1-\lambda)\,\omega_t$) and capital incomes ($\lambda\phi h_{it}^j = \phi h_t$), is equal to the total education expenditure (z_t):

$$z_t \equiv \tau_w \omega_t + \tau_y \phi h_t \tag{5a}$$

Using (4) this can be rewritten as:

$$z_t = \theta A h_t;\ \theta \equiv \tau_w (1-\alpha) + \tau_y \alpha \tag{5b}$$

where θ represents the *grant ratio* – the fraction of aggregate income that are used for public subsidy. Note that z_t is the aggregate tuition grant available at time t and could be different from the amount of tuition subsidy available per person (x_t).[15]

2.4. Optimal Education Investment

The solution for the ith household education investment is given by

[15] See Section 4.

$$e_{it}^{j*} = b\left(I_{it}^{j} - \widetilde{s}\right) - b/\beta\epsilon^{j} \tag{6a}$$

where $b \equiv \beta/(1+\beta)$ and I_{it}^{j} is defined in eq. (3a).

Three observations immediately follow, from eq. (6a). First, individuals with total income below the tuition fee, $\widetilde{s}$, *cannot afford* to send their children to college, given that they face borrowing constraints. Second, even those who could afford the fixed college tuition fee may not necessarily invest in higher education, as they may not find it optimal. Third, parents with high-ability children are more likely to send their children to college than their counterparts. Therefore, all income, tuition and ability are important factors in determining whether a child will have a college education or not.

Thus, effective college investment is given by:

$$e_{it}^{j} = \max\left(0, e_{it}^{j*}\right) \tag{6b}$$

The economy thus features two types of households. The first are those households whose consumption decision entails consuming the full amount of their income, and do not invest in education, either because their income falls short of the tuition fee, it is not optimal to invest in education, or both. The second are those who send their kids to college. It follows that the optimal human capital associated to the jth child is given by

$$h_{it+1}^{j} = \max\left(1, h_{it+1}^{j*}\right) \tag{7a}$$

Condition (7a) includes the corner solution for individuals' human capital and follows from (6b). If the individual does not join college, simply $h_{it+1}^{j} = 1-$ the human capital of any individual i and ability level j.

From (1), (3), (4) and (6a), it follows that the optimal human capital of a young individual who is born at time t and receives a college education during the same period is given by

$$h_{it+1}^{j}=\begin{cases}\epsilon^{j}b\left(A'h_t-\widetilde{s}\right)+b \text{ if } h_{it}^{j}=1\\ \epsilon^{j}b\left(B'h_t-\widetilde{s}\right)+b \text{ if } h_{it}^{j}\neq 1\end{cases} \tag{7b}$$

and[16]

$$A'\equiv(1-\alpha)(1-\tau_w)A$$
$$B'\equiv A'+A(1-\tau_y)\alpha/\lambda$$

The first and the second lines in eq. (7b) show that the human capital of a young individual with unskilled and skilled parents, respectively. In addition to their family background, they also differ in their ability to learn (as shown by the superscript j). The terms in the right hand side represent the respective incomes of the parents; a fraction of the incomes will be invested in the children education that forms their human capital whereas the rest are consumed by the household. $A'h_t-\widetilde{s}$ and $B'h_t-\widetilde{s}$ are the *average after-tax* income of non-college and college-educated parents, respectively, net of college tuition fee and subsidy.

3. Laissez-Faire

We first analyze the economy based on a laissez-faire condition while we introduce government interventions in the next section. With no government interventions, both taxes and expenditures are nil:

$$\tau_w=\tau_y=0 \text{ and } z_t=x_t=0\Leftrightarrow\widetilde{s}=s \tag{8}$$

[16]In deriving the second equation in (7b), first substitute (6a) into (1) and substitute the second equation of (3a) into that and use (4) to get

$$h_{it+1}^{j}=\epsilon^{j}\left(b\left((1-\tau_w)(1-\alpha)Ah_t+(1-\tau_y)\alpha Ah_{it}^{j}-\widetilde{s}\right)-b/\beta\epsilon^{j}\right)+1$$

Of course, since the parent itself could be gifted (regular) with probability p $(1-p)$, one could rewrite $h_{it}^{j}=ph_{it}^{g}+(1-p)h_{it}^{r}$. But, given λ number of individuals have attended higher education at time t, $h_t=\lambda h_{it}^{j}$. Substituting that into the above leads to the second equation of (7b).

3.1. Education Investment Threshold

Since household education investment is a function of their labor income, which depends on aggregate productivity, the level of aggregate human capital is what in essence determines individuals' education investment. Considering (7b) and (8), the threshold levels of aggregate human capital in the economy below which individuals do not invest in education under a laissez-faire condition are given by

$$\overline{h}_n^j(l) = \left(\frac{1}{\beta\epsilon^j} + s\right)((1-\alpha)A)^{-1} \tag{9a}$$

$$\overline{h}_c^j(l) = \left(\frac{1}{\beta\epsilon^j} + s\right)B^{-1} \tag{9b}$$

where[17]

$$B \equiv (1-\alpha)A + \alpha A/\lambda \tag{9c}$$

$$B' = B - ((1-\alpha)\tau_w + \tau_y\alpha/\lambda)A \tag{9d}$$

B is the average income share of college-educated parents at time t.

$\overline{h}_n^j(l)$ and $\overline{h}_c^j(l)$ represent the threshold levels of aggregate human capital beyond which non-college-educated and college-educated parents invest in their children's education, respectively. The superscript j shows the thresholds are different for people with different ability children. If there are no background differences among

[17] The investment threshold associated with the ith individual of ability j is derived by applying $h_{it+1}^j = 1$ in (7b), considering $\widetilde{s} = s$, and solving for h_t. For instance, (9a) – the investment threshold of the agent with non-college-educated parent – is derived as:

$$h_{nt+1}^j = 1 = \epsilon^j b(A'h_t - s) + b$$
$$\Leftrightarrow \overline{h}_n^j(l) \equiv h_t = \left(\frac{1}{\beta\epsilon^j} + s\right)((1-\alpha)A)^{-1}$$

That is, if the aggregate human capital is less than or equal to $\overline{h}_n^j(l)$, the agent will not invest in college education.

parents, those with high-ability children are more likely to invest in their children than those with lower ability ones. If there are background differences, however, both the parents' background (whether or not they are college-educated) and the children's ability is important in determining who more likely to attend college. In any of the group, parents invest in education more likely if they have high-ability children, if they are more altruistic, there is a lower tuition fee or/and a higher TFP. The higher the labor factor shares the more likely unskilled individuals invest in education.

It can be easily shown from (9) that $\overline{h}_c^g(l) < \overline{h}_c^r(l)$ and $\overline{h}_n^g(l) < \overline{h}_n^r(l)$ hold given $\epsilon^g > \epsilon^r$ but comparison between $\overline{h}_n^g(l)$ and $\overline{h}_c^r(l)$ and is rather less clear cut. But, in general, we consider the case $\overline{h}_n^j(l) > \overline{h}_c^j(l)$, which implies that regardless of their ability, poor individuals are less likely to afford a college education by themselves.[18] Therefore considering that the following relation holds:

$$\overline{h}_c^g(l) < \overline{h}_c^r(l) < \overline{h}_n^g(l) < \overline{h}_n^r(l) \tag{10}$$

Therefore, parents with a college education and with high-ability children are most likely to invest in children's education. Whereas, parents with no college education and regular ability children are least likely to invest in education. Rich parents are more likely to invest in college education than poor parents regardless of ability differences.

3.2. Aggregate Capital Dynamics

The aggregate human capital is the total human capital in the economy with regular and high-ability individuals in the population, with skilled and unskilled parents. Thus, if there are λ number of individuals (parents) who have a college

[18] Although it is a possibility that $\overline{h}_n^g(l) < \overline{h}_c^r(l)$, which implies poor but highly talented individuals are more likely to go to college than regular rich kids, it might be in contrary to intuition and empirical evidence. Furthermore, to allow such a scenario, the ability gap between gifted and regular individuals would be unrealistically high. Even with the unlikely condition of zero tuition fee ($s = 0$), the ability gap between the two should be more than three times, when calibrated with values of $\alpha = 0.33$ and $\lambda = 0.25$.

education at time t and the probability of being born with high-ability is p, then the aggregate human capital (h_{t+1}) in the economy at time $t+1$ is given by:

$$h_{t+1} = \lambda \left[p h^g_{ct+1} + (1-p)\, h^r_{ct+1} \right] + (1-\lambda) \left[p h^g_{nt+1} + (1-p)\, h^r_{nt+1} \right] \quad (11)$$

where h^g_{ct+1} and h^r_{ct+1} represent the total human capital of talented and regular individuals with skilled parents, respectively; h^g_{nt+1} and h^r_{nt+1} represent the total human capital of high and regular ability individuals with unskilled parents, respectively.

The first term (in square bracket) is the total number of skilled individuals with college-educated parents while the second is of those with parents of no college education. In each group, there are high-ability individuals with probability p and regular ability individuals with probability $1-p$. Condition (11) implicitly assumes that *all* individuals in the economy invest in education. If only part of the population invests in education, then the aggregate human capital becomes smaller, accordingly.[19] Note also that individuals are homogenous within each group and it will not be possible for some individuals from one group to invest in education while others from the same group to not.

3.3. Stage of Development and Aggregate Human Capital Dynamics

Using eqs. (7b) to (11), the dynamic system that characterizes the economy's developmental stages under a laissez-faire condition could be derived (see Appendix A.1 for details):

[19] For instance, if only parents with college education invest in education, then the second term in the square brackets will immediately disappear and the total human capital in the economy becomes: $h_{t+1} = \lambda \left[p h^g_{ct+1} + (1-p)\, h^r_{ct+1} \right]$.

$$h_{t+1}=\begin{cases} 1 \text{ if } h<\overline{h}_c^g(l) \\ b\lambda p\left(\epsilon^g\left(Bh_t-s\right)+1\right) \text{ if } \overline{h}_c^g(l)<h<\overline{h}_c^r(l) \\ b\lambda\left(\left(p\epsilon^g+\epsilon^r(1-p)\right)\left(Bh_t-s\right)+1\right) \text{ if } \overline{h}_c^r(l)<h<\overline{h}_n^g(l) \\ bp\left(\epsilon^g\left(Ah_t-s\right)+1\right)+b\lambda(1-p)\left(\epsilon^r\left(Bh_t-s\right)+1\right) \text{ if } \overline{h}_n^g(l)<h<\overline{h}_n^r(l) \\ b\left(\left(p\epsilon^g+(1-p)\epsilon^r\right)\left(Ah_t-s\right)+1\right) \text{ if } h>\overline{h}_n^r(l) \end{cases} \tag{12}$$

The developmental stage is associated with the evolution of higher education enrollment. The economy starts from an early stage where only a few elites have access to higher education, continue to evolve endogenously, and end up to a highly advanced economy where all individuals invest in higher education.

The next period aggregate human capital investment is unity if the initial aggregate capital is too small, below the threshold level $\overline{h}_c^g(l)$ (i.e., $h_{t+1}=1$ if $h<\overline{h}_c^g(l)$). Even the richest and highly talented individuals do not find it optimal to invest in education, as it yields a too low return. We see in the second line in eq. (12) some individuals, in particular, college-educated parents with high-ability children begin investing in college education. In this case, the current aggregate human capital stock should be greater than $\overline{h}_c^g(l)$ but less than $\overline{h}_c^r(l)$ (the threshold capital required for *all rich* parents to send their children to college). The total aggregate education investment is $\lambda p\epsilon^g b\left(Bh_t-s\right)$: $\lambda p\epsilon^g$ implies only the rich, with high-ability children of probability p, send their children to college. Bh_t-s is the *average* income of college-educated parents, net of the college tuition fee.

However if the current capital is greater than $\overline{h}_c^r(l)$, all rich parents regardless of child ability invest in college education (third line). If it is greater than $\overline{h}_n^g(l)$, then all rich parents and some poor parents with high-ability children invest in higher education (fourth line). Ah_t-s is the average income of *all* parents, net of the college tuition fee. The first term shows education investment by all types of parents with high-ability children while the second captures investment by the rich parents with regular children. Only then when the aggregate human capital stock passes $\overline{h}_n^r(l)$, non-college-educated parents with regular children start to invest in education (fifth line). At this stage, education investment in the economy is simply a fraction of aggregate income net of the tuition fee.

As a requirement for a growing economy, the following restriction is imposed:

$$b\lambda p\epsilon^g B - 1 > 0 \tag{13}$$

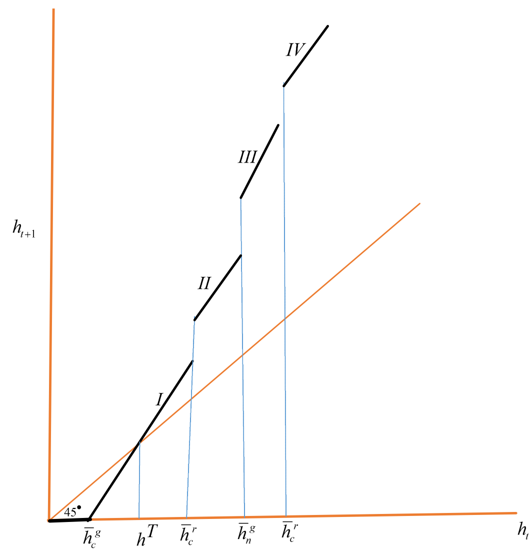
It implies that the slope of the curve in the initial stage of the economy shall be greater than unity.

Figure 2 shows the different developmental stages that the economy experiences based on eq. (12). As shown in the horizontal line, $h_{t+1} = 1$ for any initial capital $h < \overline{h}_c^g(l)$. But if $\overline{h}_c^g(l) < h < \overline{h}_c^r(l)$, the economy will be in Stage I where $h_{t+1} \neq 1$ because some individuals, in particular, parents with college education background and high-ability children begin to invest in human capital. But if the initial capital stock is not sufficiently high, the dynamic could go back to the stable equilibrium, $h_{t+1} = 1$. The economy escapes the low equilibrium only if $h_{t+1} \geq h_t$. The associated threshold could be computed from eq. (12) second line, as $h^T \equiv h_{t+1} = h_t$:

$$h^T = \frac{b\lambda p}{b\lambda p\epsilon^g B - 1}\left(s\epsilon^g - 1\right) \tag{14}$$

The economy continues to grow as long as the initial aggregate human capital is greater than this threshold level: $h_0 > h^T$. It eventually passes the thresholds required for other individuals to begin investing in human capital, through productivity spillover that boosts individual labor and human capital incomes. Stage II of development begins when $\overline{h}_c^r(l) < h < \overline{h}_n^g(l)$. At this stage, all individuals with college-educated parents invest in college education. This is followed by Stage III and Stage IV, when $\overline{h}_n^g(l) < h < \overline{h}_n^r(l)$ and $\overline{h}_n^r(l) > h$, respectively. The latter represents the long-run path of the economy where all individuals (rich and poor) invest in college education whereas the former represents a middle stage where all rich households and those poor households with talented children invest in college education.

Figure 2: Stages of Development: The economy kicks off only if the initial capital stock exceeds the threshold capital



4. Higher Education Policy

In this section, we introduce a government that engages with a provision of different types of higher education tuition grants. We let the government taxes labor income and capital income (skill premium) to finance education subsidy. We consider three higher education policies that are commonly applied: (i) a universal grant, (ii) a scholarship, or (iii) a means-tested grant scheme. The policies differ in terms of eligibility criteria that they associate with. In the first, the grant is available for any individual who enrolls in higher education. This is a case where the government has no knowledge of individuals' abilities and backgrounds and thus provides grants for anyone who joins a college or a university. In the second, tuition grant is available for individuals based on their merit. In this case, the government has knowledge of individuals' abilities but their family background. In the third scheme, the government provides tuition grants for high-ability individuals from poor family backgrounds, as it has knowledge of both individuals' ability and family background.

4.1. Per Capita Tuition Subsidy

Before we proceed to characterize the different phases of higher education development, under government interventions, we need to explicitly define the per capita tuition subsidy (x_t). The per capita tuition subsidy is the total tuition subsidy (z_t) divided by the number of *eligible individuals who are enrolled* in college at the time. The size x_t of thus depends on (i) the *enrollment rate* and (ii) *eligibility.* These, in turn, depend on the stage of the country's higher education development and the type of the grant scheme.

The per capita tuition grant could thus be different at different phases of higher education and for different grant schemes due to variation in the number of eligible individuals enrolled in higher education. For instance, consider an economy with a higher education level of Stage II (where *only* the rich invest in college education). If the tuition grant is a universal grant scheme then the number of eligible individuals with access to college is λ and the amount of tuition subsidy available to an individual is $x_t = z_t/\lambda$. However, if it is a scholarship scheme, then the number of individuals with college access who are eligible is λp and hence the per capita tuition grant is $x_t = z_t/\lambda p$. If the program is means-tested then non of the individuals who enroll in college receives grants as no one is eligible: $x_t = 0$.

In determining the values of x_t, we adopt the same enrollment trend that we have in the laissez-faire case (10).[20] That is, grant or no grant, the upper-class (λp) would most likely to invest in college, followed by the middle-class, $\lambda(1-p)$, and then the lower-middle-class, $p(1-\lambda)$ while the lower-class, $(1-\lambda)(1-p)$, are the least likely ones to invest in higher education. Table 3 below summarizes the per capita allocation of tuition grants that are available at different phases of higher education development and for different grant scheme:

[20] This makes a comparison with the laissez-faire condition possible.

Table 3: Per capita tuition subsidy provision at different stages of development and tuition grant scheme

Stages	**Grant available per person (x_t)**		
	Universal	*Scholarship*	*Means-tested*
Stage I	$z_t/\lambda p$	$z_t/\lambda p$	0
Stage II	z_t/λ	$z_t/\lambda p$	0
Stage III	z_t/ω	z_t/p	$z_t/\left(1-\lambda\right)p$
Stage IV	z_t	z_t/p	$z_t/\left(1-\lambda\right)p$

where

$$\omega \equiv \lambda + (1-\lambda)\,p$$

ω is the number of individuals who enroll in college, at Stage III.[21] One immediately sees that, during the same stage, the per capita tuition grant varies as the number of eligible individuals changes over the type of tuition grant scheme. In the universal grant scheme, z_t/ω tuition grant available for an individual who joins college whereas in the scholarship and means-tested schemes the per capita tuition grants are much higher, z_t/p and $z_t/\left(1-\lambda\right)p$, respectively.

By substituting each column x_t from Table 3 into (7b), and considering (3b), one obtains individuals' optimal human capital associated with the different grant schemes. For instance, substituting x_t from column 2 in (7b) gives the individuals' optimal human capital investment under the universal grant scheme; substituting from column 3 and column 4 give the investment under the scholarship and means-tested programs, respectively.

[21] It is the sum of lower-middle $p\,(1-\lambda)$, middle $\lambda\,(1-p)$, and upper class (λp) individuals.

5. Phases of Higher Education with Government Intervention

We characterize the different phases of higher education development, under government intervention, in a similar fashion to one in the laissez-faire. The difference from the laissez-faire is that this time the dynamics reflect the taxes that individuals pay and the tuition grant they *may* receive under alternative grant schemes such as a universal grant, scholarship, or means-tested.

5.1. Universal Grant Scheme

From (3b), (7b), (11) and Table 3 (*column 2*), the dynamics of aggregate human capital under the universal grant scheme are given by (see Appendix A.2):

$$h_{t+1}=\begin{cases}\lambda pb\left\{\epsilon^{g}\left(B'h_{t}-s\right)+1\right\}+b\epsilon^{g}z_{t}\text{ if }\overline{h}_{c}^{g}\left(u\right)<h_{t}<\overline{h}_{c}^{r}\left(u\right)\\ b\lambda\left\{\left(p\epsilon^{g}+\left(1-p\right)\epsilon^{r}\right)\left(B'h_{t}-s\right)+1\right\}+b\left(p\epsilon^{g}+\left(1-p\right)\epsilon^{r}\right)z_{t}\text{ if }\overline{h}_{c}^{r}\left(u\right)<h_{t}<\overline{h}_{n}^{g}\left(u\right)\\ b\left\{p\left(\epsilon^{g}\left(\left(1-\theta\right)Ah_{t}-s\right)+1\right)+\lambda\left(1-p\right)\left(\epsilon^{r}\left(B'h_{t}-s\right)+1\right)\right\}+\frac{\vartheta bz_{t}}{\omega}\text{ if }\overline{h}_{n}^{g}\left(u\right)<h_{t}<\overline{h}_{n}^{r}\left(u\right)\\ b\left(p\epsilon^{g}+\left(1-p\right)\epsilon^{r}\right)\left(Ah_{t}-s\right)+b\text{ if }h_{t}>\overline{h}_{n}^{r}\left(u\right)\end{cases}\quad(15)$$

where

$$\vartheta\equiv p\epsilon^{g}+\lambda\epsilon^{r}\left(1-p\right)$$

Eq. (15), which is comparable to eq. (12), characterizes the dynamics of an economy that goes through four different phases of higher education development, under a universal tuition grant government scheme. First terms, from Stage I to III, in curly brackets, show fractions of *after tax* average income invested in education; second terms show the amount of tuition grant provided.[22]

[22] In Stages I and II, λp and λ individuals invest in education while each receives $z_t/(p\lambda)$ and z_t/λ per capita tuition grants, respectively. In Stages III and IV, as more and more individuals invest in education, per capita tuition grant reduces to z_t/ω and z_t, respectively. ω is the number of eligible individuals for the grant in Stage III. And, ϑ shows that the grant is distributed to p poor and high ability and $(1-p)\lambda$ rich and regular ability individuals.

Similar to the laissez-faire case, if the initial capital at the economy level is smaller than the minimum investment threshold ($\overline{h}_c^g(u)$), then no one in the economy will enroll in higher education (i.e., $h_{t+1} = 1$ if $h < \overline{h}_c^g(u)$). However, the economy will be in Stage I if the current aggregate capital is greater than the minimum investment threshold ($h_t > \overline{h}_c^g(u)$).[23] In Stage I, only parents with a college education and highly talented children invest in education. As the economy continues growing, other families will start to join in education investment (through productivity spillover) once the economy's capital stock is sufficiently higher than the kickoff *threshold*. If not, the dynamics could go back to the stable equilibrium, $h_{t+1} = 1$.

Using similar logic as in the preceding section, we can identify the *threshold* for take off ($h^{T\prime} \equiv h_{t+1} = h_t$):

$$h^{T\prime} = \frac{b\lambda p}{b\lambda p\epsilon^g(B' + \theta A) - 1}(s\epsilon^g - 1) \tag{16}$$

Since $h^{T\prime} < h^T$, take off starts earlier under the universal tuition grant than the laissez-faire case. During the transition periods of the economy (Stages I to III), growth is relatively higher than the ones in laissez-faire.[24] The laissez-faire conditions are inferior in every stage of the higher education development process (except at the last stage where all individuals enroll in college), as seen from comparing the terms in the brackets in eq. (12) and (15). Under the universal grant, additional resources are mobilized for college education investment from individuals who are not joining college and consume the full amount of their income. Not only individuals who send their children to college bear the cost of tuition subsidy, those who do not invest in college education also share the burden.

5.2. *Scholarship Grant Scheme*

With the scholarship program, the dynamics of aggregate human capital are given by, from (3b), (7b), (11), and Table 3 (*column 3*), (see Appendix A.3):

[23] The threshold levels related to different stages of development are derived in eq. (33), Appendix B.

[24] In Stage IV, the growth rates are similar.

$$h_{t+1}=\begin{cases}\lambda pb\left\{\epsilon^{g}\left(B'h_{t}-s\right)+1\right\}+b\epsilon^{g}z_{t} \text{ if } \overline{h}_{c}^{g}(s)<h_{t}<\overline{h}_{c}^{r}(s)\\ \lambda b\left\{\left(p\epsilon^{g}+(1-p)\,\epsilon^{r}\right)\left(B'h_{t}-s\right)+1\right\}+b\epsilon^{g}z_{t} \text{ if } \overline{h}_{c}^{r}(s)<h_{t}<\overline{h}_{n}^{g}(s)\\ b\left\{p\left(\epsilon^{g}\left((1-\theta)\,Ah_{t}-s\right)+1\right)+\lambda(1-p)\left(\epsilon^{r}\left(B'h_{t}-s\right)+1\right)\right\}+b\epsilon^{g}z_{t} \text{ if } \overline{h}_{n}^{g}(s)<h_{t}<\overline{h}_{n}^{r}(s)\\ b\left\{\left(p\epsilon^{g}+(1-p)\,\epsilon^{r}\right)\left(Ah_{t}-s\right)+1\right\}+b(1-p)\left(\epsilon^{g}-\epsilon^{r}\right)z_{t} \text{ if } h_{t}>\overline{h}_{n}^{r}(s)\end{cases} \tag{17}$$

Eq. (17) represents the different phases of higher education development when the government provides scholarship – a tuition grant that targets high-ability individuals. Again, Stage I is attained only if the initial aggregate capital is greater than the minimum investment threshold ($h_t > \overline{h}_c^g(s)$) while the economy continues to evolve in a similar fashion as described above.[25] Note also that in Stage I, aggregate human capital is similar to the universal grant case, due to similarity in the amount of per capita grant available during this time ($z_t/\lambda p$). This also implies that the two economies face *similar take-off conditions*, defined in (16).

First terms in curly brackets, in Stages I to III, show the after-tax average income invested in education by λp, λ and ω individuals, respectively. The second term $b\epsilon^g z_t$ captures the total tuition grant that are provided to high-ability individuals at each stage. Unlike the previous cases, in the last stage of development (Stage IV), there is a redistribution of resources from regular ability to high-ability individuals *who invest* in college.[26]

5.3. Means-Tested Grant Scheme

Similarly, from (3b), (7b), (11), and Table 3 (*column 4*), the different stages of higher education development for the case where government provides tuition subsidy based on both merit and need basis are given by (see Appendix A.4):

[25] The thresholds related to Stages I to IV are given in (34), Appendix B.

[26] If $\epsilon^g = \epsilon^r$, aggregate investment in education becomes similar to the previous cases.

$$h_{t+1}=\begin{cases}\lambda pb\left\{\epsilon^g\left(B'h_t-s\right)+1\right\}\text{ if }\overline{h}_c^g(m)<h_t<\overline{h}_c^r(m)\\ \lambda b\left\{\left(p\epsilon^g+(1-p)\epsilon^r\right)\left(B'h_t-s\right)+1\right\}\text{ if }\overline{h}_c^r(m)<h_t<\overline{h}_n^g(m)\\ b\left\{p\epsilon^g\left((1-\theta)Ah_t-s\right)+\lambda(1-p)\epsilon^r\left(B'h_t-s\right)+\omega\right\}+b\epsilon^g z_t\text{ if }\overline{h}_n^g(m)<h_t<\overline{h}_n^r(m)\\ b\left\{\left(p\epsilon^g+(1-p)\epsilon^r\right)\left(Ah_t-s\right)+1\right\}+(1-p)b\left(\epsilon^g-\epsilon^r\right)z_t\text{ if }h_t>\overline{h}_n^r(m)\end{cases}\tag{18}$$

While the minimum threshold to be satisfied for the economy to be at Stage I is $h_t>\overline{h}_c^g(m)$, the same mechanism described above applies for the evolution of the economy.[27] $\omega\equiv\lambda(1-p)+p$

As in the previous two cases, the first terms in the curly brackets show after-tax average income while the second terms (*if any*) show total tuition grants. One may have noticed in Stages I and II, because only rich individuals are investing in education, there are no tuition grants provided by the government. These are the cases where the government collects taxes and the revenues are being "thrown to the ocean".[28] Of course, this would have the immediate effects of lowering aggregate efficiency during these stages.[29] As a result, the economy may take off much later than any of the earlier cases. The respective *threshold* to take-off can easily be computed as in the above cases:

$$h^{T\prime\prime}=\frac{b\lambda p}{b\lambda p\epsilon^g B'-1}\left(s\epsilon^g-1\right)\tag{19}$$

Only in Stage III, individuals who are eligible to the tuition grants begin to invest

[27]The threshold levels related to Stages I to IV when the grant scheme is means-tested are given in (35), Appendix B.

[28]It might be questionable, however, why the government behaves in such counterintuitive manner. An alternative will be to consider rather the case where there is no government involvement in Stages I and II but only in the later stages. In this case, in the first two Stages, aggregate capital dynamics are identical to the laissez-faire case.

[29]The term "aggregate efficiency" or "efficiency" is used throughout this paper in its loosely meaning of aggregate capital or income productivity. The use of aggregate welfare to measure efficiency may understate the need for higher education subsidy given a large section of the population cares only about its consumption (consumes the whole amount of its income).

in education. At this stage and the next, the government's revenue will be available as tuition grant for these households.

6. Efficiency

Different grant schemes may have different implications to aggregate efficiency due to differences in their eligibility criteria and their capacity to mobilize resources. There is an efficiency gain in Stages I-III when moving from laissez-faire to the universal grant scheme. These can easily be computed by taking the differences between the right-hand side equations of (12) and (15):

$$\begin{aligned}
&\textit{Stage I}\colon b\epsilon^{g}\left(1-\chi p\lambda\right)z_{t}\\
&\textit{Stage II}\colon b\left(p\epsilon^{g}+\left(1-p\right)\epsilon^{r}\right)\left(1-\lambda\chi\right)z_{t}\\
&\textit{Stage III}\colon bp\epsilon^{g}\left(1/\omega-1\right)z_{t}+b\lambda\left(1-p\right)\epsilon^{r}\left(1/\omega-\chi\right)z_{t}\\
&\textit{Stage IV}\colon\ 0
\end{aligned} \tag{20}$$

where

$$\chi \equiv \frac{\tau_w\left(1-\alpha\right)+\alpha\tau_y/\lambda}{\tau_w\left(1-\alpha\right)+\alpha\tau_y}$$

where χz_t is the tax contribution by wealthy (college-educated) individuals.[30]

The efficiency gain mainly comes from resource mobilization and redistribution from those who do not invest in college education to those who do.[31] In Stage I, for instance, the tax contribution by the $p\lambda$ elite is $p\lambda\chi z_t$ but the same individuals receive $z_t/p\lambda$ each or z_t in total. Similarly, in Stage II, the tax contribution by the wealthy λ individuals is $\lambda\chi z_t$ whereas the same individuals receive z_t/λ each or z_t in total. There are $1-p\lambda$ individuals in Stage I and $1-\lambda$ individuals in Stage II who pay taxes but do not invest in higher education and hence do not receive any grant.

[30] $\chi z_t = Bh_t - B'h_t$ where Bh_t and $B'h_t$ are before-tax and after-tax income, respectively.

[31] It is straightforward to see the first and the second equations in (20) are positive since $\lambda\chi < 1$. To see the third equation is also positive, first, rewrite it as as

$$\text{Stage III: } b\left(1-p\right)\epsilon^{r}\left(\lambda\Theta-\lambda\chi\right)z_{t}$$

where $\Theta \equiv \left(1+p\epsilon^{g}\left(1-\lambda\right)/\left(\epsilon^{r}\lambda\right)\right)/\omega$. It can then be confirmed that $\lambda\Theta-\lambda\chi>0$ if $\epsilon^{g}>\epsilon^{r}$.

Therefore, the first and second lines in (20) show the resources that are redistributed regressively to the upper and middle-class in the form of tuition grants.

In Stage III, there are $(1-\lambda)(1-p)$ individuals who pay taxes but do not have access to college. The first and second terms capture *net* grant received by p high-ability poor individuals and $(1-p)\lambda$ regular ability rich individuals, respectively. In Stage IV, all individuals who pay taxes also send their children to college. In general, the gain in efficiency would reduce when moving up of stages, which disappears eventually, as the number of college participants increases.

The scholarship program is the most efficient one due to its regressive nature. Comparing eq. (15) to eq. (17), we see the latter is greater at every stage of development, except in the first stage where they are tied. There is a constant $b(1-p)(\epsilon^g-\epsilon^r)z_t$ gain in efficiency by moving from a universal education grant to a scholarship program, from Stage II to Stage IV:

$$\begin{aligned}&\textit{Stage I}: 0\\&\textit{Stage II}\text{ to }\textit{IV}: b(1-p)(\epsilon^g-\epsilon^r)z_t\end{aligned} \tag{21}$$

The efficiency gain comes from mobilizing resources to high-ability individuals. As the skill gap $(\epsilon^g-\epsilon^r)$ widens it becomes more efficient to shift to the scholarship program; $1-p$ in (21) indicates that the extra resource comes from the regular ability individuals.

Apparently, means-tested is the least efficient grant scheme in Stages I & II as everyone pays taxes but no one qualifies to receive grants during those stages. It is interesting to note that, however, aggregate capital under means-tested is similar to that of the scholarship program in Stage III and IV. Therefore, basically, there is no difference in terms of aggregate efficiency between scholarship and means-tested grant schemes during these relatively advanced stages of higher education development.

6.1. Policy Ranking

The scholarship program is the most efficient education subsidy program regardless of the higher education developmental stage of the economy. Table 4 below ranks the public programs based on their efficiency at each of the developmental phases.

Table 4: The ranking of different higher education grant schemes based on their impacts on aggregate efficiency

	Laissez-faire	**Universal grant**	**Scholarship**	**Means-tested**
Stage I	2nd	1st	1st	3nd
Stage II	3rd	2nd	1st	4th
Stage III	3rd	2nd	1st	1st
Stage IV	2nd	2nd	1st	1st

The Proposition summarizes Table 4 and the above discussion:

Proposition 1.

1. *Universal grant and scholarship are the most efficient ones in Stage I followed by laissez-faire and means-tested.*
2. *In Stage II, means-tested are the least efficient whereas scholarship is the most efficient followed by the universal grant as the second-best efficient policy.*
3. *In Stage III and IV, the scholarship and means-tested programs are the first-best policy.*
4. *In Stage III, the universal grant is the second and laissez-faire is the last whereas they are tied in Stage IV.*

7. Equity

Inequality in this economy is identified as between-group inequality. At time t, the population can be categorized into four classes, based on ability and family background, as the lower class $(1-\lambda)(1-p)$, the lower-middle-class $(1-\lambda)p$, the middle-class $(1-p)\lambda$, and the upper-class $p\lambda$. In this section, we study how the different higher education grant schemes impact the college education investment of each of these groups at the different stages of higher education development. We first construct the Lorenz curves (for each phase of higher education development) associated with the different tuition grant schemes and then conduct a comparison accordingly.

7.1. College Tuition Grants and Equality

Table 5 shows the Lorenz curves based on the cumulative aggregate human capital ratio of the respective cumulative population, associated with Stages I-IV of the higher education development (see Appendix C for details).

Table 5: Lorenz curve ratios

	Cumulative aggregate human capital ratios			
Cumulative population ratios	*Stage I*	*Stage II*	*Stage III*	*Stage IV*
0	0	0	0	0
$(1-\lambda)(1-p)$	0	0	0	$\frac{(1-\lambda)(1-p)h^r_{nt+1}}{\lambda\left(ph^g_{ct+1}+(1-p)h^r_{ct+1}\right)+(1-\lambda)\left(ph^g_{nt+1}+(1-p)h^r_{nt+1}\right)}$
$1-\lambda$	0	0	$\frac{(1-\lambda)ph^g_{nt+1}}{\lambda\left(ph^g_{ct+1}+(1-p)h^r_{ct+1}\right)+(1-\lambda)ph^g_{nt+1}}$	$\frac{(1-\lambda)\left(ph^g_{nt+1}+(1-p)h^r_{nt+1}\right)}{\lambda\left(ph^g_{ct+1}+(1-p)h^r_{ct+1}\right)+(1-\lambda)\left(ph^g_{nt+1}+(1-p)h^r_{nt+1}\right)}$
$1-\lambda p$	0	$\frac{\lambda(1-p)h^r_{ct+1}}{\lambda\left(ph^g_{ct+1}+(1-p)h^r_{ct+1}\right)}$	$\frac{(1-\lambda)ph^g_{nt+1}+\lambda(1-p)h^r_{ct+1}}{\lambda\left(ph^g_{ct+1}+(1-p)h^r_{ct+1}\right)+(1-\lambda)ph^g_{nt+1}}$	$\frac{\lambda(1-p)h^r_{ct+1}+(1-\lambda)\left(ph^g_{nt+1}+(1-p)h^r_{nt+1}\right)}{\lambda\left(ph^g_{ct+1}+(1-p)h^r_{ct+1}\right)+(1-\lambda)\left(ph^g_{nt+1}+(1-p)h^r_{nt+1}\right)}$
1	1	1	1	1

Since we have closed-form solutions for all the values in Table 5, computing the Lorenz curves for each of the developmental stages and higher education policy is straightforward and presented in Appendix C.2. In particular, Table 7 to 9 provide the respective Lorenz curves for Stages II to IV, associated to the laissez-faire and different tuition grant schemes.

Definition 1. *Let v and w represent certain distributions and $L(v)$ and $L(w)$ are the associated Lorenz curves, respectively. If*

$$L(v) \geq L(w)$$

then we say $L(v)$ is Lorenz dominance of $L(w)$.[32]

Stage I. In the elite stage, all human capital investment is made by the top $100\lambda p$ percentile of the population; therefore, the economy will remain perfectly unequal, when only the upper-class invests in education. Regardless of the tuition subsidy program implemented at this stage at time t, Lorenz inequality will not change in the next period. But of course, compared to laissez-faire or means-tested, in the scholarship and universal grant schemes, the rich become richer but in all cases their investment represents the entire education investment in higher education.

Stage II. In this stage, education investment is made by the 100λ percentile of the population. Therefore, *basically* comparison is made between the middle-class, $100\lambda(1-p)$, and upper-class, $100\lambda p$ percentiles. Because, the rest $1-\lambda$ percentile does not invest in education.

Proposition 2. *Given $\epsilon^g > \epsilon^r$, in Stage II, the effects of higher policies in inequality can be ranked as:*

	Laissez-faire	***Universal***	***Scholarship***	***Means-tested***
Rank	*2nd*	*3rd*	*4th*	*1st*

Means-tested tops in terms of reducing inequality followed by laissez-faire and then universal grant. Means-tested leaves everyone worse off, as non of the groups

[32]See Davies and Hoy (1995) for more in the subject.

who invest in education at this stage qualify for the program. Even though all pay tax and none are qualified for the grant scheme, taxation seems to hurt the high-ability individuals more. At this early phase of higher education development, even if all groups are equally benefitted from the universal grant scheme, laissez-faire would be relatively better when reducing inequality is the policy target. This is because although universal grant benefits the middle-class ($100\lambda(1-p)$ percentile), it benefits more the upper-class ($100\lambda p$ percentile). Apparently, a scholarship program is quite regressive and benefits only those individuals with the high-ability (the top $100\lambda p$ percentile). The higher the difference in ability, the higher the regressivity of the policy becomes.

Stage III. In this stage, both poor and rich individuals invest in higher education; analytical comparison can be made between the means-tested and scholarship schemes where the former is found to be a better policy in terms of reducing inequality. This can be seen by comparing the respective columns in Table 8 where, in all cases, the nominators are relatively greater for means-tested while the denominators are smaller.

Stage IV . At this advanced stage, all individuals in the economy invest in higher education. We can easily make a comparison between the universal-grant scheme and laissez-faire, and, between the means-tested and scholarship scheme as one Lorenz dominates the other, for every percentile of the population. The following Propositions hold in Stage IV:

Proposition 3.
1. *The distribution under the means-tested grant scheme Lorenz dominates the distribution in the scholarship grant scheme.*
2. *The distribution under the universal grant scheme Lorenz dominates the one in laissez-faire.*

These results are quite intuitive. Under the universal grant scheme, all individuals are eligible for tuition subsidy and they have access to college education in Stage IV. In such a case, labor tax has basically *no effect* on individual human capital investment but capital tax has a direct redistributive effect. This is because each individual pays a labor tax and receives it back in the form of tuition grant. We can see that immediately by rewriting (7b), using (3b), (5b) and noting that $x_t = z_t$ under the universal grant scheme in Stage IV (see Table 3):

$$h^j_{it+1} = \begin{cases} \epsilon^j b \left(\omega_t + \tau_y \phi h_t - s\right) + b \text{ if } h^j_{it} = 1 \\ \epsilon^j b \left(\omega_t + \left[1 - \left(1-\lambda\right)\tau_y\right] \phi h_t / \lambda - s\right) + b \text{ if } h^j_{it} \neq 1 \end{cases} \tag{22}$$

where[33]

$$\psi \equiv 1 - \left(1 - \lambda\right) \tau_y$$

As we see in (22), the labor tax has disappeared. We also see each of the $1-\lambda$ non-educated parents are now subsidized by an amount of $\tau_y \phi h_t$ (first line of the equation). This is paid by λ number of college-educated household heads, in the amount of $\tau_y \phi h_t \left(1-\lambda\right)/\lambda$ per head (second line of the equation). The term in the square bracket is the left over of a dollar of a skill premium after tax deduction. Therefore, subsidizing tuition fee under the universal grant scheme at Stage IV is nothing but redistribution of income from skilled to unskilled households where the government budget constraint is given by:

$$\underbrace{1-\lambda}_{\substack{\text{Number of}\\\text{unskilled parents}}} \times \underbrace{\tau_y \phi h_t}_{\substack{\text{Subsidy received}\\\text{per head}}} = \underbrace{\tau_y \phi h_t \left(1-\lambda\right)/\lambda}_{\substack{\text{Skill premium}\\\text{paid per head}}} \times \underbrace{\lambda}_{\substack{\text{Number of}\\\text{skilled parents}}}$$

Table 6 below makes comparison between the rest of the programs based on their Lorenz dominance:

[33] To get (22), substitute $\widetilde{s} = s - x_t$ into (7b) but $x_t = z_t$ in Stage IV and under the universal grant scheme (Table 3). Then substitute for z_t from (5b).

Table 6: Ranking of the public programs based on Lorenze dominance in Stave IV

	Laissez-faire	**Universal**	**Scholarship**	**Means-tested**
$(1-\lambda)(1-p)$	2nd	1st	3rd	3rd
$1-\lambda$	3rd	2nd	1st	1st
$1-\lambda p$	4th	3rd	2nd	1st

Note: 1st implies the greatest Lorenze dominance while 4th is the least.

We see immediately from Table 6 that Proposition 3 holds. When comparing and contrasting the universal grant scheme with the scholarship one in Stage IV, we have the following Propositions:

Proposition 4.
1. *Universal grant Lorenz dominates scholarship for the bottom* $100(1-\lambda)(1-p)$ *percentile of the population.*
2. *But scholarship Lorenz dominates universal grant for the poor & middle-class,* $100(1-\lambda)$ *percentile of the population.*
3. *For the* $100(1-p\lambda)$ *percentile of the population, scholarship Lorenz dominates universal grant if* $\epsilon^g(1-\lambda) > \epsilon^r$.

The following Corollary follows from Proposition 3 and 4:

Corollary 1.
1. *For the* $100(1-p\lambda)$ *percentile of the population, the same relationship holds as in Proposition 4 when comparing the universal grant scheme with the means-tested program except that a more weaker condition than* $\epsilon^g(1-\lambda) > \epsilon^r$ *may be required.*
2. *Both the scholarship and means-tested grant schemes Lorenz dominate laissez-faire for* $100(1-\lambda)$ *and* $100(1-p\lambda)$ *percentile of the population while they are both Lorenz dominated for the poorest section of the society, or the* $100(1-\lambda)(1-p)$ *percentile of the population.*

Therefore, at the more advanced stage, the poorest section of the society is relatively better off from a universal grant. Both the means-tested and scholarship grant schemes seem to benefit disproportionately the poor & the lower-middle-class (the $100(1-\lambda)$ percentile of the population). When the target is to narrow the gap between the upper-class ($p\lambda$) and the rest of the population ($1-p\lambda$), means-tested seems the most effective policy.

8. Enrollment

By comparing the education investment thresholds associated with the different grant schemes to laissez-faire, we can study how different higher education policies affect the college enrollment rate. Because college access is categorized based on class in each phase of higher education development, we make a comparison of the thresholds associated with each policy for the group of individuals who have access to a college education for the first time at that specific stage. In Stage I, for instance, the elite will have access to higher education for the first time; thus, we examine how a given policy (in comparison to laissez-faire) affects their likelihood to enroll in higher education. Similarly, in Stage II, individuals with regular ability but from affluent families will have access to higher education for the first time. Thus, the question will be: how does each policy affect the investment thresholds of this group of individuals? In Stages III and IV, high and regular ability individuals from poor families, respectively, will have college access for the first time. We thus examine how the investment thresholds of high and regular ability individuals will be affected by each policy in Stages III and IV.

The investment thresholds related to the different grant schemes at different stages of higher education development are derived in Appendix B, by combining Table 3, (3b) and eq. (7b). The following Propositions make a comparison of these thresholds to the investment threshold associated with laissez-faire:

Proposition 5.
1. *Stage I: The universal and scholarship programs have a similar positive effect on enrollment rate; means-tested has a negative effect.*
2. *Stage II: The enrollment rate increases in the universal grant scheme but decreases in other policies.*
3. *Stage III: Means-tested is the first best in increasing the college enrollment rate; scholarship and universal grants are the second- and third-best, respectively.*
4. *Stage IV: The enrollment rate increases in the universal grant scheme but decreases in other policies.*

In Stage I, individuals who are likely to enroll in college do not qualify for the means-tested grant scheme despite they pay taxes. In the universal and scholarship programs, those individuals who have access to a college education are better off compared to the laissez-faire because the tuition grants that they receive are higher than the taxes that they pay. Note that in Stage II and IV, the investment thresholds are associated with regular ability individuals and these individuals are not qualified for the scholarship and means-tested programs despite they pay taxes. They are thus better off with the universal grant scheme, in which the grants that they receive are higher than the taxes that they pay. In Stage II, the additional fund comes from those who do not enroll in college; in Stage IV, it comes from individuals with a rich background (from the capital tax revenue). In Stage III, the investment thresholds are associated with high-ability individuals but poor family background. The means-tested program is the most effective one when it comes to boosting the enrollment rates of this group of individuals, as the whole fund is available for them. Whereas, in the scholarship or the universal grant schemes, the fund is distributed among a larger section of the society.

9. Conclusion

This paper has made a comprehensive analysis of the effects of alternative higher education financing policies on efficiency, equity, and enrollment rates. It has ranked different higher education grant schemes based on their impact on efficiency and equity, *vis-à-vis* a laissez-faire condition. What makes the work unique is that all the analysis, comparisons, and contrasts are made while considering the different phases of higher education development that countries may have faced. Many of today's industrialized economies, more or less, have gone through, what is well-known in the education literature, Trow's phases of higher education development – the transformation of the higher education system from elite, to mass and the universal system – since the Second World War. The simple model employed herein has accounted for the "massification" of higher education while it has resulted in closed-form solutions and provided a rich analysis in many aspects of higher education grants. The work, in particular, has captured the four phases and endogenous transition of the higher education system that starts from an early stage where only a few elites have access to higher education, and evolves endogenously and ends up eventually to a highly advanced economy where all individuals invest in higher education.

The analysis was conducted both under government intervention and laissez-faire systems. In the former, the dynamics and equilibrium reflect the taxes that individuals must pay and the tuition grants that they may receive under alternative grant schemes such as the universal, scholarship, and means-tested. Different grant schemes are found to have different implications to efficiency, equity and enrollment due to differences in their eligibility criteria and capacity to mobilize resources from, individuals who do not invest in college to those who do and, from the low ability to high-ability individuals.

Some of the main results include that a scholarship program is the most efficient higher-education-subsidy program at all stages of higher education development due to its highly regressive nature. The universal grant scheme Lorenz dominates laissez-faire in the late stages of development, and vice versa in the early phases of development. Higher education subsidy could thus be regressive in developing countries but progressive in advanced economies. At the later stages of higher education development, the enrollment rate increases in the universal grant scheme but decreases in other policies. The results imply that the recent shift away from the universal grant scheme in the UK could go wrong on at least two fronts: it could lead to a decline in college enrollment rate and aggravate some of the equity issues in higher education.

Acknowledgements

The author is grateful to Zhang Jie for extensive comments on a previous draft of the paper and thanks participants to various conferences (*WORLD CONGRESS 2020: The Econometric Society*; *The 2019 African Meeting of the Econometric Society*; *2019 Biennial Conference of the Economic Society of South Africa; Brown Bag Seminar, Economics Department, University of Pretoria*) for fruitful discussions and suggestions. The usual disclaimers apply.

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A. Aggregate Dynamics

A.1. Laissez-Faire

A.1.1. Stage I

To derive the second line of (12), substitute the second line from (7b) into the first term of (11) and use (8) to obtain:

$$\begin{aligned} h_{t+1} &= \lambda p h^{g}_{ct+1} \\ &= b\lambda p \left(\epsilon^{g} \left(Bh_t - s\right) + 1\right) \end{aligned} \tag{23}$$

A.1.2. Stage II

To derive the third line of equation (12), substitute the second line from (7b) into the first two terms of (11) and use again (8) to obtain:

$$\begin{aligned} h_{t+1} &= \lambda p h^{g}_{ct+1} + \lambda \left(1-p\right) h^{r}_{ct+1} \\ &= \lambda p \left(b \left(\epsilon^{g} \left(Bh_t - s\right) + 1\right)\right) + \lambda \left(1-p\right) \left(b \left(\epsilon^{r} \left(Bh_t - s\right) + 1\right)\right) \\ &= b\lambda \left\{\left(p\epsilon^{g} + \left(1-p\right)\epsilon^{r}\right) \left(Bh_t - s\right) + 1\right\} \end{aligned} \tag{24}$$

A.1.3. Stage III

To derive the fourth line of equation (12), add the third term from (11) into the above and use (7b) and (8):

$$\begin{aligned} h_{t+1} &= b\lambda \left(\left(p\epsilon^{g} + \left(1-p\right)\epsilon^{r}\right) \left(Bh_t - s\right) + 1\right) + \left(1-\lambda\right) p h^{g}_{nt+1} \\ &= b\lambda \left(\left(p\epsilon^{g} + \left(1-p\right)\epsilon^{r}\right) \left(Bh_t - s\right) + 1\right) \\ &+ \left(1-\lambda\right) p \left(\epsilon^{g} b \left(\left(1-\alpha\right) Ah_t - s\right) + b\right) \\ &= bp \left(\epsilon^{g} \left(Ah_t - s\right) + 1\right) + b\lambda \left(1-p\right) \left(\epsilon^{r} \left(Bh_t - s\right) + 1\right) \end{aligned} \tag{25}$$

A.1.4. Stage IV

To derive the fifth line of equation (12), add the fourth term from (11) into the above and use (7b) and (8):

$$\begin{aligned} h_{t+1} &= bp\left(\epsilon^g\left(Ah_t - s\right) + 1\right) + b\lambda\left(1-p\right) \\ &\left(\epsilon^r\left(Bh_t - s\right) + 1\right) + \left(1-\lambda\right)\left(1-p\right)h^r_{nt+1} \\ &= bp\left(\epsilon^g\left(Ah_t - s\right) + 1\right) + b\lambda\left(1-p\right)\left(\epsilon^r\left(Bh_t - s\right) + 1\right) \\ &+ \left(1-\lambda\right)\left(1-p\right)\left(\epsilon^r b\left(\left(1-\alpha\right)Ah_t - s\right) + b\right) \\ &= b\left(\left(p\epsilon^g + \left(1-p\right)\epsilon^r\right)\left(Ah_t - s\right) + 1\right) \end{aligned} \tag{26}$$

A.2. Universal Grant

A.2.1. Stage I:

In deriving the first line of (15), note that in Stage I, only λp number of high-ability individuals from college-educated parents have access to higher education. Therefore, from (11), the aggregate human capital, in Stage I, is given by

$$h_{t+1} = \lambda p h^g_{ct+1}$$

Then, substitute the second equation from (7b) into the above to obtain:

$$h_{t+1} = \lambda p\left[\left(\epsilon^g b\left(B'h_t - \widetilde{s}\right) + b\right)\right]$$

Under the universal grant scheme anyone who enrolls in college is eligible for tuition grants; therefore, given (3b), $\widetilde{s} = s - x_t$ where $x_t = z_t/\lambda p$ (which is, from Table 3, the value of x_t for Stage I & the universal grant scheme). Substituting that into the above gives the first equation in (15):

$$\begin{aligned} h_{t+1} &= \lambda p\left[\left(\epsilon^g b\left(B'h_t - s + z_t/\lambda p\right) + b\right)\right] \\ &= \lambda p b\left\{\epsilon^g\left(B'h_t - s\right) + 1\right\} + b\epsilon^g z_t \end{aligned} \tag{27}$$

A.2.2. Stage II:

In Stage II, from Table 3, the value of x_t for Stage II & the universal grant scheme is $x_t = z_t/\lambda$. To derive the second line of equation (15), substitute the second line from (7b) into the first two terms of (11) to obtain:

$$\begin{aligned} h_{t+1} &= \lambda \left(p \left(\epsilon^g b \left(B' h_t - \widetilde{s}\right) + b\right) + (1-p)\left(\epsilon^r b \left(B' h_t - \widetilde{s}\right) + b\right)\right) \\ &= b\lambda \left(\left\{p\epsilon^g + (1-p)\,\epsilon^r\right\}\left(B' h_t - s + z_t/\lambda\right) + 1\right) \\ &= b\lambda \left\{\left(p\epsilon^g + (1-p)\,\epsilon^r\right)\left(B' h_t - s\right) + 1\right\} + b\left(p\epsilon^g + (1-p)\,\epsilon^r\right) z_t \end{aligned} \tag{28}$$

Similar procedure can be used to derive the rest of the equations.

A.3. Scholarship

Under the scholarship program, in State I and II, λp individuals are eligible for the tuition grants while in Stage III and IV, p individuals are eligible for the grants. Therefore, the respective per capita tuition grants are, considering (3b), $x_t = z_t/\lambda p$ and $x_t = z_t/p$ (see Table 3, Scholarship).

A.3.1. Stage I

This also implies that aggregate capital dynamics in Stage I, under the scholarship and universal grants schemes are similar since in both cases $x_t = z_t/\lambda p$.

A.3.2. Stage II

Note that in Stage II, under the scholarship grant schemes, while λ individuals invest in higher education only λp are eligible for grants. To derive the aggregate dynamics for this stage, substitute the second equation from (7b) into the first two terms of (11), and use $x_t = z_t/\lambda p$:

$$\begin{aligned} h_{t+1} &= \lambda \left(p h^g_{ct+1} + (1-p)\, h^r_{ct+1}\right) \\ &= \lambda \left(p \left(\epsilon^g b \left(B' h_t - \widetilde{s}\right) + b\right) + (1-p)\left(\epsilon^r b \left(B' h_t - s\right) + b\right)\right) \\ &= \lambda b \left(p\epsilon^g \left(B' h_t - s + z_t/\lambda p\right) + (1-p)\,\epsilon^r \left(B' h_t - s\right) + 1\right) \\ &= \lambda b \left\{\left(p\epsilon^g + (1-p)\,\epsilon^r\right)\left(B' h_t - s\right) + 1\right\} + b\epsilon^g z_t \end{aligned} \tag{29}$$

Similar procedure can be applied to derive the rest of the equations.

A.4. Means-tested

Under the means-tested grant scheme, in State I and II, no individual with access to college is eligible for the grants while in Stage III and IV, $p(1-\lambda)$ individuals are eligible. Therefore, the respective per capita tuition grants are, considering (3b), $x_t = 0$ and $x_t = z_t / ((1-\lambda)p)$ (Table 3, *column 4*).

A.4.1. Stage I & II

Therefore, at the early stages, aggregate dynamics are similar to laissez-faire except that disposable income is income less taxes. That is, aggregate dynamics in Stage I is,

$$h_{t+1} = b\lambda p \left\{ \epsilon^g \left(B' h_t - s \right) + 1 \right\} \tag{30}$$

and in Stage II is given by,

$$h_{t+1} = b\lambda \left\{ \left(p\epsilon^g + (1-p)\,\epsilon^r \right) \left(B' h_t - s \right) + 1 \right\} \tag{31}$$

A.4.2. Stage III

To derive the aggregate dynamics in Stage III, add the third term from (11) into the above, substitute the first line from (7b), and use $x_t = z_t / (p(1-\lambda))$:

$$\begin{aligned} h_{t+1} &= b\lambda \left\{ \left(p\epsilon^g + (1-p)\,\epsilon^r \right) \left(B' h_t - s \right) + 1 \right\} + (1-\lambda)\, p h^g_{nt+1} \\ &= b\lambda \left\{ \left(p\epsilon^g + (1-p)\,\epsilon^r \right) \left(B' h_t - s \right) + 1 \right\} \\ &\quad + (1-\lambda)\, pb \left(\epsilon^g A' h_t - s + z_t / ((1-\lambda)\, p) + 1 \right) \\ &= b \left\{ p \left(\epsilon^g \left((1-\theta) A h_t - s \right) + 1 \right) + \lambda (1-p) \left(\epsilon^r \left(B' h_t - s \right) + 1 \right) \right\} + b\epsilon^g z_t \end{aligned} \tag{32}$$

Follow similar procedure to derive the dynamics for Stage IV.

B. Investment Thresholds under Higher Education Policy

We derive the investment threshold levels, which determine individuals' college education investment, associated with the different grant schemes in a similar fashion to the laissez-faire case (see (9)). That is, the investment threshold associated with the ith individual of ability j is derived by substituting $h^j_{it+1} = 1$ and $\widetilde{s} = s - x_t$ into (7b) and solving for h_t. The value of x_t is determined from (5b) and Table3, accordingly.

But note that the per capita tuition grant (x_t) is different, not only for different grant schemes but also at different phases of higher education development, which is shown in Table3. It may also differ among individuals due to differences in eligibility. Therefore, in contrast to the laissez-faire, we may end up having different threshold levels for different phases of higher education development when applying the same policy.

B.1. Universal Grant

The investment threshold of the j ability agent with non-college-educated parent ($\overline{h}^j_n(u)$) is derived by substituting $h^j_{it+1} = 1$ and $\widetilde{s} = s - x_t$ into the first line of (7b) to get

$$h^j_{it+1} = 1 = \epsilon^j b\,(A' h_t - s + x_t) + b \tag{33a}$$

But from Table3, column 2, and (5b) $x_t = z_t/\kappa^u = \theta A h_t/\kappa^u$ where $\kappa^u \in \{\lambda p,\, \lambda,\, \omega, 1\}$ is the number of eligible individuals at Stages I, II, III and IV, respectively.[34] Substituting that into the above and solving for h_t gives

$$\overline{h}^j_n(u) \equiv h_t = \left(\frac{1}{\beta \epsilon^j} + s\right) \left(A' + A\theta/\kappa^u\right)^{-1} \tag{33b}$$

[34]Recall that the enrollment rate at each stage is similar to the number of eligible individuals because, in the universal grant scheme, anyone who enrolls in college is automatically eligible for the tuition grants.

The investment threshold of the j ability agent with college-educated parent ($\overline{h}_c^j(u)$) is derived similarly from (3b), (5b), the second equation of (7b) and Table3, column 2:

$$\overline{h}_c^j(u) \equiv \left(\frac{1}{\epsilon^j \beta} + s\right)(B' + A\theta/\kappa^u)^{-1} \tag{33c}$$

We follow similar steps to derive the investment thresholds associated with the scholarship and means-tested grant schemes.

B.2. Scholarship

From (3b), (5b), (7b) and Table3, column 3, one derives the investment thresholds associated with means-tested:

$$\overline{h}_n^g(s) \equiv \left(\frac{1}{\beta\epsilon^g} + s\right)(A' + A\theta/\kappa^s)^{-1} \tag{34a}$$

$$\overline{h}_n^r(s) \equiv \left(\frac{1}{\beta\epsilon^r} + s\right)A'^{-1} \tag{34b}$$

$$\overline{h}_c^g(s) \equiv \left(\frac{1}{\epsilon^g \beta} + s\right)(B' + A\theta/\kappa^s)^{-1} \tag{34c}$$

$$\overline{h}_c^r(s) \equiv \left(\frac{1}{\epsilon^r \beta} + s\right)B'^{-1} \tag{34d}$$

$\overline{h}_i^j(s)$ is the threshold associated to the ith person of j ability if the grant scheme is scholarship where $\kappa^s \in \{\lambda p, \lambda p, p, p\}$ is the number of eligible individuals for the scholarship grants at Stages I to IV, respectively. The number of eligible individuals for the tuition grants are different from the college enrollment rate as the scheme naturally excludes some individuals. In Stage I, λp individuals enroll in college where all are eligible for the tuition grants. In Stage II, λ individuals enroll in college but only the λp high-ability individuals are eligible for the tuition grants. In Stage III and IV, ω and 1 individuals enroll in college respectively, but only the p high-ability individuals are eligible for the tuition grants.

B.3. Means-tested

From (3b), (5b), (7b) and Table3, column 4, one derives the investment thresholds associated with the means-tested program:

$$\overline{h}_n^r(m) = \left(\frac{1}{\beta\epsilon^r} + s\right) A'^{-1} \tag{35a}$$

$$\overline{h}_n^g(m) = \left(\frac{1}{\beta\epsilon^g} + s\right) \left[A' + A\theta/\left((1-\lambda)p\right)\right]^{-1} \tag{35b}$$

$$\overline{h}_c^j(m) = \left(\frac{1}{\epsilon^j\beta} + s\right) B'^{-1} \tag{35c}$$

where $\overline{h}_i^j(m)$ is the threshold associated with the ith person of j ability if the grant scheme is means-tested. No one is eligible for this scheme in Stage I and II. But, in Stage III and IV, there are $(1-\lambda)p$ high-ability individuals who are eligible to the program.

C. Equity

In this section, we first construct the Lorenz curves (shown in Table 5). We then compute the corresponding values for each stage using eqs. (3b), (5b), (7b), (8), Table 3 and Table 5. These are given by Tables 7, 8, and 9.

C.1. Constructing the Lorenz Curves

Table 5 shows the Lorenz curves based on the cumulative aggregate human capital ratios of the respective cumulative populations, associated with Stages I-IV of the higher education development. The first column shows the cumulative ratios of the population. The rest of the columns show the corresponding cumulative ratios of aggregate human capital in Stages I to IV.

The cumulative population ratios are constructed between 0 and 1. We start from the lower-class $(1-\lambda)(1-p)$. Adding the lower-middle-class $(1-\lambda)p$ to that gives the cumulative population ratio $1-\lambda$. When adding the middle-class $(1-p)\lambda$, it leads to the cumulative ratio $1-\lambda p$. Finally, adding the upper-class λp gives the cumulative population ratio 1.

In Stage I, investment is made only by the upper-class λp while investment by the rest of the population is zero; therefore, the cumulative aggregate human capital ratio is either 0 or 1.[35]

In Stage II, $\lambda\left(ph^{g}_{ct+1}+(1-p)\,h^{r}_{ct+1}\right)$ human capital investment is made by the middle- and upper-class λ while the rest $1-\lambda$ individuals do not invest in college education. Thus, the share of the middle-class $\lambda(1-p)$ who invests in college education is given by the second last row. This is also the cumulative capital ratio for the $1-\lambda p$ size of the population, as the $1-\lambda$ individuals below them do not invest.

In Stage III, the total investment is the sum of investments made by the middle- and upper-class λ and lower-middle-class $(1-\lambda)\,p$.[36] The third last row shows the cumulative investment ratio of the $1-\lambda$ individuals, which comes from the investment made by the lower-middle-class $(1-\lambda)\,p$. Adding to that the investment made by the middle-class $\lambda(1-p)$ gives the cumulative investment ratio of the $1-\lambda p$ individuals (the second last row).[37]

In Stage IV, all individuals invest in college education. The fourth last row shows the total investment ratio by the lower-class $(1-\lambda)(1-p)$. The third last row shows the cumulative investment ratio by the lower- and middle-class $1-\lambda$. The second last row captures the cumulative ratio by the lower-, middle- and upper-class $1-\lambda p$.

C.2. Computing the Lorenz Curves

The next step is to compute the values of the Lorenz curves in Table 5 at each stage and make a comparison between laissez-faire and the different higher education policies.

[35] That is, the cumulative capital ratio is simply $\lambda ph^{g}_{ct+1}/\lambda ph^{g}_{ct+1}=1$.
[36] That is, the total investment is $\lambda\left[ph^{g}_{ct+1}+(1-p)\,h^{r}_{ct+1}\right]+(1-\lambda)\,ph^{g}_{nt+1}$.
[37] Apparently, if we add the investment by the upper-class λp to that we get 1.

C.2.1. Stage I & II

Apparently, in Stage I, there is a perfect inequality between the classes, with a unity of Gini coefficient, as all investments are made by the top $100\lambda p$ percentile of the population.

We compute the values of the Lorenz curves for Stage II, which is shown in Table 7, as follows: First note that in Stage II, from Table 5 (column 3), the cumulative aggregate capital ratio is given by

$$\frac{(1-p)\,h^r_{ct+1}}{ph^g_{ct+1}+(1-p)\,h^r_{ct+1}} \tag{36}$$

Under laissez-faire, $h^r_{ct+1}=\epsilon^r b\,(Bh_t-s)+b$ and $h^g_{ct+1}=\epsilon^g b\,(Bh_t-s)+b$, considering eqs. (3b), (7b) and (8). Substituting these back in (36) gives column 2 of Table 7.

Under the universal grant scheme, from eqs. (3b), (7b) and Table 3, we have:

$$\begin{aligned} h^r_{ct+1} &= \epsilon^r b\,(B'h_t-\widetilde{s})+b=\epsilon^r b\,(B'h_t-s+x_t)+b \\ &= \epsilon^r b\,(B'h_t-s+z_t/\lambda)+b \qquad (37) \\ h^g_{ct+1} &= \epsilon^g b\,(B'h_t-\widetilde{s})+b=\epsilon^g b\,(B'h_t-s+x_t)+b \\ &= \epsilon^g b\,(B'h_t-s+z_t/\lambda)+b \qquad (38) \end{aligned}$$

where z_t is defined in (5b) and allocated to λ individuals with college access. Substituting (37) and (38) into (36) leads to column 3, Table 7.

Under the scholarship grant scheme, from eqs. (3b), (7b) and Table 3, we have:

$$\begin{aligned} h^r_{ct+1} &= \epsilon^r b\,(B'h_t-s)+b \qquad (39) \\ h^g_{ct+1} &= \epsilon^g b\,(B'h_t-\widetilde{s})+b=\epsilon^g b\,(B'h_t-s+x_t)+b \\ &= \epsilon^g b\,(B'h_t-s+z_t/(\lambda p))+b \qquad (40) \end{aligned}$$

The difference from the universal grant scheme is that individuals with regular ability are not eligible for the tuition grants and the per capita grants available for eligible individuals are higher $(z_t/(\lambda p))$. Then, Substituting (39) and (40) into (36) leads to column 4, Table 7.

Under the means-tested program, again using eqs. (3b), (7b) and Table 3, we have:

$$h^r_{ct+1} = \epsilon^r b\left(B' h_t - s\right) + b \tag{41}$$

$$h^g_{ct+1} = \epsilon^g b\left(B' h_t - s\right) + b \tag{42}$$

This time none of the individuals are eligible for tuition grants and substituting the above into (36) leads to column 5, Table 7.

C.2.2. Stage III

We compute the values in Table 8 by the substituting the corresponding values from eqs. (3b), (5b), (7b), (8) and Table 3 into Table 5, column 4.

Under laissez-faire, $h^r_{ct+1} = \epsilon^r b\left(B h_t - s\right) + b$, $h^g_{ct+1} = \epsilon^g b\left(B h_t - s\right) + b$ and $h^g_{nt+1} = \epsilon^g b\left(\left(1-\alpha\right) A h_t - s\right) + b$ considering eqs. (3b), (7b) and (8). Substituting that into column 4 of Table 5 gives column 2 of Table 8.

Under the universal grant scheme where all individuals with college access are eligible for the tuition grants $x_t = z_t/\omega$ (Table 3), we have from (3b) and (7b):

$$h^r_{ct+1} = \epsilon^r b\left(B' h_t - s + z_t/\omega\right) + b \tag{43}$$

$$h^g_{ct+1} = \epsilon^g b\left(B' h_t - s + z_t/\omega\right) + b \tag{44}$$

$$h^g_{nt+1} = \epsilon^g b\left(A' h_t - s + z_t/\omega\right) + b \tag{45}$$

Substituting that into column 4 of Table 5 and using (5b) and the definitions for B' and A' gives column 3 of Table 8.

Under the scholarship program, only high-ability individuals are eligible for the tuition grants $x_t = z_t/p$ (Table 3). We have from (3b) and (7b):

$$h_{ct+1}^{r} = \epsilon^{r} b \left(B' h_t - s\right) + b \tag{46}$$

$$h_{ct+1}^{g} = \epsilon^{g} b \left(B' h_t - s + z_t/p\right) + b \tag{47}$$

$$h_{nt+1}^{g} = \epsilon^{g} b \left(A' h_t - s + z_t/p\right) + b \tag{48}$$

Substituting that into column 4 of Table 5, using (5b) and the definitions for B' and A' gives column 4 of Table 8.

Under the means-tested grant scheme, only high-ability individuals from poor family background are eligible for the tuition grants. Therefore, $x_t = z_t / \left(\left(1 - \lambda\right) p\right)$ (Table 3) and from (3b) and (7b), we have

$$h_{ct+1}^{r} = \epsilon^{r} b \left(B' h_t - s\right) + b \tag{49}$$

$$h_{ct+1}^{g} = \epsilon^{g} b \left(B' h_t - s\right) + b \tag{50}$$

$$h_{nt+1}^{g} = \epsilon^{g} b \left(A' h_t - s + z_t / \left(\left(1 - \lambda\right) p\right)\right) + b \tag{51}$$

Again, substituting that into column 4 of Table 5, using (5b) and the definitions for B' and A' gives column 5 of Table 8.

C.2.3. Stage IV

Similar procedures can be followed to compute the Lorenz curves for Stage IV. One may notice we have already computed the denominators of column 5 of Table 5 in Sections 3.3 and 5. For instance, the denominators under laissez-faire and the universal grant scheme are given by the last equations of (12) and (15). And, the denominators under the scholarship and means-tested programs are similar to the last equations of (17) and (18).

Under laissez-faire, $h_{ct+1}^{r} = \epsilon^{r} b \left(Bh_t - s\right) + b$, $h_{ct+1}^{g} = \epsilon^{g} b \left(Bh_t - s\right) + b$ and $h_{nt+1}^{g} = \epsilon^{g} b \left(\left(1 - \alpha\right) Ah_t - s\right) + b$ and $h_{nt+1}^{r} = \epsilon^{r} b \left(\left(1 - \alpha\right) Ah_t - s\right) + b$ considering eqs. (3b), (7b) and (8). Substituting that into column 5 of Table 5 gives column 2 of Table 9.

Under the universal grant scheme where all individuals with college access are eligible for the tuition grants $x_t = z_t$ (Table 3), we have from (3b) and (7b):

$$h^r_{ct+1} = \epsilon^r b\left(B' h_t - s + z_t\right) + b \tag{52}$$

$$h^g_{ct+1} = \epsilon^g b\left(B' h_t - s + z_t\right) + b \tag{53}$$

$$h^g_{nt+1} = \epsilon^g b\left(A' h_t - s + z_t\right) + b \tag{54}$$

$$h^r_{nt+1} = \epsilon^r b\left(A' h_t - s + z_t\right) + b \tag{55}$$

Substituting that into column 5 of Table 5 and using (5b) and the definitions for B' and A' gives column 3 of Table 9.

Under the scholarship program, only high-ability individuals are eligible for the tuition grants $x_t = z_t/p$ (Table 3), we have from (3b) and (7b):

$$h^r_{ct+1} = \epsilon^r b\left(B' h_t - s\right) + b \tag{56}$$

$$h^g_{ct+1} = \epsilon^g b\left(B' h_t - s + z_t/p\right) + b \tag{57}$$

$$h^g_{nt+1} = \epsilon^g b\left(A' h_t - s + z_t/p\right) + b \tag{58}$$

$$h^r_{nt+1} = \epsilon^r b\left(A' h_t - s\right) + b \tag{59}$$

Substituting that into column 5 of Table 5 and using (5b) and the definitions for B' and A' gives column 4 of Table 9.

Under the means-tested grant scheme, only high-ability individuals from poor family background are eligible for the tuition grants. Therefore, $x_t = z_t/\left(\left(1-\lambda\right)p\right)$ (Table 3) and from (3b) and (7b), we have

$$h^r_{ct+1} = \epsilon^r b\left(B' h_t - s\right) + b \tag{60}$$

$$h^g_{ct+1} = \epsilon^g b\left(B' h_t - s\right) + b \tag{61}$$

$$h^g_{nt+1} = \epsilon^g b\left(A' h_t - s + z_t/\left(\left(1-\lambda\right)p\right)\right) + b \tag{62}$$

$$h^r_{nt+1} = \epsilon^r b\left(A' h_t - s\right) + b \tag{63}$$

Substituting that into column 5 of Table 5, using (5b) and the definitions for B' and A' gives column 5 of Table 9.

D. Proofs for Propositions

This section provides the proofs for the Propositions.

D.1. Proposition 1

Proof.

It is straightforward that it follows from Table 4 and the preceding discussion.

■

D.2. Proposition 2

Proof.

Let the Lorenz curves associated with the laissez-faire, universal, scholarship and means-tested grant schemes are defined as $L(l)$, $L(u)$, $L(s)$ and $L(m)$, respectively. Then from Table 7, given, $\epsilon^g > \epsilon^r$, we want to prove that for the $100\,(1-\lambda p)$ percentile (since the rest are zero):

$$L^{II}(m) > L^{II}(l) > L^{II}(u) > L^{II}(s) \tag{64}$$

where the superscript ('II') denotes the stage of development (Stage II),

$$\begin{aligned}
L^{II}(l) &= \frac{(1-p)\left(\epsilon^r\left(Bh_t - s\right)+1\right)}{\left(p\epsilon^g + (1-p)\,\epsilon^r\right)\left(Bh_t - s\right)+1} \\
L^{II}(u) &= \frac{(1-p)\left(\epsilon^r\left(B'h_t - s + \frac{z_t}{\lambda}\right)+1\right)}{\left(p\epsilon^g + (1-p)\,\epsilon^r\right)\left(B'h_t - s + \frac{z_t}{\lambda}\right)+1} \\
L^{II}(s) &= \frac{(1-p)\left(\epsilon^r\left(B'h_t - s\right)+1\right)}{\left(p\epsilon^g + (1-p)\,\epsilon^r\right)\left(B'h_t - s\right)+\epsilon^g\frac{z_t}{\lambda}+1} \\
L^{II}(m) &= \frac{(1-p)\left(\epsilon^r\left(B'h_t - s\right)+1\right)}{\left(p\epsilon^g + (1-p)\,\epsilon^r\right)\left(B'h_t - s\right)+1}
\end{aligned}$$

Now let's define, for convenience:

$$a_1 \equiv Bh_t - s; a_2 \equiv B'h_t - s;\ b_1 \equiv p\epsilon^g + (1-p)\,\epsilon^r \tag{65a}$$

$$c_1 \equiv Ah_t - s; c_2 \equiv A'h_t - s \tag{65b}$$

Substituting these into the above we get:

$$L^{II}(l) = \frac{(1-p)\left(\epsilon^r a_1 + 1\right)}{b_1 a_1 + 1}$$

$$L^{II}(u) = \frac{(1-p)\left(\epsilon^r\left(a_2 + \frac{z_t}{\lambda}\right) + 1\right)}{b_1\left(a_2 + \frac{z_t}{\lambda}\right) + 1}$$

$$L^{II}(s) = \frac{(1-p)\left(\epsilon^r a_2 + 1\right)}{b_1 a_2 + \epsilon^g \frac{z_t}{\lambda} + 1}$$

$$L^{II}(m) = \frac{(1-p)\left[\epsilon^r a_2 + 1\right]}{b_1 a_2 + 1}$$

First, note that

$$L^{II}(m) > L^{II}(l) \Rightarrow \frac{\epsilon^r a_2 + 1}{b_1 a_2 + 1} > \frac{\epsilon^r a_1 + 1}{b_1 a_1 + 1}$$

$$\Rightarrow b_1 > \epsilon^r \tag{66a}$$

since $\epsilon^g > \epsilon^r \Rightarrow \epsilon^r < b_1$. Second, we can easily verify:

$$L^{II}(u) > L^{II}(s) \Rightarrow \frac{\epsilon^r a_2 + \frac{z_t}{\lambda}\epsilon^r + 1}{b_1 a_2 + b_1 \frac{z_t}{\lambda} + 1} > \frac{\epsilon^r a_2 + 1}{b_1 a_2 + \epsilon^g \frac{z_t}{\lambda} + 1} \tag{66b}$$

The numerator in the left hand side is higher (since $\frac{z_t}{\lambda}\epsilon^r > 0$) while the denominator is smaller (since $b_1 < \epsilon^g$) implying (66b) holds. Third, one can also show that:

$$L^{II}(l) > L^{II}(u) \Rightarrow \frac{\epsilon^r a_1 + 1}{b_1 a_1 + 1} > \frac{\epsilon^r\left(a_2 + \frac{z_t}{\lambda}\right) + 1}{b_1\left(a_2 + \frac{z_t}{\lambda}\right) + 1}$$

$$\Rightarrow a_1 < a_2 + \frac{z_t}{\lambda} \tag{66c}$$

As long as $0 < \lambda < 1$, the last relation holds. Therefore, given (66a), (66b) and (66c), (64) holds.

■

D.3. Proposition 3

Proof.

We see immediately from Table 9:

1. $L^{IV}(u) > L^{IV}(l)$, because the numerators associated with the universal grant scheme are greater than that of the laissez-faire for each cumulative population ratio whereas the denominators remain equal. Particularly, when comparing the numerators of the two columns row-by-row one finds the Lorenz curve for the universal grants strictly dominates that of the one for laissez-faire since $A'h_t + z_t > (1 - \alpha) Ah_t$.
2. $L^{IV}(m) \geq L^{IV}(s)$, because while the denominators for the Lorenz curves associated with the means-tested and scholarship programs remain identical, the numerators for the means-tested program are equal to or higher than that of the one for the scholarship program. When comparing the numerators of the two columns row-by-row, one finds that they are tied for the $100 (1 - \lambda) (1 - p)$ percentile of the population. For any other percentile of the population, the Lorenz curve associated with the means-tested program dominates.

■

D.4. Proposition 4

Proof.

1. For the $100 (1 - \lambda) (1 - p)$ percentile of the population, it is straightforward to see the universal grant scheme Lorenz-dominates the scholarship program ($L^{IV}(u) > L^{IV}(s)$) from Table 9, as the numerator (denominator) related to the former is greater (lesser) than that of the latter.

2. But for the $100(1-\lambda)$ percentile of the population, the scholarship program Lorenz-dominates the universal grant scheme ($L^{IV}(s) > L^{IV}(u)$). To see that first note, given Table 9, we have:

$$L^{IV}(u) = (1-\lambda)\frac{(p\epsilon^g + (1-p)\epsilon^r)(A'h_t - s) + 1 + (p\epsilon^g + (1-p)\epsilon^r) z_t}{(p\epsilon^g + (1-p)\epsilon^r)(Ah_t - s) + 1} \quad (67)$$

$$L^{IV}(s) = (1-\lambda)\frac{(p\epsilon^g + (1-p)\epsilon^r)(A'h_t - s) + 1 + \epsilon^g z_t}{(p\epsilon^g + (1-p)\epsilon^r)(Ah_t - s) + 1 + (1-p)(\epsilon^g - \epsilon^r) z_t} \quad (68)$$

Then, define,

$$d \equiv (p\epsilon^g + (1-p)\epsilon^r)(A'h_t - s) + (p\epsilon^g + (1-p)\epsilon^r) z_t + 1$$
$$f \equiv (p\epsilon^g + (1-p)\epsilon^r)(Ah_t - s) + 1$$
$$g \equiv (1-p)(\epsilon^g - \epsilon^r) z_t = \epsilon^g z_t - (p\epsilon^g + (1-p)\epsilon^r) z_t > 0$$

and rewrite (67) and (68) as

$$L^{IV}(u) = (1-\lambda)\frac{d}{f}$$
$$L^{IV}(s) = (1-\lambda)\frac{d+g}{f+g}$$

One confirms that

$$f > d \Leftrightarrow \frac{d+g}{f+g} > \frac{d}{f}$$

which immediately implies $L^{IV}(s) > L^{IV}(u)$ holds.

3. For the $100(1-\lambda p)$ percentile of the population, we also see the distribution under the scholarship program Lorenz-dominates the one in the universal program or

$$L^{IV}(s) > L^{IV}(u)$$

To see that we first note that from Table 9,

$$L^{IV}(u) = \frac{(1-p)\,\epsilon^r\,(Ah_t - s) + (1-\lambda)\,p\epsilon^g\,(A'h_t - s) + 1 - \lambda p + (1-\lambda)\,p\epsilon^g z_t}{(p\epsilon^g + (1-p)\,\epsilon^r)\,(Ah_t - s) + 1} \tag{69}$$

$$L^{IV}(s) = \frac{(1-p)\,\epsilon^r\,(Ah_t - s) + (1-\lambda)\,p\epsilon^g\,(A'h_t - s) + 1 - \lambda p + \Omega z_t}{(p\epsilon^g + (1-p)\,\epsilon^r)\,(Ah_t - s) + (1-p)\,(\epsilon^g - \epsilon^r)\,z_t + 1} \tag{70}$$

Again we make the following definitions,

$$\begin{aligned}
q &\equiv (1-p)\,\epsilon^r\,(Ah_t - s) + (1-\lambda)\,p\epsilon^g\,(A'h_t - s) + 1 - \lambda p + (1-\lambda)\,p\epsilon^g z_t \\
f &\equiv (p\epsilon^g + (1-p)\,\epsilon^r)\,(Ah_t - s) + 1 \\
m &\equiv (1-p)\,(\epsilon^g - \epsilon^r)\,z_t \\
r &\equiv \lambda\epsilon^g\,(1-p)\,z_t \\
m - r &= \Omega z_t - (1-\lambda)\,p\epsilon^g z_t = ((1-\lambda)\,\epsilon^g - \epsilon^r)\,(1-p)\,z_t
\end{aligned}$$

Then, we rewrite (69) and (70) using our definitions:

$$\begin{aligned}
L^{IV}(u) &= \frac{q}{f} \\
L^{IV}(s) &= \frac{q + m - r}{f + m}
\end{aligned}$$

It follows that

$$\frac{q + m - r}{f + m} > \frac{q}{f} \Leftrightarrow L^{IV}(s) > L^{IV}(u)$$

if

$$m - r > 0 \Leftrightarrow \epsilon^g\,(1-\lambda) > \epsilon^r$$

■

D.5. Proposition 5

Proof.

Comparing the investment thresholds associated with the universal $\overline{h}_i^j(u)$, scholarship $\overline{h}_i^j(s)$, and means-tested $\overline{h}_i^j(m)$ grant schemes in (33), (34) and (35), respectively, to the thresholds associated with laissez-faire $\overline{h}_i^j(l)$ in (10), we see:

1. In Stage I: $\overline{h}_c^g(m) > \overline{h}_c^g(l) > \overline{h}_c^g(u) = \overline{h}_c^g(s)$, the investment threshold associated with the means-tested program is the largest followed by the one for laissez-faire.
2. In Stage II: $\overline{h}_c^g(s) = \overline{h}_c^g(m) > \overline{h}_c^r(l) > \overline{h}_c^r(u)$, the investment threshold for the universal grant scheme is the smallest followed by the one for laissez-faire.
3. In Stage III: $\overline{h}_n^g(l) > \overline{h}_n^g(u) > \overline{h}_n^g(s) > \overline{h}_n^g(m)$, the investment threshold associated with the means-tested program is the smallest followed by the one for the scholarship program. The threshold associated with laissez-faire is the largest.
4. In Stage IV: $\overline{h}_n^r(s) = \overline{h}_n^r(m) > \overline{h}_n^r(l) > \overline{h}_n^r(u)$, the threshold for the universal grant scheme is the smallest followed by the one for laissez-faire.

■

Table 7: Lorenz curves associated with laissez-faire and different grant schemes in Stage II

	Cumulative aggregate human capital ratios			
Cumulative population ratios	**Laissez-faire**	**Universal**	**Scholarship**	**Means-tested**
0	0	0	0	0
$(1-\lambda)(1-p)$	0	0	0	0
$1-\lambda$	0	0	0	0
$1-\lambda p$	$\frac{(1-p)[\epsilon^r(Bh_t-s)+1]}{[p\epsilon^g+(1-p)\epsilon^r](Bh_t-s)+1}$	$\frac{(1-p)\left[\epsilon^r\left(B'h_t-s+\frac{z_t}{\lambda}\right)+1\right]}{[p\epsilon^g+(1-p)\epsilon^r]\left(B'h_t-s+\frac{z_t}{\lambda}\right)+1}$	$\frac{(1-p)[\epsilon^r(B'h_t-s)+1]}{[p\epsilon^g+(1-p)\epsilon^r](B'h_t-s)+\epsilon^g\frac{z_t}{\lambda}+1}$	$\frac{(1-p)[\epsilon^r(B'h_t-s)+1]}{[p\epsilon^g+(1-p)\epsilon^r](B'h_t-s)+1}$
1	1	1	1	1

Table 8: Lorenz curves associated with laissez-faire and different grant schemes in Stage III

	Cumulative aggregate human capital ratios			
Cumulative population ratios	**Laissez-faire**	**Universal**	**Scholarship**	**Means-tested**
0	0	0	0	0
$(1-\lambda)(1-p)$	0	0	0	0
$1-\lambda$	$\frac{(1-\lambda)p[\epsilon^g((1-\alpha)Ah_t-s)+1]}{p(\epsilon^g(Ah_t-s)+1)+\lambda(1-p)(\epsilon^r(Bh_t-s)+1)}$	$\frac{(1-\lambda)p[\epsilon^g(A'h_t-s)+1]+(1-\lambda)p\epsilon^g z_t/\omega}{p[\epsilon^g(Ah_t-s)+1]+\lambda(1-p)[\epsilon^r(B'h_t-s)+1]+\varrho z_t/\omega}$	$\frac{(1-\lambda)p[(A'h_t-s)+1]+(1-\lambda)\epsilon^g z_t}{p[\epsilon^g(Ah_t-s)+1]+\lambda(1-p)[\epsilon^r(B'h_t-s)+1]+(1-p)\epsilon^g z_t}$	$\frac{(1-\lambda)p[\epsilon^g(A'h_t-s)+1\ ]+\epsilon^g z_t}{p[\epsilon^g(Ah_t-s)+1]+\lambda(1-p)[\epsilon^r(B'h_t-s)+1]+(1-p)\epsilon^g z_t}$
$1-\lambda p$	$\frac{(1-\lambda)p(\epsilon^g((1-\alpha)Ah_t-s)+1)+\lambda(1-p)(\epsilon^r(Bh_t-s)+1)}{p(\epsilon^g(Ah_t-s)+1)+\lambda(1-p)(\epsilon^r(Bh_t-s)+1)}$	$\frac{(1-\lambda)p(\epsilon^g(A'h_t-s)+1)+\lambda(1-p)(\epsilon^r(B'h_t-s)+1)+\psi z_t/\omega}{p[\epsilon^g(Ah_t-s)+1]+\lambda(1-p)[\epsilon^r(B'h_t-s)+1]+\varrho z_t/\omega}$	$\frac{(1-\lambda)p(\epsilon^g(A'h_t-s)+1)+\lambda(1-p)(\epsilon^r(B'h_t-s)+1)+(1-\lambda)\epsilon^g z_t}{p[\epsilon^g(Ah_t-s)+1]+\lambda(1-p)[\epsilon^r(B'h_t-s)+1]+(1-p)\epsilon^g z_t}$	$\frac{(1-\lambda)p(\epsilon^g b(A'h_t-s)+1)+\lambda(1-p)(\epsilon^r b(B'h_t-s)+1)+\epsilon^g z_t}{p[\epsilon^g(Ah_t-s)+1]+\lambda(1-p)[\epsilon^r(B'h_t-s)+1]+(1-p)\epsilon^g z_t}$
1	1	1	1	1

Table 9: Lorenz curves associated with laissez-faire and different grant schemes in Stage IV

Cumulative population ratios	**Cumulative aggregate human capital ratios**			
	Laissez-faire	**Universal**	**Scholarship**	**Means-tested**
0	0	0	0	0
$(1-\lambda)(1-p)$	$\frac{(1-\lambda)(1-p)(\epsilon^r((1-\alpha)Ah_t-s)+1)}{(p\epsilon^g+(1-p)\epsilon^r)(Ah_t-s)+1}$	$\frac{(1-\lambda)(1-p)(\epsilon^r(A'h_t-s)+1+\epsilon^r z_t)}{(p\epsilon^g+(1-p)\epsilon^r)(Ah_t-s)+1}$	$\frac{(1-\lambda)(1-p)(\epsilon^r(A'h_t-s)+1)}{(p\epsilon^g+(1-p)\epsilon^r)(Ah_t-s)+(1-p)(\epsilon^g-\epsilon^r)z_t+1}$	$\frac{(1-\lambda)(1-p)(\epsilon^r(A'h_t-s)+1)}{(p\epsilon^g+(1-p)\epsilon^r)(Ah_t-s)+(1-p)(\epsilon^g-\epsilon^r)z_t+1}$
$1-\lambda$	$\frac{(1-\lambda)((p\epsilon^g+(1-p)\epsilon^r)((1-\alpha)Ah_t-s)+1)}{(p\epsilon^g+(1-p)\epsilon^r)(Ah_t-s)+1}$	$\frac{(1-\lambda)((p\epsilon^g+(1-p)\epsilon^r)(A'h_t-s)+1+(p\epsilon^g+(1-p)\epsilon^r)z_t)}{(p\epsilon^g+(1-p)\epsilon^r)(Ah_t-s)+1}$	$\frac{(1-\lambda)(((p\epsilon^g+(1-p)\epsilon^r)(A'h_t-s)+1)+\epsilon^g z_t)}{(p\epsilon^g+(1-p)\epsilon^r)(Ah_t-s)+(1-p)(\epsilon^g-\epsilon^r)z_t+1}$	$\frac{(1-\lambda)((p\epsilon^g+(1-p)\epsilon^r)(A'h_t-s)+1)+\epsilon^g z_t}{(p\epsilon^g+(1-p)\epsilon^r)(Ah_t-s)+(1-p)(\epsilon^g-\epsilon^r)z_t+1}$
$1-\lambda p$	$\frac{(1-p)\epsilon^r(Ah_t-s)+(1-\lambda)p\epsilon^g((1-\alpha)Ah_t-s)+1-\lambda p}{(p\epsilon^g+(1-p)\epsilon^r)(Ah_t-s)+1}$	$\frac{(1-p)\epsilon^r(Ah_t-s)+(1-\lambda)p\epsilon^g(A'h_t-s)+1-\lambda p+(1-\lambda)p\epsilon^g z_t}{(p\epsilon^g+(1-p)\epsilon^r)(Ah_t-s)+1}$	$\frac{(1-p)\epsilon^r(Ah_t-s)+(1-\lambda)p\epsilon^g(A'h_t-s)+1-\lambda p+\Omega z_t}{(p\epsilon^g+(1-p)\epsilon^r)(Ah_t-s)+(1-p)(\epsilon^g-\epsilon^r)z_t+1}$	$\frac{(1-p)\epsilon^r(Ah_t-s)+(1-\lambda)p\epsilon^g(A'h_t-s)+1-\lambda p+\Omega' z_t}{(p\epsilon^g+(1-p)\epsilon^r)(Ah_t-s)+(1-p)(\epsilon^g-\epsilon^r)z_t+1}$
1	1	1	1	1

where

$$\begin{aligned}
\varrho &\equiv p\epsilon^{g}\left(1-\omega\right)+\lambda\epsilon^{r}\left(1-p\right) \\
&= \left(1-p\right)\left[\left(1-\lambda\right)p\epsilon^{g}+\lambda\epsilon^{r}\right] \\
\psi &\equiv p\epsilon^{g}\left(1-\lambda\right)+\lambda\epsilon^{r}\left(1-p\right) \\
\Omega &\equiv \left(1-\lambda\right)\epsilon^{g}-\left(1-p\right)\epsilon^{r}=\Omega'-\lambda\epsilon^{g} \\
\Omega' &\equiv \epsilon^{g}-\left(1-p\right)\epsilon^{r}
\end{aligned}$$