'Quantitative Easing' and central bank asset purchases in South Africa: A DSGE approach

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Abstract

This paper develops a small open-economy (SOE) dynamic stochastic general equilibrium (DSGE) model to evaluate the effect of the temporary emergency purchases of government bonds by the South African Reserve Bank (SARB) during 2020. The model is constructed in the portfolio balancing framework, in which the non-bank private sector holds a portfolio of imperfectly substitutable domestic government bonds of different maturities. This allows bond purchases by the central bank, through changing the composition of household bond portfolios, to influence the macroeconomy. The model is calibrated and simulated on South African data. Consistent with similar models of Quantitative Easing simulated for the US and the UK, the results here illustrate that bond purchases by the SARB could have a broader stimulatory macroeconomic impact, over and above the SARB's primary objective of providing liquidity to domestic financial markets. This includes an expansion in the money supply, a fall in long-term government bond yields, and an increase in consumption, inflation and output. However, given the relatively small scale of the SARB's bond purchases, the stimulus effect is modest.

Keywords: open-economy DSGE; central bank asset purchases; quantitative easing; portfolio balance theory
JEL codes: E12, E17, E44, E52

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1. Introduction

The rapid worldwide spread of the Covid-19 pandemic during early 2020 has disrupted global supply chains and sparked fears of a global recession. When several countries shut down non-essential services and industries in the hope of curbing the spread, it became obvious that the world economy would contract significantly. In response to the consequent panic on global financial markets, central banks around the world attempted to ease monetary conditions and support markets and economies by aggressively cutting interest rates. The South African Reserve Bank (SARB) followed suit, and aggressively cut interest rates to provide relief to households and businesses and support domestic financial markets. This accommodation was drastic: The repo rate (the rate at which the SARB lends to commercial banks) was cut by 100 basis points in early March 2020.[1] However, this was deemed insufficient to calm distressed financial markets. On 25 March 2020, the SARB announced that they would commence "a programme of purchasing government securities in the secondary market" (SARB, 2020a:1). The SARB would "create money to buy assets" (SARB, 2020b:2); these bond purchases would therefore be financed by money creation (i.e. unsterilised), with the amount and maturity at the discretion of the SARB. The SARB did not announce the scope and time horizon of the purchases, save that it will be "at the discretion of the SARB and dependent on market conditions", and that the purchases will be "on a bilateral basis, with a variety of market players" (SARB, 2020b:2–3). Dates and sizes of intended purchases would also not be pre-announced (SARB, 2020b).

The stated aim of this bond purchase programme (BPP) was to provide liquidity to promote the smooth functioning of domestic financial markets. Indeed, central banks around the world have again embarked on aggressive asset purchase programmes in response to the current crisis (Haas and Neely, 2020). Previously, these type of programmes were pursued mainly by central banks in developed economies, notably the Fed, Bank of England, the European Central Bank and the Bank of Japan. However, several other emerging economies also embarked upon similar programmes in response to the Covid-19 pandemic and the resultant bond market instability (Arslan, Drehmann, and Hofmann, 2020; Hartley and Rebucci, 2020).

[1] Rates have since been cut by a further 150 basis points, bringing the repo rate to 3.75%, its lowest level since 1973.

To market commentators and analysts it appeared as though the SARB had finally embarked on *quantitative easing* (QE). However, even though the SARB's intervention shares many properties of QE, the SARB has countered that their BPP should not be viewed as QE since they are simply "injecting liquidity into the market [to ensure] a smoothly functioning market, rather than for economic stimulus purposes" (SARB, 2020b). Outright QE has generally been applied where interest rates are close to zero in order to stimulate economic activity and raise inflation, such as the Fed and Bank of England's interventions in response to the global financial crisis, or the Bank of Japan's attempts to prevent deflation since the late-1990s. QE is also associated with targeted purchases of specific assets in order to raise asset prices and lower long-term yields. South African interest rates are not close to the zero lower-bound (ZLB), and the SARB's aim for this programme was explicitly not stimulus. It appears therefore that the SARB's intervention is not strictly 'QE' – and should therefore not be labelled as such – but rather just a programme of asset purchases focused on the temporary dislocation in the government bond market.

The BPP is also not what is traditionally known as 'monetizing' government debt, where newly created central bank money is used to finance government spending, buy newly-issued government bonds, or pay off existing government debt. Even though the SARB Act 90 of 1989 (RSA, 1989:S13(f)) allows the SARB to hold a small amount of government securities, the SARB is not permitted to "lend directly to government or to print money to finance the government deficit" (SARB, 2020b:2). The SARB's intervention was limited to the secondary market, and it therefore does not put newly created money directly in the hands of the fiscus.

However, while the SARB's stated aim was to provide liquidity to the bond market, the announcement of these interventions has already had an additional impact on the macroeconomy. Immediately following the announcement of the programme, yields on long-term government bonds fell. Given that the size of the intervention was unknown at the time, as was the frequency and maturities of purchases, the longer-term impacts on macroeconomic variables are therefore also uncertain. In addition to shoring up financial markets, the injection of cash should influence the money stock and could ultimately spill over to consumption, output and inflation. And what would the impact of additional bond holdings be on the SARB's balance sheet?

As a first step to consider these broader potential economic effects of the

SARB's BPP, this paper constructs a small open-economy DSGE model for the South African economy. It applies the portfolio balance theory through which changes in the private sector's portfolio composition – i.e. relative holdings of short- and long-term bonds – influence the private sector's consumption decision. A novel contribution is the inclusion in the model of this mechanism which allows central bank bond purchases to influence the macroeconomy, something which has not been done in the South African context before. Because the SARB's BPP is not outright QE, the remainder of this paper uses the nomenclature 'asset purchases' or 'bond purchases' to refer to the SARB's purchases of domestic long-term government bonds.

The paper is structured as follows: Section 2 briefly reviews the relevant South African data for the period December 2019 – June 2020. It also considers the theory behind QE and balance sheet policies and the portfolio balance theory. A small open-economy (SOE) DSGE model is then constructed in Section 3. The standard SOE model is augmented by the central bank's balance sheet and a household bond portfolio, through which central bank bond purchases influence the household's optimisation problem. The model is calibrated to the South African economy in Section 4. The simulation results are presented and discussed in Section 5. This section also includes sensitivity analyses. Section 6 concludes.

2. Data and theoretical background

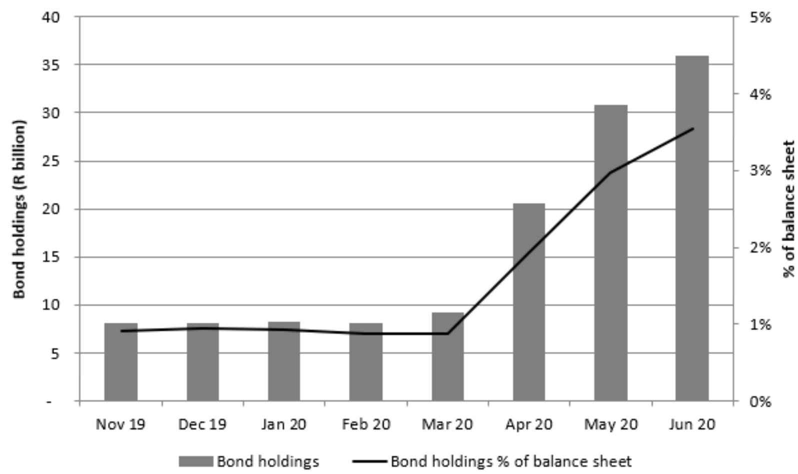
2.1. The South African data

Figure 1 illustrates that the SARB's holdings of government bonds were stable between December 2019 and March 2020, but then spiked sharply from April 2020 onwards. The SARB had in fact already started its bond purchases on 20 March (Kganyago, 2020), five days before announcing the programme. Recent data releases indicate that the SARB purchased around R11.4 billion worth of government securities during April, a further R10.1 billion during May and an additional R5 billion during June (SARB, 2020c). This implies that the SARB's holdings of *total* government bonds just more than doubled after the first month relative to its holdings around the time of the announcement of its BPP, and more than quadrupled by June.[2] Government bond holdings consequently increased from around 0.9% to more than

[2] At the end of February 2020 the SARB held R8.1 billion in government securities, compared to R35.9 billion by the end of May (SARB, 2020c).

3% of the SARB's balance sheet (Figure 1). However, while the SARB only purchased R1.1 billion in bonds during March, its balance sheet expanded by almost 15% in the same month. A large part of this increase can be ascribed to a sharp increase in repurchase agreements, presumably to provide additional accommodation to banks in response to the turmoil in financial markets at the time.

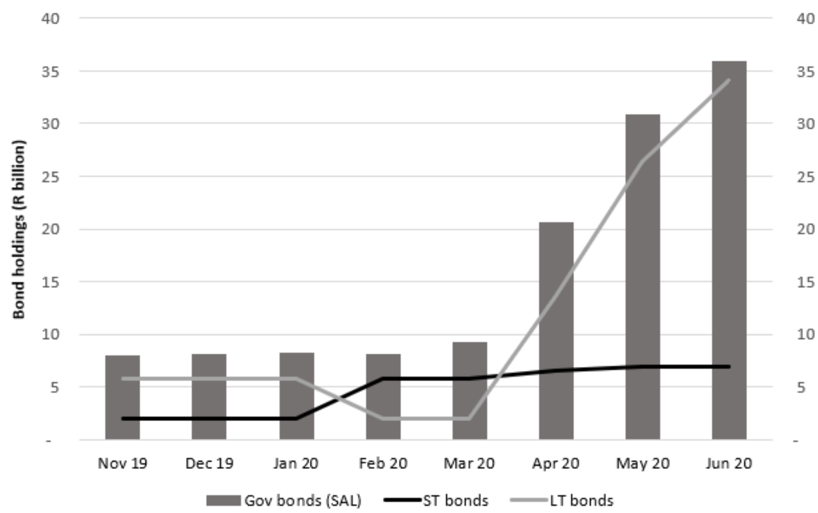
Figure 1: Government bonds (all maturities) as % of the SARB's balance sheet



Source: SARB Statement of Assets and Liabilities (various releases)

The maturity composition of these purchases are not detailed in the SARB's Statement of Assets and Liabilities. However, Quarterly Bulletin data (Table D1) provides a breakdown of short- and long-term government debt held by the central bank. The SARB held about R5.8 billion in long-term government bonds in December 2019. This represented 72% of the SARB's total holdings of government securities at the time. By June 2020 this ratio had spiked to 95% by virtue of an additional R28.2 billion of long-term bond purchases. In addition, the SARB's March-April 2020 bond purchases saw their fraction of outstanding domestic long-term bonds held just about double, while the non-bank private sector's holdings fell by almost a full percentage point.

Figure 2: Maturity composition of SARB's government bond holdings



Source: SARB Quarterly Bulletin, Statement of Assets and Liabilities (various releases)

Consistent with the SARB's statement that bond purchases would be financed with money creation, the monetary base[3] also grew rapidly between February and June 2020. Total government bond holdings increased across all maturities on the SARB balance sheet (i.e. the net effect was non-zero), but especially in longer maturities (Figure 2), supporting the notion that bond purchases were indeed unsterilised.

2.2. The theory of quantitative easing

While the SARB's BPP cannot strictly be called 'QE', it shares two main characteristics with it: Under both (i) long-term fixed income securities are purchased, (ii) financed through expanding the monetary base. As such, the two programmes could be analysed using similar mechanisms. In contrast to the BPP, however, outright QE purchases are directly aimed at influencing "economic activity by altering the structure of private sector balance sheets" (Borio and Disyatat, 2009:8). Since such policies essentially expand "the central bank's balance sheet through asset purchases" (Joyce, McLaren, and Young, 2012:672), it is often referred to as 'balance sheet' policies.

[3] The M1(A) money supply, consisting of notes and coin in circulation plus cheque and transmission deposits of the domestic private sector with monetary institutions.

The mechanics of such balance sheet policies can be illustrated from the perspective of the balance sheets of the central bank, non-bank private sector and the banking sector (Table 1, adapted from Bowdler and Radia (2012)). In this example, the central bank purchases securities in the secondary market from the non-bank private sector. The private sector's holdings of these assets falls, while the central bank's holdings increases. The transaction is financed through the central bank "issuing base money in the form of reserves held by commercial banks" (Bowdler and Radia, 2012:607). The banking sector's balance sheet therefore also expands by these newly created central bank reserves, which are matched against the increased deposits of the non-bank private sector. Given its sheer volume, such an intervention "unusually increases the monetary base" (Fawley and Neely, 2013:52), and is expected to ultimately stimulate economic activity through massive injections of liquidity via the banking sector.

Table 1: Balance sheet effects of QE

Non-bank private sector

Assets	Liabilities
– Securities	
+ Deposits	

Central bank

Assets	Liabilities
+ Securities	+ Reserves

Banking sector

Assets	Liabilities
+ Reserves	+ Deposits

A further objective of outright QE is that of "the central bank seeking to directly affect asset prices" (Bowdler and Radia, 2012:604), such as lowering the yields on longer-term government bonds in order to lower long-term borrowing costs, increase investment spending in other asset classes and ultimately stimulate the economy. Subsequent to the initial announcement of the programme, the SARB indicated that their BPP was not aimed at stimulating demand, but that it should help to "reduce excessive volatility in the price of government bonds" (SARB, 2020b). The potential impact on long-term yields is not a primary goal of the intervention but merely a side-effect or (unintended?) consequence.[4]

However, within an hour after the SARB's announcement of its bond purchase programme (BPP), the yield on South African 10-year government bonds dropped by an astounding 150 basis points (Arslan et al., 2020). At the end of the day of the announcement the 10-year yield was down by 66 basis points (Hartley and Rebucci, 2020). This suggests that just the *announcement* of the programme already played an important signalling role, and likely reduced the risk premium component of the long-term yield.

2.3. The portfolio balance theory

Based on Markowitz (1952) and Sharpe's (1964) seminal theories of portfolio selection and asset pricing, rational investors would adjust their portfolios in response to changes in risk and returns in a certain asset or asset class. Their views were subsequently expanded by Tobin (1969:26), who argues that "when the supply of any asset is increased, the structure of rates of return, on this and other assets, must change in a way that induces the public to hold the new supply". Extending this argument, a change in the supply of one asset would affect both the yield on that specific asset, as well as the spread between returns on that asset and alternative assets (Andrés, López-Salido, and Nelson, 2004). This view has come to be known as the 'portfolio balance theory' (PBT). In the context of central bank balance sheet policies, the PBT suggests that central bank asset purchases, by removing e.g. longer-term government bonds from the secondary market, reduces the supply of these bonds. As a result, the private sector "is left holding money in the form of bank deposits" (Bowdler and Radia, 2012:609). Since long-term government bonds are generally higher-yielding instruments than money, money cannot be viewed as a close substitute for such bonds. Therefore, "changes in relative holdings of the two will induce portfolio rebalancing and movements in asset prices" (Ibid.).

Based on the PBT, central bank asset purchases change investors' portfolios by essentially exchanging (mainly long-term) bonds with (short-term)

[4] A reduction in government's financing costs, in the form of lower government bond yields, would of course offer the fiscus some welcome relief.

money holdings.[5] Investors now have to rebalance their portfolios by investing these increased money holdings elsewhere. To the extent that they could regard other assets as closer substitutes for bonds than money, they would subsequently "reduce their increased money holdings resulting from QE purchases and buy those other assets" (Joyce et al., 2012:694), which would put upward pressure on the prices of those assets. In the absence of substitute assets, the additional cash could simply be allocated towards consumption, or kept as additional money balances.

There is a growing literature studying the macroeconomic effects of central bank asset purchases in general equilibrium models. A dominant paradigm is that of imperfect substitutability between asset classes (notably between short- and long-term bonds); in this context portfolio rebalancing influences bond yields, asset prices broadly, exchange rates and the private sector's saving/consumption optimisation decision. This is therefore the transmission channel through which the SARB's BPP is postulated to affect the macroeconomy, and which the following model aims to capture.

3. A small open-economy DSGE model

South Africa is a small open emerging economy (SOE). New-Keynesian SOE models, for example Ortiz and Sturzenegger (2007) and Steinbach, Mathuloe, and Smit (2009), were previously estimated for South Africa, yet the South African empirical literature on central bank balance sheet policies is virtually non-existent. Moreover, internationally, analyses of balance sheet policies are predominantly performed in a closed-economy setting, with limited application to the SOE setting. This paper therefore aims to contribute to both these gaps.

The model simulated here is an augmented version of the workhorse Galí and Monacelli (2005) small open-economy (SOE) model. The specification developed here draws on models of QE constructed for the US and UK economies following the GFC, including Falagiarda (2014), Harrison (2012) and Chen, Cúrdia, and Ferrero (2012). It is assumed that the *home* (South

[5] In some cases, e.g. the Fed's Operation Twist of 2012, the intervention is sterilised by way of long-term bonds exchanged for short-term bonds instead of cash or reserves. Since the SARB indicated that their BPP would be financed by money creation, and this is supported by the data, sterilised asset purchase programmes are not considered further here.

African) economy is a small emerging economy (i.e. domestic factors do not influence the rest of the world), while the *foreign* economy is advanced. Complete international financial markets, the law of one price (Galí, 2015), and staggered price setting by domestic firms (sticky prices) à la Calvo (1983) are assumed. The home household can purchase both domestic and foreign consumption goods, modeled as a single world good produced both at home and abroad. There are no transaction costs in the international goods market. The home household saves for future consumption by purchasing domestic financial assets.

A novel property of the model constructed here is the addition of a household bond portfolio, in which the home household holds a combination of imperfectly substitutable domestic short- and long-term government bonds. The central bank is allowed to purchase long-term bonds[6] directly from the household, in an attempt to replicate the central bank's purchases of government debt in the secondary market. As in the workhorse SOE model, as well as Falagiarda (2014), Harrison (2012) and Chen et al. (2012), there is no financial sector or intermediary;[7] the central bank therefore trades directly with the home household, representing the non-bank private sector.

3.1. Households

The home economy is populated by a representative household who derives utility from consumption C_t and real money holdings M_t/P_t, and supplies labour N_t to the firm. The household maximises utility according to a standard money-in-the-utility specification

$$\max_{C_t, \frac{M_t}{P_t}, N_t} \quad E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} + \frac{1}{1-\delta} \left(\frac{M_t}{P_t} \right)^{1-\delta} - \frac{N_t^{1+\phi}}{1+\phi} \right] \tag{3.1}$$

where β is the discount rate. σ, δ and ϕ are parameters which respectively represent the coefficient of relative risk aversion (CRRA, equal to the inverse of the elasticity of intertemporal substitution), elasticity of money demand and the inverse of the Frisch elasticity of labour supply. Total consumption

[6]Based on the data (Section 2.1), the SARB's interventions were clearly focused on bonds of longer maturities.

[7]This is also the approached followed in the domestic literature, e.g. Ortiz and Sturzenegger (2007) and Steinbach et al. (2009).

consists of consumption of both home and foreign goods, with C_t representing a composite consumption index given by

$$C_t \equiv \left((1-\upsilon)^{\frac{1}{\eta}} C_{H,t}^{1-\frac{1}{\eta}} + \upsilon^{\frac{1}{\eta}} C_{F,t}^{1-\frac{1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \tag{3.2}$$

where $C_{H,t}$ and $C_{F,t}$ represent consumption of home and foreign goods, respectively (Galí, 2015:225). $\upsilon > 0$ is a measure of openness and $\eta > 0$ measures the degree of substitutability between foreign and domestic goods.[8]

Two types of securities are traded in domestic financial markets: short-term (one-period) and long-term bonds, both issued by the home government. New issues of long-term bonds can be purchased by home households (representing the domestic private sector), the home central bank[9] and other (including foreign) investors, while there exists a secondary market for trading these securities. The home household alone purchases short-term bonds, and therefore holds a portfolio of domestic short- and long-term government bonds (B_t and $B_{L,t}^H$, respectively) which generates returns to be used mainly for future consumption. The household's real budget constraint is given by

$$\begin{aligned} C_t + T_t + \frac{M_t}{P_t} + \frac{B_t}{P_t R_t} + \frac{B_{L,t}^H}{P_t R_{L,t}}(1 + AC_t) \\ = \frac{W_t}{P_t} N_t + \frac{M_{t-1}}{P_t} + \frac{B_{t-1}}{P_t} + \frac{B_{L,t-1}^H}{P_t R_t} \end{aligned} \tag{3.3}$$

The LHS of equation 3.3 shows that the household allocates their wealth among consumption C_t, payment of a lump-sum tax T_t, real money holdings $\frac{M_t}{P_t}$, and bond purchases. The household holds two types of zero-coupon bonds: short-term (one-period) bonds B_t and long-term bonds $B_{L,t}^H$. The nominal price of these bonds in time t are given by the yields R_t and $R_{L,t}$, respectively.[10] There is an adjustment cost (AC_t) attached to purchasing

[8] It is easy to see that in the extreme case of $\upsilon \to 0$ (i.e. a closed economy), total consumption consists of domestic consumption only.

[9] The SARB is not permitted to excessively subscribe to new issues of Treasury stocks (RSA, 1989:13(f)). However, as the government's banker it is not unreasonable to assume that the SARB holds a small amount of government bonds on its books.

[10] Bonds are priced or valued at their gross interest rate (Falagiarda, 2014), which is the standard treatment in the literature of zero-coupon bonds. For example, if the face value is R1,000 (B_t = R1,000) and the discount rate is 4%, $R_t = 1 + 0.04 = 1.04$ and $\frac{B_t}{R_t}$ $\approx$ R961. The household therefore purchases a one-period bond for R961 in period t that will pay out R1,000 in period $t + 1$.

long-term bonds, which prevents market participants from taking full advantage of arbitrage opportunities (Chen et al., 2012). Households therefore have to allocate their discretionary spending between contemporaneous consumption and bond purchases (investment in financial assets). Intuitively, higher expected returns on bonds might encourage households to forgo current consumption in order to invest in bonds which would enable higher future consumption.

The RHS of equation 3.3 represents the household's real wealth or total income in time t. Following Andrés et al. (2004), households enter period t with a certain level of real money holdings and a bond portfolio, consisting of maturing one-period bond holdings, purchased in period $t-1$, and net holdings of long-term bonds following transactions[11] in period $t-1$. They earn a nominal wage of W_t during period t. The final two terms in equation 3.3 capture the *ex post* returns on short- and long-term bonds (Harrison, 2012). Total income thus consists of real wage income $\frac{W_t}{P_t}N_t$, real money holdings brought forward from the previous period $\frac{M_{t-1}}{P_t}$ and real earnings realised from holding bonds in the previous period.

$\frac{B_{t-1}}{P_t}$ and $\frac{B^H_{L,t-1}}{P_t R_t}$ represent real earnings on holdings of short-term bonds B_t and long-term bonds $B^H_{L,t}$ brought forward from the previous period.[12] At time t, the returns (or bond prices or values) R_t and $R_{L,t}$ are known. However, in the presence of a secondary market for bond trading, long-term bonds are "subject to price risk prior to maturity" (Ljungqvist and Sargent, 2012:375) and therefore $R_{L,t}$ cannot be known at time $t-1$. An agent who brings a bond portfolio into time t would therefore be uncertain about its value at the start of the period. If B_t and $B^H_{L,t}$ are viewed as the household's net holdings of short- and long-term bonds between periods t and $t+1$, that each pay a unit of the consumption good at time $t+j$, it follows from a simple arbitrage argument that in period t long-term bonds represent "identical sure claims to consumption goods at the time of the end of the maturity as newly issued one-period bonds in period t" (Falagiarda, 2014:309). Subsequently, at the beginning of period t, long-term bonds in the household's portfolio are valued at the same rate as one-period bonds, namely the money market rate

[11]Such transactions could include new bond purchases from the government and/or net sales to the central bank.

[12]Recall that the short-term bond was purchased at a discount in the previous period based on the prevailing interest rate R_{t-1}. It now pays out its nominal value of unity.

R_t.[13]

Finally, portfolio adjustment frictions are imposed to allow segmentation in financial markets, which represent "impediments to arbitrage behaviour that would equalise asset returns" (Falagiarda, 2014:309). Households are subsequently assumed to face bond transaction (portfolio adjustment) costs given by

$$AC_t = \left[\frac{\phi_L}{2} \left(\kappa_L \frac{B_t}{B^H_{L,t}} - 1 \right)^2 \right] Y_t \tag{3.4}$$

where κ_L is a parameter describing the steady state ratio of long-term to short-term bond holdings ($\bar{B}^H_L/\bar{B}$) and ϕ_L is a parameter for portfolio adjustment frictions. Households therefore incur a cost whenever their portfolio allocation between short- and long-term bonds deviates from its steady state allocation, which is paid in terms of income Y_t. This cost can be intuitively explained by the preferred-habitat theory, where agents could have set preferences for certain bond maturities over others (Vayanos and Vila, 2009; Chen et al., 2012; Kabaca, 2016). It would be costly for the household to return to this preferred habitat in the event of a disturbance or shock. Others have interpreted it as reflecting a liquidity risk attached to holding long-term bonds (Andrés et al., 2004), or simply the cost of managing the household's bond portfolio (Falagiarda, 2014). Adjustment costs are zero in steady state, as the household's portfolio will be allocated optimally.[14]

This adjustment cost framework allows for asset purchases by the central bank to directly influence the private sector's (households') spending decisions: Insofar as asset purchases remove long-term bonds from the household's portfolio it will disturb the ratio $B_t/B^H_{L,t}$. The larger the disturbance, the larger to cost to households. This could be illustrated by a simple hypothetical example. If we assume the household's 'preferred habitat' is a (steady state) ratio of long-term to short-term bonds of 2:1, we have $\kappa_L = \bar{B}^H_L/\bar{B} = 2$. Central bank asset purchases now removes a significant portion of long-term bonds from the portfolio of households in exchange for cash balances (money holdings). That is, $B^H_{L,t}$ falls and M_t increases, while B_t remains unchanged.

[13] See also Ljungqvist and Sargent (2012:375–376). In period t, j-term bonds are traded at the price R_t, since R_{t+j} is not known at time t.

[14] In steady state $\kappa_L \frac{B_t}{B^H_{L,t}} = \frac{\bar{B}^H_L}{\bar{B}} \frac{\bar{B}}{\bar{B}^H_L} = 1$ and equation 3.4 thus collapses to zero.

The immediate effect of this central bank action is that the ratio of long-term to short-term bonds $B^H_{L,t}/B_t$ falls to, say, 1:1, and as a result $AC_t = \frac{\phi_L}{2} Y_t > 0$. If the central bank's asset purchase programme is even larger and the ratio $B^H_{L,t}/B_t$ thus falls even further, say to 1:2, $AC_t = 9\frac{\phi_L}{2} Y_t > \frac{\phi_L}{2} Y_t > 0$. Clearly, the larger the asset purchase programme, the higher the adjustment cost to households. Conversely, the more weighted the steady state portfolio is toward long-term bonds (i.e. a larger κ_L), the relatively smaller the adjustment cost following identical central bank bond purchases is likely to be.

This framework also illustrates how asset purchases could have an expansionary effect on the economy. Following the central bank's intervention, households need to decide how to allocate their new-found liquid reserves. Clearly, there is limited scope for the household to immediately restore its preferred habitat. Immediately allocating the cash to buying short-term bonds will distort the preferred habitat even more, while there is limited availability elsewhere of long-term bonds. The only realistic source of long-term bonds is the government,[15] but the contemporaneous supply (and price) of long-term bonds might not be adequate to fully restore the household's preferred habitat. It will now be more expensive[16] for the household to purchase long-term bonds, which, coupled with the inelastic supply of long-term bonds in the short run, makes it highly unlikely that the household will be able to simply restore their portfolio. It follows that a portion of this new-found liquidity will be allocated towards additional consumption, ultimately contributing to higher output and inflation. Therefore, the household's return to its preferred habitat is likely to be gradual, rather than immediate.

The household's optimality conditions are given by

$$1 = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{R_t}{\pi_{t+1}} - \frac{R_t}{R_{L,t}} \kappa_L \phi_L \left(\kappa_L \frac{B_t}{B^H_{L,t}} - 1 \right) Y_t \tag{3.5}$$

$$\frac{W_t}{P_t} = \frac{N_t^{\phi}}{C_t^{-\sigma}} \tag{3.6}$$

[15] In an open economy setting there are of course other options available, such as foreign long-term bonds. This model could also be expanded to include investment in firm capital or other (substitute) financial assets such as equities.

[16] When bond yields fall, bond prices increase.

$$\left(\frac{M_t}{P_t}\right)^{-\delta} = C_t^{-\sigma}\left[1 - \frac{1}{R_t} - \frac{\kappa_L \phi_L Y_t}{R_{L,t}}\left(\kappa_L \frac{B_t}{B_{L,t}^H} - 1\right)\right] \tag{3.7}$$

$$R_{L,t} = R_t R_{t+1}\left[1 - \frac{1}{2}\phi_L\left(\kappa_L \frac{B_t}{B_{L,t}^H} - 1\right)^2 Y_t\right] \tag{3.8}$$

Equation 3.5 is the consumption Euler equation, 3.6 is the household's labour supply condition and 3.7 is the real money demand schedule. In the absence of portfolio adjustment frictions ($\phi_L = 0$) there is no wedge between the short- and long-term interest rates, and the Euler and money demand equations collapse to their familiar expressions.[17] Equation 3.8 is the first-order condition for long-term bond holdings, which gives an expression for the long-term interest rate. Gross inflation is defined as $\pi_t \equiv P_t/P_{t-1}$, while in steady state $R = 1/\beta$ and $R_L = 1/\beta^2$.

The Euler equation and term structure can be further considered by log-linearising[18] 3.5 and 3.8 and solving for $\hat{c}_t$ and $\hat{r}_{L,t}$.

The Euler equation:

$$\hat{c}_t = E_t[\hat{c}_{t+1}] - \frac{1}{\sigma}(\hat{r}_t - E_t[\hat{\pi}_{t+1}]) - \frac{\Psi_1}{\sigma}(\hat{b}_{L,t}^H - \hat{b}_t) \tag{3.9}$$

The term structure:

$$\hat{r}_{L,t} = \hat{r}_t + E_t[\hat{r}_{t+1}] + \Psi_2(\hat{b}_{L,t}^H - \hat{b}_t) \tag{3.10}$$

$\Psi_1 = \beta\kappa_L\phi_L\bar{Y}$ and $\Psi_2 = \phi_L\bar{Y}(1 + \beta\kappa_L)$ are convolutions of the steady state parameters.

3.1.1. Euler equation

The first two terms of equation 3.9 are virtually identical to the workhorse model (Galí and Monacelli, 2005:54), which indicates that the household's consumption decision is a function of expected consumption and the expected

[17] $1 = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{R_t}{\pi_{t+1}}$ and $\left(\frac{M_t}{P_t}\right)^{-\delta} = C_t^{-\sigma}\left(1 - \frac{1}{R_t}\right)$. See Galí (2015) for the text book derivation.

[18] The full log-linearised system is given in Appendix C.

real interest rate in the next period. In addition to the canonical model, however, the ratio of the household's long- to short-term bond holdings also influences the consumption decision. An increase in this ratio $(\hat{b}^H_{L,t} - \hat{b}_t)$ implies that the household now holds relatively more long-term bonds in its portfolio. Intuitively, by foregoing current consumption, households could invest in additional holdings of long-term bonds, which – through the future payoffs of said bond holdings – could generate higher future consumption. Conversely, a fall in this ratio (see the earlier hypothetical example) implies the removal of long-term bonds from the household's portfolio in exchange for more liquid assets (e.g. money), which are likely to be used to finance current consumption.

3.1.2. Term structure

Theories of the term structure suggest that the long-term interest rate is a function of the expected future path of short-term interest rates and a term premium which captures factors such as investors' preferred habitat and a liquidity premium (Mishkin, 2013). The first two terms of equation 3.10 represent the expected future path of short-term interest rates, while the final term is a measure of the household's preferred habitat. In the absence of portfolio adjustment frictions ($\phi_L = 0$), the liquidity premium disappears and the long-term interest rate is equal to the expected future path of short-term rates. The long-term interest rate here depends positively on the household's holdings of long-term bonds and negatively on the household's holdings of short-term bonds, due to the imperfect substitutability between the two asset classes. Therefore, consistent with Tobin's (1969) theory, equation 3.10 suggests that asset purchases by the central bank, by reducing the supply of long-term bonds available to the household, would reduce the long-term interest rate. This is neatly summarised by Falagiarda (2014:314), who states that "to get agents to accept the fact that holding a larger (smaller) fraction of short-term bonds in their portfolio, the spread between the two rates has to decrease (increase)". The implication is therefore also that central bank asset purchase will flatten the yield curve by pushing down long-term interest rates.

3.2. Government

The home government issues short- and long-term bonds, while raising a lump-sum tax on the home household. Short-term (one-period) bonds are held by the household only, while long-term bonds can be held by the household, central bank and other investors. Taxes raised on the household T_t plus new bond issuance finance the government's debt financing costs. The government's real budget constraint is therefore given by

$$T_t + \frac{B_t}{P_t R_t} + \frac{B_{L,t}}{P_t R_{L,t}} + \frac{\Delta_t}{P_t} = \frac{B_{t-1}}{P_t} + \frac{B_{L,t-1}}{P_t R_t} \tag{3.11}$$

The LHS of equation 3.11 represents the government's income, consisting of taxes and the total value of bond issuance (price × quantity of bonds issued). Δ_t represents the change in the central bank's balance sheet and is discussed in detail in section 3.3 below. The RHS represents the debt burden at the beginning of time t.[19] Similar to the household's problem above, new long-term bonds are priced at $R_{L,t}$, while the outstanding long-term debt burden at the start of time t is valued at R_t.

The government's supply of long-term bonds follows a stochastic AR(1) process of the form

$$\log\left(\frac{B_{L,t}}{\bar{B}_L}\right) = \phi^{BL} \log\left(\frac{B_{L,t-1}}{\bar{B}_L}\right) + \varepsilon_t^{BL} \tag{3.12}$$

where ε_t^{BL} is a stochastic error term with mean of zero and standard deviation of σ^{BL}. The total issuance (supply) of long-term bonds is taken up between households, the central bank and other investors, that is, $B_{L,t} = B_{L,t}^H + B_{L,t}^{CB} + B_{L,t}^F$.

The total amount of taxes raised T_t is a function of the government's outstanding liabilities and is expressed as (Falagiarda, 2014:312)

$$T_t = \bar{T} + \psi_1 \left[\frac{B_{t-1}}{P_t} - \frac{\bar{B}}{\bar{P}}\right] + \psi_2 \left[\frac{B_{L,t-1}}{R_t P_t} - \frac{\bar{B}_L}{\bar{R}\bar{P}}\right] \tag{3.13}$$

where $\bar{T}$ represents the steady state level of T_t.[20] Taxes therefore react to deviations of government liabilities from its steady state levels, and respond to shortfalls from the previous period.

[19] Equation 3.11 can be rearranged to show that net bond issuance plus the change in the central bank balance sheet finances net transfers to/from households T (Harrison, 2012:9).

[20] In steady state $B_{t-1} = \bar{B}$ and $B_{L,t-1} = \bar{B}_L$, while $P_t = \bar{P}$ and $R_t = \bar{R}$. Therefore equation 3.13 reduces to $T_t = \bar{T}$.

3.3. Monetary policy

The central bank plays two roles in the economy. First, it conducts monetary policy through a standard Taylor rule. Second, it trades in long-term government bonds through its balance sheet.

3.3.1. Taylor rule

The nominal interest rate is set according to a Taylor rule with interest rate smoothing:

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}}\right)^{\alpha_r} \left(\left(\frac{\pi_t}{\bar{\pi}}\right)^{\phi^\pi} \left(\frac{Y_t}{\bar{Y}}\right)^{\phi^y}\right)^{1-\alpha_r} e^{\varepsilon_t^R} \tag{3.14}$$

where R_t, π_t and Y_t denote the short-term interest rate, inflation rate and output, respectively. ε_t^R is a stochastic error term with mean of zero and standard deviation of σ^R. The parameters $\bar{R}$, $\bar{\pi}$ and $\bar{Y}$ are steady state values for the interest rate, inflation rate and GDP. The central bank responds to deviations in the inflation rate and GDP from their steady state values (the inflation gap and output gap, respectively) in proportion ϕ^π and ϕ^y. The smoothing parameter α_r represents the weight on the lagged interest rate. Given the nominal interest rate chosen, the central bank adjusts the money supply to ensure equilibrium in the money market.

3.3.2. Central bank balance sheet

The central bank's simplified balance sheet is represented by the money supply M_t on the liabilities side and its holdings of long-term government bonds $B_{L,t}^{CB}$ on the assets side, and is given by

$$M_t = B_{L,t}^{CB}$$

If the central bank purchases long-term government bonds from the non-bank private sector (the household), the household's holdings of these assets falls, while the central bank's holdings increases ($\downarrow B_{L,t}^{H} = \uparrow B_{L,t}^{CB}$). The total long-term bond supply $B_{L,t}$ (the current stock of long-term bonds in circulation) remains unchanged. The transaction is financed through the central bank "issuing base money in the form of reserves held by commercial banks" (Bowdler and Radia, 2012:607).

Absent an explicit banking sector, asset purchases can be modelled as the central bank transferring money balances directly to households (representing the non-bank private sector in Table 1) in exchange for some of the households' holdings of long-term government bonds. This is a strong simplification, however, since, for example, much of the liquidity injected into the financial system by the Fed during their QE operations were accumulated by banks as excess reserves[21] and did not make its way to households via a commensurate increase in the money supply. Moreover, the SARB's asset purchases are likely to be from banks and institutional investors, and not South African households directly. However, as a first step towards pinning down a tractable model, the substantial literature where central bank balance sheet policies are modelled *without* a financial sector is followed here.

Setting Δ_t to represent the change in the central bank balance sheet, equal to money creation and net asset purchases (Falagiarda, 2014:311), the central bank's nominal balance sheet can be represented by

$$\Delta_t = [M_t - M_{t-1}] - \left[\frac{B^{CB}_{L,t}}{R_{L,t}} - \frac{B^{CB}_{L,t-1}}{R_t} \right] \tag{3.15}$$

where a change in the amount of long-term government bonds held by the central bank is balanced by a commensurate change in the money supply. According to Harrison (2012:11), the level of Δ is dictated by fiscal commitments. Therefore "additional purchases of debt by the central bank, which increase the asset side of its balance sheet, must be financed by an expansion of the liabilities side of the balance sheet via money creation." If asset purchases are fully financed by money creation, Δ_t will be zero and there will be no externality or spillover to the government's budget constraint (equation 3.11). However, if the money supply does not expand in line with net asset purchases, Δ_t will be negative. Central bank asset purchases not fully financed by money creation would therefore mechanically impose a negative externality on the government's income. $\Delta_t < 0$ implies that the government's income is less than its expenditure (see equation 3.11). This could be interpreted as the monetary authority passing the burden of financing their asset purchases to the fiscal authority. In such a case, asset purchases today comes at the cost of the future need to raise additional taxes. Conversely, if $\Delta_t > 0$, asset purchases by the central bank acts as a seignorage-type

[21]According to Mishkin (2013:428), during the financial crisis "the huge increase in the monetary base led primarily to a massive rise in excess reserves and bank lending did not increase". This could also have been due to regulatory changes, uncertainty, and constraints on credit demand.

mechanism, representing an additional source of revenue for the government through an expansion in the money supply.[22]

The central bank's holdings of long-term government bonds are a fraction d_t of the total amount issued, i.e. $B^{CB}_{L,t} = d_t B_{L,t}$. The remaining proportion $(1 - d_t)$ is available to households (h_t) and other investors (f_t).[23] We assume that the central bank transacts only with the home household, so that $\Delta d_t = -\Delta h_t$ (assuming no change in f_t). That is, any secondary market transaction that changes the fraction of total domestic long-term bonds held by the central bank must have the home household as counterparty. For example, a hypothetical increase in d_t from 1% to 2% (the fraction of long-term bonds held by the central bank) must be "financed" by a commensurate fall in h_t from e.g. 60% to 59% of the total.

Asset purchases by the central bank can be performed by varying the fraction d_t, modelled as the following AR(1) process:

$$\log\left(\frac{d_t}{\bar{\mathcal{D}}}\right) = \phi^{\mathcal{D}} \log\left(\frac{d_{t-1}}{\bar{\mathcal{D}}}\right) + \varepsilon^{\mathcal{D}}_t \tag{3.16}$$

where $\bar{\mathcal{D}}$ is the steady state fraction of long-term bonds held by the central bank $\left(\bar{B}^{CB}_L / \bar{B}_L\right)$ (Falagiarda, 2014), and $\varepsilon^{\mathcal{D}}_t$ is a shock to asset purchases with a mean of zero and standard deviation of $\sigma^{\mathcal{D}}$. This is the mechanism through which asset purchases by the central bank are transmitted through the model: If the central bank changes the fraction d_t, it changes the supply of long-term bonds available to the household, *ceteris paribus*. Based on Tobin (1969:26)'s logic, this will necessarily require that the "rates of return, on this and other assets, must change in a way that induces the public [households] to hold the new supply". Note, however, that the supply of long-term bonds (equation 3.12) is a simple AR(1) process which evolves *independently* from the central bank's choice of d_t. This restriction is deliberately imposed to prevent the moral hazard of the government over-issuing long-term bonds, expecting that the central bank will simply carry the cost. Asset purchase shocks $\varepsilon^{\mathcal{D}}_t$ are therefore assumed to affect "only the composition of outstanding government liabilities" (Falagiarda, 2014:312), and not the quantity of bonds to come into circulation.

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[22]rrespective bond purchases, money creation could also 'inflate away' government debt.

[23]$h_t + d_t + f_t = 1$. f_t includes foreign investors, commercial banks and the Public Investment Corporation (PIC).

The remaining model blocks – firms, inflation dynamics, exports, goods market clearing and the foreign sector – are identical to the standard SOE specification of Galí and Monacelli (2005) and are relegated to Appendix A.

3.4. Bond market clearing

3.4.1. Long-term bonds

The total amount of long-term bonds issued by the home government is held between the home household, home central bank and other investors. That is,

$$B_{L,t} = B_{L,t}^{H} + B_{L,t}^{CB} + B_{L,t}^{F} \tag{3.17}$$

where $B_{L,t}^{H} = h_t B_{L,t}$, $B_{L,t}^{CB} = d_t B_{L,t}$ and $B_{L,t}^{F} = f_t B_{L,t}$ represent, respectively, home household, central bank and other holdings of domestic long-term bonds at time t. Therefore, a varying in the fraction d_t through the central bank's asset purchase programmes would introduce a wedge in the household's optimisation problem by disturbing his preferred portfolio mix. A shock to d_t increases the central bank's demand for domestic long-term bonds. Given a finite supply of long term bonds,[24] the central bank can only purchase such securities from the non-bank private sector, represented by the home household, which pushes up the price and lowers the yield on domestic long-term bonds. The home household's additional liquidity as a result of these transactions can now be allocated towards additional consumption, real money holdings or further investment. The home household is, however, restricted from investing in foreign securities, which requires that the additional liquidity be allocated domestically.

The steady state stock of long-term bonds is distributed according to

$$\begin{aligned} \bar{B}_L &= \bar{B}_L^{H} + \bar{B}_L^{CB} + \bar{B}_L^{F} \\ &= (\bar{\mathcal{H}} + \bar{\mathcal{D}} + \bar{\mathcal{F}})\bar{B}_L \end{aligned} \tag{3.18}$$

where $\bar{\mathcal{H}}$, $\bar{\mathcal{D}}$ and $\bar{\mathcal{F}}$ represent the steady state fractions of long-term bonds held by the home household, home central bank and other investors, respectively. In equilibrium, each sector maintains its steady state share. The long-term bond market clears when the home household, home central bank and foreign investor demand all issued bonds.

[24] If the debt is monetized, the government would issue additional securities in response to the central bank's demand, and the central bank will not transact with the private sector.

3.4.2. Short-term bonds

Short-term bond issues closes the consolidated government block of the model. The government's budget constraint and the central bank's balance sheet (equations 3.11 and 3.15) are represented (in nominal terms) by

$$B_{t-1} + \frac{B_{L,t-1}}{R_t} = P_t T_t + \frac{B_t}{R_t} + \frac{B_{L,t}}{R_{L,t}} + \Delta_t \tag{3.19}$$

$$\Delta_t = [M_t - M_{t-1}] - \left[\frac{B^{CB}_{L,t}}{R_{L,t}} - \frac{B^{CB}_{L,t-1}}{R_t}\right] \tag{3.20}$$

The government's debt burden is financed by tax income, new bond issuance and the residual arising from the central bank's balance sheet operations. Substituting the central bank's balance sheet (Δ_t) into the government's budget constraint and simplifying yields

$$P_t T_t = \left[B_{t-1} - \frac{B_t}{R_t}\right] + \left[\frac{B_{L,t-1}}{R_t} - \frac{B_{L,t}}{R_{L,t}}\right] + \left[\frac{B^{CB}_{L,t}}{R_{L,t}} - \frac{B^{CB}_{L,t-1}}{R_t}\right] - [M_t - M_{t-1}] \tag{3.21}$$

This shows that nominal taxes result from the net short-term debt burden, the net long-term debt burden, asset transactions by the central bank, and the change in the money supply. If $\Delta = 0$ (i.e. asset purchases fully financed by money creation) the final two terms drop out and the tax is simply determined by the government's net total debt burden. But if $\Delta \neq 0$ central bank asset operations represent either an income or cost to the government.

By substituting $B^{CB}_{L,t} = d_t B_{L,t}$ and $B^{CB}_{L,t-1} = d_{t-1} B_{L,t-1}$ (see section 3.3.2) the consolidated budget constraint can be represented by

$$P_t T_t + \frac{B_t}{R_t} + \frac{B_{L,t}}{R_{L,t}} + M_t - M_{t-1} - \frac{d_t B_{L,t}}{R_{L,t}} + \frac{d_{t-1} B_{L,t-1}}{R_t} = B_{t-1} + \frac{B_{L,t-1}}{R_t} \tag{3.22}$$

Solving the above for B_t closes the model.

4. Model calibration

4.1. Parameters

The complete log-linearised model equations are given in Appendix C. Model parameters are calibrated (Table 2) based on South African data and standard DSGE literature.[25] The discount factor $\beta = 0.99$, the import share (or openness index) $\upsilon = 0.2$ and the inverse elasticity of labour supply $\phi = 3$ are adopted from Steinbach et al. (2009). The constant elasticity of substition (CES) parameter of $\varepsilon = 6$ implies a steady state markup of 1.2 and is taken from Galí and Monacelli (2005). The Calvo probability of firms not being able to change price $\theta = 0.75$ is taken from Harrison (2012:17), and is guided by the assumption that "firms change prices on average once a year". The CRRA $\sigma = 1.026$ and the elasticity of substitution between home and foreign goods $\eta = 0.591$ are based on Steinbach et al. (2009)'s estimations, while the three Taylor rule parameters ϕ^{π}, ϕ^{y} and α_r are taken from Ortiz and Sturzenegger (2007)'s estimations for South Africa.

The fiscal response to debt ψ_1 and ψ_2 are taken from Falagiarda (2014), as are the portfolio adjustment frictions ϕ_L and the elasticity of money demand δ. The persistence of the central bank's asset purchases $\phi^{\mathcal{D}}$ is uncertain, as the SARB did not indicate the time horizon over which the transactions will take place, nor the unwinding of its position.[26] As a starting point we assume $\phi^{\mathcal{D}} = 0.83$, which assumes that the central bank's asset purchases are unwound in about 24 quarters (Falagiarda, 2014).[27]

4.2. Steady states

To calculate the steady states, data was sourced from the SARB's Quarterly Bulletin online time-series facility. Total debt to GDP is the ratio of the total amount of marketable government debt to GDP. Long-term debt is the total supply of domestic long-term government bonds. Short-term debt is the obtained by subtracting long-term debt from total debt. The ratios of total debt to GDP, short- and long-term debt to total debt, and the ratios of short- to long-term debt held by households and the central bank are taken as close as possible to the SARB's announcement of its bond market intervention. Given the lag in publishing data, this implies that the steady

[25] We are only interested in the asset purchase shock. Parameters which govern other shocks, or variables which do not respond to an asset purchase shock, are not described here.

[26] Since the start of the BPP total government bonds held by the SARB has kept increasing every month, although it has seemingly plateaued since July 2020. As of September 2020 its holdings is nearly 5 times as big as it was in February 2020 (SARB, 2020c).

[27] The sensitivity of the model to this persistence parameter is considered in Section 5.2.

Table 2: Calibration of standard parameters

Parameter	**Value**	**Description**
Preferences and technology		
β	0.99	Discount rate
σ	1.026	CRRA
υ	0.2	Trade openness
η	0.591	Elasticity of substitution between home and foreign goods
ϕ	3	Inverse elasticity of labour supply
δ	7	Elasticity of money demand
θ	0.75	Calvo sticky price parameter
ε	6	CES
Fiscal and monetary policy		
ψ_1	0.3	Fiscal response to short-term debt
ψ_2	0.3	Fiscal response to long-term debt
ϕ^{π}	1.11	Monetary policy response to inflation
ϕ^{y}	0.27	Monetary policy response to output
α_r	0.73	Monetary policy smoothing
Bond market parameters		
ϕ_L	0.01	Portfolio adjustment frictions
$\phi^{\mathcal{D}}$	0.83	Persistence of SARB asset purchases

states, including the key ratios κ_L and $\bar{B}_L^{CB}$, are calculated as the ratios during December 2019.[28] The notable steady state ratios are reported in Table 3.[29] The money supply is the ratio of high-powered money (M1A) to GDP. Following Falagiarda (2014) $\bar{N} = 0.3/1.3$, and output, foreign output and the price level are normalised to 1.

Table 3: Steady states

Steady state	Value	Description
$\bar{B} + \bar{B}_L$	0.622	Total debt to GDP
$\bar{B}$	0.176	ST debt on total debt
$\bar{B}_L$	0.446	LT debt on total debt
$\bar{B}_L^H$	0.289	LT debt held by households
$\bar{B}_L^F$	0.156	LT debt held by foreign and other investors
$\bar{B}_L^{CB}$	0.001	LT debt held by the SARB
κ_L †	1.645	Ratio of LT to ST bonds in household portfolio
$\bar{T}$	0.252	Taxes to GDP
$\bar{M}$	0.176	Money supply (M1A) to GDP

† Calculated as $\kappa_L = \bar{B}_L^H/\bar{B}$

The debt and bond ratios are expressed in terms of output normalised to 1 in December 2019. The debt-to-GDP ratio was 0.622. This 0.622 consists of 0.446 in long-term and 0.176 in short-term debt. The 0.446 in turn consists of 0.289 held by the non-bank private sector, 0.001 held by the SARB and the remaining 0.156 held by foreign and other investors.

5. The impact of SARB asset purchases

5.1. Impulse response functions

The effect of an increase in the amount of long-term bonds purchased by the SARB is illustrated by the impulse response functions in Figure 3. Data published since the start of the BPP shows that the SARB's total holdings of

[28] While a point estimate of this kind is not quite a steady state, we wish to pin down the dynamics arising from this shock given the *most recent* distribution of government debt before the crisis hit. The March 2020 figures were likely already affected by the crisis.

[29] The data series used are reported in Table D1.

long-term government bonds increased nearly *seventeen*-fold between March and July 2020 (section 2.1). Given additional developments in the bond market, this represents a seven-fold increase in the fraction of total domestic long-term government bonds held by the SARB (d_t). The magnitude of the asset purchase shock is therefore set equal to 6, implying a 600% increase in the amount of long-term bonds held by the central bank.

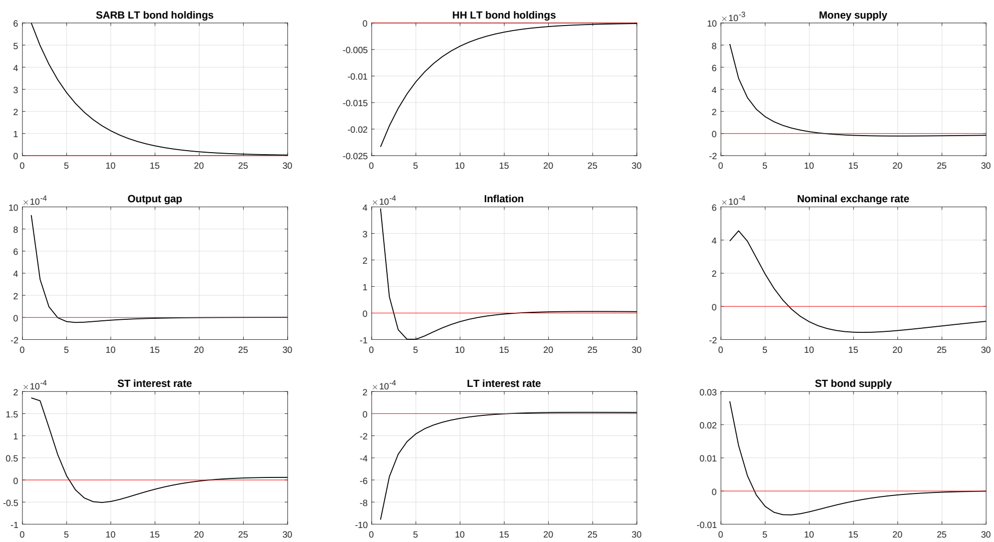
Central bank bond purchases result in a fall in the amount of long-term bonds held by the household. The liquidity created by virtue of the household now holding fewer long-term bonds – having been partially relieved thereof by the central bank – can now be allocated toward additional consumption (including imported goods), money holdings and investment in short-term bonds. Output and inflation duly increases, which is in turn countered by the central bank increasing the short-term interest rate through the Taylor rule. A very mild and short-lived contraction in output and inflation occurs after about one year as the economy returns to equilibrium, which is counteracted by an expansionary fall in the short-term interest rate.

The increased demand for long-term bonds by the central bank drives up its price, which is illustrated by a fall in the long-term interest rate. The lower interest rate on long-term bonds lowers the government's future debt burden, which allows the government to gradually issue fewer short-term bonds to finance this debt. The supply of short-term bonds also mirrors the short-term interest rate, and is consistent with Harrison (2012:3)'s suggestion that "reductions [increases] in the short-term nominal interest rate reduce [increase] the relative supply of short-term bonds". The supply of long-term bonds evolves according to an AR(1) process, and essentially remains fixed. The yield curve flattens as short rates increase and long rates fall. The initial depreciation of the currency (represented by an increase in e_t) is the result of higher home demand, which leads to an increased demand for foreign goods.

Clearly, in addition to its direct effect on liquidity in bond markets, asset purchases increase both output and inflation. The money supply increases, while long-term bond yields fall and the term spread narrows. This suggests that the SARB's bond market interventions should, over and above its credit market support, qualitatively have a stimulatory impact on the broader South African economy.

However, this effect is quantitatively very small. A seven-fold increase in the central bank's long-term bond holdings only translates into a 0.09% increase in output. Similarly, its impact on inflation, interest rates and the exchange rate is minute. This is hardly surprising, given that the SARB is

Figure 3: SARB asset purchase shock



Variables are expressed in % deviations from the steady state on the vertical axis. Time elapsed on the horizontal axis is in quarters.

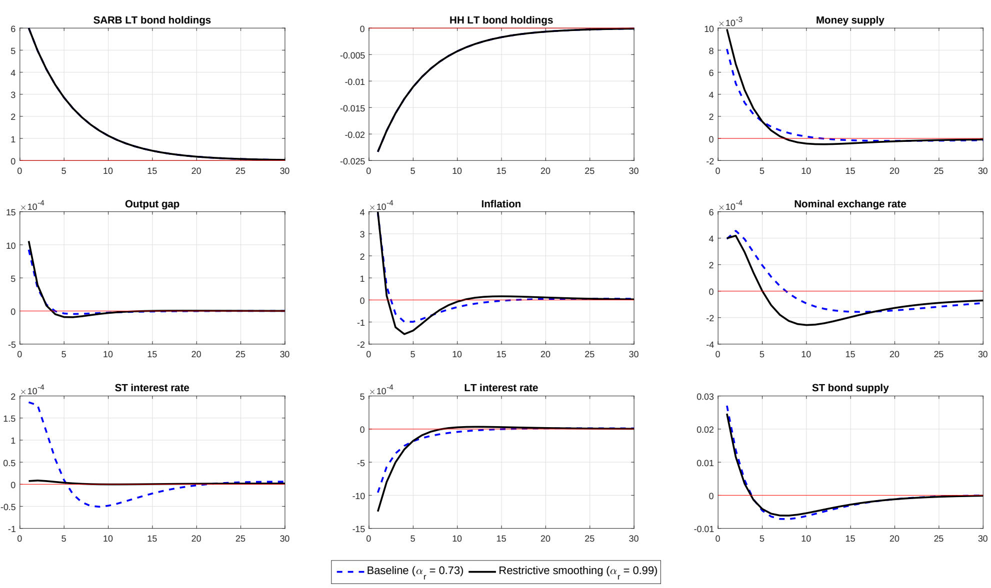
a comparatively small player in the long-term government bond market. In December 2019 the SARB held roughly R5.8bn in long-term bonds, equal to about 0.1% of all bonds issued (Table 3). Thus for the SARB's BPP to have a substantial expansionary impact on the macroeconomy it has to either start off from a bigger base, or be numerically much larger.

5.2. Sensitivity analyses

5.2.1. Taylor rule response

Figure 3 demonstrated that output and inflation increase in response to asset purchases. However, the central bank mechanically responds through the Taylor rule by increasing the short-term interest rate in order to counteract the 'overheating' economy. This action perversely tightens credit markets, which counteracts the SARB's very intention to provide liquidity to these markets. In order to prevent this, the Taylor rule response is shut down by setting the smoothing parameter extremely high, e.g. $\alpha_r = 0.99$.

Figure 4: Sensitivity: Taylor rule smoothing



Variables are expressed in % deviations from the steady state on the vertical axis. Time elapsed on the horizontal axis is in quarters.

Figure 4 suggests that the initial stimulus is virtually unaffected by the interest rate response. However, the magnitude of the contraction one year in is marginally larger. This is due to the fact that the Taylor rule will be unresponsive, and the short-term interest rate is therefore unable to counter the contraction. In addition, the fall in long-term yields will be more pronounced. However, the difference between the two parameterisations is very small, reflecting the extremely small stimulus effect of the BPP in the first place.

5.2.2. Persistence of asset purchases

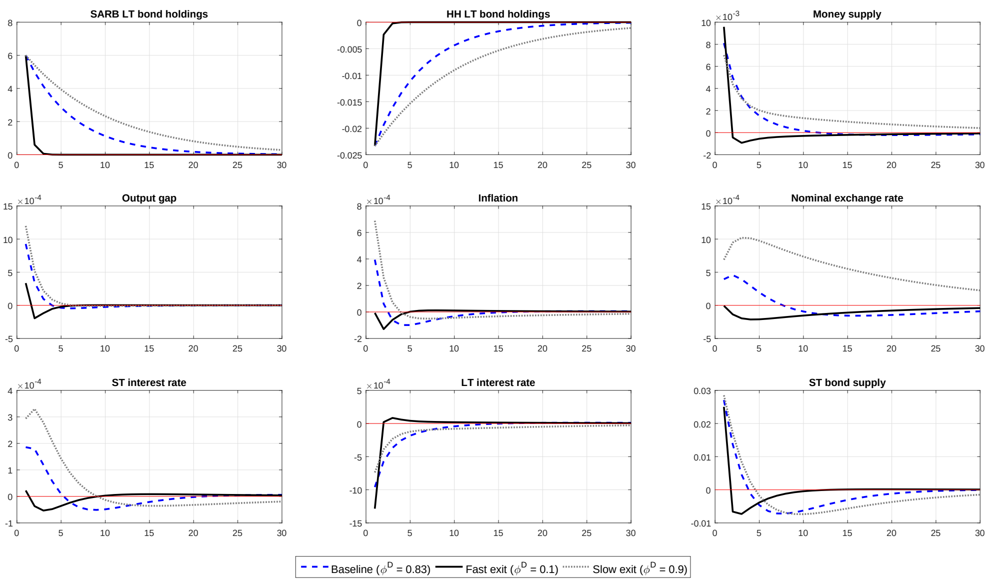
In the calibration of the benchmark model, it is assumed that the central bank follows a medium-term exit strategy following asset purchases. That is, the assets purchased by the central bank are gradually sold over the following five–six years (illustrated by the duration of the return of the central bank's long-term bond holdings to steady state of just under 24 quarters according to figure 3). A decrease in the parameter $\phi^{\mathcal{D}}$ represents a lower persistence in asset purchases, which is tantamount to a faster exit strategy. Conversely, a slower exit strategy is represented by a higher parameter $\phi^{\mathcal{D}}$.

The faster the central bank's exit from its asset purchase programme, the smaller the stimulus. If the SARB's position is unwound within a year ($\phi^{\mathcal{D}} = 0.1$), the stimulus is extremely short-lived and output starts to contract within two quarters. The fall in domestic demand also sees the currency appreciate, as demand for foreign consumption goods is now smaller. The short-term interest rate quickly falls in response to the slowdown. Conversely, a slow exit is associated with a larger stimulus effect, without the later contraction in output observed under other parameterisations. The short-term interest rate response is necessarily larger. The longer the SARB's position takes to unwind, the more persistent is the increase in money balances held by the private sector, hence the larger impact on consumption and output. To date, the SARB is showing no signs of unwinding as it is maintaining its balance sheet at the current high levels.

5.2.3. Household portfolio composition

The steady state ratio of short- to long-term bonds in the household's asset portfolio is calculated here from recent South African data, and is equal to $\kappa_L = 1.645$. This implies that the household holds more long-term than short-term bonds in its steady state portfolio. However, in the event that the household's steady state portfolio is weighted more towards short-term bonds, e.g. $\kappa_L = 0.8$, the impact of the same asset purchase

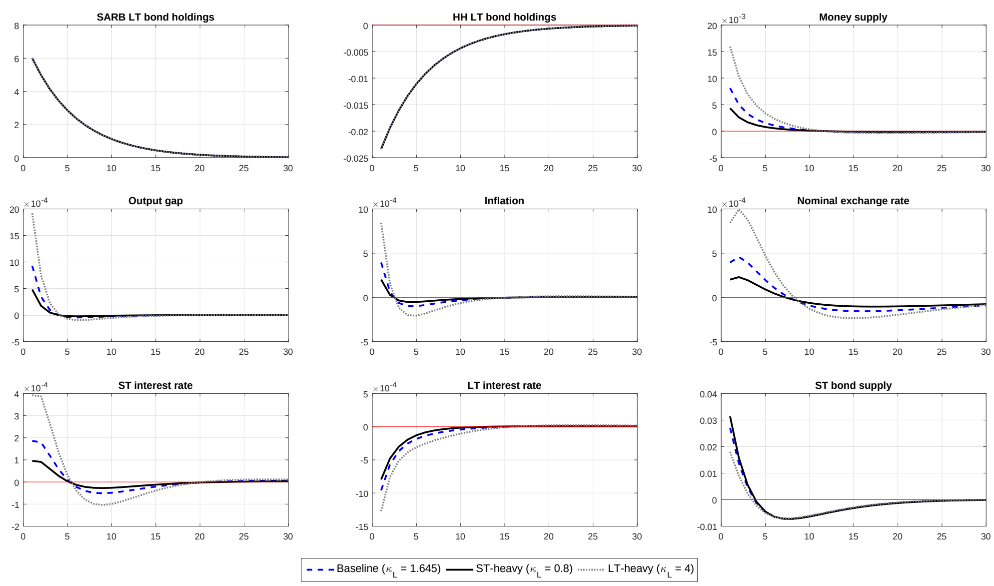
Figure 5: Sensitivity: Asset purchase persistence



Variables are expressed in % deviations from the steady state on the vertical axis. Time elapsed on the horizontal axis is in quarters.

shock diminishes. If the household holds relatively more short-term bonds, a change in its holdings of long-term bonds will have a proportionally smaller effect. Conversely, if the household's portfolio is heavily weighted to long-term bonds, e.g. $\kappa_L = 4$, an asset purchase shock will have a much larger impact. The latter is also associated with a relatively smaller adjustment cost (section 3.1) in terms of output.

Figure 6: Sensitivity: Household portfolio composition



Variables are expressed in % deviations from the steady state on the vertical axis. Time elapsed on the horizontal axis is in quarters.

6. Conclusion

The SARB recently announced that it would be purchasing long-term South African government bonds in the secondary market, financed by money creation. The SARB's primary objective is to provide liquidity and support domestic financial markets, with a secondary stated objective to attempt to reduce volatility in government bond prices and yields. This paper considers the potential knock-on effects that such asset purchases could have on the broader South African economy, over and above the liquidity support to financial markets.

To this end, it constructs a small open-economy DSGE model which is calibrated and simulated based on recent South African data and existing estimations of the deep structural parameters of the South African economy. The model utilises the portfolio balance theory to pin down a transmission mechanism through which central bank asset purchases influence the macroeconomy. It is assumed that the household, representing the non-bank private sector, holds a portfolio of imperfectly-substitutable short- and long-term domestic government bonds. The central bank could purchase long-term bonds from the household in exchange for money balances. The household's additional liquidity increases consumption, and subsequently output and inflation. It is concluded that the SARB's asset purchases would have a stimulatory – although very small – effect on the South African economy, even though the SARB indicated that this was not an aim of the programme.

Sensitivity analyses were performed on some parameters of interest. First, the initial stimulus is virtually unaffected if the short-term interest rate response is muted, while the subsequent evolution of variables are not significantly different. Second, the longer the SARB takes to unwind its position, the larger and more persistent the stimulus. It would be interesting to see how this plays out, as the SARB is currently maintaining its larger balance sheet. Finally, the smaller the steady state weight on long-term bonds in the household's portfolio, the smaller the stimulus.

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Appendix A. Standard SOE blocks and closing the model

Appendix A.1. Firms

Following Galí and Monacelli (2005:715) we assume no capital share in production. The representative home firm j therefore produces a homogenous good according to the production function

$$Y_t(j) = Z_t N_t(j) \tag{A.1}$$

where Z_t is a stochastic productivity shock, with $\hat{z}_t \equiv \log Z_t$, which evolves over time according to

$$\hat{z}_t = \rho_z \hat{z}_{t-1} + \varepsilon_t^z \tag{A.2}$$

Real marginal cost is identical across domestic firms (Galí and Monacelli, 2005:715), and is given by

$$\hat{mc}_t^r = \hat{w}_t - \hat{p}_{H,t} - \hat{z}_t \tag{A.3}$$

Total labour demand is given by $N_t = \int_0^1 N_t(j)dj$, while aggregate domestic output is defined by

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{A.4}$$

where $\frac{\varepsilon}{\varepsilon-1}$ is the steady state price markup (Andrés et al., 2004).

Domestic firms set prices following Calvo's (1983) staggered pricing framework. That is, in each period t a fraction $1-\theta$ of producers can reset their prices, while the remaining fraction θ keep their prices unchanged. The optimal price setting decision of the domestic firm is represented by (Galí and Monacelli, 2005:715)

$$\bar{p}_{H,t} = (1-\theta\beta) E_t \sum_{k=0}^{\infty} \theta^k \beta^k \left[\hat{mc}_{t+k|t}^r + \hat{p}_{H,t+k} \right] \tag{A.5}$$

As $\theta \to 0$ (the flexible price limit), the home firms' optimal price setting rule becomes $\bar{p}_{H,t} = \hat{mc}_t^r + \hat{p}_{H,t}$ (i.e. a fixed markup equal to real marginal cost over the domestic price level).

Appendix A.2. Exports

Following Galí (2015:234), the home economy's exports in a two-economy model are given by

$$X_t = \upsilon \left(\frac{P_{H,t}}{\mathcal{E} P_t^\star} \right)^{-\eta} Y_t^\star \\ = \upsilon \mathcal{S}_t^\eta Y_t^\star \tag{A.6}$$

where $\mathcal{S}_t = \frac{\mathcal{E}_t P_t^\star}{P_{H,t}}$ represents the terms of trade (i.e. the ratio of the price of foreign to home goods). This result holds "under the assumption that the preferences of households in the rest of the world are identical to those of domestic households" and the fact that global goods market clearing requires that $C_t^\star = Y_t^\star$ (Galí, 2015:234). The home economy's exports are therefore a function of trade openness, the terms of trade, the degree of substitutability between home and foreign goods, and world output (equivalent to aggregate world demand). The terms of trade, in turn, is a positive function of the nominal exchange rate ($\mathcal{E}_t$) and the world price level, and a negative function of the domestic price level. Intuitively, a weaker domestic currency (captured by an increase in $\mathcal{E}_t$) and higher world prices would increase foreign demand for home goods, while higher home production prices would lower foreign demand for home goods.

In the symmetric steady state, where $\bar{\mathcal{S}} = 1$, steady state exports is given by $\bar{X} = \upsilon \bar{Y}^\star$. Log-linearising equation A.6 around this steady state yields

$$\hat{x}_t = \eta \hat{s}_t + \hat{y}_t^\star \tag{A.7}$$

Furthermore, since in steady state $\bar{C}_F = \upsilon \bar{C}$ (Appendix B.2) and $\bar{C} = \bar{Y}^\star$ (from the risk sharing condition, Appendix B.5) it follows that $\bar{C}_F = \upsilon \bar{C} = \upsilon \bar{Y}^\star = \bar{X}$ (i.e. imports = exports). Therefore, "trade is balanced at the steady state" (Galí, 2015:234).

Net exports, in terms of domestic output and expressed as a fraction of steady state output (Galí, 2015:236), is given by

$$nx_t \equiv \left(\frac{1}{\bar{Y}} \right) \left(Y_t - \frac{P_t}{P_{H,t}} C_t \right) \tag{A.8}$$

Steady state net exports can therefore be expressed as

$$\overline{nx} = \frac{1}{\bar{Y}} \left(\bar{Y} - \frac{\bar{P}}{\bar{P}_H} \bar{C} \right) = \frac{\bar{Y} - \bar{C}}{\bar{Y}} \tag{A.9}$$

Equation A.9 shows that steady state net exports is equal to the fraction of steady state output not consumed domestically. Under the assumption that "trade is balanced in the steady state" (Galí, 2015:236), net exports is equal to zero and it follows from equation A.9 that $\overline{nx} = 0 \Rightarrow \bar{Y} = \bar{C}$. Log-linearising equation A.8 around this steady state, noting that $\hat{p}_t - \hat{p}_{H,t} = \upsilon \hat{s}_t$ (equation B.3), yields

$$\hat{nx}_t = \hat{y}_t - \hat{c}_t - \upsilon \hat{s}_t \tag{A.10}$$

Appendix A.3. Goods market clearing

The domestic goods market will clear when local production is fully consumed. That is, all locally-produced goods are either consumed by the home household or exported. Following Galí (2015:234) and including our portfolio adjustment friction, the resource constraint therefore becomes

$$Y_t = C_{H,t} + X_t + \frac{B^H_{L,t}}{R_{L,t}} AC^L_t \tag{A.11}$$

Substituting in the expressions we know for $C_{H,t}$ and X_t,[30] the resource constraint can be expressed as

$$Y_t = (1-\upsilon)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \upsilon \mathcal{S}_t^{\eta} Y_t^{\star} + \frac{b^H_{L,t}}{R_{L,t}} AC^L_t \tag{A.12}$$

In the symmetric steady state, where $\bar{\mathcal{S}} = 1$ and $\bar{P} = \bar{P}_H$, the resource constraint becomes[31]

$$\bar{Y} = (1-\upsilon)\bar{C} + \upsilon \bar{Y}^{\star} \tag{A.13}$$

Under the assumption of balanced trade we have $\bar{Y} = \bar{C}$ (eq. A.9). It then follows from equation A.13 that $\bar{Y}^{\star} = \bar{Y} = \bar{C}$. Utilising this, $\bar{P} = \bar{P}_H$ and $\hat{p}_t - \hat{p}_{H,t} = \upsilon \hat{s}_t$ (eq. B.3), we can log-linearise the resource constraint around the symmetric steady state:

$$\hat{y}_t = (1-\upsilon)\hat{c}_t + \upsilon\eta(2-\upsilon)\hat{s}_t + \upsilon \hat{y}_t^{\star} \tag{A.14}$$

Combining equation A.14 (the linearised resource constraint) with equation B.12 (the risk sharing condition linking domestic consumption with

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[30] $C_{H,t} = (1-\upsilon)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t$ (Appendix B.2) and $X_t = \upsilon \mathcal{S}_t^{\eta} Y_t^{\star}$ (equation A.6).

[31] Recall that in steady state AC^L_t collapses to zero (see the discussion of equation 3.4).

world output and the terms of trade) yields the following expression for the terms of trade:

$$\hat{s}_t = \sigma_\upsilon(\hat{y}_t - \hat{y}^\star_t) \tag{A.15}$$

where $\sigma_\upsilon = \frac{\sigma}{(1-\upsilon)^2+\sigma\upsilon\eta(2-\upsilon)} > 0$. Following Galí (2015:235), this can be simplified to $\sigma_\upsilon \equiv \sigma\Phi > 0$, with $\Phi \equiv \frac{1}{1+\upsilon(\varpi-1)} > 0$ and $\varpi \equiv \sigma\eta + (1-\upsilon)(\sigma\eta - 1)$.

Finally, combining equation A.10 (net exports) with equations A.14 (the resource constraint) and B.12 (the risk sharing condition) allows net exports to be expressed as a function of the terms of trade (Galí, 2015:236):

$$\hat{nx}_t = \upsilon\left(\frac{\varpi}{\sigma} - 1\right)\hat{s}_t \tag{A.16}$$

Appendix A.4. Dynamic IS curve

The open-economy consumption Euler equation (3.9) is identical to the closed-economy specification (Galí, 2015:228). However, domestic inflation in the open economy is now a function of both the rate of change in domestic goods prices and the change in the terms of trade ($\hat{\pi}_t = \hat{\pi}_{H,t} + \upsilon\Delta\hat{s}_t$, eq. B.4), and not simply the former as in the closed-economy scenario. The expanded open-economy Euler equation is given by

$$\hat{c}_t = E_t[\hat{c}_{t+1}] - \frac{1}{\sigma}(\hat{r}_t - E_t[\hat{\pi}_{H,t+1}]) + \frac{\upsilon}{\sigma}E_t[\Delta\hat{s}_{t+1}] - \frac{\Psi_1}{\sigma}(\hat{b}^H_{L,t} - \hat{b}_t) \tag{A.17}$$

The difference between the closed- and open-economy Euler equations is therefore the addition of one term which captures the terms of trade and trade openness, which both influence aggregate domestic inflation, and therefore the home household's consumption decision.

The dynamic IS curve can now be derived by combining the Euler equation (A.17) with the resource constraint (A.14) and the terms of trade, expressed as a function of domestic and foreign output (A.15), and is given by

$$\hat{y}_t = E_t\hat{y}_{t+1} + \Omega_1\left[E_t\Delta\hat{y}^\star_{t+1}\right] - \Omega_2(\hat{r}_t - E_t[\hat{\pi}_{H,t+1}]) - \Omega_2\Psi_1(\hat{b}^H_{L,t} - \hat{b}_t) \tag{A.18}$$

where $\Omega_1 = \left(\frac{-\upsilon-\Omega}{1+\Omega}\right) > 0$ and $\Omega_2 = \frac{1}{\sigma}\left(\frac{1-\upsilon}{1+\Omega}\right) > 0$ are convolutions of the parameters. $\Omega = \frac{\upsilon(1-\upsilon)\sigma_\upsilon}{\sigma} - \upsilon\eta(2-\upsilon)\sigma_\upsilon < 0$, $(1+\Omega) > 0$, $(1-\upsilon) > 0$ and $(-\upsilon - \Omega) > 0$. Similar to Galí and Monacelli (2005), domestic demand is a function of expected future domestic and foreign demand and the expected real interest rate, as well as the wedge introduced by bond market transactions.

It is easy to verify that for $\upsilon \to 0$ we will have $\Omega \to 0$, and thus $\Omega_1 \to 0$ and $\Omega_2 \to \frac{1}{\sigma}$. Therefore, if the degree of openness is set equal to zero, the IS curve collapses to the familiar closed-economy representation. Applying some algebra on the Ω_i coefficients shows that $\Omega_1 = \upsilon(\varpi - 1)$ and $\Omega_2 = \frac{1}{\sigma_\upsilon}$. This yields the equivalent IS curve with the more familiar open-economy coefficients (Galí, 2015:235):

$$\hat{y}_t = E_t\hat{y}_{t+1} + \upsilon(\varpi - 1)\left[E_t\Delta\hat{y}^\star_{t+1}\right] - \frac{1}{\sigma_\upsilon}(\hat{r}_t - E_t[\hat{\pi}_{H,t+1}]) - \frac{\Psi_1}{\sigma_\upsilon}(\hat{b}^H_{L,t} - \hat{b}_t) \quad \text{(A.19)}$$

Appendix A.5. Inflation dynamics and the Phillips curve

Following Galí and Monacelli (2005:717) domestic inflation can be expressed in terms of real marginal cost as

$$\hat{\pi}_{H,t} = \beta E_t[\hat{\pi}_{H,t+1}] + \lambda\hat{mc}^r_t \quad \text{(A.20)}$$

where $\lambda \equiv \frac{(1-\theta)(1-\theta\beta)}{\theta}$.

From the production function A.1, the marginal product of labour is equal to $MPN_t = \frac{\partial Y_t(j)}{\partial N_t(j)} = Z_t$,[32] while the log-linearised aggregate production function can be expressed as

$$\hat{y}_t = \hat{z}_t + \hat{n}_t \quad \text{(A.21)}$$

Given the assumption that domestic conditions do not affect world output, it can be shown that domestic real marginal costs (eq. A.3) are related to the output gap according to

$$\begin{aligned}\hat{mc}^r_t &= (\hat{w}_t - \hat{p}_{H,t}) - \hat{z}_t \\ &= (\hat{w}_t - \hat{p}_t) + (\hat{p}_t - \hat{p}_{H,t}) - \hat{z}_t \\ &= (\sigma\hat{c}_t + \phi\hat{n}_t) + \upsilon\hat{s}_t - \hat{z}_t\end{aligned}$$

From equations B.12 and A.21 we have $\hat{c}_t = \hat{y}^\star_t + \frac{1-\upsilon}{\sigma}\hat{s}_t$ and $\hat{n}_t = \hat{y}_t - \hat{z}_t$. Substituting these into the previous expression and simplifying yields

$$\hat{mc}^r_t = \sigma\hat{y}^\star_t + \phi\hat{y}_t + \hat{s}_t - (1+\phi)\hat{z}_t$$

[32]In log-linear terms this becomes $\hat{mpn}_t = \hat{z}_t$.

Real domestic marginal cost is therefore a positive function of the terms of trade and domestic and world output. The domestic real wage is influenced by both these variables, "through the wealth effect on labour supply resulting from their impact on domestic consumption" (Galí and Monacelli, 2005:718), while "changes in the terms of trade have a direct effect on the product wage, for any given real wage" (*Ibid.*). Finally, substituting in equation A.15 for $\hat{s}_t$ allows real marginal cost to be expressed in terms of world output, domestic output and local technology:

$$\hat{m}c_t^r = (\sigma_\upsilon + \phi)\hat{y}_t + (\sigma - \sigma_\upsilon)\hat{y}_t^\star - (1 + \phi)\hat{z}_t \tag{A.22}$$

For $\upsilon \to 0$ we will have $\sigma_\upsilon \to \sigma$, and real marginal cost will be identical to the canonical closed-economy specification. Following Galí and Monacelli (2005:718), the output gap and real marginal cost are related according to

$$\hat{m}c_t^r = (\sigma_\upsilon + \phi)\hat{y}_t \tag{A.23}$$

which can be combined with equation A.20 to obtain the open-economy specification of the New-Keynesian Phillips curve

$$\hat{\pi}_{H,t} = \beta E_t[\hat{\pi}_{H,t+1}] + \kappa_\upsilon \hat{y}_t \tag{A.24}$$

where $\kappa_\upsilon \equiv \lambda(\sigma_\upsilon + \phi)$.[33] It can be verified that for $\upsilon \to 0$ the slope coefficient will be analogous to the closed-economy specification.

Appendix B. Open-economy identities

The standard open-economy identities, as described in Galí (2015:229–232) and Galí and Monacelli (2005:712), insofar relevant to this model, are briefly discussed below.

Appendix B.1. Terms of trade

The terms of trade is defined as the ratio of the price of foreign goods ($P_{F,t}$) to home goods ($P_{H,t}$)[34] in terms of domestic currency:

[33] For $\alpha \neq 0$ this becomes $\kappa_\upsilon \equiv \lambda(\sigma_\upsilon + \frac{\phi+\alpha}{1-\alpha})$ (Galí, 2015:238).

[34] Steinbach et al. (2009:231) defines the terms of trade as the "relative price of imports to domestically produced goods".

$$\mathcal{S}_t = \frac{P_{F,t}}{P_{H,t}}$$

Log-linearising around the steady state of $\bar{\mathcal{S}} = \frac{\bar{P}_F}{\bar{P}_H}$ yields

$$\hat{s}_t \approx \hat{p}_{F,t} - \hat{p}_{H,t} \tag{B.1}$$

An increase in $\hat{s}_t$ represents a depreciation in the terms of trade, since foreign goods are now relatively more expensive than domestic goods.

Appendix B.2. Home price level

According to Galí (2015:226), the "optimal allocation of expenditures between domestic and imported goods" can be given by

$$C_{H,t} = (1-\upsilon)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \ ; \ C_{F,t} = \upsilon\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t$$

where the domestic price level (CPI) is given by

$$P_t^{1-\eta} = (1-\upsilon)P_{H,t}^{1-\eta} + \upsilon P_{F,t}^{1-\eta}$$

In steady state, this becomes $\bar{P}^{1-\eta} = (1-\upsilon)\bar{P}_H^{1-\eta} + \upsilon\bar{P}_F^{1-\eta}$. Log-linearising around a symmetric steady state with $\bar{\mathcal{S}} = 1$[35] yields

$$\hat{p}_t = (1-\upsilon)\hat{p}_{H,t} + \upsilon\hat{p}_{F,t} \tag{B.2}$$

Finally, substituting in the log-linearised terms of trade (equation B.1) yields an expression for the domestic CPI price level

$$\hat{p}_t = \hat{p}_{H,t} + \upsilon\hat{s}_t \tag{B.3}$$

Domestic inflation is defined as $\pi_{H,t} \equiv p_{H,t} - p_{H,t-1}$ (Galí, 2015:229). Utilising equation B.3, it follows that CPI (aggregate) inflation can be expressed as a function of domestic inflation and the terms of trade:

$$\hat{\pi}_t = \hat{\pi}_{H,t} + \upsilon\Delta\hat{s}_t \tag{B.4}$$

[35] This implies that $\bar{P} = \bar{P}_H = \bar{P}_F$. See Galí (2015:228).

Appendix B.3. Law of one price

The assumption of the law of one price dictates that if a good has a price P_t in the domestic market, its price in the foreign market will be equal to this price times the nominal exchange rate. That is, $P_{F,t} = \mathcal{E}_t P_t^\star$ (Galí, 2015:229), where $\mathcal{E}_t$ is the nominal exchange rate, defined as the price of foreign currency in terms of domestic currency. An increase in $\mathcal{E}_t$ therefore represents a depreciation in the domestic currency (foreign currency is now more expensive in terms of domestic currency). $P_t^\star$ is the price of foreign goods expressed in foreign currency.[36] Plugging this into the definition of the terms of trade yields $\mathcal{S}_t = \frac{\mathcal{E}_t P_t^\star}{P_{H,t}}$, which can be log-linearised as

$$\hat{s}_t = \hat{e}_t + \hat{p}_t^\star - \hat{p}_{H,t} \tag{B.5}$$

The terms of trade can therefore be expressed in terms of the (nominal) exchange rate and the difference between world and home price levels.

Appendix B.4. Real exchange rate and UIP

The real exchange rate is defined as the ratio of world to domestic CPI, expressed in domestic currency (Galí, 2015:230), and is given by

$$\mathcal{Q}_t \equiv \frac{P_{F,t}}{P_t} \tag{B.6}$$

Log-linearising equation B.6, utilising the results from equations B.1 and B.3 and noting that in steady state $\bar{\mathcal{Q}} = \frac{\bar{P}_F}{\bar{P}_t}$, yields

$$\hat{q}_t = (1 - \upsilon)\hat{s}_t \tag{B.7}$$

Finally, following Galí and Monacelli (2005:714), the uncovered interest parity (UIP) condition can be expressed as

$$\hat{r}_t - \hat{r}_t^\star = E_t[\Delta \hat{e}_{t+1}] \tag{B.8}$$

which could then be used to "relate the interest rate differential to the terms of trade or real exchange rate" (Kotzé, 2014:76).

[36] $P_t^\star$ can also be interpreted as a world price index, since the "size of the small open economy is assumed to be negligible relative to the rest of the world" (Galí, 2015:229).

Appendix B.5. International risk sharing

The first-order condition from which the Euler equation (3.9) is derived is a simplified version of equation the household's intertemporal optimality condition, and is given by

$$\beta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right)\right] = \frac{1}{R_t} + \frac{\kappa_L \phi_L Y_t\left(\kappa_L \frac{B_t}{B^H_{L,t}} - 1\right)}{R_{L,t}} \tag{B.9}$$

Following Galí (2015:230), we assume a "complete set of state-contingent securities traded internationally" (i.e. complete markets for international securities). This implies that a condition analogous to equation B.9 must also hold for foreign households. That is

$$\beta E_t\left[\left(\frac{C^\star_{t+1}}{C^\star_t}\right)^{-\sigma}\left(\frac{P^\star_t}{P^\star_{t+1}}\right)\left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right)\right] = \frac{1}{R_t} + \frac{\kappa_L \phi_L Y_t\left(\kappa_L \frac{B_t}{B^H_{L,t}} - 1\right)}{R_{L,t}} \tag{B.10}$$

where the presence of the exchange rate terms reflects the fact that the securities' payoffs are "expressed in the currency of the small open economy" (Galí, 2015:230). Combining the law of one price ($P_{F,t} = \mathcal{E}_t P^\star_t$) with the real exchange rate ($\mathcal{Q}_t \equiv \frac{P_{F,t}}{P_t}$, eq. B.6) gives $\mathcal{Q}_t = \frac{\mathcal{E}_t P^\star_t}{P_t}$. This result, combined with equations B.9 and B.10 yields

$$C_t = \vartheta C^\star_t \mathcal{Q}_t^{\frac{1}{\sigma}} \tag{B.11}$$

where $\vartheta = \frac{C_{t+1}}{C^\star_{t+1}\mathcal{Q}_{t+1}^{\frac{1}{\sigma}}}$ is a "constant which will generally depend on initial conditions regarding relative net asset positions" (Galí, 2015:231).[37] Assuming symmetric initial conditions it follows that $\vartheta = 1$. Moreover, since we have assumed an infinitesimally small home economy, it follows that $C^\star_t = Y^\star_t$, where $Y^\star_t$ represents world output. Log-linearising this condition around the steady state where $\bar{C} = \vartheta \bar{Y}^\star \bar{\mathcal{Q}}^{\frac{1}{\sigma}} = \bar{Y}^\star$ and $\bar{\mathcal{Q}} = 1$, and noting that $\hat{q}_t = (1-\upsilon)\hat{s}_t$ (eq. B.7), yields the risk sharing condition

$$\hat{c}_t = \hat{y}^\star_t + \frac{1-\upsilon}{\sigma}\hat{s}_t \tag{B.12}$$

[37] See also Devereux and Sutherland (2009:187) for the theoretical considerations regarding the steady state net foreign asset position.

Appendix B.6. World variables

Finally, following Galí (2015:230), we assume that world demand and the world price level are "taken as exogenous to the small open economy". That implies that these two variables evolve according to the following stochastic processes:

$$\hat{y}_t^{\star} = \phi^{y\star}\hat{y}_{t-1}^{\star} + \varepsilon_t^{y\star} \tag{B.13}$$

$$\hat{p}_t^{\star} = \phi^{p\star}\hat{p}_{t-1}^{\star} + \varepsilon_t^{p\star} \tag{B.14}$$

Appendix C. Log-linearised model equations

(1) **IS curve:** $\hat{y}_t = E_t[\hat{y}_{t+1}] + \upsilon(\varpi - 1)\left[E_t \Delta \hat{y}^\star_{t+1}\right] - \frac{1}{\sigma_\upsilon}(\hat{r}_t - E_t[\hat{\pi}_{H,t+1}]) - \frac{\Psi_1}{\sigma_\upsilon}(\hat{b}^H_{L,t} - \hat{b}_t)$

(2) **Taylor rule:** $\hat{r}_t = \alpha_r r_{t-1} + (1-\alpha_r)(\phi^\pi \hat{\pi}_t + \phi^y \hat{y}_t) + \varepsilon^R_t$

(3) **NK Phillips curve:** $\hat{\pi}_{H,t} = \beta E_t[\hat{\pi}_{H,t+1}] + \kappa_\upsilon \hat{y}_t$

(4) **Term structure:** $\hat{r}_{L,t} = \hat{r}_t + E_t[\hat{r}_{t+1}] + \Psi_2(\hat{b}^H_{L,t} - \hat{b}_t)$

(5) **LT bond supply:** $\hat{b}_{L,t} = \phi^{BL}\hat{b}_{L,t-1} + \varepsilon^{BL}_t$

(6) **Tax rule:** $\hat{t}_t = \frac{1}{\bar{T}\bar{P}}\left[\psi_1 \bar{B}(\hat{b}_{t-1} - \hat{p}_t) + \beta\psi_2 \bar{B}_L(\hat{b}_{L,t-1} - \hat{r}_t - \hat{p}_t)\right]$

(7) **CB LT bonds:** $\hat{b}^{CB}_{L,t} = \hat{d}_t + \hat{b}_{L,t}$

(8) **Asset purchases:** $\hat{d}_t = \phi^{\mathcal{D}}\hat{d}_{t-1} + \varepsilon^{\mathcal{D}}_t$

(9) **HH LT bonds:** $\bar{\mathcal{H}}\hat{b}^H_{L,t} = \hat{b}_{L,t} - \bar{\mathcal{D}}\hat{b}^{CB}_{L,t} - \bar{\mathcal{F}}\hat{b}^F_{L,t}$

(10) **ST bonds:** $\hat{b}_t = \frac{1}{\beta}\hat{b}_{t-1} + (1-(1+\bar{\mathcal{D}})\frac{\bar{B}_L}{\bar{B}})\hat{r}_t + (1-\beta\bar{\mathcal{D}})\frac{\bar{B}_L}{\bar{B}}\hat{r}_{L,t} + (1-\bar{\mathcal{D}})\frac{\bar{B}_L}{\bar{B}}\hat{b}_{L,t-1}$

$+ (\beta\bar{\mathcal{D}} - 1)\frac{\bar{B}_L}{\bar{B}}\hat{b}_{L,t} + \bar{\mathcal{D}}\frac{\bar{B}_L}{\bar{B}}(\beta\hat{d}_t - d_{t-1}) - \frac{\bar{M}}{\beta\bar{B}}(\hat{m}_t - \hat{m}_{t-1}) - \frac{\bar{P}\bar{T}}{\beta\bar{B}}(\hat{p}_t + \hat{t}_t)$

(11) **Production function:** $\hat{y}_t = \hat{z}_t + \hat{n}_t$

(12) **Technology:** $\hat{z}_t = \rho_z \hat{z}_{t-1} + \varepsilon^z_t$

(13) **Wages:** $\hat{w}_t - \hat{p}_t = \phi\hat{n}_t + \sigma\hat{c}_t$

(14) **Resource constraint:** $\hat{y}_t = (1-\upsilon)\hat{c}_t + \upsilon\eta(2-\upsilon)\hat{s}_t + \upsilon\hat{y}^\star_t$

(15) **Money demand:** $\hat{m}_t - \hat{p}_t = \frac{1}{\delta}\frac{1}{1-\beta}(\sigma\hat{c}_t - \beta E_t[\sigma\hat{c}_{t+1} + \hat{\pi}_{t+1}])$

(16) **CPI price level:** $\hat{p}_t = \hat{p}_{H,t} + \upsilon\hat{s}_t$

(17) **CPI inflation:** $\hat{\pi}_t = \hat{\pi}_{H,t} + \upsilon\Delta\hat{s}_t$

(18) **Home price level:** $\hat{p}_{H,t} = \hat{\pi}_{H,t} + \hat{p}_{H,t-1}$

(19) **World output:** $\hat{y}^\star_t = \phi^{y\star}\hat{y}^\star_{t-1} + \varepsilon^{y\star}_t$

(20) **Terms of trade:** $\hat{s}_t = \sigma_\upsilon(\hat{y}_t - \hat{y}^\star_t)$

(21) **Nominal ER:** $\hat{e}_t = \hat{s}_t - (\hat{p}^\star_t - \hat{p}_{H,t})$

(22) **Real ER:** $\hat{q}_t = (1-\upsilon)\hat{s}_t$

(23) **World prices:** $\hat{p}^\star_t = \phi^{p\star}\hat{p}^\star_{t-1} + \varepsilon^{p\star}_t$

Because the foreign economy is not affected by domestic developments, world variables (output/demand and prices) are assumed to be exogenously determined according to an AR(1) process. These equations are not all essential to simulating the SARB's asset purchases. They can, however, be used to simulate other shocks, and are reported here for completeness and replicability.

Appendix D. Data sources

Table D1: Data

Variable	Identifier	Description
GDP	KBP6006L	GDP at current prices, seasonally adjusted
Taxes	KBP4595F	Total national government tax revenue (net, fiscal year)
Monetary base	KBP1371M	M1A money supply
Long-term bond supply	KBP4167M	Ownership distribution of domestic marketable bonds: Total long term
HH LT bond holdings	KBP4562M	Domestic long term marketable bonds of national government held by the private sector
SARB LT bond holdings	KBP4161M	Domestic marketable long term national government bonds held by the SARB
Debt to GDP	KBP4116F	Total loan debt of national government: Total gross loan debt as percentage of GDP

Source: South African Reserve Bank Quarterly Bulletin