Demographic changes and asset prices in an overlapping generations model

Beatrice D. Simo-Kenge, Frank Riedel and Ghislain H. Demeze-Jouatsa

ERSA Working Paper 884

February 2023

Demographic changes and asset prices in an overlapping generations model

Beatrice D. Simo-Kengne[1], Frank Riedel[2] and Ghislain H. Demeze-Jouatsa[3]

Abstract

We examine the effect of demographic shifts on asset prices in an overlapping generations model with endogenous population dynamics. We establish a robust inverse relationship between returns and the old dependency ratio. We document the absence of a simple monotonic relationship between asset prices and demographic parameters. Returns depend on the joint evolution of fertility, mortality, and lifetime work in a complex way that we quantify. We carry out an extensive empirical study involving 55 countries. Both theoretical and empirical findings reconcile existing propositions on the population age structure and asset returns for riskless and short-lived risky assets.

Keywords: Demography, Asset prices, OLG, Panel cointegration, Granger causality

JEL classification: D9, E44

1 Introduction

Demographic developments have an important impact on the economy as the intergenerational overlap continuously shapes the labor market and the saving behavior of economic agents with consequences on financial equilibria. For instance, an increasing proportion of elderly population stirs up concerns on the sustainability of the social security scheme while a higher young cohort implies health care and education challenges, all of which are to be carried by the working age generation with significant implications on their saving capacity and hence on asset demand.

This paper studies how changes in population structure influence asset returns in equilibrium. To this end, we combine a structured population model with an overlapping generations model. We derive a robust dynamic equilibrium equation that shows that the old dependency ratio (ODR) is inversely related to the long-run population growth as determined by the dominant eigenvalue of the demographics. The analysis further allows for global characterization he equilibrium return based on different demographic

[1] School of Economics, University of Johannesburg, South Africa, Email: bdsimokengne@uj.ac.za, Corresponding author.

[2] Center for Mathematical Economics, Bielefeld University, Germany, and School of Economics, University of Johannesburg, South Africa. Email: frank.riedel@uni-bielefeld.de

[3] Center for Mathematical Economics, Bielefeld University, Germany, Email: demeze.jouatsa@unibielefeld.de support through The World Academy of Sciences and German Research Foundation (TWAS/DFG) is gratefully acknowledged. We are indebted to Prof. Volker Bo¨hm for ample discussions.

manifestations (such as fertility, mortality, lifetime work etc.) which determine the ODR. We describe a variety of equilibria depending on the evolution of different demographic constellations.

We carry out an extensive empirical assessment of our theoretical hypotheses involving data from 55 countries. Our empirical findings confirm the theoretical predictions that the asset demography nexus is best characterized by a variety of equilibria depending on the demographic context. Favorable to the meltdown hypothesis, the cross-country results depict a positive long-term response of equity prices to an increase in the old de- pendency ratio, im- plying a decrease in equity returns in the long run while bond yields exhibit a similar long-term decrease following an increase in the old dependency ratio. Estimation outputs from individual countries show that the old dependency ratio has no significant effect on both asset types in most countries, with only a few countries displaying either a positive or negative long-term response of equity prices and bond yields to an increase in the old dependency ratio. Furthermore, the old dependency ratio and mortality rate appear to Granger-cause equity prices while only population density Granger-causes bond yields.

The debate about the relationship between asset prices and demographics remains controversial since the seminal work of Mankiw and Weil (1988) predicting an asset meltdown in the US housing market due to the retirement of the baby boom generation. Favorable to this melt down hypothesis, Brooks (2000); Geanakoplos et al. (2004); Damato (2012) and Kang (2013) further confirm these predictions for financial assets. However, several studies have refuted the meltdown expectation; suggesting either a slight increase in asset prices as the population ages (Green and Hendershott (1996); Brooks et al. (2006); Kedar-Levy (2006)) or a marginal to no decline effect of mass retirement on asset prices (Poterba (2001); Lim and Weil (2003); Cai (2004); Bovbjerg and Scott (2006); Santoro (2010); Wallick et al. (2013)). In addition, Helmenstein et al. (2002) vindicate the role of various economic and demographic manifestations in alleviating the negative effect of collective retirement on the financial asset. Accordingly, Cornell (2012) emphasizes the mitigating role of global economic and financial integrations in canceling out the potential effect of demographic changes on asset prices.

Despite these appealing conclusions, previous studies are confined to testing the validity of the meltdown assumption with the main finding that risk and uncertainty play a key role in driving the joint dynamics between demographics and asset markets. Conceptually, this tradition calls for a macroeconomic model and namely an overlapping generation framework in which optimal saving decision (with certain and/or uncertain payoffs) is analyzed under the influence of either an exogenous deterministic or a stochastic population development. Accordingly, various OLG models with financial assets exist, that have identified the source of fluctuations of economic and financial variables. Nakata

(2007) develops an OLG model with heterogeneous beliefs to underline the role of communication in explaining the volatility of economic variables and concludes that information asymmetry does not necessary drive large economic fluctuations when allowing for heterogeneous belief. Krueger and Ludwig (2007) employs an overlapping generations model to quantify the impact of the demographic transition towards an older population in industrialized countries on world-wide rates of return.

Kikuchi (2008) points to the importance of financial integration in determining the increase volatility of market using an OLG framework with international asset market which allows for both inter and intra-generational risk diversification. Under the assumption that asset market is the only market where transactions occur between two countries and in the presence of short selling, the author shows the existence of steady states in which risk-adjusted returns for different capital stocks are equal between two countries. Gârleanu and Panageas (2015) document the role of preference heterogeneity in isolating the variation in interest rate from the variation of risk-premium in a continuous time overlapping generations framework. Feng and Hoelle (2017) develop a stochastic OLG model to show that long run effects of indeterminacy is quantitatively more important than endowment shocks in explaining consumption in asset price volatility.

While these studies provide a partial characterization of equilibria for very specific cases of population dynamics, if the goal is to analyze the interaction between demographic manifestations that ensures adjustment of financial portfolio, endogenous population models are required for a global characterization of equilibria. A few exceptions of dynamic optimization with endogenous demographics include Liang and Ma (2015) and Carvalho et al. (2016). In a continuous-time utility framework, Liang and Ma (2015) solve an optimal asset liabilities management problem using a stochastic dynamics programming approach to reveal the influence of salary risk and mortality risk on the optimal investment strategy. Unlike Liang and Ma (2015) who focus on a single demographic characteristic (mortality), Carvalho et al. (2016) build a dynamic general equilibrium model with stochastic population development to show that demographic transition through life expectancy is responsible for about one-third to half of the decline in real interest rates. Particularly, they stress out that aging exerts a downward pressure on long-term real interest rate while population growth has two opposing effects. On one hand, the reduction in population growth rises capital-per worker which depresses the marginal capital per worker and induces the decline in real interest rate. On the other hand, as population growth shrinks, the increase in old dependency ratio lowers aggregate savings which eventually leads to higher real interest rate since retirees save less than workers.

However, the dynamic general equilibrium framework, though more tractable, is computationally less intensive with the cost of lacking flexibility in controlling for empirical age distribution (Carvalho et al. (2016)). Therefore, in line with this reasoning and in order to allow for cross framework comparison, we investigate the effect of population dynamics on asset prices using an OLG model which offers a distinguished feature of accommodating a richer set of demographic characteristics.

Unlike Carvalho et al. (2016), we use an overlapping generations model with three generations, young, middle-aged, and retired, to analyze the impact of demographics on returns. Savings decision is made by individuals from the middle-aged generation who choose the level of savings in the form of asset owned by the retirees' cohort in an exchange economy. The young cohort is assumed to be renewed from the middle-aged population at an exogenous birth rate with a fixed proportion transiting from young to middle-aged each period. Similarly, retirement occurs at different time horizons but follows a constant exogenous aging rate while the old generation disappears at a constant old mortality rate.

These three generations evolve endogenously to characterize the steady-state age structures which determine the adjustment mechanism between asset demand and supply through the intergenerational exchange and this, in turn, shapes the dynamics of asset prices. The rest of the paper is set up as follows. The next section sets up the benchmark model. Sections 3 and 4 present our main results and discuss some limitations which give rise to further extensions explored in Sections 5. Section 6 provides a practical application. The paper ends with concluding remarks and some policy recommendations.

2 An Overlapping Generations Model with Population Dynamics

In this section, we combine a typical overlapping generations model with simple, yet robust demographic dynamics. Agents live for three periods, covering childhood, middle age, and retirement. For simplicity, we consider consumption in middle age and retirement only. The typical middle-aged agent works and earns an exogenous income during the first period and decides how much to save in the form of asset to fund her future consumption in the next period.

Let $U(C_{t+1}^{t}, C_{t+2}^{t})$ be the lifetime utility function of an individual born in period *t* consuming C_{t+1}^{t} in period *t+1* and C_{t+2}^{t} in period *t+2* when retired. We consider a standard time-additive utility function of the form

$$U(C_{t+1}^{t}, C_{t+2}^{t}) = \mathrm{u}(C_{t+1}^{t}) + \theta u(C_{t+2}^{t}) \qquad (1)$$

where $0 < \theta < 1$ is the discount factor. We impose the usual assumptions on period utility, that is, u is strictly increasing, strictly concave, and twice continuously differentiable over the interior of the positive real numbers with $u'(C) > 0, u''(C) < 0$, and $\lim_{c\to 0} u'(C) = +\infty$.

Let W_{t+1} be the exogenous wage income earned by a typical working agent born time *t* in period

$t + 1$, and S_{t+1}^{t} her saving in the form of riskless asset expected to pay off interests at a rate r_{t+1} in the retirement period. The wage income is assumed to grow at an exogenous growth rate a_{t+1} so that

$$w_{t+2}^{t} = (1 + a_{t+1})W_{t+1}. \qquad (2)$$

This agent becomes inactive in the retirement period and therefore consumes the saving proceeds, i.e. $C_{t+2}^{t} = (1 + r_{t+1})S_{t+1}^{t}$. Assuming a utility function with additive preferences and taking consumption as a numeraire, the decision problem of the working agent t consists to maximize her lifetime utility subject to the budget constraint

$$C_{t+1}^{t} + S_{t+1}^{t} = W_{t+1}^{t}\ and C_{t+2}^{t} = (1 + r_t + 1) \qquad (3)$$

For the population dynamics, we use a matrix population model as introduced by Leslie (1945). The cohorts of young, middle-aged, and retired change over time according to the following recursive dynamics.

$$\begin{aligned} Y_T + 1 &= (1 - \gamma)Y_t + \alpha M_t \\ M_t + 1 &= \gamma Y_t + (1 - \beta)M_t \qquad (4) \\ R_t + 1 &= \beta M_t + (1 - \delta)R_t \end{aligned}$$

where Y, M and R are the size of the young, middle-aged and retired cohorts, respectively; α is the fertility captured by the birth rate of middle-aged people, β is the aging rate (i.e., the rate at which adults transit to retirees), γ is the maturity rate from childhood to adult, and δ is the mortality rate. The economy starts with strictly positive cohorts' sizes (Y_0, M_0 and R_0) with demographic parameters $\alpha,\ \beta,\ \gamma,\ \delta$ between 0 and 1.

In equilibrium, at time t, market clearing requires

$$M_t C_t^{t-1} + R_t C_t^{t-2} - M_t W_t \qquad (5)$$

Equations (1), (2), (3), (4) and (5) describe our dynamic economy. We will investigate the effects of demographics on equilibrium interest rates.

Definition 1 *The consumption plans* $\{(C_{t-1}^{t}, C_{t+2}^{t})\}\ t \geq 0$ *and the Interest rate process* $(r_t)t \geq 0$ *form an equilibrium if for all* $t \geq 0$ $(C_{t+1}^{t}, C_{t+2}^{t})$ *maximizes the life utility function U of agents born at time t subject to the budget constraints(3), and markets clear, i.e., (5) holds true.*

The population dynamics can be rewritten as

$$X_{t+1} = A.X_t \quad (6)$$

for the transition matrix (also called Leslie matrix)

$$A = \begin{pmatrix} 1-\gamma & \alpha & 0 \\ \gamma & 1-\beta & 0 \\ 0 & \beta & 1-\delta \end{pmatrix}$$

and the population vector $\begin{pmatrix} Y_t \\ M_t \\ R_t \end{pmatrix}$ The dynamics can be computed quite explicitly; we refer to the appendix A.1 for details. For our setup, the long-run growth rate of the middle-aged

$$MCG = \lim_{t\to\infty} \frac{M_{t+1}}{M_t}$$

cohort plays an important role. The next lemma shows that the limit exists and can be computed explicitly. Its value corresponds to an eigenvalue of the transition matrix.

Lemma 1 The long-run growth rate of the middle-aged cohort is

$$MCG = \frac{1}{2}\left(2-\gamma-\beta+\sqrt{(\beta-\gamma)^2+4\alpha\gamma}\ \right)$$

Another important demographic factor for the equilibrium interest rate is the long-run old dependency ratio,

$$ODR = \lim_{t\to\infty} \frac{R_t}{M_t}$$

which we will characterize now.

Theorem 1 If *$1-\delta < MCG$, the* old dependency ratio (ODR), *the ratio of middle aged to retired* $\frac{R_t}{M_t}$ converges to

$$ODR = \frac{2\beta}{2\delta-\gamma-\beta+\sqrt{(\beta-\gamma)^2+4\alpha\gamma}}$$

The old dependency ratio is related to the long-run growth rate of the middle cohort via

$$MCG = \frac{\beta}{ODR} + 1 - \delta$$

In the following proposition, we record comparative statics for the old dependency ratio and the long-run growth rate of the middle cohort as these properties will play an important role in deriving our hypotheses for the relationship between returns and demographics.

Proposition 1 *The following properties hold for the long-run value of the old dependency ratio (ODR) and the growth of the middle cohort (MCG).*

1. *MCG is increasing in fertility,*

$$\frac{\partial(\mathrm{MCG})}{\partial\alpha} = \frac{\gamma}{\sqrt{(\beta-\gamma)^2+4\alpha\gamma}}$$

2. *ODR is decreasing with fertility,*

$$\frac{\partial(\mathrm{ODR})}{\partial\beta} = -\frac{4\beta\gamma}{\sqrt{(\beta-\gamma)^2+4\alpha\gamma}\,(2\delta-\gamma-\beta+\sqrt{(\beta-\gamma)^2+4\alpha\gamma})^2} < 0.$$

3. *(MCG) is decreasing in the aging rate,*

$$\frac{\partial(\mathrm{MCG})}{\partial\beta} + \frac{\beta-\gamma-\sqrt{(\mathrm{B}-\mathrm{Y})^2+4\alpha\gamma}}{2\sqrt{(\beta-\gamma)^2+4\alpha\gamma}} < 0.$$

4. *ODR is increasing in the aging rate,*

$$\frac{\partial(ORD)}{\partial\beta} = \frac{2\left[\beta\gamma-4\alpha\gamma-\gamma^2+(\gamma-2\delta\sqrt{(\beta-\gamma)^2+4\alpha\gamma}\right]}{\sqrt{(\beta-\gamma)^2+4\alpha\gamma}\,(2\delta-\gamma-\beta+\sqrt{(\beta-Y)^2+4\alpha\gamma})}$$

Let us now tackle the equilibrium. As a benchmark, we consider the logarithmic utility function defined as

$$U(C_{t+1}^t, C_t^t) = \log(C_{t+1}^t) + \theta\log(C_{t+2}^t). \qquad (7)$$

Let $^* = \lim_{t\to\infty} r_t$ be the long-run value of the equilibrium interest rate and $a = \lim_{t\to\infty} a_t$ be the long-run value of the growth rate of the economy.

Theorem 2 *The long-run value of the equilibrium returns on riskless assets is inverse proportional to ODR and proportional to long-run growth:*

$$1 + r^* = \frac{1+\alpha}{ODR} \qquad (8)$$

In particular, the long-run returns

- *decrease with fertility α,*
- *increase in the aging rate β,*
- *decreases in the mortality rate δ.*

The previous theorem generalizes Samuelson's theorem on the biological market interest rate to our setup (Samuelson (1958)). In particular, we derive here the efficient dynamic equilibrium of our overlapping generations model to get a simple, yet robust relation between demographics as a basic hypothesis for our empirical study[4].

The long-term interest rates are increasing in growth and fertility rates but decreasing in aging rate. With respect to the demographic factors, Equation (8) establishes an inverse relationship between equilibrium returns and (ODR), which induces a negative effect of aging on asset returns given the positive association between (*ODR*) and aging rate. This finding is consistent with the results of

[4] On the theoretical side, a number of natural extensions of the basic model are possible, that we leave to future work. In particular, it would be interesting to include our more specific demographics in the model of Krueger and Ludwig (2007) who have only two cohorts and only one demographic parameter.

Carvalho et al. (2016) that long-term real interest rates are subject to downward pressure as longevity increases. Similarly, as fertility is negatively related to (ODR), the equilibrium return is increasing in fertility suggesting a positive impact of population development on returns. This finding partially corroborates Carvalho et al. (2016) who highlight two counteracting effects of population development on long-term real interest rates. On the one hand, a reduction of population growth induces a rise in capital per worker which, in turn, lowers real interest rates through the decline in the marginal product. On the other hand, as population growth shrinks, (ODR) increases which lowering aggregate savings resulting in upward pressure on real interest rates.

Next to the ceteris paribus effects that we get from the comparative statics, it is also interesting to explore the combined effects of demographic parameters on returns. Figure 1 shows the impact on returns of the pair fertility–mortality and fertility–aging rate, respectively. The blue line characterizes the situation of financial neutrality, i.e. a combination of fertility and mortality (resp. aging rate) that leave the returns unchanged. Based on this numerical analysis, we conclude that the impact of demographic variation on asset prices depends on the joint evolution of different demographic manifestations. Demographic changes can produce a positive effect in the presence of a high fertility rate or when a low fertility rate coincides with a high mortality or a high aging rate.

3 Extension to Risky Returns and General Preferences

We are now going to extend the benchmark model to allow for risky assets and more general utility functions. A robustness analysis with respect to more general demographics is performed in the Appendix, Section A.5. The period utilities now exhibit general constant relative risk aversion $u(x) = \frac{x^{1-p}}{1-p}$ for a coefficient of relative risk aversion ρ. The agent born in period t makes consumption and saving decisions in period $t+1$to maximize

$$u(C_{t+1}^t, C_{t+2}^t) = \frac{1}{1-\rho}(C_{t+1}^t)^{1-\rho} + \frac{\theta}{1-\rho}(C_{t+2}^t)^{1-\rho} \tag{9}$$

Using the budget constraints in Equation (3), we can rewrite the objective function of agent t in terms of his wage income and saving. We obtain the following optimal saving and consumption decisions.

$$S_{t+1}^t = \frac{\theta\frac{1}{\rho}(1+r_{t+1})\frac{1-\rho}{\rho}}{1+\theta\frac{1}{\rho}(1+r_t+1)\frac{1-\rho}{\rho}}W_{t+1},$$

$$C_{t+1}^t = \frac{1}{1+\theta\frac{1}{\rho}(1+r_t+1)\frac{1-\rho}{\rho}}W_{t+1}, \tag{10}$$

$$C_{t+2}^{t} = \frac{\theta\frac{1}{\rho}(1+r_t+1)^{\frac{1}{\rho}}}{1+\theta\frac{1}{\rho}(1+r_t+1)^{\frac{1-\rho}{\rho}}} W_{t+1}.$$

Putting the optimal consumption bundles $\{C^{t-1}, Ct^{-2}\}$ into the market clearing condition (see Equation (5)), we get

$$\frac{1}{1+\theta\frac{1}{\rho}(1+r_{t)})^{\frac{1-\rho}{\rho}}} M_t W_t + \frac{\theta\frac{1}{\rho}(1+r_t-1)^{\frac{1}{\rho}}}{1+\theta\frac{1}{\rho}\,(1+r_t-1)^{\frac{1-\rho}{\rho}}} R_t W_{t-1} = M_t W_t$$

Rearranging the later equation and solving for the long-run equilibrium returns on riskless asset $(1+r^*)$, we get

$$(1+r^*) = \frac{1+\alpha}{ODR}.$$

Consistently with the benchmark model, the equilibrium asset prices converge with economic and demographic factors hence revealing different effect depending on the demographic proxy and/or the interaction of demographic manifestations. Therefore, Theorem 2 still applies to the case of general constant relative risk aversion.

Let us now introduce uncertainty. Unlike the previous scenarios, let wage income follow a stochastic process with dynamics.

$$\frac{W_{t+1}}{W_t} = \exp\left(a + b\tilde{\varepsilon}_{t+1}\right)$$

where $(\widetilde{\varepsilon_t})$ are i.i.d. normally distributed random variables with mean 0 and variance 1. Agents born at t put their savings, which we now denote by $I_t^t + 1$, into a one period risky asset with return $1+r_{t+1}$ that is realized at time *t+2*. The agent thus maximizes

$$\max_{C_{t+1}^t + I_{t+1}^t = W_{t+1};\, C_{t+2}^t = (1+r_{t+2}) I_{t+1}^t} E_{t+1}\left[U\left(C_{t+1}^t, C_{t+2}^t\right)\right] \quad (11)$$

where for all $t \geq 0, I_{t+1}^t$ is agent *t*'s investment the risky asset and $U(C_{t+1}^t, C_{t+2}^t)$ is given by Equation (9). The consumption and investment plans $\{(C_{t+1}^t, I_{t+1}^t)\}_{t\geq 0}$ and the risky return $t \geq 0, \{C_{t+1}^t, C_{t+2}^t = (1+r_{t+1}) I_{t+1}^t\}$solve the life-time problem of agent *t* for each $t \geq 0$ (see Equation 11) and markets clears, i.e. (1) holds true.

Theorem 3 *In equilibrium, the risky return is approximately given by*

$$(1+r_t^*) \approx \frac{exp(\alpha + b\tilde{\varepsilon}_t)}{ODR} \quad (12)$$

Demographic changes affect the mean of equilibrium asset returns in the same way they effect the interest rate in the deterministic model. The volatility of growth induces volatility in the risky returns. Once again, the demographic impact on the mean returns is not clear cut. Figure 2 illustrates the existence of multiple response types of mean returns to demographics applied to the zero young and middle aged mortality case, that is

$$ODR = \frac{2\beta}{-\gamma - \beta + 2\delta + \sqrt{(\beta - \gamma)^2 + 4\alpha\gamma}}$$

Since the equilibrium mean returns$\mu *= \alpha - I_n(ODR)$ is determined by four demographic factors, its potential response to any of them depends on the interaction among these factors. This interaction can be null, positive or negative depending on how the expression *ln*(*ODR*) compares with zero. For given numerical values of α and γ, the representation of the lines *ln*(*ODR*) = 0 in the two-dimensional space (β, δ) provides an overview of the later interaction.

For positive values of *ODR* demarcated by the red line, Figure 2 depicts three possible solution areas: null on the blue line, positive in green areas and negative in gray zones. Therefore, consistent with the equilibrium with non-stochastic returns, it emerges that, for given values of fertility (α) and maturity (γ) rates, demographic changes prompted by the interaction between aging and degeneration are likely to exhibit different effects on the long term mean returns. In sum, our model predicts the existence of various types of equilibria depending on the inter- action between different demographic parameters with three possible responses of long-term returns to demographics: positive, negative and null. This finding, in turn, reconciles divergent predictions from previous modeling frameworks.

4 Empirical Evidence

We now take the results of our theoretical analysis to the data. From the equilibrium with non-stochastic returns, we conjecture that the demographic impact on asset prices depends on the trade-off between mortality and fertility rate. Similarly, when uncertainty is accounted for, the mean returns on risky assets has different responses to demographics while the volatility of asset returns is totally driven by business cycle fluctuations.

These theoretical predictions are empirically assessed based on data from 55 countries covering the period 2000–2019. From the seminal studies of the effect of the so-called baby boom on asset returns (such as Geanakoplos et al. (2004)) to the recent evidence of demographic shifts on asset prices (Hettihewa et al. (2018), Chen et al. (2020)), the existing empirical assessment of the life cycle hypothesis has mainly focused on single country analysis.

However, there are substantial variations, both within and across countries, in demographic and economic dynamics with divergent policy responses to these processes, which may ex- plain the predicted various forms of equilibria between asset prices and demographic changes. In addition, contrasting conclusions reported thus far from the empirical literature have often been alluded to economic dynamism as well as political and social factors (Hettihewa et al. (2018)). Unlike previous studies, this study emphasizes the role of demographic rather than socioeconomic and political dynamism in explaining the interplay between asset prices and changes in demographic structure. We use the panel cointegration approach which provides both individual country and panel level inquiries thus allowing for comparative analysis. A cointegration relationship between demographic development and asset prices suggests the existence of a possible long-term convergence to a common trajectory despite their tendency to move randomly in the short run. Furthermore, a causality analysis is carried out to assess the demographic predictability of asset prices. In fact, while cointegrated variables can exhibit causal relationships, cointegration does not imply causation. Beyond identifying the degree of sensitivity between demographic changes and asset prices (cointegration), it is important to ascertain whether knowledge of demographic factors helps predict asset prices, that is, whether demographic factors exert a causal inference on asset prices. This will be achieved following the Granger non-causality test in heterogeneous panels proposed by Dumitrescu and Hurlin (2012). The empirical strategy is explained in detail in the Appendix, Section A.6.

4.1 Data and preliminary analysis

The empirical investigation uses yearly data on 55 countries. The risky and riskless returns were approximated by equity returns and long-term government bond yields, obtained from Thomson Reuters database and the Federal Reserve of Saint Louis database, respectively. Un- like equity prices (full sample), government bond yields were available for only 30 countries (short sample). The demographic and economic variables were drawn from the World Bank's World Development Indicators (WDI) database. They include the old age dependency ratio (ODR), the fertility rate, the mortality rate, the life expectancy (LE), the population growth (POP), and the per capita GDP (PCGDP). In addition, the study controls for the last financial crisis captured by a dummy variable that takes the value 1 for the years 2007, 2008, and 2009 and 0 otherwise. To minimize the scale variation, equity returns, bond yields, and PCGDP are used in their logarithmic form while the rest of the variables remain unchanged.

The graphical illustration in Figure 3 depicts a downward sloping OLS fitted line between asset returns and ODR, suggesting a negative association between asset returns and population aging. However, such a relationship is subject to the existence of confounding factors and thus requires refined econometric modeling. We thus start by testing for stationarity.

This study uses the Im et al. (2003) (IPS) panel unit root test which is suitable for heterogeneous panels and does not require a large time dimension. We test the null hypothesis that all panels have a unit root against the alternative that some panels are stationary. In line with the structure of our dataset, the test accommodates panels with fixed time series and delivers critical values when the number of cross sections is fixed or large. The IPS panel unit root test summarized in Table 1 suggests that all the variables are non-stationary in level and stationary in their first differences, with the exception of ODR which remains non-stationary in its first difference. It is, therefore, possible to find a stationary linear combination of these variables, referred to as cointegration relation.

4.2 Empirical results and discussion

Three blocks of panel cointegration tests are carried out for each asset type with and without the inclusion of the financial crisis dummy, see Table 2. The results reject the null hypothesis of no cointegration for both asset types. However, some variants of the cointegration tests fail to or weakly reject the null hypothesis, which might be attributed to the heterogeneity in cross-country convergence. The estimation of the cointegration equation is therefore required to characterize the short and long run dynamics of the studied variables.

To some extent, the estimation output of the cointegration equation remains consistent in the presence or not of the structural changes and this is valid for both asset types. The ECT term (Panel B of Tables 3 and 4) is negative and significant, confirming the existence of the long term equilibrium between asset prices, demographic and economic developments as expected equity returns and bond yields respond differently to population aging. Under the homogeneity assumption of the sample countries (DFE model), there appears to be no relationship between population aging and equity returns but a negative association between old dependency and bond yields. However, assuming the heterogeneity of the sample countries leads to a different outcome for the equity but not for the bond. The estimation outputs un- der full heterogeneity assumption (MG model) and semi-heterogeneity assumption (PMG) are very close, but the Hausman test of selection between MG and PMG favours the PMG output, which will therefore guide the statistical inference.

Besides the economic factor, population growth and population aging have driven the cross-country growth of equity prices while mortality and the financial crisis have had the opposite effect. These effects generally occur in the long run with the exception of the economic factor, which is equally significant in the short run. As the saving and investment propensity of people increases with their wealth, the improvement in economic performance is expected to fuel the demand for equity and hence the rise in equity prices. Likewise, equity prices in- crease with population aging, resulting in a decrease in asset return. The same pattern emerges from the bond market where population aging tends to reduce bond yields. Note the fast speed of adjustment to short run disequilibrium in the equity market compared to the bond market. Equity returns display at least 97% of adjustment to the previous period's short term disequilibrium against less than 54% for bond prices. As bonds are generally more conservative than stocks, the observed difference could be attributed to the relative stability of the bond market associated with less speculative or profitable opportunities.

A major difference is observed between panel results and individual country output, possibly illustrating the cross-country heterogeneity in the level of demographic as well as economic development. Although of expected sign, the speed of adjustment in individual countries is mostly greater than 1 in absolute value, illustrating a rather oscillatory convergence between asset prices, demographic and economic factors. In addition, population aging appears to be of marginal effect on asset prices in most countries, with only a few of them displaying either a negative or a positive long-term response of asset prices to the rise of the old dependency ratio. Consistently with Table 5, when crisis is controlled for (Table 6) the equity meltdown hypothesis is evidenced in a few countries, namely Denmark, France, India and Indonesia while the opposite effect is shown in Chile, Czech Republic, Estonia, Malta, Portugal and Sri Lanka. Conversely, the meltdown hypothesis in the bond market holds for Czech Republic, Latvia, Finland, Portugal, Russia and Slovakia (Table 7, last column). These findings confirm the theoretical predictions put forth that the asset–demography nexus is best characterized by a variety of equilibria depending on the demographic context rather than a one-size-fits-all equilibrium.

The causal analysis further demonstrates that demographic factors are important predictors of asset returns besides economic output. Particularly, Table 8 shows that the old dependency ratio and the mortality rate Granger-cause equity prices while only population density Granger causes bond yields.

5 Conclusion

We study the impact of demographic changes on asset returns in an overlapping generations model with population dynamics. We establish a robust inverse relationship between returns and the old dependency ratio. Our study also shows how returns depend on the combination of fertility, mortality, and lifetime work time in a complex way that we are able to quantify.

We carry out an extensive empirical assessment of our theoretical hypotheses involving data from

55 countries. Our empirical findings confirm the theoretical predictions that the asset demography nexus is best characterized by a variety of equilibria depending on the demographic context. Favorable to the meltdown hypothesis, the cross-country results depict a positive long-term response of equity prices to an increase in the old dependency ratio, implying a decrease in equity returns in the long run while bond yields exhibit a similar long-term decrease following an increase in the old dependency ratio. Estimation outputs from individual countries show that the old dependency ratio has no significant effect on both asset types in most countries, with only a few countries displaying either a positive or negative long-term response of equity prices and bond yields to an increase in the old dependency ratio. Furthermore, the old dependency ratio and mortality rate appear to Granger-cause equity prices while only population density Granger-causes bond yields.

References

1. Barbara D. Bovbjerg and George A. Scott. *Baby Boom Generation: Retirement of Baby Boomers is Unlikely to Precipitate Dramatic Decline in Market Returns, But Broader Risks Threaten Retirement Security: Report to Congressional Committees*. US Government Accountability Office, 2006.
2. Bradford Cornell. Demographics, gdp, and future stock returns: The implications of some basic principles. *The Journal of Portfolio Management*, 38(4): 96–99, 2012.
3. Carlos Carvalho, Andrea Ferrero, and Fernanda Nechio. Demographics and real interest rates: Inspecting the mechanism. *European Economic Review*, 88: 208–226, 2016.
4. Chihwa Kao. Spurious regression and residual-based tests for cointegration in panel data. *Journal of econometrics*, 90(1): 1–44, 1999.
5. Christian Helmenstein, Alexia Prskawetz, Yuri Yegorov, et al. Wealth and cohort size: stock market boom or bust ahead? *Max-Planck–Institute Working Paper WP*, 51: 2002, 2002.
6. Clive WJ Granger. Investigating causal relations by econometric models and cross-spectral methods. *Econometrica: journal of the Econometric Society*, pages 424–438, 1969.
7. Daniel W. Wallick, Julieann Shanahan, and Christos Tasopoulos. *Baby Boomers and Equity Returns: Will a Boom in Retirees Lead to a Bust in Equity Returns?* Vanguard Research, 2013.
8. Dirk Krueger and Alexander Ludwig. On the consequences of demographic change for rates of returns to capital, and the distribution of wealth and welfare. *Journal of monetary Economics*, 54(1): 49–87, 2007.
9. Elena-Ivona Dumitrescu and Christophe Hurlin. Testing for granger non-causality in heterogeneous panels. *Economic modelling*, 29(4): 1450–1460, 2012.
10. Haim Kedar-Levy. Can baby-boomers' retirement increase stock prices? *The Quarterly Review of Economics and Finance*, 46(2): 284–299, 2006.
11. Hiroyuki Nakata. A model of financial markets with endogenously correlated rational beliefs.
12. James M Poterba. Demographic structure and asset returns. *Review of Economics and Statistics*, 83(4):565–584, 2001.
13. Jauer Chen, Rajarshi Mitra, et al. Demographic shifts and asset returns in Japan". *Economics Bulletin*, 40(2): 1570–1582, 2020.
14. Joakim Westerlund. New simple tests for panel cointegration. *Econometric Reviews*, 24(3): 297–316, 2005.

15. John Geanakoplos, Michael Magill, and Martine Quinzii. Demography and the long-run predictability of the stock market. *Brookings Papers on Economic Activity*, 2004(1): 241–307, 2004.
16. Jong Man Kang. Retirement of baby boomers and its impact on the financial market. *Korean Economic and Financial Review*, 18(3): 57–61, 2013.
17. Junning Cai. Baby boom, asset market meltdown and liquidity trap. *Department of Economics, University of Hawaii*, 2004.
18. Karen Damato. Bad news for boomers. *The Wall Street Journal*, 2012.
19. Kyung So Im, M Hashem Pesaran, and Yongcheol Shin. Testing for unit roots in heterogeneous panels. *Journal of econometrics*, 115(1): 53–74, 2003.
20. Kyung-Mook Lim and David N Weil. The baby boom and the stock market boom. *Scandinavian Journal of Economics*, 105(3): 359–378, 2003.
21. Luciano Lopez and Sylvain Weber. Testing for granger causality in panel data. *The Stata Journal*, 17(4): 972–984, 2017.
22. M Hashem Pesaran and Ron Smith. Estimating long-run relationships from dynamic heterogeneous panels. *Journal of econometrics*, 68(1): 79–113, 1995.
23. M Hashem Pesaran, Yongcheol Shin, and Ron P Smith. Pooled mean group estimation of dynamic heterogeneous panels. *Journal of the American statistical Association*, 94(446): 621–634, 1999.
24. Marika Santoro. *Will the Demand for Assets Fall When the Baby Boomers Retire?* DIANE Publishing, 2010.
25. N Gregory Mankiw and David N Weil. The baby boom, the baby bust, and the housing market, 1988.
26. Nicolae Gaˆrleanu and Stavros Panageas. Young, old, conservative, and bold: The implications of heterogeneity and finite lives for asset pricing. *Journal of Political Economy*, 123(3): 670–685, 2015.
27. Patrick H Leslie. On the use of matrices in certain population mathematics. *Biometrika*, 33 (3): 183–212, 1945.
28. Paul A Samuelson. An exact consumption-loan model of interest with or without the social contrivance of money. *Journal of political economy*, 66(6): 467–482, 1958.
29. Peter Pedroni. Critical values for cointegration tests in heterogeneous panels with multiple regressors. *Oxford Bulletin of Economics and statistics*, 61(S1): 653–670, 1999.
30. Peter Pedroni. Panel cointegration: asymptotic and finite sample properties of pooled time series tests with an application to the ppp hypothesis. *Econometric theory*, 20(3): 597–625, 2004.
31. Richard Green and Patric H Hendershott. Age, housing demand, and real house prices. *Regional Science and Urban Economics*, 26(5): 465–480, 1996.
32. Robin Brooks et al. Demographic change and asset prices. In *Demography and financial markets*, volume 2006, pages 235–261. Reserve Bank of Australia, 2006.
33. Robin Brooks. Life cycle portfolio choice and asset market effects of the baby boom. *International Monetary Fund*, 2000.
34. Samanthala Hettihewa, Shrabani Saha, and Hanxiong Zhang. Does an aging population influence stock markets? evidence from new zealand. *Economic modelling*, 75: 142–158, 2018.
35. Tomoo Kikuchi. International asset market, nonconvergence, and endogenous fluctuations. *Journal of Economic Theory*, 139(1): 310–334, 2008.
36. Zhigang Feng and Matthew Hoelle. Indeterminacy in stochastic overlapping generations mod- els: real effects in the long run. *Economic Theory*, 63(2): 559–585, 2017.
37. Zongxia Liang and Ming Ma. Optimal dynamic asset allocation of pension fund in mortality and salary risks framework. *Insurance: Mathematics and Economics*, 64: 151–161, 2015.

Table 1: Panel stationarity test (IPS unit root)

	Level	First Difference	Decision
Panel A. Full sample			
Equity	0.3562	0.000	I(1)
Old Dependency	1.000	0.9846	I(2)
Fertility	0.9807	0.000	I(1)
Mortality	0.6235	0.000	I(1)
Population growth	0.4116	0.000	I(1)
Life expectancy	0.9590	0.000	I(1)
Economic growth	0.3037	0.000	I(1)
Panel B. Short sample			
Bond	0.0609	0.000	I(1)
Old Dependency	1.000	0.9984	I(2)
Fertility	0.9442	0.000	I(1)
Mortality	0.8981	0.000	I(1)
Population growth	0.4120	0.000	I(1)
Life expectancy	0.1264	0.000	I(1)
Economic growth	0.2874	0.000	I(1)

Note. Figures displayed are p.value of the IPS panel unit root test statistics, which controls for the trend.

Table 2: Panel cointegration tests

Panel A. Kao cointegration tests

H0: no cointegration; Ha: cointegration in all countries

	Risky asset		Riskless asset	
	Without crisis	With crisis	Without crisis	With crisis
Modified Dickey-Fuller *t*	-5.0589***	-5.2567***	-1.1553	-1.7528**
	(0.0000)	(0.0000)	(0.1240)	(0. 0398)
Dickey-Fuller *t*	-5.5181***	-5.5805***	1.5889*	1.267
	(0.0000)	(0.0000)	(0.0560)	(0.1024)
Augmented Dickey-Fuller *t*	-4.1135***	-4.5861***	1.1338	1.1163
	(0.0000)	(0.0000)	(0.1284)	(0.1322)
Unadjusted Modified Dickey-Fuller *t*	-6.2901 ***	-6.3380***	-1.3214*	-1.8410**
	(0.0000)	(0.0000)	(0.0932)	(0.0328)
Adjusted Dickey-Fuller *t*	-6.0324***	-6.0255***	1.4802*	1.2142
	(0.0000)	(0.0000)	(0.0694)	(0.1123)

Panel B. Pedroni Cointegration Test.

H0: no cointegration; Ha: cointegration in all countries

	Risky asset		Riskless asset	
	Without crisis	With crisis	Without crisis	With crisis
Modified Phillips-Perron *t*	6.6011***	7.5833***	5.7868***	6.9543***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Phillips-Perron *t*	-16.8399***	-23.7716***	-8.4571***	-9.7022***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Augmented Dickey-Fuller *t*	-15.6894***	-20.3129***	-9.5424***	-10.8349 ***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)

Panel C. Westerlund Cointegration tests
H0: no cointegration; Ha: cointegration in some (Ha1) or in all (Ha2) countries are cointegrated

	Risky asset		Riskless asset	
	Without crisis	With crisis	Without crisis	With crisis
Variance ratio (Ha1)	-4.9671***	-4.5456***	-2.8095 ***	-2.2534 **
	(0.0000)	(0.0000)	(0.0025)	(0.0121)
Variance ratio (Ha2)	-2.7670***	-2.5482***	-1.3428*	-0.8040
	(0.0028)	(0.0054)	(0.0897)	(0.2107)

Note. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The reported figures are the test statistics with p-values in brackets. "Without crisis" does not control for the great depression. "With crisis" includes the crisis dummy.

Table 3: Long- and short-term effects of population aging on asset prices across countries (without crisis)

	Risky asset			Riskless asset		
	DFE	MG	PMG	DFE	MG	PMG
Panel A. Long-term effects						
Old Dependency	0.0380	0.5328	0.0588***	-0.312***	-0.307	-0.333***
Fertility	-0.0510*	-1.4480	0.00444	0.00803	0.843	-0.0660**
Mortality	0.03410	0.7809	-0.170***	-0.139	2.716*	0.349***
Population growth	0.0366	-12.5826	0.00959	-0.658*	2.015	0.104**
Life expectancy	0.0786	2.1395	-0.0166	-0.313	1.460*	0.160***
Economic growth	0.5956	-10.62227	1.828***	3.525*	-2.553	1.236***
Panel B. Short run effects						
ECT	-0.2636***	-1.4852***	-1.010***	-0.165***	-0.942***	-0.529***
Old Dependency	-0.0375	-0.3374	0.143	0.00208	-0.267	-0.216
Fertility	0.0250	1.8174	-0.550	-0.0706	-0.963**	-0.200*
Mortality	-0.0234	1.3165	-3.555	0.302***	-2.372	-0.451
Population growth	-0.0126	1.55647	0.470	-0.0195	-1.666	-0.113
Life expectancy	-0.0086	4.6218	-7.347*	0.356***	0.428	0.731
Economic growth	1.1654***	6.7116***	3.967***	-0.300	-2.021	0.249
Constant	-1.0014	83.6187	-6.304***	-0.454	-147.5**	-10.11***
Observations	1045	1045	1045	521	521	521
Hausman Test		0.28 (Pr=0.9996)			0.17 (Pr=0.9999)	

Note. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. This estimation output does not account for the great depression, that is, "Without crisis" scenario. Hausman test compares MG and PMG under the null hypothesis that the estimated coefficients are consistent under PMG. This test is chi squared distributed.

Table 4: Long- and short-term effects of population aging on asset prices across countries (with crisis)

	Risky asset			Riskless asset		
	DFE	MG	PMG	DFE	MG	PMG
Panel A. Long-term effects						
Old Dependency	0.0363	1.339*	0.0414***	-0.297***	0.240	-0.354***
Fertility	-0.0343	-1.479	-0.00286	-0.0228	-1.073	-0.0390
Mortality	0.00859	-3.883	-0.106***	-0.0811	-2.701	0.499***
Population growth	0.0128	-0.397	0.0524***	-0.586*	-0.188	0.193***
Life expectancy	0.0714	-1.701	0.0110	-0.251	-1.660	0.280***
Economic growth	0.753*	-3.020	2.043***	2.544	5.027	0.259
Great depression	-0.477***	-0.524	-0.257***	0.707**	0.184	0.104***
Panel B. Short run effects						
ECT	-0.251***	-1.473***	-0.974***	-0.177***	-0.801***	-0.539***
Old Dependency	-0.0723	-2.054**	-0.361	0.0325	1.095	-0.241
Fertility	0.0385	2.275	-0.839	-0.101**	-2.154	-0.284**
Mortality	-0.0110	12.78*	-6.193	0.287***	-7.479	0.0266
Population growth	-0.0104	1.360	0.554	-0.0264	-6.518	-0.151
Life expectancy	0.00450	5.965	-9.614*	0.349***	-2.610	0.789
Economic growth	0.712**	5.246**	2.066***	0.442	4.298	1.175*
Great depression	0.0907***	0.381***	0.215***	-0.00558	-0.229	0.0151
Constant	-1.147	95.62	-9.415***	0.297	-438.7	-10.77***
Observations	1045	1045	1045	521	521	521
Hausman Test		1.75(Pr=0.9722)			0.33 (Pr=0.9999)	

Note. $***p < 0.01$, $**\ p < 0.05$, $*p < 0.1$. This estimation output controls for the great depression, that is, "With crisis" scenario. Hausman test compares MG and PMG under the null hypothesis that the estimated coefficients are consistent under PMG. This test is chi squared distributed.

Table 5: Long term effects of population aging on equity prices by country (without crisis)

Countries	ECT	ODR	Countries	ECT	ODR
Argentina	-1.115**	-2.473	Luxembourg	-1.382***	0.380
Australia	-1.972***	0.0757**	Malaysia	-2.215***	0.434
Austria	-1.796	-0.146	Malta	-0.863**	-0.0578
Belgium	-1.452**	0.0871	Mexico	-0.429	-4.598
Brazil	-1.939***	6.074	New Zealand	-1.454***	0.265*
Bulgaria	-1.230	-0.308	Norway	-1.124***	-0.371
Canada	-1.377***	0.259	Oman	-1.653***	-1.790
Chile	-1.463***	-5.365	Pakistan	-1.512***	-17.82
China	-1.200***	0.353	Peru	-1.196**	-5.089
Croatia	-1.748***	-0.214	Philippines	-1.543***	7.435*
Czech Republic	-1.012**	-0.378**	Poland	-1.088*	-0.103
Denmark	-2.312***	0.214**	Portugal	-1.805***	-0.226***
Egypt	-1.311**	1.801	Romania	-0.698	-1.425
Estonia	-3.013**	-0.0470	Russia	-3.179***	0.263
Finland	-1.468***	0.725**	Singapore	-1.792***	0.0472
France	-1.909***	0.517***	Slovakia	-0.00342	17.44
Germany	-1.339***	0.165	South Africa	-1.930***	3.105**
Hong Kong	-1.491***	0.110	South Korea	-1.539	0.370
Hungary	0.274	1.198	Spain	-1.003	0.0795
India	-1.464***	8.249***	Sri Lanka	-1.445*	-2.832
Indonesia	-0.883*	12.30	Sweden	-1.871*	-0.0111
Israel	-1.710***	-0.123	Switzerland	0.412	0.707
Japan	-2.106***	0.143***	Thailand	-2.215***	-1.204
Jordan	-1.644***	5.194*	Tunisia	-1.286***	2.462
Kenya	-1.474***	6.142*	Turkey	-2.188***	-3.844
Kuwait	-1.362***	-0.218	UK	-1.481***	0.250
Latvia	-1.408***	0.000422	USA	-2.215***	0.194
Lebanon	-2.061***	0.909			

Note. ***$p < 0.01$, **$p < 0.05$, *$p < 0.1$. This output does not account for great depression referring to the “Without crisis” scenario

Table 6: Long term effects of population aging on equity prices by country (with crisis)

Countries	ECT	ODR	Countries	ECT	ODR
Argentina	-1.439***	-1.349	Luxembourg	-1.395	0.572
Australia	-1.961***	0.0836	Malaysia	-0.628	30.48
Austria	-0.543	1.361	Malta	-0.611***	-2.288**
Belgium	-0.806	1.159	Mexico	-0.325	-6.403
Brazil	-1.999***	3.988	New Zealand	-1.440*	0.229
Bulgaria	-0.226	1.460	Norway	-1.376**	0.00377
Canada	-0.137	2.555	Oman	-1.023***	-2.883
Chile	-1.803***	-3.237**	Pakistan	-1.406***	-0.665
China	-0.827*	-0.888	Peru	-1.773***	-4.632
Croatia	-2.337***	-0.0771	Philippines	-1.966***	8.991**
Czech Republic	-1.210*	-0.486***	Poland	-1.604***	-0.0788
Denmark	-2.241***	0.296***	Portugal	-1.807***	-0.232***
Egypt	-0.970	10.81	Romania	-0.881	-1.064
Estonia	-3.390***	-0.208*	Russia	-2.194*	0.352
Finland	-2.242***	0.364	Singapore	-1.624***	0.0745
France	-1.915**	0.506***	Slovakia	-0.118	0.938
Germany	-1.052**	0.256	South Africa	-1.876***	0.334
Hong Kong	-1.686**	0.0887	South Korea	-1.491	0.562
Hungary	-0.216	-1.078	Spain	-2.054***	-0.301
India	-1.501***	16.48**	Sri Lanka	-1.428***	-3.623***
Indonesia	-1.654***	10.97***	Sweden	-0.948	0.0996
Israel	-2.437***	-0.0911	Switzerland	0.115	-2.480
Japan	-1.596***	-0.0693	Thailand	-2.303*	0.902
Jordan	-2.423***	-0.282	Tunisia	-1.951***	0.110
Kenya	-1.402***	12.45	Turkey	-2.089***	-2.039
Kuwait	-1.305***	-0.419	UK	-1.599***	0.114
Latvia	-1.686***	0.114	USA	-2.219***	0.301
Lebanon	-1.993**	1.525			

Note. ***$p < 0.01$, **$p < 0.05$, *$p < 0.1$. This output controls for great depression, that is, "With crisis" scenario

Table 7: Long term effect of population aging on bond yields by country

Countries	Without crisis		With crisis	
	ECT	ODR	ECT	ODR
Australia	-1.693*	-0.160	-2.035	-0.371
Austria	4.155***	0.854**	4.134***	0.711
Belgium	-1.322**	-0.526	-1.364**	0.644
Canada	-1.210***	-0.240	-1.302***	-0.325
Chile	-2.674***	-0.986**	-2.967	-0.981
Czech Republic	-1.797	-2.338***	-2.556	-2.443***
Denmark	-0.472*	1.074	-0.583	0.636
Finland	-2.005***	-0.200	-2.774***	-0.673**
France	-0.0761	6.169	0.310	-1.910
Germany	-0.799*	-0.437	-0.904**	-0.690
Hungary	-1.162**	0.0186	-1.056	-0.0339
Israel	-0.865*	-0.109	-1.289***	-0.207
Japan	-0.292	-0.577	-0.985	0.0998
Latvia	-0.812	-2.015*	-1.980***	-1.539***
Lebanon	2.142	-1.064	2.142	-1.064
Luxembourg	-0.708	-0.809	2.226	8.033
Mexico	-1.293**	-0.0220	-0.707	-0.0736
New Zealand	-0.554	0.0107	-0.341	-0.224
Norway	-1.361*	-0.0748	-1.436	-0.104
Poland	-1.442**	-0.144	-1.520***	-0.191
Portugal	-1.560***	-0.366***	-1.889***	-0.399***
Russia	-0.987***	-0.776	-1.098***	-1.366**
Slovakia	-2.033***	-0.613	-1.934***	-2.791***
South Africa	-0.992*	-1.354	0.221	14.51
South Korea	-0.723	-2.294	-1.308	-1.625
Spain	-1.063	-1.354	-1.059	-1.368
Sweden	-1.693*	-0.160	-1.614	-0.474
Switzerland	4.155***	0.854**	1.814**	1.234***
UK	-1.322**	-0.526	-1.368***	-0.0583
USA	-1.210***	-0.240	-2.035	-0.371

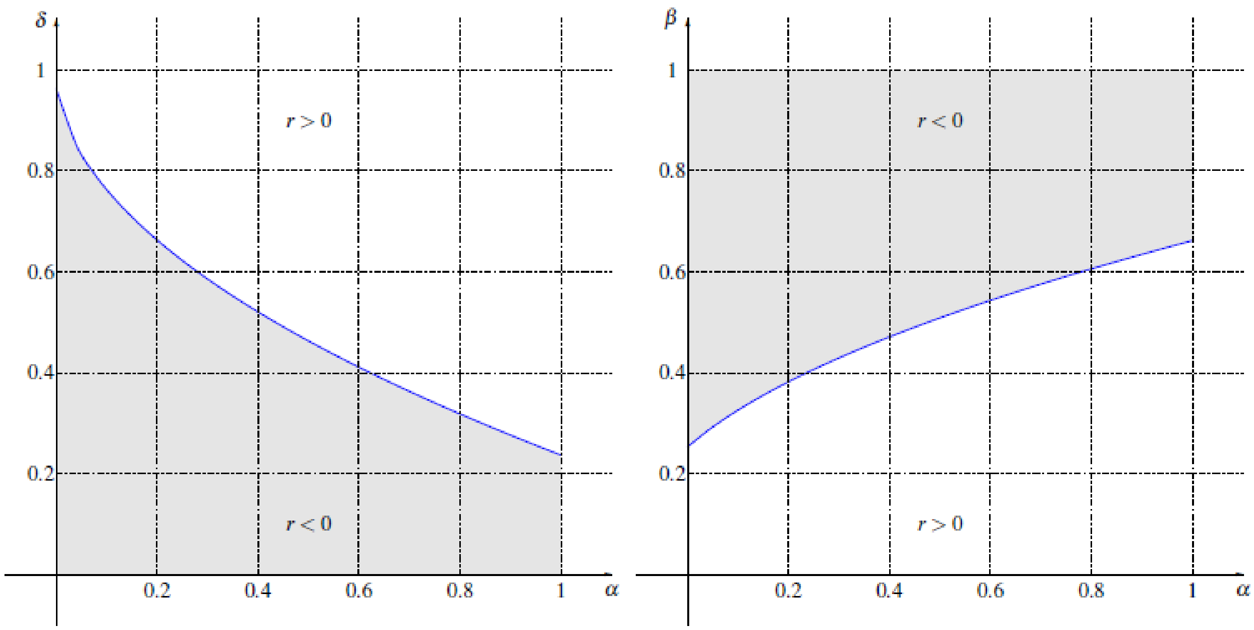
Note. ***$p < 0.01$, ** $p < 0.05$, *$p < 0.1$. “Without crisis” does not control for the great depression. “With crisis” includes the crisis dummy.

Table 8: Causal impact on asset prices

	Equity Prices	Bond yields
Old Dependency	-3.2975***	0.9732
Fertility	1.3522	0.7259
Mortality	2.7536**	0.0532
Population growth	0.2068	2.5673***
Life expectancy	-1.1423	0.2247
Economic growth	1.9151**	0.9294

Note. Figures displayed are Z-bar tilde. The optimal lag length was determined by BIC information criteria and ranges between 1 and 4. Because the Stata command used to perform the causality test does not accommodate missing values, the sample countries with balanced bond yields data was reduced to 20 countries. Because odr is an I(2) variable, the first two years were excluded from the sample period due to the twice difference transformation necessary to achieve stationary odr. This takes the sample period for causal analysis to 2002-to 2019. $***p < 0.01$, $**p < 0.05$, $*p < 0.1$.

Figure 1: impact of pairs of demographic factors on returns and the line of financial neutrality



(a) Fertility and mortality rate for $a = 0.08$; $\beta = 0.5$; $\gamma = 0.6$

(b) Fertility and mortality rate for $a = 0.08$; $\gamma = 0.6$; $\delta = 0.5$

Figure 2: Possible responses of mean returns to the interaction of aging and degeneration rates

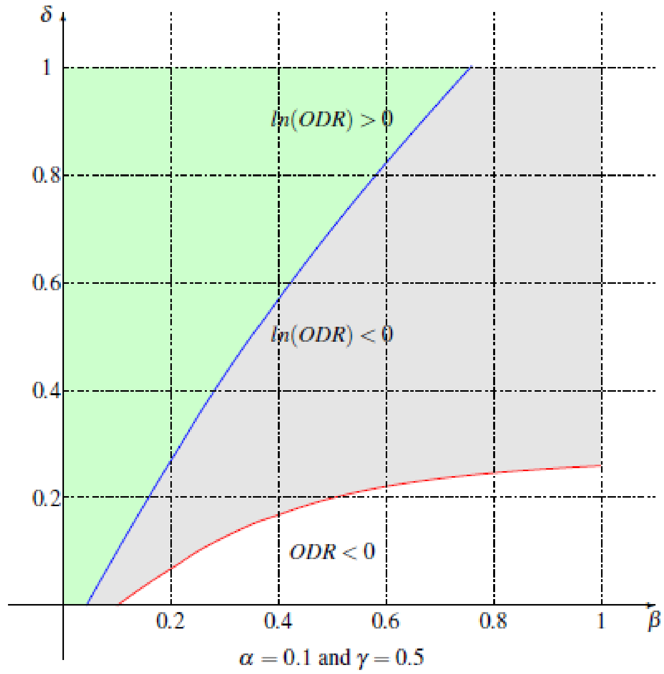
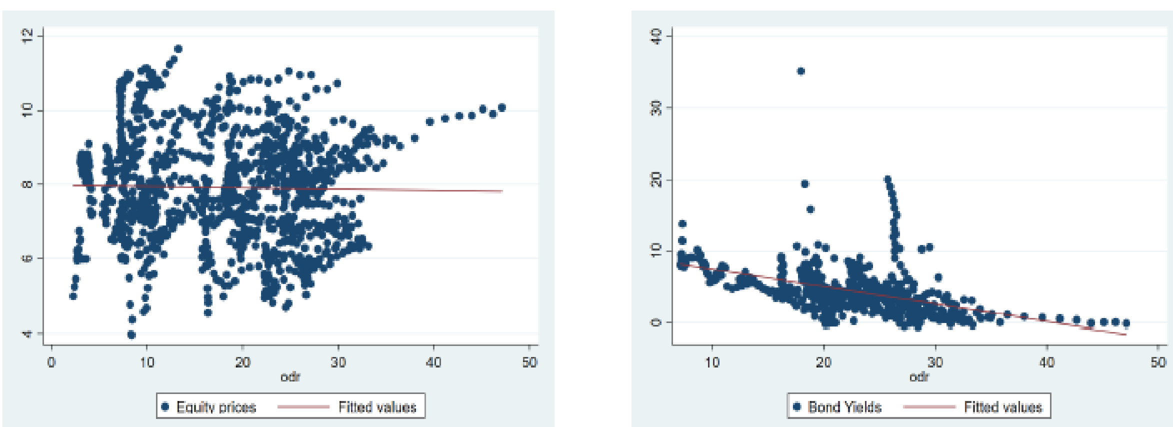


Figure 3: Asset prices and population aging



Note. OLS regression line between equity prices (left Figure)/ government bond yields (right Figure) and old dependency ratio across countries.

A Appendix

A.1 Proof of Lemma 1

The matrix A has the three eigenvalues

$$\lambda_1 = \frac{1}{2}\left(2-\gamma-\beta-\sqrt{(\beta-\gamma)^2+4\alpha\gamma}\right),$$
$$\lambda_2 = \frac{1}{2}\left(2-\gamma-\beta+\sqrt{(\beta-\gamma)^2+4\alpha\gamma}\right),$$
$$\lambda_3 = 1-\delta.$$

The associated eigenvectors are

$$e_1 = \begin{pmatrix} u_1 \\ v_1 \\ w_1 \end{pmatrix} \text{ and } e_2 = \begin{pmatrix} u_2 \\ v_2 \\ w_2 \end{pmatrix}, \; e_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix},$$

with

$$\begin{aligned}
u_1 &= \left(\gamma-\beta+\sqrt{(\beta-\gamma)^2+4\alpha\gamma}\right)\left(\gamma+\beta-2\delta+\sqrt{(\beta-\gamma)^2+4\alpha\gamma}\right)\\
v_1 &= -2\gamma\left(\gamma+\beta-2\delta+\sqrt{(\beta-\gamma)^2+4\alpha\gamma}\right)\\
w_1 &= 4\beta\gamma\\
u_2 &= \left(\gamma-\beta-\sqrt{(\beta-\gamma)^2+4\alpha\gamma}\right)\left(\gamma+\beta-2\delta-\sqrt{(\beta-\gamma)^2+4\alpha\gamma}\right)\\
v_2 &= -2\gamma\left(\gamma+\beta-2\delta-\sqrt{(\beta-\gamma)^2+4\alpha\gamma}\right)\\
w_2 &= 4\beta\gamma.
\end{aligned}$$

Let θ_1, θ_2 and θ_3 be defined by the relation

$$\begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{pmatrix} = P^{-1} \cdot \begin{pmatrix} Y_0 \\ M_0 \\ R_0 \end{pmatrix} \tag{13}$$

where P^{-1} is the inverse of the matrix

$$P = \begin{pmatrix} u_1 & u_2 & 0 \\ v_1 & v_2 & 0 \\ w_1 & w_2 & 1 \end{pmatrix}. \tag{14}$$

The solution of the dynamical system (6) is given by

$$X_t = \theta_1 \lambda_1^t e_1 + \theta_2 \lambda_2^t e_2 + \theta_3 \lambda_3^t e_3. \tag{15}$$

In particular, we have for the middle aged cohort

$$M_t = \theta_1 \lambda_1^t v_1 + \theta_2 \lambda_2^t v_2. \tag{16}$$

Therefore, the growth rate of the middle aged cohort is

$$\frac{M_{t+1}}{M_t} = \frac{\theta_1 \lambda_1^{t+1} v_1 + \theta_2 \lambda_2^{t+1} v_2}{\theta_1 \lambda_1^t v_1 + \theta_2 \lambda_2^t v_2} \tag{17}$$

which converges to λ_2 since $\lambda_1 < \lambda_2$. We conclude

$$\lim_{t\to\infty} \frac{M_{t+1}}{M_t} = \lambda_2. \tag{18}$$

A.2 Proof of Theorem 1

From Lemma 1, the sequence $\frac{M_{t+1}}{M_t}$ converges to the strictly positive eigenvalue $\lambda_2 (= MCG)$. Let $t_0 > 0$ and $p_0 > 0$ such that

$$0 < \frac{1}{\lambda_2} - \frac{1}{p} \leq \frac{M_t}{M_{t+1}} \leq \frac{1}{\lambda_2} + \frac{1}{p} \tag{19}$$

for all $p \geq p_0$ and $t \geq t_0$. For all $t \geq t_0$, let x_t and y_t be defined by

$$x_t = \begin{cases} \frac{R_{t_0}}{M_{t_0}} & \text{if } t = t_0 \\ \beta\left(\frac{1}{\lambda_2} - \frac{1}{p}\right)\left(1 + \frac{1-\delta}{\beta} x_{t-1}\right) & \text{if } t > t_0 \end{cases} \tag{20}$$

and

$$y_t = \begin{cases} \frac{R_{t_0}}{M_{t_0}} & \text{if } t = t_0 \\ \beta\left(\frac{1}{\lambda_2} + \frac{1}{p}\right)\left(1 + \frac{1-\delta}{\beta} y_{t-1}\right) & \text{if } t > t_0 \end{cases} \tag{21}$$

As $1-\delta$ and $\frac{1}{\lambda_2} - \frac{1}{p}$ are positive numbers, it holds by induction that

$$x_t \leq \frac{R_t}{M_t} \leq y_t \tag{22}$$

for all $t \geq t_0$. Now, observe that $\{x_t, t \geq t_0\}$ and $\{y_t, t \geq t_0\}$ are arithmetico-geometric sequences and can be rewriten as follows.

$$x_t = (1-\delta)^{t-t_0}\left(\frac{1}{\lambda_2}-\frac{1}{p}\right)^{t-t_0}\left(\frac{R_{t_0}}{M_{t_0}}-\frac{\beta\left(\frac{1}{\lambda_2}-\frac{1}{p}\right)}{1-(1-\delta)\left(\frac{1}{\lambda_2}-\frac{1}{p}\right)}\right)+\frac{\beta\left(\frac{1}{\lambda_2}-\frac{1}{p}\right)}{1-(1-\delta)\left(\frac{1}{\lambda_2}-\frac{1}{p}\right)} \tag{23}$$

and

$$y_t = (1-\delta)^{t-t_0}\left(\frac{1}{\lambda_2}+\frac{1}{p}\right)^{t-t_0}\left(\frac{R_{t_0}}{M_{t_0}}-\frac{\beta\left(\frac{1}{\lambda_2}+\frac{1}{p}\right)}{1-(1-\delta)\left(\frac{1}{\lambda_2}+\frac{1}{p}\right)}\right)+\frac{\beta\left(\frac{1}{\lambda_2}+\frac{1}{p}\right)}{1-(1-\delta)\left(\frac{1}{\lambda_2}+\frac{1}{p}\right)}. \tag{24}$$

If $1-\delta < \lambda_2$, then

$$0 < (1-\delta)\left(\frac{1}{\lambda_2}+\frac{1}{p}\right) < 1 \tag{25}$$

and

$$0 < (1-\delta)\left(\frac{1}{\lambda_2}-\frac{1}{p}\right) < 1 \tag{26}$$

for sufficiently high p, and the sequences $\{x_t, t \geq t_0\}$ and $\{y_t, t \geq t_0\}$ converges, and respectively to $x(p)$ and $y(p)$. As $\{x(p), p \geq p_0\}$ and $\{y(p), p \geq p_0\}$ converges to the same limit, it follows that the ratio $\frac{R_t}{M_t}$ converges.
If $1-\delta > \lambda_2$, the sequences $\{x_t, t \geq t_0\}$ and $\{y_t, t \geq t_0\}$ diverge to $+\infty$ for sufficiently high p. If follows that the ratio $\frac{R_t}{M_t}$ diverges.

From Equation (4), we have

$$R_{t+1} = \beta M_t + (1-\delta) R_t. \tag{27}$$

Dividing the later equation by βM_{t+1}, we get

$$\frac{M_t}{M_{t+1}} = \frac{1}{\beta}\frac{R_{t+1}}{M_{t+1}} - \frac{1-\delta}{\beta}\frac{R_t}{M_t}\frac{M_t}{M_{t+1}}. \tag{28}$$

Solving Equation (28) for $\frac{M_{t+1}}{M_t}$, we get

$$\frac{M_{t+1}}{M_t} = \frac{\beta}{\frac{R_{t+1}}{M_{t+1}}}\left(1+\frac{1-\delta}{\beta}\frac{R_t}{M_t}\right). \tag{29}$$

It follows that

$$\lim_{t\to\infty}\frac{M_{t+1}}{M_t} = \frac{\beta}{ODR} + 1 - \delta. \tag{30}$$

The reader can check that

$$ODR = \frac{2\beta}{-\gamma-\beta+2\delta+\sqrt{(\beta-\gamma)^2+4\alpha\gamma}} \tag{31}$$

A.3 Proof of Theorem 2

Subject to the budget constraints in Equation (3), the problem of a middle aged agent t (7) can be rewritten as a function of saving and wage income,

$$\max_{S_{t+1}^t} U(W_{t+1}-S_{t+1}^t,(1+r_{t+1})S_{t+1}^t) = \max_{S_{t+1}^t} \log(W_{t+1}-S_{t+1}^t)+\theta\log((1+r_{t+1})S_{t+1}^t).$$

From the resulting first-order conditions, we obtain the optimal savings and consumption decisions

$$S_{t+1}^t = \left(\frac{\theta}{1+\theta}\right)W_{t+1},\ C_{t+1}^t = \left(\frac{1}{1+\theta}\right)W_{t+1},\ C_{t+2}^t = \left(\frac{\theta}{1+\theta}\right)(1+r_{t+1})W_{t+1}.$$

Substituting the optimal consumptions in the market clearing equation (see Equation (5)) leads to

$$M_t\left(\frac{1}{1+\theta}\right)W_t + R_t\left(\frac{\theta}{1+\theta}\right)(1+r_{t-1})W_{t-1} = M_tW_t \tag{32}$$

inducing with Equation (2) the equilibrium returns of

$$(1+r_{t-1}) = \frac{W_t/W_{t-1}}{R_t/M_t} = \frac{(1+a_{t-1})}{R_t/M_t}. \tag{33}$$

Taking the limit as t increases, we get

$$(1+r^*) = \frac{(1+a)}{ODR}.$$

A.4 Proof of Theorem 3

Let us write the risky return in the form $1+r_{t+1} = \exp(\mu+\sigma\tilde{\varepsilon}_{t+2})$ for μ and σ to be determined in equilibrium. The expected utility of agent t then results as

$$\begin{aligned}
\mathbb{E}_{t+1}[U(C_{t+1}^t,C_{t+2}^t)] &= \mathbb{E}_{t+1}\left[\frac{1}{1-\rho}(W_{t+1}-I_{t+1}^t)^{1-\rho}+\frac{\theta}{1-\rho}(I_{t+1}^t)^{1-\rho}(1+r_{t+1})^{1-\rho}\right] \\
&= \frac{1}{1-\rho}(W_{t+1}-I_{t+1}^t)^{1-\rho}+\frac{\theta}{1-\rho}(I_{t+1}^t)^{1-\rho}\mathbb{E}_{t+1}\left[(1+r_{t+1})^{1-\rho}\right] \\
&= \frac{1}{1-\rho}(W_{t+1}-I_{t+1}^t)^{1-\rho}+\frac{\theta}{1-\rho}(I_{t+1}^t)^{1-\rho}\mathbb{E}_{t+1}\exp[\mu(1-\rho)+\sigma(1-\rho)\varepsilon_{t+2}] \\
&= \frac{1}{1-\rho}(W_{t+1}-I_{t+1}^t)^{1-\rho}+\frac{\theta}{1-\rho}(I_{t+1}^t)^{1-\rho}\exp\left[\mu(1-\rho)+\frac{\sigma^2(1-\rho)^2}{2}\right].
\end{aligned}$$

Optimal decisions then result as

$$I_{t+1}^t = \frac{x_0}{1+x_0} W_{t+1}; \;\; C_{t+1}^t = \frac{1}{1+x_0} W_{t+1} \text{ and } C_{t+2}^t = \frac{x_0 \exp(\mu + \sigma \varepsilon_{t+2})}{1+x_0} W_{t+1}$$

where

$$x_0 = \theta^{\frac{1}{\rho}} \exp\left(\frac{1-\rho}{\rho}\mu + \frac{\sigma^2(1-\rho)^2}{2\rho}\right).$$

Substituting the optimal consumption plans $\{C_t^{t-1}, C_t^{t-2}\}$ into the market clearing condition (see Equation (5)), we obtain

$$\frac{R_t}{M_t} \exp(\mu + \sigma \varepsilon_t) = \exp(a + b\tilde{\varepsilon}_t) \tag{34}$$

which induces

$$\mu = a - ln\left(\frac{R_t}{M_t}\right) \text{ and } \sigma = b. \tag{35}$$

The equilibrium return is therefore given by Equation (12).

A.5 Population dynamics with young and middle aged mortality

This section checks the benchmark results for robustness with respect to the introduction of young and middle aged mortality rates. With the introduction of young and middle aged mortality rates, the modified version of the population dynamics becomes:

$$\begin{pmatrix} Y_{t+1} \\ M_{t+1} \\ R_{t+1} \end{pmatrix} = \begin{pmatrix} 1-\gamma & \alpha & 0 \\ \gamma - \delta_Y & 1-\beta & 0 \\ 0 & \beta - \delta_M & 1-\delta_R \end{pmatrix} \begin{pmatrix} Y_t \\ M_t \\ R_t \end{pmatrix} \tag{36}$$

where $\delta_Y, \delta_M, \delta_R$ represent the mortality rate for young, middle aged and retired generations, respectively.

Lemma 2 *The long-run growth rate of the middle-aged cohort is*

$$MCG = \frac{1}{2}\left(2 - \gamma - \beta + \sqrt{(\beta-\gamma)^2 + 4\alpha(\gamma - \delta_Y)}\right).$$

Proof. The corresponding eigenvalues of the matrix of the dynamic system (36) are

$$\lambda_1 = \frac{1}{2}\left(2 - \gamma - \beta - \sqrt{(\beta-\gamma)^2 + 4\alpha(\gamma - \delta_Y)}\right)$$

$$\lambda_2 = \frac{1}{2}\left(2 - \gamma - \beta + \sqrt{(\beta-\gamma)^2 + 4\alpha(\gamma - \delta_Y)}\right)$$

and

$$\lambda_3 = 1 - \delta_R.$$

Consistently with the proof of Lemma 1, the long-run growth rate of the middle aged cohort equals the second eigenvalue, that is $\lim_{t\to\infty} \frac{M_{t+1}}{M_t} = \lambda_2$. ■

Theorem 4 *If* $1-\delta < MCG$*, the* old dependency ratio (ODR)*, the ratio of middle aged to retired* $\frac{R_t}{M_t}$*, converges to*

$$ODR = \frac{2(\beta-\delta_M)}{-\gamma-\beta+2\delta_R+\sqrt{(\beta-\gamma)^2+4\alpha(\gamma-\delta_Y)}}.$$

The old dependency ratio is related to the long-run growth rate of the middle cohort via

$$MCG = \frac{\beta-\delta_M}{ODR} + 1 - \delta_R.$$

Proof.

The growth rate of the middle aged cohort can be expressed as

$$\frac{M_{t+1}}{M_t} = \frac{\beta-\delta_M}{\frac{R_{t+1}}{M_{t+1}}}\left(1+\frac{1-\delta_R}{\beta-\delta_M}\frac{R_t}{M_t}\right). \tag{37}$$

It follows that

$$\lim_{t\to+\infty} \frac{M_{t+1}}{M_t} = \frac{\beta-\delta_M}{ODR} + 1 - \delta_R. \tag{38}$$

Solving for ODR we get the desired expression.

■ Similarly, to the benchmark scenario with zero mortality for young and middle aged cohorts, these dynamics lead to the same comparative statics of the long-run values of the middle cohort growth (MCG) and old dependency ratio (ODR). We have

$$\frac{\partial(ODR)}{\partial\alpha} = -\frac{4(\beta-\delta_M)(\gamma-\delta_Y)}{\sqrt{(\beta-\gamma)^2+4\alpha(\gamma-\delta_Y)}(2\delta_R-\gamma-\beta+\sqrt{(\beta-\gamma)^2+4\alpha(\gamma-\delta_Y)})^2} < 0$$

and

$$\frac{\partial(ODR)}{\partial\beta} > 0$$

since

$$\frac{\partial(MCG)}{\partial\beta} = \frac{1}{2}\left(-1+\frac{\beta-\gamma}{\sqrt{(\beta-\gamma)^2+4\alpha(\gamma-\delta_Y)}}\right) > 0.$$

The results of Theorem 1 are thus robust with respect to the introduction of young and middle aged mortality rates. The expression of the long-run equilibrium return is slightly modified, but leading to comparable impacts with respect to fertility and aging rates. Formally:

$$(1+r^*) = \frac{(1+a)}{ODR}$$

where ODR is given by Theorem 4;

$$\frac{\partial r^*}{\partial \alpha} = \frac{\partial r^*}{\partial (ODR)} \cdot \frac{\partial (ODR)}{\partial \alpha} > 0 \quad (39)$$

and

$$\frac{\partial r^*}{\partial \beta} = \frac{\partial r^*}{\partial (ODR)} \cdot \frac{\partial (ODR)}{\partial \beta} < 0. \quad (40)$$

A.6 Empirical strategy

A.6.1 Panel cointegration test

There are a number of cointegration tests applied to panel dataset, which include Kao (1999), Pedroni (1999), Pedroni (2004), and Westerlund (2005) panel cointegration tests. These tests are built on the following panel-data model:

$$r_{it} = \beta_i X_{it} + Y_{it}'\gamma_i + \varepsilon_{it} \quad (41)$$

where r_{it} is the interest rate and X_{it} denotes the vector of explanatory variables including the demographic, economic and financial variables in country i at time t; ε_{it} is the error term Y_{it} is panel specific means and/or time trend, or nothing, depending of the test options. r_{it} is required to be a $I(1)$ series. $Y_{it} = 1$ by default so the term ${}_{it}^{\prime}\gamma_i$ represents country specific means (fixed effects) with the empirical specification of the form:

$$r_{it} = \beta_i X_{it} + \gamma_i + \varepsilon_{it}. \quad (42)$$

While these tests have the same null hypothesis of no cointegration, the alternative hypothesis has varying formulations. Kao test and Pedroni test have the alternative hypothesis that there is cointegration in all countries. However, Westerlung test has two versions of the alternative hypothesis that cointegration exists either in some countries (Ha1), or in all the countries (Ha2). These residual based tests essentially test whether ε_{it} is nonstationary; the rejection of the null of no cointegration corresponding to ε_{it} being stationary.

A.6.2 Panel cointegration estimation

Besides its flexibility in modelling both short term and long term dynamics, panel cointegration accommodates a wide range of estimation techniques therefore improving the quality of estimates. This ensure low-collinearity and high degree of freedom benefits derived from pooling both cross section and time characteristics. The first generation of panel cointegration estimation include, the Mean Group (MG) the Pooled Mean Group (PMG) and the dynamic fixed effect (DFE). Unlike the DFE estimator which is homogeneous by construction, the MG technique assumes heterogeneity while the PMG method combines both pooling and averaging by imposing equal long-run estimates across countries with all other estimates remaining country-specific (Pesaran and Smith (1995), Pesaran et al. (1999)). Assuming the long-run interest rate model, the one lag dynamic panel specification of (42) is:

$$r_{it} = \gamma_i + \delta_{10i}X_{it} + \delta_{11i}X_{i,t-1} + \lambda_i r_{i,t-1} + \varepsilon_{it}. \tag{43}$$

The error correction representation of (43) is given by:

$$\Delta r_{it} = \phi_i(\lambda_i r_{i,t-1} - \theta_{0i} - \theta_{1i}X_{it}) + \delta_{11i}\Delta X_{i,t-1} + \varepsilon_{it} \tag{44}$$

$$\phi_i = -(1-\lambda_i);\ \theta_{0i} = \frac{\gamma_i}{1-\lambda_i};\ \theta_{1i} = \frac{1}{1-\lambda_i}(\delta_{10i}+\delta_{11i})$$

The parameters are estimated by maximum likelihood. ϕ_i is the error correction term, also known as speed of adjustment parameter and the long-run coefficient, θ_{0i} and θ_{1i} are the key estimates of the interest rate. The presence of θ_{0i} ensures a nonzero mean of the cointegrating relationship. If r_{it} and X_{it} exhibit a return to long-term equilibrium, they are said to be cointegrated and ϕ_i is negative. Equation (44) is, therefore, identical to a fixed effect model with lag dependent variable $(r_{i,t-1})$.

For the sake of robustness, different setups are considered in the estimation of the cointegration equation. Assuming that structural changes may alter the consistency of the estimates, the empirical investigation compares and contrasts the studied phenomenon with and without controlling for the great depression, which appears to be the major structural break for the selected sample period. This is referred to as "Without crisis" and "With crisis" scenarios. In addition, we analyse both panel and individual country effects to account for possible heterogeneity across the panel.

A.6.3 Panel Granger causality test

Dumitrescu and Hurlin (2012) extended the seminal causality model by Granger (1969) to panel data where the existence of causality alludes to the significant effect of the past values of a predictor variable on the current value of the predicted variable. Thus, a demographic variable is assumed to exert a causal effect on asset prices if its past values exhibit a significant effect on the current values of asset prices; resulting in the following formal representation:

$$r_{it} = a_i + \sum_{k=1}^{K} \alpha_{ik} r_{i,t-k} + \sum_{k=1}^{K} \sigma_{ik} x_{i,t-k} + u_{i,t} \text{ with } i = 1, \cdots, N; \ t = 1, \cdots, T; \text{ and } k = 1, \cdots, K; \tag{45}$$

where K is the optimal lag length assumed to be identical across countries, r_{it} is asset prices (either equity or bond price) and $x_{i,t-k}$ is a single demographic factor, both of which are assumed to be stationary. Coefficients are all heterogeneous across countries but time-invariant.

Identifying the existence of a causal effect of x on r amounts to testing the null hypothesis that all coefficients of x are equal to zero. That is: $H_0 : \sigma_{i1} = \cdots = \sigma_{iK} = 0 \ \forall i = 1, \cdots, N$. The null hypothesis is associated to the absence of causality from x to r for all countries in the panel against the alternative hypothesis that there is causality at least for some countries in the panel. That is: $H_1 : \sigma_{i1} = \cdots = \sigma_{iK} = 0 \ \forall i = 1, \cdots, N_1$.
$\sigma_{i1} \neq 0$ or, $\cdots, \sigma_{iK} \neq 0 \ \forall i = N_1 + 1, \cdots, N$ where $N_1 \in [0, N-1]$ is unknown. The causality for all countries in the panel exists when $N_1 = 0$ and $N_1 < N$. If $N_1 = N$, the alternative hypothesis collapses into H_0, implying that there is no causality for all the countries in the panel (Lopez and Weber (2017)).

The testing procedure consists of running the N individual regressions encompassed in Equation 45 after which a F-test of K linear hypothesis $\sigma_{i1} = \cdots = \sigma_{iK} = 0$ allows to extract the individual Wald statistics W_i. Finally, the average statistic (W) can be computed, which has proved to be asymptotically normally distributed. Because W_i are independent and identically distributed, the standardised statistics can be obtained from W, which then follow the standardised normal distribution. Two statistics are proposed based on the relative dimensions of N and T. This study uses $Z-$bar tilde the approximated standardised statistic that accommodates our panel data-like structure with large N relative to T. . If this statistic is greater than the critical value, one should reject H_0 and conclude that x Granger causes r.