A New Keynesian DSGE model for Low Income Economies with Foreign Exchange Constraints

Bertha C. Bangara

ERSA working paper 795

September 2019

A New Keynesian DSGE model for Low Income Economies with Foreign Exchange Constraints

Bertha C. Bangara*

Abstract

The existing literature is clear that low-income economies tend to suffer from foreign exchange shortages exacerbated by their exports. Most importantly, the concentration of their exports renders these countries susceptible to international price fluctuations. This frequently affects the level of foreign exchange, causing an excess demand for foreign exchange leading to foreign exchange shortages. Using a four-sector New Keynesian dynamic stochastic general equilibrium (DSGE) model with foreign exchange constraints faced by importing firms, we calibrate the model to the Malawian economy to investigate the implications of foreign exchange constraints on key macroeconomic variables in low-income import-dependent economies. We demonstrate that imports are a vital part of the production process for LIEs and determine the response and direction of output and consumption. Second, the degree of the foreign exchange constraint determines the degree of variability of the shock but does not change the direction of the shock. Third, increasing imports in an effort to increase productivity reduce output and consumption and induces a depreciation of the exchange rate. Fourth, the model illustrates that the domestic contractionary monetary policy produces the conventional results on output, consumption and other variables.

Keywords: Low income economies, Foreign Exchange Constraint, DSGE, Malawi
JEL Classification: E32, F31, F35, O55

*Corresponding Author: University of Cape Town, Rondebosch 7701, Cape Town, South Africa. Email: berthabangara@gmail.com

1 Introduction

In the 1980s, most developing countries suffered a decline in foreign exchange reserves that led to a decline in total imports (Moran, 1989). Sub-Saharan African countries imports fell by 9%, leading to a fall in exports and per capita output, while non-oil exporting economies remained stagnant. The same period experienced a major shift in the composition of total imports in developing countries from consumer goods to intermediate goods and capital. This pushed research into models that relate output growth with foreign exchange resources via the aggregate production function but considering foreign exchange as a scarce resource and imports as factors of production and not final products (Lensink, 1995; Moran, 1989). This has led to a growing concern about foreign exchange imbalances and short-run macroeconomic policy in developing economies (Porter and Ranney, 1982). While most developing countries have experienced a substantial rise in foreign reserves, many low income[1] Sub-Saharan African (SSA) countries have by contrast experienced declining levels of foreign reserves (ibid). The accumulation of reserves in emerging economies attracted a wide range of research (see for example, Fukuda and Kon, 2008). However recently, a debate emerged on the macroeconomic implications of declining levels of foreign exchange reserves in SSA countries, arguing that although it is justifiable by fundamentals to hold minimum levels of foreign reserves (say three months of import cover); holding too low levels can have serious implications on the economy (McCormack, 2015). For example, continuous shortage of foreign exchange in a country can be a signal of some macroeconomic and financial stress which constrain economic growth, and may further entail balance of payments pressures in the economy (McCormack, 2015; Porter and Ranney, 1982).

The growing concerns for problems of foreign exchange shortages in SSA sparked a research focusing on the the dynamics of monetary and fiscal policies in low income economies, particularly in SSA where the problem of declining foreign exchange reserves has been observed since the 1980s. This is because SSA countries are prone to weak international prices of their export commodities that leads to fluctuations in foreign exchange earnings, capital flows and unstable macroeconomic policies (Addison and Ghoshray, 2013; Claessens and Ghosh, 2013). Furthermore, understanding the dynamics of monetary policy in LIEs can generally provide insights into whether some responses of macroeconomic variables may be used as early warning signals for potential instability in the economy that may require instant intervention. This paper seeks to contribute to the available literature on the dynamic responses of macroeconomic variables in LIEs by providing a

[1] In our analysis a 'typical' low income economy (LIE) is defined using the World Bank definition of a LIE with PPP-adjusted income per capita of less than US$1,000 per year since we take into consideration that some countries with higher incomes per capita share many characteristics with typical LIEs.

case study of Malawi. The paper investigates the dynamic responses of various macroeconomic variables to different policies in an economy facing foreign exchange problems when importing intermediate inputs. Using a four sector DSGE framework, we model the Malawian economy with foreign exchange constraints.

The paper is divided as follows. Section 2 discusses the role of foreign exchange in Malawi and provides some macroeconomic developments that were put in place as interventions for the recurring problem of foreign exchange unavailability. Section 3 provides a review of the empirical literature on the problems of declining foreign exchange reserves in most developing economies and section 4 builds the New Keynesian DSGE model for a foreign exchange constrained small open economy. Section 5 calibrates the model and describes the choice of the parameters that are included in the model as well as a discussion of the results and section 6 concludes the paper.

2 The Role of Foreign Exchange in Malawi

Foreign exchange reserves help maintain confidence in a country's currency by allowing the central bank to intervene in the foreign exchange market when there is a need to influence the exchange rate (Williams, 2006). However, holding very low levels of reserves erodes the soundness and confidence of the economy and economic growth may seem elusive (ibid). In addition, foreign exchange shortages can cause instability in firms by limiting their ability to purchase adequate factors of production to stabilise production across the years in the same way a rise in the world price of imports, or a decline in export earnings impacts firms (Senbeta, 2013; Moran, 1989). For countries with undiversified exports, fluctuations in world prices lead to fluctuations in export earnings (Agénor and Montiel, 2008b). Malawi for example, has experienced declining levels of foreign reserves for a long period of time, worsened by the fluctuations in international prices of the country's single export crop of tobacco leaf. As such, the role of foreign exchange availability in most Sub-Saharan African countries including Malawi cannot be overemphasized empirically (Senbeta, 2013; Mathisen, 2003; Moran, 1989; Marquez, 1985).

Malawi experiences a seasonal nature of foreign exchange earnings which is directly related to agricultural activities. With tobacco exports accounting for 60 percent of total foreign exchange earnings, the Kwacha appreciates during the tobacco market season (April to August) reflecting an increased supply of foreign exchange and normally depreciates in off season (September to March), reflecting increased demand of foreign exchange on the market as the economy imports farm inputs such as fertlizer (Simwaka and Mkandawire, 2008). This pattern however, tend to vary if the country received a substantial inflow of

foreign aid or has received less aid than anticipated.

Malawi floated the exchange rate in 1994 to reduce the pressure created by the insatiable demand for imports. However, the Reserve Bank of Malawi (RBM) intervenes continuously in the foreign exchange market (Simwaka and Mkandawire, 2008). More often, these interventions in foreign exchange market affect the value of the Malawi currency, the Malawi Kwacha (MK) by overvaluing it (ibid). This is because research on optimal exchange rate policies seem to lie between the theoretical extremes of complete flexibility and fixity of exchange rate (Doroodian and Caporale, 2001). In this case, optimal policy responses of the shocks to the economy are a function of the nature of the shocks and the degree of capital mobility in the economy (Simwaka and Mkandawire, 2008; Simwaka, 2004; Doroodian and Caporale, 2001).

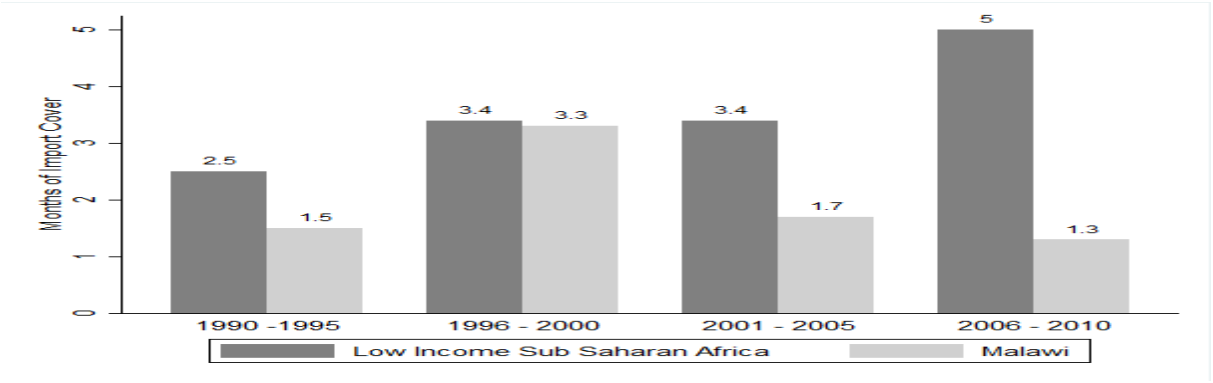
Malawi has for more than four decades practised a fixed exchange rate regime and a managed float to keep the Kwacha strong (Simwaka and Mkandawire, 2008). The country's monetary policy is conducted in an environment characterised by fiscal dominance, excessive dependence on donor aid, a non-competitive banking structure coupled with exogenous shocks where central bank independence is lacking (Mangani, 2011; Simwaka and Mkandawire, 2008). In addition, the political and institutional set-up for Malawi has made implementation of monetary policy difficult, which has also worsened the macroeconomic environment of the country (IMF, 2012). As such, the environment does not permit the policies to yield the desired results. Although the country's main exports are tobacco, tea, cotton and sugar; tobacco is the major export crop, making Malawi the largest producer of tobacco in Africa. The tobacco industry in Malawi is by far the largest employer after the government and the crop earns 20 times more than the value of tea (WHO, 2001).

The problem of shortage of foreign reserves seems to be a recurrent phenomenon. From the 1980s after the debt crisis that engulfed practically all developing countries to the present, the Malawian economy continues to experience tough macroeconomic imbalances (Simwaka and Mkandawire, 2008; Simwaka, 2006). Fluctuations in the level of foreign exchange reserves have always coincided with fluctuations in tobacco revenues every year because prices of tobacco on the auction floors are quoted in United States Dollars. As such, when tobacco fetches low prices, it also entails low levels of foreign exchange generated and therefore low foreign reserves. This indicates that the country's heavy dependence on tobacco exports contributes immensely to fluctuations in export revenues when the country experiences weak tobacco prices which lead to unreliable flow of foreign exchange revenues.

Malawi's total reserves (minus gold) did not rise when reserves in the SSA region were

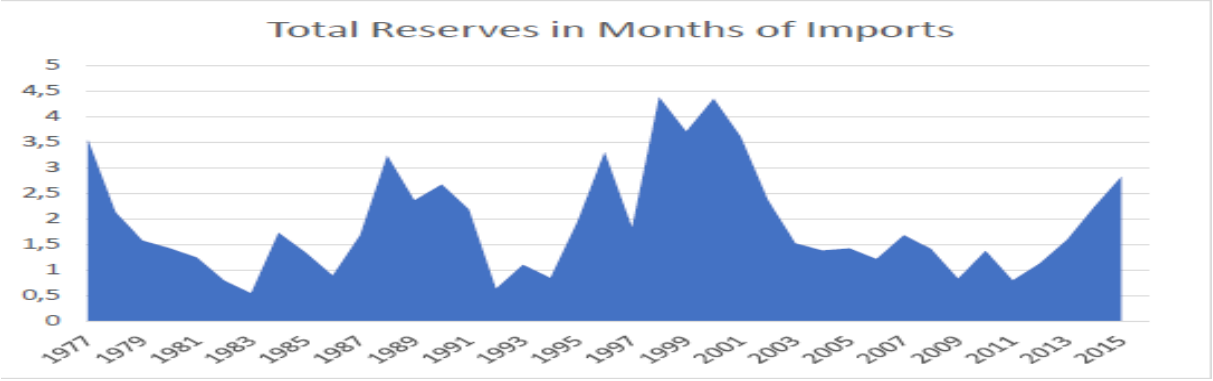
rising. Of all the countries in the SSA, Malawi has the lowest levels of foreign exchange and the difference with the SSA reserves is large (see Figure 1).

Figure 1: Import Cover for Low Income SSA and Malawi



In a bid to revive the declining economic growth, the country implemented the World Bank's structural adjustment reforms from 1981 to liberalise the economy, broaden and diversify the production base towards non-primary products and allocate resources to more productive sectors. However, the country failed to realise what the reforms intended to achieve which resulted in more macroeconomic problems. The failure to meet key economic stabilisation targets of low inflation, low interest rates and prudent spending led the country to incessant volatility in foreign reserves in the early 1990s when Malawi's traditional donors withheld economic reform funds. This led to the first ever floatation and a 74% depreciation of the exchange rate in 1994 to resolve the foreign exchange crisis that was exacerbated by the drought in 1992/93 (Simwaka, 2006; Munthali, 2004). Between 1990 and 1995, Malawi constantly experienced declining foreign reserves with about 1.5 months of import cover when low income economies and SSA countries had 2.5 months of import cover (see Figure 1). Between 1995 and 1997, the country implemented a fixed exchange rate which was maintained by running down foreign reserves. The low inflation that was attained at the end of 1997 was achieved at the expense of huge reserves (see figure 1) (Simwaka, 2006).

Figure 2: Reserves (months of import cover)



Source: The IMF 2012 Presentation on Economic Reforms and Economic Recovery Plan (Malawi)

However, the country experienced high levels of foreign exchange inflows between 1996 and

2000 after the first referendum when a new multiparty government was ushered into power. The year saw most donors who withdrew their aid restoring their provision. Consequently, the nominal exchange rate appreciated and created a current account imbalance. This, coupled with the fixed exchange rate worsened the foreign reserves status. By August 2003, the Kwacha stabilised at MK103 to 1 US$ in response to serious economic disequilibrium that followed after the suspension of the first IMF Poverty Reduction Growth Facility (PRGF) (Simwaka, 2006).

The Kwacha has been free-falling since 2003, depreciating more than 300% with a worsening of the foreign exchange problem in 2010. The country witnessed an increasingly larger trade deficit due to declining prices of tobacco exports and rising prices of imports which put a lot of pressure on low foreign reserves. In addition, the constant withdrawal of foreign aid due to poor macroeconomic policies, poor leadership and misappropriation of government and donor funds worsened the condition (MCC, 2012, Malawi-Government-Publications, 2011). The country experienced extreme shortages of foreign exchange from 2011 to 2012 when parallel market prices of the US dollar were 200% more than the official prices of the foreign currency, leading to devaluation of the Kwacha by 20% in September 2011 and another 50% in May 2012 (IMF, 2012).

3 Related Literature

The two key stylised facts of LIEs are that they rely on imported intermediate inputs and physical capital; and they frequently experience shortages of foreign exchange (Senbeta, 2011b). The shortage of foreign exchange constrains growth by limiting a country's ability to finance external payment bv to smooth current consumption but also delaying them in finalising laid down programs (Craigwell et al., 2003). In addition, LIEs fail to intervene in the foreign exchange market and fail to provide a buffer to cushion the economy against future fluctuations in the exchange rate, thereby creating lack of confidence in investors leading to capital flight. More importantly, the ability of LIEs to import intermediate inputs and capital determines the performance of the firms that operate in these countries, such that the availability and cost of foreign exchange play a huge role in the production process of LIEs.

Most empirical studies on the implications of foreign exchange on LIEs macroeconomic performance are cross country studies and have found a strong link between the availability and cost of foreign exchange and macroeconomic performance (see Agénor and Montiel, 2008a, Stiglitz et al., 2006, Lensink, 1995, Porter and Ranney, 1982, Moran, 1989). For example, using a 'standard LIE model' of aggregate demand and aggregate supply, Porter

and Ranney (1982) found that standard macroeconomic policy prescriptions often produce non-standard results where expanding output without increasing costs of production in the short-run is possible only if foreign exchange is located to purchase the needed imported raw materials. In addition, Lensink (1995), Moran (1989), and Marquez (1985) argue that the availability of foreign exchange in LIEs define the dynamics of macroeconomic policy of LIEs that heavily depend on imported intermediate inputs. For instance, Moran (1989) observed that declines in foreign lending and declines in terms of trade and debt service costs in the 1980s reduced foreign exchange availability and limited the import capacity of most developing countries. SSA countries experienced a significant fall in imports which in turn led to a decline in investment and per capita output. Furthermore, Moran (1989) argues that improvement in economic growth in LIEs depends on the availability of foreign exchange and imported intermediate inputs such that declines in foreign exchange inflows constrain production by limiting the amount of imported intermediate inputs and therefore growth.

A few recent empirical works underscore the importance of foreign exchange by stressing that imported intermediate inputs, the costs and the availability of foreign exchange are important determinants of private investment behaviour in LIEs (Agénor and Montiel, 2008a; Stiglitz et al., 2006; Polterovich and Popov, 2003). Specifically, Agénor and Montiel (2008a) argue that the specifications of relative factor prices in LIEs should not be restricted to wage rate and capital costs, but should take into account the domestic currency price as well as the availability of imported inputs because a domestic depreciation may result in higher revenues for exports and higher costs of imports, resulting in an ambiguous net effect depending on the degree of reliance on imported inputs. This is the reason Stiglitz and Charlton (2006) tend to argue that the principal limiting factor of economic activity in LIEs is the availability of foreign exchange because shortages of foreign exchange (supply constraint), outweighs demand constraints and force firms to produce below capacity.

However, three studies have recognised the importance of imported intermediate inputs in macroeconomic fluctuations of LIEs using a DSGE framework (Senbeta, 2013, 2011a; Kose and Riezman, 2001). Kose and Riezman (2001) use the real business cycle (RBC) model [2]to examine the role of external shocks in explaining macroeconomic fluctuations in African countries. They conclude that trade shocks account for half of the fluctuations in aggregate outputs such that adverse trade shocks cause prolonged recessions by inducing a significant decrease in aggregate investment. However, Kose and Riezman (2001) failed to recognise the importance of foreign exchange availability to determine the amount

[2]Fluctuations in export prices of primary commodities, imported capital goods and intermediate inputs.

of imported inputs which Senbeta (2013) and Senbeta (2011a) managed to incorporate. Senbeta (2013) and Senbeta (2011a) compared the variability of the standard and modified foreign exchange constrained DSGE models calibrated to the Ethiopian economy for specific monetary policy shock. Senbeta (2013) finds that contractionary monetary policy leads to an expansion in output and consumption and a contraction in employment. Furthermore, the impulse responses of the two models in Senbeta (2013) show that the modified model generates more variability than the standard model.

We add to the evidence provided in Senbeta (2013) and Senbeta (2011a) in the following ways: First, we examine the effects of aid shocks on the macroeconomic fluctuations of LIEs because Adam et al. (2009) and Mwabutwa et al. (2013) argue that erratic flows of aid to LIEs have serious economic implications in achieving the broader macroeconomic objectives of stable exchange rates and sustainable economic growth. Second, we incorporate stylised features of a small open economy LIE which depend on a single agricultural export commodity where fluctuations in international prices worsens its economic conditions. Third, we consider Malawi, which is an ideal country to analyse the effects of foreign exchange constraints on macroeconomic fluctuations due to its economic make-up.

The next section presents the DSGE model for a foreign exchange constrained low income economy.

4 A DSGE Model of Foreign Exchange Constraints

The model structure builds on the standard small open economy New Keynesian models of Monacelli (2005) and Justiniano and Preston (2004) with four sectors in the economy: households, firms, monetary authorities and the external sector. The household maximises inter-temporal utility function separable in consumption and labour with its financial resources coming from labour income and returns from holding bonds.

The firms consist of domestic producers and importing firms and their price setting behaviour follows Calvo, 1983 where the price setting mechanism allows for partial indexation of domestic and imported prices to their past inflation to provide additional nominal rigidities to the staggered price setting framework as in (Justiniano and Preston, 2004). In addition, we assume incomplete pass-through of exchange rate movement while habit formation provides real rigidity in the model.

Due to the small open economy properties of the model, we postulate that the relative size of the foreign economy is so large that it is not affected by any developments in

the Malawian economy and therefore approximates a closed economy. This work adopts most of its presentation and notation from Senbeta (2011a), Galí and Monacelli (2005), Galí (2008), and Peiris and Saxegaard (2007) and extends the Senbeta (2011a) model by including foreign aid and export earnings in the evolution of foreign exchange equation to capture some salient features that are specific to most LIEs that depend heavily on commodity exports and aid as the main sources of foreign financial inflow.

4.1 Household Behaviour

The infinitely lived representative household maximises inter-temporal utility subject to an inter-temporal budget constraint. The objective function is:

$$E_0 \sum_{t=0}^{\infty} \beta^t U_t \tag{1}$$

where

$$U_t = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{(C_t - hC_{t-1})^{1-\upsilon}}{1-\nu} - \chi \frac{(N_t)^{1+\varphi}}{1+\varphi} \right\} \tag{2}$$

E is the expectation operator and β is the subjective discount factor of the representative household's utility. The household derives utility from consumption of a composite good C_t, and disutility from labour effort N_t. The parameter ν is the inverse of the elasticity of inter-temporal substitution in consumption, h is the coefficient of habit persistence (where $0 < h < 1$), φ is the inverse of the elasticity of labour supply and χ is the marginal disutility of participating in the labour market. Since consumption is a composite good comprising the home and foreign goods, it is given by a CES aggregator:

$$C_t = \left[(1-\alpha_1)^{\frac{1}{\rho 1}} C_{H,t}^{\frac{(\rho 1-1)}{\rho 1}} + \alpha_1^{\frac{1}{\rho 1}} C_{F,t}^{\frac{(\rho 1-1)}{\rho 1}} \right]^{\rho_1/(\rho_1-1)} \tag{3}$$

Where $C_{H,t,'}C_{F,t}$ are consumption of home and foreign goods respectively, ρ_1 measures the elasticity of intra-temporal substitution of consumption between the home and foreign goods. The paraeter α_1 measures the proportion of foreign goods in the household's consumption. The household maximises the utility from consumption of both goods. The overall consumer price index is given as:

$$P_t = \left[(1-\alpha_1)(P_{H,t})^{1-\rho_1} + \alpha_1 (P_{F,t})^{1-\rho_1} \right]^{1/(1-\rho_1)} \tag{4}$$

Where$P_{H,t}$,$P_{F,t}$,P_t are price indices of domestic goods, foreign goods and overall consumer goods respectively. The total expenditure becomes:

$$P_t C_t = P_{F,t} C_{F,t} + P_{H,t} C_{H,t} \tag{5}$$

Both home and foreign goods are composite bundles of differentiated products. Solving the problem of allocation by the household, given the overall price index yields the following demand functions:

$$C_{H,t} = (1 - \alpha_1) \left(\frac{P_{H,t}}{P_t} \right)^{-\rho_1} C_t \tag{6}$$

$$C_{F,t} = \alpha_1 \left(\frac{P_{F,t}}{P_t} \right)^{-\rho_1} C_t \tag{7}$$

The household's income comes from wages and dividends. However, households have imperfect access to the financial market and as such, they hold foreign bonds earning interest rate r^*. Following Justiniano and Preston (2010) and Schmitt-Grohé and Uribe (2003), a debt elastic interest rate premium is introduced to close the model. The debt elastic interest premium takes the form: $\phi_t \; B_t^*$where ϕ_t is a function that is increasing in foreign debt (d_t); $\phi_t = exp\left[-\eta(d_t + \omega_t)\right]$. ϕ_t represents a risk premium shock and foreign debt is defined as $d_t \equiv \frac{\varepsilon_{t-1} B^*_{t-1}}{\bar{Y} P_{t-1}}$ [3] as in Justiniano and Preston (2010).

The household maximises lifetime utility subject to a budget constraint:

$$P_t C_t + B_t + \varepsilon_t B_t^* \leq W_t N_t + D_t + r_{t-1} B_{t-1} + \varepsilon_{t-1} r^*_{t-1} B^*_{t-1} \phi_t + \varepsilon_t P_t^* A_t \tag{8}$$

Where B_t is government bonds earning interest r_t. Thus the household expenditure consists of expenditure on consumption C_t and purchases of government bonds B_t and foreign bonds B_t^*. Income is comprised of dividends, D_t, wages W_t, returns from previous holdings of bonds, r_{t-1}, and returns from previous foreign bond holdings r^*_{t-1}; while ε_t is the nominal exchange rate. A_t represents aid and it captures all net foreign transfers both institutional and private. It is important to note here that a large percentage of household income in LIEs is transfers. Its log linearised function follows an AR(1) process as follows:

$$a_t = \rho_a a_{t-1} + \epsilon_{a,t}, 0 < \epsilon_{a,t} < 1 \tag{9}$$

[3] which is the real outstanding foreign debt expressed in terms of domestic currency as a fraction of steady-state output. Also see Benigno (2004) for a good discussion on how households face additional costs of borrowing in the international markets. One might also look at Schmitt-Grohé and Uribe (2003) on how to close small open economies and how this term ensures stability of the model.

where $\epsilon_{a,t} \sim i.i.d.N(0, \sigma_\epsilon)$. The first order conditions are as follows:

$$(C_t - hC_{t-1})^{-\nu} = \lambda_t P_t \tag{10}$$

$$\chi(N_t)^{\varphi} = \lambda_t W_t \tag{11}$$

$$\beta E_t \lambda_{t+1} r_t = \lambda_t \tag{12}$$

$$\beta E_t \lambda_{t+1} \varepsilon_{t+1} r_t^* \phi_{t+1} = \lambda_t \varepsilon_t \tag{13}$$

Combining equations (10) and (11) gives the marginal rate of substitution between consumption and labour:

$$\chi(N_t)^{\varphi}(C_t - hC_{t-1})^{\nu} = \frac{W_t}{P_t} \tag{14}$$

while (11) and (12) provides the consumption Euler equation:

$$\beta E_t \frac{(C_{t+1} - hC_t)^{-\nu}}{(C_t - hC_{t-1})^{-\nu}} \frac{P_t}{P_{t+1}} = \frac{1}{r_t} \tag{15}$$

Furthermore, the combination of (12) and (13) yields the uncovered interest parity (UIP) condition as: $\beta E_t \lambda_{t+1} r_t = \beta E_t \lambda_{t+1} \frac{\varepsilon_{t+1}}{\varepsilon_t} r_t^* \phi_{t+1}$

which simplifies to

$$\frac{r_t}{r_t^* \phi_{t+1}} = E \frac{\varepsilon_{t+1}}{\varepsilon_t} \tag{16}$$

4.2 Firms

4.2.1 Domestic Production

There exists a continuum of identical monopolistic competitive firms which produce domestic goods using capital, labour and imported intermediate inputs. We introduce the foreign exchange constraint in the model by assuming that importation of intermediate inputs by firms solely depends on the availability of foreign exchange. As such, when the country faces declining levels of foreign exchange, it is unable to provide the required amount of the needed foreign exchange for the importation of inputs, and this acts as a

constraint to the importing firms. We assume the free mobility of capital and labour in the economy for simplicity and these inputs are therefore homogeneous.

Firms use labour (N) and imported intermediate inputs (M) to produce tradable goods as in Senbeta (2011a) while capital is assumed fixed. As such, capital is excluded from the model. We assume a linear technology with constant returns to scale and the firm's production function is given as:

$$Y_{H,t} = A_{H,t} N_{H,t}^{\sigma_1} M_{H,t}^{\sigma_2} \tag{17}$$

where $\sigma_1, \sigma_2 > 0$ and $\sigma_1 + \sigma_2 = 1$. In addition, $A_{H,t}$ represents total factor productivity and its logarithm follows a first order autoregression process as follows:

$$lnA_{H,t} = \rho_H lnA_{H,t-1} + e_{H,t} \tag{18}$$

Where $0 < \rho_H < 1$. The term $e_{H,t}$ is i.i.d N(0, σ_{eH}). The cost minimisation problem by the representative firm given the production level is:

$$Min_{N_{H,t}, M_t}(W_t N_t + P_{F,t} M_t), s.t. Y_{H,t} = A_{H,t} N_{H,t}^{\sigma_1} M_{H,t}^{\sigma_2} \tag{19}$$

The resulting input demand functions are as follows:

$$N_t = \left(\frac{\sigma_1}{\sigma_2}\right)^{\sigma_2} P_{F,t}^{\sigma_2} W_t^{-\sigma_2} Y_{H,t} A_{H,t}^{-1} \tag{20}$$

$$M_t = \left(\frac{\sigma_2}{\sigma_1}\right)^{\sigma_1} P_{F,t}^{-\sigma_1} W_t^{\sigma_1} Y_{H,t} A_{H,t}^{-1} \tag{21}$$

Because $\sigma_1 + \sigma_2 = 1$, substituting the input demand functions into the objective function, and differentiating with respect to output, we obtain the marginal cost function:

$$MC_{H,t} = \left[\left(\frac{\sigma_2}{\sigma_1}\right)^{\sigma_1} + \left(\frac{\sigma_1}{\sigma_2}\right)^{\sigma_2}\right] \frac{P_{F,t}^{\sigma_2} W_t^{\sigma_1} A_{H,t}^{-1}}{P_t} \tag{22}$$

Equation (22) gives us the real marginal cost function in terms of total productivity, output, input prices and the share parameters.

4.2.2 Price Setting Behaviour

Domestic firms follow Calvo (1983) to set their prices, with each firm having the probability $1-\theta$ of being able to change the price of goods that are produced. For those prices that have been changed, we use $P^*_{H,t}$. Therefore, θ_H is used to describe the proportion of goods with a current price, $P_{H,t}$, equal to that of the previous period (i.e.$P_{H,t-1}$) as in Justiniano and Preston (2010). All firms adjust their prices according to an indexation rule ζ_H where $0 \leq \zeta_H \leq 1$ and ζ_H measures the degree of the firm's indexation to a past period's inflation rate. The re-optimising firm's price index evolves according to:

$$P_{H,t}(j) = \left[(1-\theta_H) P^{*(1-\rho_1)}_{H,t} + \theta_H \left(P_{H,t-1} \left(\frac{P_{H,t-1}}{P_{H,t-2}} \right)^{\zeta_H} \right)^{1-\rho_1} \right]^{1/(1-\rho_1)} \tag{23}$$

Assuming that the preferences are symmetrical between domestic and foreign goods, then the demand curve for the firm in period $t+k$ setting its price in period t becomes:

$$C_{H,t+k} = \left(\frac{P^*_{H,t}}{P_{H,t+k}} \left(\frac{P_{H,k-1}}{P_{H,t-1}} \right)^{\zeta_H} \right)^{-\rho_1} (C_{H,t+k} + C^*_{H,t+k}) \tag{24}$$

where $C^*_{H,t+k}$ is the foreign consumption (domestic exports). Then the price setting firm aims to maximise the expected discounted profits given by:

$$E_t \sum_{k=t}^{\infty} \theta_H^{k-t} \beta_{t,t+k} C_{H,t+k} \left[P^*_{H,t} \left(\frac{P_{H,t+k-1}}{P_{H,t-1}} \right)^{\zeta_H} - P_{H,t+k} MC_{H,t+k} \right] \tag{25}$$

β_{t+k} is the usual stochastic discount factor and MC_{t+k} is the real marginal cost function for each firm. The firm's first order condition is the aggregate price index for the traded goods that are produced domestically and is presented as:

$$E_t \textstyle\sum_{k=0}^{\infty} \theta_H^k \beta_{t,t+k} C_{H,t+k} \left[P_{H,t} \left(\frac{P_{H,k-1}}{P_{H,t-1}} \right)^{\zeta_H} - \frac{\rho_H}{\rho_H - 1} P_{H,t+k} MC_{H,t+k} \right] = 0$$

Solving for the domestic price of the traded goods yields:

$$P^*_{H,t} = \frac{\rho_H}{\rho_H - 1} \frac{E_t \sum_{k=0}^{\infty} \theta_H^k [\beta_{t,t+k} C_{H,t+k} P_{H,t+k} MC_{H,t+k}]}{E_t \sum_{k=0}^{\infty} \theta_H^k [\beta_{t,t+k} C_{H,t+k} (P_{H,k-1}/P_{H,t-1})^{\zeta_H}]} \tag{26}$$

4.2.3 Importing Firms

Importing firms import two types of goods: First, they use foreign currency to import final goods which are sold to domestic retailers in domestic currency and are consumed directly by the consumers. Second, these firms import intermediate goods which are used in the production of other final products. This representation is as in Justiniano and Preston (2010) and Christiano et al. (2011) however, the only difference is that firms in Christiano et al. (2011) do not face a foreign exchange constraint. In practice, the central bank is often not able to supply the required amount of foreign exchange to importers, and this creates an excess demand for foreign currency. We assume at the beginning of the period, the central bank has to distribute a certain amount of foreign exchange $P_t^* Y_{F,t}$ equally to identical importing firms who also face the same foreign exchange constraint $P_t^* Y_{jF,t}$. Therefore, at the aggregate level, $P_t^* Y_{F,t} = \int_0^1 P_t^* Y_{jF,t} di$. Due to insufficient flow of foreign exchange in the country, an importing firm may requests to import $Y_{F,t}(j)$ but the central bank provides only a fraction of the total amount of foreign exchange demanded by a firm, $\varrho_{F,t}$, where $0 \leq \varrho_{F,t} \leq 1$.

The amount of foreign exchange provided to each importing firm during foreign exchange constraints is given as $Y_{F,t}(i) = \varrho_{F,t} P_t^* Y_{jF,t}(i)$, causing the firms to import fewer final goods and intermediate inputs and to raise their prices because they sourced the additional foreign exchange at a premium in parallel markets. We differ from Senbeta (2013) in the way import demand is defined, and we move away from estimating the central bank's loss function and limit our nalysis to estimating a foreign exchange constrained import demand that defines the imported consumption goods and the intermediate inputs into the production process. We define the aggregate quantity of imported intermediate inputs and consumption goods demanded as: $Y_{F,t} = C_{F,t} + M_T$, where C is the imported consumption goods and M is the imported intermediate inputs. The foreign exchange constraint exists when the central bank can only satisfy a fraction $\varrho_{F,t}$ of the demanded quantity of foreign exchange. We also assume that the constraint is binding except at the steady state where the quantities are equal and $\varrho_{F,t} = 1$. The actual constrained import demand is denoted:

$$Y_{F,t} = \varrho_{F,t}(Y_{F,t}), \quad 0 < \varrho_{F,t} < 1 \tag{27}$$

Since we assume that the importing firm j faces the same foreign exchange constraint $P_t^* Y_{F,t}$, its production function is given as:

$$Y_{F,t} = \left[\int_0^1 Y_{F,t}(j)^{1-\frac{1}{\rho 1}} di \right]^{\frac{\rho 2}{\rho 2 - 1}}$$

Solving the profit maximisation problem for a perfectly competitive importing firm that imports $Y_{F,t}(j)$ gives the demand function for each input as:

$$Y_{F,t}(j) = \left(\frac{P_{F,t}(j)}{P_{F,t}}\right)^{-\rho_2} Y_{F,t} \tag{28}$$

To assess the effects of the foreign exchange constraint on the firm and the quantity imported, we compare the outcomes under the two conditions by first outlining the optimal price of an importing firm without the constraint which takes the price and the demand of its imports $Y_{F,t}(j)$ to maximise its profits, such that:

$$\underset{P_{F,t(j)}}{Max}(P_{F,t}(j) - \varepsilon_t P_t^*)Y_{F,t}(j) \tag{29}$$

subject to 28 . Solving this problem, the optimal mark-up price that the unconstrained firm charges for its imports is given as:

$$P_{F,t}(j) = \frac{\theta_F}{\theta_F - 1}\varepsilon_t P_F^* \tag{30}$$

However, when a firm faces a foreign exchange constraint, the quantity of their imports is less than what they would like to import without the foreign exchange constraint. The objective of the importing firm that faces the foreign exchange constraint is to maximise profits with respect to the foreign price of imports, the demand function it faces and the foreign exchange constraint it is experiencing;

$$\underset{P_{F,t(j)}}{Max}(P_{F,t}(j) - \varepsilon_t P_t^*)Y_{F,t}(j) s.t. \quad Y_{F,t}(j) \leq \varrho_{F,t}(Y_{F,t})$$

gives us the optimal price that the firm under constraint charges, and thus;

$$P_{F,t}(j) = \frac{\theta_F}{\theta_F - 1}\varepsilon_t P_F^*(1 + \varpi_{F,t}) \tag{31}$$

where $\varpi_{F,t}$ is the additional mark-up on the price that the firm charges as a result of a change in foreign exchange quantity. This is equal to zero if there are no foreign exchange constraints and is greater than 0 when constraint is binding, indicating that the optimal price that the firm charges under constraint is always greater than the price the firm

charges without the foreign exchange constraint; a result which is consistent with intuition and economic theory (Senbeta, 2013). In addition, when the constraint is binding, the quantity restriction imposed by the foreign exchange constraint allows the importers to charge a higher price compared to the price that they would charge when the constraint is not binding. This happens because we assume that the demand for the imported goods that the firms face remains unchanged in foreign exchange constraint and unconstrained times.

4.2.4 Price Setting by Importing Firms

Price setting by importing firms is the same for domestic producers. As was the case for domestic firms, the importing firms set their prices according to Calvo (1983) where $(1-\theta)$ represents the proportion/fraction of firms that can reset their prices and θ is the fraction of firms that index their prices to the past period's inflation as follows:

$$P_{F,t}(i) = P_{F,t-1}(i)\left(\frac{P_{F,t-1}}{P_{F,t-2}}\right)^{\zeta_F} \tag{32}$$

where $0 \leq \zeta_H \leq 1$ measures the degree of the firm's indexation to the past period's inflation rate. The firm's price index evolves according to:

$$P_{F,t}(i) = \left[(1-\theta_F)P_{F,t}^{*(1-\rho_1)} + \theta_F\left(P_{F,t-1}\left(\frac{P_{F,t-1}}{P_{F,t-2}}\right)^{\zeta_F}\right)^{1-\rho_2}\right]^{1/(1-\rho_2)} \tag{33}$$

Assuming that the preferences are symmetrical between domestic and foreign goods, then the demand curve firm in period $t+k$ setting its price in period t becomes:

$$y_{F,t+k} = \left(\frac{P_{F,t}^*}{P_{F,t+k}}\left(\frac{P_{F,k-1}}{P_{F,t-1}}\right)^{\zeta_F}\right)^{-\rho_2}(Y_{F,t+k}) \tag{34}$$

Therefore, the price setting firm aims to maximise the expected discounted profits

$$E_t\sum_{k=t}^{\infty}\theta_F^{k-t}\beta_{t,t+k}y_{F,t+k}\left[P_{F,t}^*\left(\frac{P_{F,t+k-1}}{P_{F,t-1}}\right)^{\zeta_F} - P_{F,t+k}MC_{F,t+k}\right] \tag{35}$$

where β_{t+k} is the usual stochastic discount factor and MC_{t+k} is the real marginal cost function for each firm. The firm's first order condition which is the aggregate price index for the traded goods that are produced domestically is given as:
$E_t\sum_{k=0}^{\infty}\theta_F^k\beta_{t,t+k}Y_{F,t+k}\left[P_{F,t}\left(\frac{P_{F,k-1}}{P_{F,t-1}}\right)^{\zeta_F} - \frac{\rho_F}{\rho_F-1}P_{F,t+k}MC_{F,t+k}\right] = 0$
and solving for the domestic price of the traded goods yields:

$$P_{H,t}^* = \frac{\rho_F}{\rho_F-1}\frac{E_t\sum_{k=0}^{\infty}\theta_F^k[\beta_{t,t+k}Y_{F,t+k}P_{F,t+k}MC_{F,t+k}]}{E_t\sum_{k=0}^{\infty}\theta_F^k[\beta_{t,t+k}Y_{F,t+k}(P_{F,k-1}/P_{F,t-1})^{\zeta_F}]} \tag{36}$$

It is worth remembering that firms in this section charge a mark-up on the original prevailing price to realise their goal of profit maximisation and cover their marginal costs. This is because the foreign exchange for importing the goods can be obtained from entities other than the Central Bank or other formal financial institutions (FFIs) albeit at a higher prices. It should be noted that these firms use foreign currency to import final goods, which are consumed directly by the consumers and are sold to domestic retailers in domestic currency. In addition, the firms import intermediate goods as factors of production in the economy. This representation is similar to that of Justiniano and Preston (2010) and Christiano et al. (2011), however, a key difference is that firms in Christiano et al. (2011) do not face foreign exchange constraints. In practice, in many LIEs, the central bank is often not able to supply the required amount of foreign exchange to importers, thereby creating an excess demand for foreign currency in the economy.

4.3 International Risk Sharing

The law of one price indicates that commodities will have the same price when exchange rates are taken into consideration. The ratio of the foreign price to the domestic price should equal 1; on the other hand, the law of one price gap reveals that the law of one price fails to hold. For example, Monacelli (2005) states that the law of one price fails to hold when the ratio of two currencies is not equal to one. Therefore, the gap is given by the ratio of the index of foreign price in terms of domestic currency to the domestic currency price of imports (which is not equal to 1) as:

$$\Psi_t = \frac{\varepsilon_t P_t^*}{P_{F,t}} \tag{37}$$

where ε_t is nominal exchange rate, P_t^* is domestic price of exported goods and $P_{F,t}$ is the foreign price. The real exchange rate is the ratio of the rest of the world price index in terms of domestic currency to the domestic price index as:

$$Q_t = \frac{\varepsilon_t P_t^*}{P_t} \tag{38}$$

And terms of trade is defined as $s_t = \frac{P_{F,t}}{P_{H,t}}$. However, apart from the risk sharing assumptions, we introduce a country specific risk premium shock which follows an AR(1) process as $risk_t = \chi_{risk} risk_{t-1} + \epsilon_{risk}$. This shock captures time-varying country risk premia as in Alpanda et al. (2010).

4.4 Monetary Policy

Most recent research in DSGE literature indicates that LIEs employ monetary policy regimes that are very different from HIEs and therefore different from the standard simple Taylor rule (Mwabutwa et al., 2013; Senbeta, 2013). Most central banks in developing economies especially LIEs use foreign exchange market intervention as a macroeconomic stabilisation policy tool that enables them to buy foreign exchange and build up reserves that help moderate the exchange rate fluctuations apart from their main roles of stabilising inflation and promoting output growth (Simwaka, 2006).

According to Mangani (2011) and Mwabutwa et al. (2013), the Reserve Bank of Malawi (RBM) targets broad money and reacts to inflation while moderating the exchange rate in setting the monetary base; with the bank rate determination being influenced by the desire to correct the disequilibria rather than economic developments. Clearly, the central bank managed the exchange rate in the period under analysis resulting in numerous foreign exchange unavailability problems and constant depreciation of the Kwacha. This indicates that the central bank should also respond to changes in the foreign exchange rate. Therefore, we modify the Taylor rule to incorporate the fact that changes in the foreign exchange rate affects key macroeconomic variables and the central bank reacts to changes in the real exchange rate apart from the standard reactions of deviations in inflation and output. Therefore, the monetary authority is assumed to stabilise inflation, output and exchange rate. In log-linearised form it is given as:

$$r_t = \rho_r r_{t-1} + (1-\rho_r)(\phi_{r\pi}\pi_t + \phi_{ry}y_t + \phi_{re}\Delta e_e) + \epsilon_{r,t} \tag{39}$$

where $\phi_{r\pi}, \phi_{ry}, \phi_{re}$ are weights that allow the monetary authorities to control inflation, output and nominal exchange rate. ρ_r is the smoothing parameter which indicates the persistence of interest rate. The lagged interest rate is for interest rate smoothing while $\epsilon_{r,t}$ captures the monetary policy shock. $\epsilon_{r,t}$ is i.i.d $(0_{\epsilon r}, \sigma_{\epsilon r})$.

Reserve Accumulation

The central bank accumulates foreign reserves as an important instrument in its implementation of monetary policy since it continuously face foreign exchange problems. The country relies heavily on a single agricultural export commodity which makes the economy vulnerable to international prices and creates an unstable flow of export earnings. The country also relies on the constant flow of developmental aid which supplements the inflow of foreign exchange. This means any withdrawal of aid coupled with fluctuation in international prices of agricultural commodities reduce the inflow of foreign exchange and

result in serious implications in the economy. As such, the RBM influences the quantity of the imported goods as it decides on how much is allocated to imports every month. The current account for the country is represented by

$$\varepsilon_t B_t^* = r_{t-1}^* \varepsilon_t B_{t-1}^* \phi_t + \varepsilon_t A_t + P_t C_{H,t}^* - \varepsilon_t P_t^* Y_{F,t} \tag{40}$$

with $\varepsilon_t B_t^*, r_{t-1}^* \varepsilon_t B_{t-1}^* \phi_t, \varepsilon_t A_t$ being net foreign assets and $P_t C_{H,t}^* - \varepsilon_t P_t^* Y_{F,t}$ are net exports. $P_t^* C_{H,t}^*$ represents domestic exported goods, or foreign consumption of domestically produced goods. The variables are defined in the previous sections of the model. Therefore, the foreign exchange holdings evolve according to:

$$res_t = res_{t-1} + r_{t-1}^* \varepsilon_t B_{t-1}^* \phi_t + \varepsilon_t A_t + P_t^* C_{Ht}^* - \varepsilon_t P_t^* Y_{F,t} \tag{41}$$

where res_t is the foreign exchange holdings this year. Thus, the reserves this year depend on last year's reserves, returns on last year's foreign bond, foreign aid, export earnings minus import expenditures. We assume that the central bank has an operational target for foreign exchange reserves, and if the reserves are below that target, the country fails to import the required amount of intermediate inputs and capital needed for production and therefore reducing growth.

4.5 The External Sector

The domestic economy is relatively small, therefore, we model it as a closed economy since it cannot affect the world prices, but the foreign economy is modelled as exogenous with the foreign interest rate r_t^*, foreign inflation π_t^*, foreign output or income y_t^* being determined by a vector of autoregression processes of order one, i.e. AR(1) process (Monacelli, 2005). The processes are given as:

$$y_t^* = \rho_{y^*} y_{t-1}^* + \epsilon_{y^*,t} \tag{42}$$

$$\pi_t^* = \rho_{\pi^*} \pi_{t-1}^* + \epsilon_{\pi^*,t} \tag{43}$$

$$r_t^* = \rho_{r^*} r_{t-1}^* + \epsilon_{r^*,t} \tag{44}$$

where $0 < \rho_{y^*}, \rho_{\pi^*}, \rho_{r^*} < 1$, and y_t^*, π_t^*, r_t^* are foreign output, inflation and interest rate respectively in log-deviations from the steady state. All disturbance terms are distributed normally as follows: $\epsilon_{i,t} \sim N\ (0, \sigma_i^2)$.

4.6 General Equilibrium

The model equilibrium is where households maximise utility subject to the budget constraint, producers of home produced and importers of intermediate inputs maximise profits. Thus equilibrium is where the domestic goods market requires output to equal the sum of domestic and foreign consumption for it to clear. Therefore

$$Y_t = Y_{H,t} = C_{H,t} + C^*_{H,t} \tag{45}$$

but

$$C_{H,t} = (1-\alpha_1)\left(\frac{P_{H,t}}{P_t}\right)^{-\rho_1} C_t \tag{46}$$

which simplifies to

$$c_{H,t} = -\rho_1(p_{H,t} - p_t) + c_t) \tag{47}$$

then the consumption of foreign residents (domestic exports) is given as:

$$C^*_{H,t} = \alpha_1\left(\frac{P_{H,t}}{P_t}\right)^{-\rho_1} C^*_t \tag{48}$$

but using the fact that $p_t - p_{H,t} = \alpha_1 s_t$, then $C_{H,t} = \rho_1\left(\frac{P_{H,t}}{\varepsilon_t P^*_t}\right)^{-\rho_1} C^*_t$, thus $C_{H,t} = \alpha_1\left(\frac{P_{H,t}}{\varepsilon_t P^*_t}\right)^{-\rho_1} C^*_t$ which can be simplified to $c^*_{H,t} = \rho_1\alpha_1 s_t + \rho_1 q_t + c^*_t$.
While assuming that all households in the economy face the same budget constraint, the aggregate foreign assets in the economy minus net exports is provided as:
$\varepsilon_t B^*_t = r^*_{t-1}\varepsilon_t B^*_t\phi_t + \varepsilon_t A_t + P^*_t C^*_{H,t} - \varepsilon P^*_t Y_{F,t}$
the variables are as defined in the previous sections.

4.7 Model Solution

The model solution for the system of linear equations presented in the previous section, and the detailed representations in Appendix characterise the model to consist of both lagged variables (for example c_{t-1}) and expected future endogenous variables (e.g. $E_t c_{t+1}$). This indicates that the model consists of backward looking and forward looking variables. Therefore we can write the unique solution of the model in matrix representation as follows:

$$Ay_t = BE_t(y_{t+1}) + Cy_{t-1} + Dz' \tag{49}$$

$$z_t = Kz_{t-1} + v_t \tag{50}$$

where y_t is a vector of endogenous variables, z_t is a vector of shocks, A, B, C, D and K are

matrices of structural (unknown) parameters which are functions of deep parameters. In addition, v_t is a vector of disturbances in the model. Therefore, the closed-form solution is given by:

$$y_t = Py_{t-1} + Qz_t \tag{51}$$

$$z_t = Kz_{t-1} + v_t \tag{52}$$

where P, Q are reduced form parameters of the model and equation (48) simply states that the model, though in its compact form is a function of lagged variables or backward looking patrameters and a vecor of shocks. we can rewrite the whole model in compact form as:

$$x_t = \Phi x_{t-1} + \Theta\varepsilon_t \tag{53}$$

which is a VAR(1) model where $x_t = (y_t', z_t')'$ and $\varepsilon_t = (0', v_t')'$ is a vector of errors.

5 Calibration

DeJong and Dave (2007) state that calibration is the quickest way to estimate the usefulness of successive extensions or modifications of a model and to be able to compare the dynamics of some fundamental macroeconomic variables in response to various shocks affecting the economy. Apart from knowing whether the modifications that we introduce to an otherwise standard model are supported by facts about the economies under analysis and their overarching theories, parameters of the model are calibrated to simulate the model and compare the responses of these variables. However, the problem that exists in many low income studies is the lack of data to carry out calculations specific for a single economy. As such, studies rely on calibrations made by other studies which are close or provide a close representation of the country under analysis. We use parameters that have been used in other studies similar to ours and where a specific parameter is unavailable it is then calculated using Malawian data.

Several parameter values have been adopted from the literature where the values are fairly standard. Following Mwabutwa et al. (2013), the consumer discount factor β is approximated at 0.99 which is also supported by Alpanda et al. (2010), Peiris and Saxegaard (2009) and Adam et al. (2009). The common value for intertemporal elasticity of substitution for low income Sub-Saharan African countries is 0.34 ($\nu = 2.96$) as estimated by Ogaki and Park (1997) which was also adopted by Berg et al. (2012) and Senbeta (2013) while the elasticity of labour supply φ is assumed to be 2 as in Berg et al. (2010) supported by Mwabutwa et al. (2013). Since evidence from many countries indicates that

time spent working does not vary dramatically, we adopt the share of labour in the production of home produced goods and the share of intermediate inputs in the production of home produced goods σ_1, σ_2 , to be 0.74 and 0.26 respectively following Mwabutwa et al. (2013).

Some parameter values are obtained from the quarterly data under analysis to obtain the steady state values. For example, χ_r , the ratio of imports to foreign exchange reserves is approximated at 1 since we assume that the Central Bank's aim is to at least have an operational target that is fixed at 3 months of import cover, as recommended by the IMF. We take the most plausible level of 1 month as reported by the World Bank's Economic Indicators which states that the RBM has been struggling at 1.3 months of import cover for most of the years. In addition, consumption to GDP ratio χ_g is assumed to be 0.8, total imports to GDP ratio χ_f is approximated to be 0.4750, ratio of imported consumption goods in the total imports χ_c is 0.8 and the ratio of aid to imports χ_a is 0.12 following Mwabutwa et al. (2013), Ngalawa and Viegi (2013) and IMF Country Reports.

A full summary of the parameter values is presented in Table 1 and the sources are provided in Appendix.
Data used for calculation of some of the parameters and the steady state values is sourced from the IMF's International Financial Statistics, World Bank, the Reserve Bank of Malawi and National Statistics Office of Malawi. DYNARE[4] is used to solve the model numerically and generate impulse response functions to domestic and external shocks. The parameters used are listed in the appendix and were selected on the basis that they are estimated for LIEs such as Mozambique as in Peiris and Saxegaard (2007), for Malawi estimated by Mwabutwa et al. (2013), Sub-Saharan African countries as in Berg et al. (2010), Ogaki and Park (1997) and other LIE studies (Senbeta, 2011a).

5.1 Simulations, Results and Inferences

To analyse the foreign exchange problem as a constraint to importing firms in an otherwise standard DSGE model, and to assess the dynamics of certain macroeconomic variables, the model is simulated. An analysis is made of the impact of shocks to: aid, terms of trade, domestic monetary policy, foreign monetary policy, domestic productivity and imports on output, consumption, marginal costs, CPI inflation, domestic inflation, imported inflation, nominal depreciation, real exchange rate and imports. We examine the dynamics of the variables in response to these shocks by analysing the impulse response functions and forecast error variance decomposition of the shocks to the variables. We assume that all

[4]Dynare is a free software that is used for the analysis of dynamic stochastic general equilibrium (DSGE) and overlapping generations (OLG) models which runs in Matlab. Dynare can be downloaded at http://www.dynare.org/download.

Table 1: Calibration of the Model

Parameter	Value	Description
χ_a	0.12	Ratio of net aid to imports
χ_r	1	Ratio of imports to foreign exchange reserves
χ_c	0.3	Ratio of imported consumption goods in total imports
χ_g	0.8	Consumption to GDP ratio
χ_f	0.48	Total imports to GDP ratio
χ_m	0.7	Ratio of imported intermediate inputs in total imports
β	0.99	Household discount factor
ν	0.34	Intertemporal elasticity of substitution in consumption
φ	2	Elasticity of labour
χ	0.24	Marginal disutility of working
ρ^1	0.83	Elasticity of substitution between imported and home goods
θ_f	0.56	Elasticity of substitution between varieties of imported goods
α_1	0.5	Share of imported goods in consumption
h	0.25	Coefficient of habit persistence
χ_{risk}	0.92	Risk premium parameter
σ_1	0.74	Share of labour in production of home goods
σ_2	0.26	Share of intermediate inputs in production of home goods
ς_F	0.5	Weight attached to past inflation for importing firms
ς_H	0.5	Weight attached to past inflation by domestic producers
θ_F	0.5	Fraction of importing firms that index their prices
θ_H	0.5	Fraction of domestic producers that index prices
ρ_H	0.66	Persistence of total productivity shock
ρ_r	0.73	Persistence of interest rate
ϕ_π	1.6	Inflation stabilisation
ϕ_y	0.057	Output stabilisation
ϕ_e	0.18	Exchange rate stabilisation
ρ_a	0.12	Persistence of aid shock
ρ_{y^*}	0.54	Persistence of foreign income shock
ρ_{π^*}	1.2771	Persistence of foreign inflation shock
ρ_{r^*}	0.8	Persistence of foreign interest rate shock

the shocks are temporary and show percentage deviations of the variables from their steady state levels. The next section provides the analysis of the impulse response functions.

5.1.1 Foreign Monetary Policy Shock

Figure 3: Impulse Responses to Foreign Monetary Policy Shock

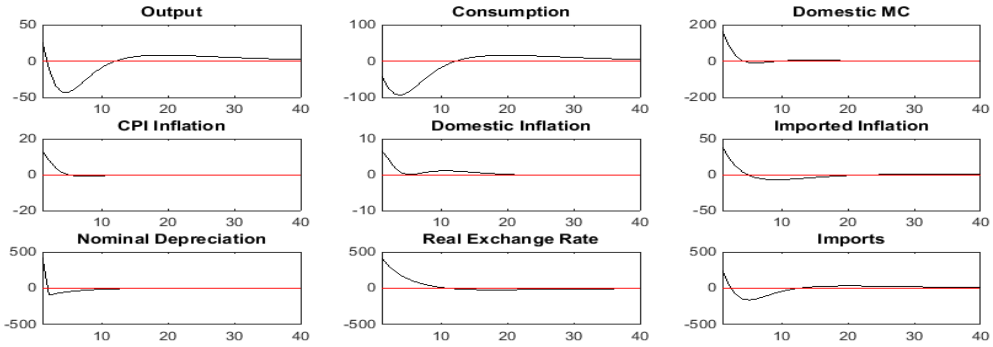
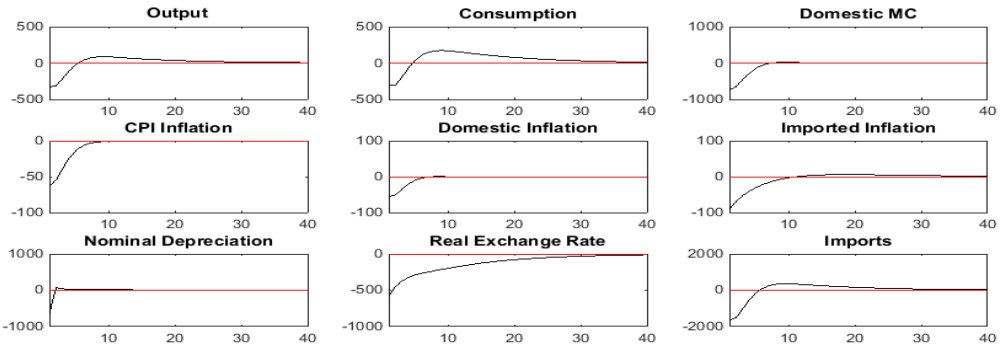


Figure 4 shows a 100 basis point innovation in the foreign monetary policy. From the figure, we note that a contractionary foreign monetary policy increases domestic output on impact which then falls and later begins to recover after some time similar to the impulse response of output in Senbeta (2011a). However, consumption falls at impact and continues to fall for a certain period, but later starts to rise and converges to its steady state level. An increase in the foreign policy interest rate results in a nominal depreciation of the domestic currency and depreciates the real exchange rate at impact but gradually appreciates to its steady state level. This effect makes exports competitive and imports expensive. This can be seen by a fall in consumption and output with an increase in imported inputs and consumption goods.

A rise in imports at impact raises both imported inflation and domestic inflation which leads to a rise in the overall CPI inflation in the economy. However, an increase in imported intermediate inputs and consumption goods lead to an increase in productivity and therefore output starts to rise and consumption also begins to rise. Total imported intermediate inputs and consumption goods start to fall as exchange rate appreciates gradually. This also decreases the domestic marginal costs which reduces imported inflation, domestic and CPI inflation at the same time. The outcome of a tight foreign monetary policy differs slightly from Senbeta (2013) where both output and consumption falls when tight monetary policy leads to a depreciation of the domestic currency, raising the costs of imported intermediate inputs and consumption goods. This makes both output and consumption fall after the impact, but gradually recovers as they move to their steady state levels.

5.1.2 Domestic Monetary Policy Shock

Figure 4: Impulse Responses to Domestic Monetary Policy Shock

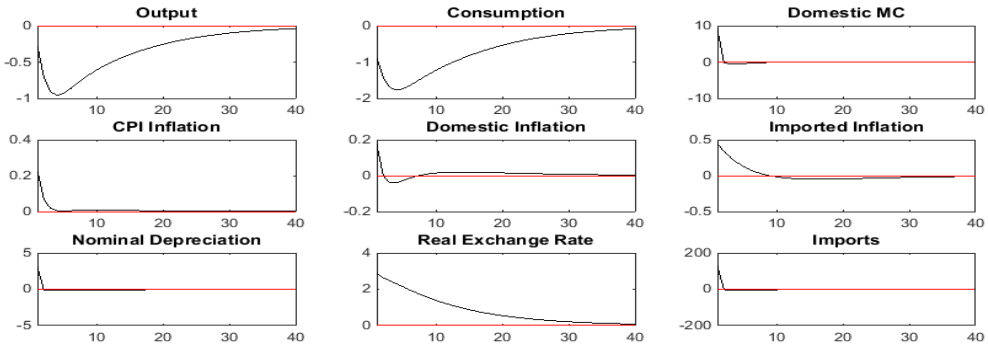


A 100 basis point decrease in domestic monetary policy shock (contractionary monetary policy) decreases output, consumption, marginal costs, imported inflation, domestic inflation and CPI inflation in the economy on impact but all gradually rise and converge to their steady state levels. In addition, the currency appreciates by more than 500 basis points while imports also decrease. This is in line with theory and it achieves what a contractionary monetary policy is set to achieve. In addition, the foreign exchange problems seem to amplify the shocks but does not change the direction of the shock. This is the reason we see a conventional response of monetary policy shock while currenc appreciates by more than 500 basis points in response to a 100 basis point decrease in domestic monetary policy. As a result, a 60 basis point contractionary policy changes output, consumption and all other variables by more than 60 basis points as shown in Figure 5.

5.1.3 Import Shock

Figure 6 presents impulse response functions for a positive innovation in imports (both intermediate inputs and consumption goods). A 100 basis point increase to a shock to imports of both intermediate inputs and consumption goods decreases output and consumption which is an expected result. A positive shock to import increases the real exchange rate causing a real depreciation and improves trade competitiveness for the country. But since the country is import dependent, a real depreciation increases the costs of imported intermediate inputs and consumption goods thereby increasing production costs, evidenced by the rise in the domestic marginal costs.

Figure 5: Impulse Responses to Import Shock

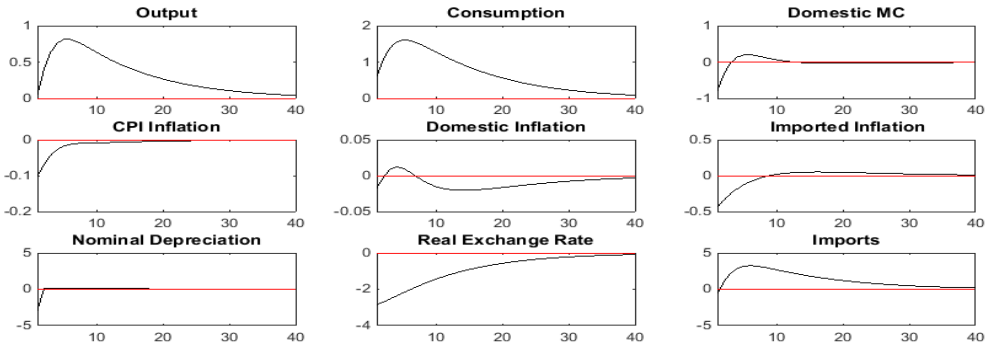


With a rise in marginal costs, production is supposed to fall. This effect impacts negatively on unemployment and wages in the economy, therefore, unemployment increases and income fall. Consequently, this leads to a fall in private consumption as imported consumption goods become expensive. Due to a rise in imported inflation, domestic inflation and CPI inflation rise. As a result, output and consumption fall in the process with consumption falling more than output at impact (1 basis point fall in consumption as compared to 0.5 basis points fall in output).

5.1.4 Aid Shock

A one percent increase to a shock to aid results in an increase in output and consumption in line with the standard theory of aid which postulates that the effect of aid surges in an economy depends on whether the aid is absorbed or accumulated as reserves. Berg et al. (2010) state that when aid is fully absorbed and spent in the economy, a situation which is rare in LIEs, government increases investment, and aid finances the resulting rise in net imports without creating a balance of payments problem. A real appreciation is required to enable the reallocation of resources. Therefore, in line with this theory, the responses of the variables to a positive shock to aid are strikingly surprising as they reproduce the theoretical responses: output and consumption increases as government increases expenditure, but this positive shock to aid also results in nominal and real appreciation of the exchange rate leading to cheaper imports and low imported inflation, reducing the domestic marginal costs, as firms import the inputs at a lower price.
Due to lower imported inflation and lower marginal costs, domestic inflation and CPI inflation fall. But because the country depends more on its exports for foreign exchange inflows, the real appreciation of the exchange rate could worsen the current account position of the country as it threatens the competitiveness of the export sector which may be critical for long-run economic growth. A fall in Malawian exports could mean a fall in foreign exchange inflow available for importation of the intermediate inputs and consumption goods and this could also cause imports to fall in the long run. But because this effect results in low inflation and interest rates, it creates a good economic

Figure 6: Impulse Responses to Aid Shock

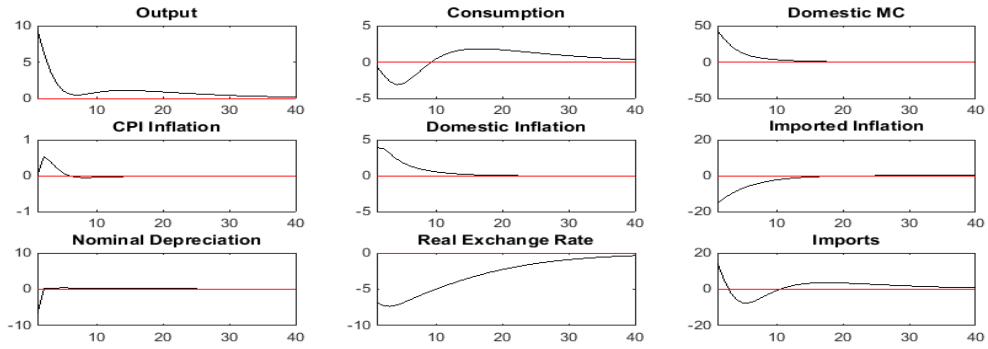


environment and therefore a good outcome for both the donors and the government. Since this is in the short-run, the impact may not be visible in the output and consumption in Figure 7 as explained earlier on by Mwabutwa et al. (2013). However, the result may be different in the long run.

In addition, this result is consistent with the conclusions reached by Rajan and Subramanian (2011) who argue that aid inflows harm recipient nations by reducing the relative growth rate of exportable industries. They find evidence that inflows of aid cause real exchange rate appreciation and ultimately find little evidence supporting the views that aid leads to economic growth. This indicates that the response of the economy to aid can be contrary to conventional responses, because aid is mostly associated with exchange rate depreciation in most LIEs (see Mwabutwa et al. (2013), Berg et al., 2010).

5.1.5 Terms of Trade Shock

Figure 7: Impulse Responses to Terms of Trade Shock

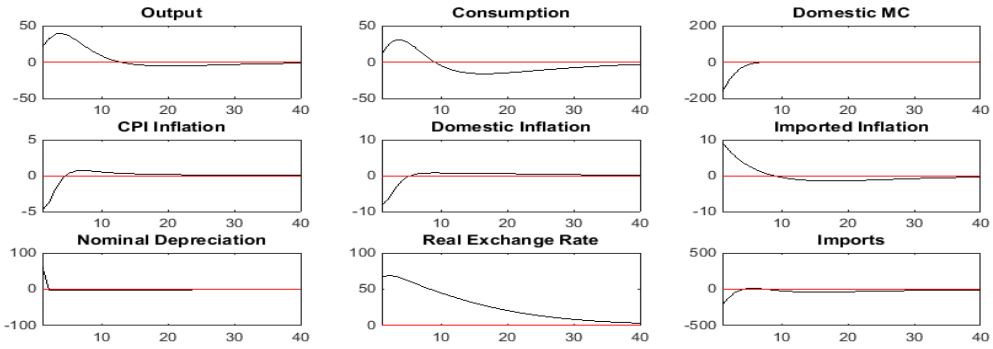


Likewise, an improvement in terms of trade induces an increase in output as the country increases imports of intermediate inputs and therefore production. Terms of trade is defined as a ratio of domestic currency price of home produced tradable goods to the domestic currency price of imports. An increase in terms of trade in Figure 8 leads to real and nominal appreciation of the exchange rates which make exports expensive.

This makes imports cheaper and the country increases the importation of inputs and consumables. This appreciation of the exchange rate does not put pressure on imported goods thereby reducing imported inflation, while at the same time putting pressure on domestic goods and hence raising the domestic inflation and CPI inflation altogether. This therefore makes consumption fall. Marginal costs also increase as imports increase, due to high dependency of the country on imported intermediate inputs.

5.1.6 Productivity Shock

Figure 8: Impulse Responses to Domestic Productivity Shock



Finally, a positive shock to domestic productivity increases employment and therefore output and consumption and decreases marginal costs. Both output and consumption increase on impact. They then start declining as rising demand for labour increases wages which results in rising production costs, leading to declining consumption and output after impact as marginal costs begins to rise. This is complemented by a fall in inflation on impact which gradually increases. As in conventional monetary policy, the increase in productivity leads to an appreciation of the real exchange rate, thereby hurting the export sector and generating less foreign exchange necessary for the flow of imports into Malawi. Therefore imports decline on impact. However, as the changes in the whole economy start to take shape, imports begin to rise and gradually converge. Imported inflation begins to rise as consumption rises, while output starts falling as domestic inflation and CPI inflation falls, with the real exchange rate deprecating gradually before all the variables return to their steady state level in figure 9.

A robustness check of the results in the model is carried out to verify the validity of the results presented in the previous section. First, we assume that the ratio of net foreign aid inflows to imports is 0.2 and the elasticity of substitution between imported and home produced goods is 0.3. The model indicates that the impulse response functions to all the shocks analysed in the model do not change the direction of the shock but the magnitude is amplified.

Second, we assessed the impulse responses of innovations to some selected shocks in the model when the constraint is not binding. The innovations only changed the magnitudes of the responses of the macroeconomic variables under investigation, as the responses of the variables became smaller when compared with the responses of the same variables in a foreign exchange constrained model. For example, a foreign contractionary monetary policy leads to real exchange depreciation and a competitiveness in the export sector, but because the economy is import dependent, imports fall. But a real depreciation still makes imports expensive and raises marginal costs and imported inflation, which leads to a rise in domestic inflation and CPI inflation, while output and consumption fall. Despite the changes being the same, the magnitudes differ, thus the IRFs for the unconstrained model have less variability than the constrained model, as seen with the rise in inflation of 3 basis points in the unconstrained model compared to 7 basis points in the constrained model.

Therefore, we conclude that the IRFs for unconstrained model still predict almost the same directions but differ in magnitudes with the response to shocks of the assessed macroeconomic variables in a foreign exchange constrained economy of Malawi thereby reaching almost the same conclusions that were reached in the model analysis (IRFs are provided in Appendix).

Having analysed the impulse response functions of the variables to different shocks in the model, the next section discusses the forecast error variance decomposition of the shocks to the variables.

5.1.7 Forecast Error Variance Decomposition (FEVD)

Forecast error variance decomposition[5] measures the contribution of each type of shock to the forecast variance by providing information on how shocks to economic variables reverberate through the system. The shocks that provide the cyclical fluctuations through the propagation of macroeconomic mechanisms are at the same time the sources of forecast uncertainty. Therefore, FEVD determines how much of the forecast error variance of each variable is explained by exogenous shocks to the other variables. Table 2 provides the results.

The output in the model is more affected by shocks to domestic monetary productivity, domestic monetary policy, terms of trade and foreign monetary policy; with minimal ef-

[5]The IRFs provide all the shocks that are analysed in the model. However, some of the shocks contribution to the forecast error variance decomposition of the variables are minimal and they are left out in the analysis.

fect emanating from shocks to aid and imports. Domestic monetary policy shocks account for about 59% of the total variations in the one quarter ahead forecast of output which happens to be the maximum as their contribution diminished in longer horizons although substantial, 34% in the five period ahead forecast, 32% in the 10th and hovering around 32% and 31% in the remaining periods up to the 40th period. The importance of domestic productivity shocks in explaining the variation in output cannot be overemphasised, contributing about 16% in the one quarter ahead forecast but increased significantly in the following quarters with the highest being about 58%. Thereafter the contribution remained above 50%. The terms of trade shocks contribute 23% to the variations in output in the first quarter but their contribution diminished over the years. The results are in line with the effects of monetary policy on domestic economy (Senbeta, 2013). However, since Malawi has an import dependent and also aid dependent economy, we expected imports and aid to contribute significantly to fluctuations in output with the economy, which is not the case with these results.

Table 2: Forecast Error Variance Decomposition of 10,30,40 Quarters Ahead (in %)

Shock / Variable	*Aid*	*Domestic Productivity*	*Domestic M Policy*	*Foreign M Policy*	*TOT*
(*10 quarters*) Output	0.04	58.62	30.26	3.53	7.42
Consumption	0.18	33.81	42.84	20.26	2.3
Marginal Cost	0.00	46.56	20.43	2.15	30.56
CPI Inflation	0.01	22.44	68.4	7.29	1.77
Domestic Inflation	0.00	21.7	17.71	0.65	59.92
Imported Inflation	0.01	3.55	5.11	2.43	88.89
Nominal Depreciation	0.05	18.34	27.73	52.23	1.51
Real Exchange Rate	0.07	46.93	21.22	27.82	3.77
Imports	0.04	27.13	43.71	4.21	1.39
(*30 Quarters*) Output	0.05	56.84	31.82	3.57	7.55
Consumption	0.18	40.96	40.98	14.21	3.14
Marginal Cost	0.00	46.49	20.41	2.15	30.66
CPI Inflation	0.01	22.76	68.07	7.27	1.78
Domestic Inflation	0.00	22.32	17.56	0.68	59.42
Imported Inflation	0.01	4.03	5.11	2.55	88.27
Nominal Depreciation	0.05	18.54	27.7	52.05	1.53
Real Exchange Rate	0.07	51.66	20.54	23.35	4.19
Imports	0.05	31.33	42.37	3.97	1.64
(*40 Quarters*) Output	0.05	56.92	31.72	3.6	7.54
Consumption	0.17	41.73	40.33	14.03	3.21
Marginal Cost	0.00	46.49	20.41	2.15	30.66
CPI Inflation	0.01	22.77	68.07	7.27	1.78
Domestic Inflation	0.00	22.37	17.56	0.68	59.37
Imported Inflation	0.01	4.1	5.12	2.55	88.2
Nominal Depreciation	0.05	18.55	27.69	52.04	1.53
Real Exchange Rate	0.07	31.77	20.5	23.27	4.2
Imports	0.05	31.67	42.14	3.99	1.68

Likewise, domestic productivity and monetary policy shocks contribute immensely to fluctuations in consumption with about 9% and 85% in the first quarter and 54% and 34% in the 5th quarter respectively where they continued in the same line, contributing above 50% and 30% respectively. Foreign monetary policy shocks lagged behind, explaining

only 6% of variation in consumption in the first quarter but increased substantially in the following quarters reaching above 20% in the 5th and 10th quarter and then dropping down to 14% while terms of trade, imports and aid shocks insignificantly explained fluctuations in consumption.

At most, 88% of variation in the imported inflation is explained by terms of trade shocks, and the contribution remains larger even at longer horizons. This result is unsurprising for a small open economy where its imports and exports are a function of terms of trade, and unsurprising for the economy under analysis, because most of the intermediate inputs and capital are imported. The high imported inflation is also a function of terms of trade. In addition, terms of trade shocks explain significantly the variation in domestic inflation ranging between 47% in the one quarter ahead forecast, 58% in the five quarters ahead forecast, and about 60% in the remaining forecasted quarters while domestic monetary policy shocks seem to explain almost 66% of all the fluctuations in CPI inflation, a value that is maintained throughout the forecasted quarters, even at longer horizons. Again this result is unsurprising as inflation is one of the monetary policy tools used to influence output in the economy. Imported inflation influences the domestic and CPI inflation of the economy all the time as the economy is import dependent.

Finally, the variations in nominal depreciation and the real exchange rate are largely explained by domestic monetary policy in the one period ahead forecast, providing 59% and 25% respectively while foreign monetary policy shocks explained 49% of the variations in both variables in the same quarter. In addition, domestic productivity shocks explain 19% and 24% respectively while contributions of terms of trade, imports and aid to variations in nominal depreciation and real exchange rate are very minimal. However, at longer horizons, foreign monetary policy and domestic productivity contributed significantly to fluctuations in nominal depreciation and the real exchange rate as foreign monetary policy shocks explained over 50% of the variations in nominal depreciation and above 20% of the variations in the real exchange rate. Domestic productivity shocks explained above 50% of the variation in the exchange rate and about 20% of variation in real exchange rate, a result that is not surprising given the Malawian economy. However, this result is worth noting since almost all the variations in nominal depreciation are explained by the deviations from UIP, but the introduction of the foreign exchange constraints has attributed a huge variation in nominal depreciation and exchange rate to productivity shocks.

6 Conclusion

We aimed to establish the importance of the availability of foreign exchange to an import dependent economy such as Malawi. We calibrated a New Keynesian DSGE model of foreign exchange constraints where firms faced foreign exchange constraints in importation of intermediate inputs and consumption goods. We introduced a country specific risk in addition to the assumption of risk sharing with the foreign economy in the Senbeta (2013) model. Using impulse response functions we showed that imports constituted a vital part of the production process in Malawi and the unavailability of foreign exchange amplified the variability of the macroeconomic shocks in the Malawian economy. In addition, the model indicated that the effect of the additional risk is so small such that it has no clear effect on the dynamic paths of the variables. However, the country risk rises when the domestic currency appreciates and falls when the currency depreciates.

We showed that contrary to the findings by Senbeta (2013), our model produced the conventional results of domestic contractionary monetary policy. A contractionary monetary policy in our model led to a decline in both output and consumption, especially because increasing interest rates leads to an appreciation of the exchange rate, which worsens the terms of trade conditions for the economy. The worsening terms of trade leads to a decline in exports and foreign exchange earnings from exports. Since Malawi depends on imports for most of its production processes, as long as exports are less than imports, thus (X-M) is lower, then aggregate demand will fall which will lead to low economic growth. This may be the reason why stable and strong economic growth has been elusive in Malawi for a long time. With an overvalued exchange rate as indicated by Simwaka and Mkandawire (2008), followed by a decline in export demand and greater spending on imports for production - the economy has completely turned into an import dependent economy that constantly faces foreign exchange shortages. Until recently when the exchange rate was floated, the strong Kwacha has always worsened the country's terms of trade (ibid). Therefore, policy makers should let market forces determine the value of the Kwacha and only intervene when necessary. This would help the currency to stabilise and reduce the foreign exchange problems.

We also reveal that for low income economies such as Malawi, an aid shock has the same effect as net foreign transfers because it eases the foreign exchange problems in the economy by providing the much needed foreign exchange. This works to decrease marginal costs and CPI inflation when both imported and domestic inflation fall. Aid has a remarkable effect in economies such as Malawi where about 40% of national government budget is donor funded annually. Therefore, we conclude that the exchange rate appreciates by aid inflow when the constraint is binding, but, it does no worsen the economy because

it increases government expenditure and imports, raising production, output and private consumption.

Policy makers in LIEs need to understand that for economies dependent upon imported intermediate inputs and consumption goods, both a depreciation and appreciation of the exchange rate affects the economy negatively. An appreciation worsens the macroeconomic environment more than a depreciation. Despite making imports cheaper, an appreciation makes exports expensive leading to a decline in export earnings and therefore, low levels of foreign exchange. As a result, foreign exchange problems occur, worsening the macroeconomic environment further. On the other hand, although a depreciation of the exchange rate makes imports expensive, it leads to favourable terms of trade conditions which are necessary for demand for exports. When exports improve, the economy improves due to an inflow of foreign exchange from export earnings, but consumers feel the effect of the depreciation through the high costs of acquiring goods and services - which are only passed on to consumers from importers and producers. Therefore, it is important that policy makers realise that increasing imports to increase production in the economy may only result in high prices of goods and services for the consumers. However, it is necessary that the Central Bank allows market forces to determine the value of the currency since politically motivated interventions into the foreign exchange markets only worsens the economy.

References

Adam, C., O'Connell, S., Buffie, E., and Pattillo, C. (2009). Monetary policy rules for managing aid surges in Africa. *Review of Development Economics*, 13(3):464–490.

Addison, T. and Ghoshray, A. (2013). Agricultural commodity price shocks and their effect on growth in Sub-Saharan Africa. Technical report, WIDER Working Paper.

Agénor, P.-R. and Montiel, P. J. (2008a). *Development macroeconomics*. Princeton University Press.

Agénor, P.-R. and Montiel, P. J. (2008b). Monetary policy analysis in a small open credit-based economy. *Open Economies Review*, 19(4):423–455.

Alpanda, S., Kotzé, K., and Woglom, G. (2010). The role of the exchange rate in a New Keynesian DSGE model for the South African economy. *South African Journal of Economics*, 78(2):170–191.

Benigno, P. (2004). Optimal monetary policy in a currency area. *Journal of international Economics*, 63(2):293–320.

Berg, A., Mirzoev, T., Portillo, R., and Zanna, L. (2010). The Short Run Macroeconomics of Aid Inflows: Understanding the Interaction of Fiscal and Reserve Policy. Technical report, nIMF Working Paper 10/65.

Berg, M. A., Portillo, R., Buffie, M. E. F., Pattillo, M. C. A., and Zanna, L.-F. (2012). *Public investment, growth, and debt sustainability: Putting together the pieces*. Number 12-144. International Monetary Fund.

Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of monetary Economics*, 12(3):383–398.

Christiano, L. J., Trabandt, M., and Walentin, K. (2011). Introducing financial frictions and unemployment into a small open economy model. *Journal of Economic Dynamics and Control*, 35(12):1999–2041.

Claessens, S. and Ghosh, S. R. (2013). Capital flow volatility and systemic risk in emerging markets: the policy toolkit. *Dealing with the Challenges of Macro Financial Linkages in Emerging Markets*, page 91.

Craigwell, R., Greenidge, K., Codrington, H., and Worrell, M. D. (2003). *Economic resilience with an exchange rate peg: The Barbados experience, 1985-2000*. Number 3-168. International Monetary Fund.

DeJong, D. and Dave, C. (2007). Structural macroeconomics.

Doroodian, K. and Caporale, T. (2001). Central bank intervention and foreign exchange volatility. *International Advances in Economic Research*, 7(4):385–392.

Fukuda, S.-i. and Kon, Y. (2008). Macroeconomic impacts of foreign exchange reserve accumulation: A theory and some international evidence.

Galí, J. (2008). Monetary Policy. *Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*.

Galí, J. and Monacelli, T. (2005). Monetary policy and exchange rate volatility in a small open economy. *Review of Economic Studies*, 72(3):707–734.

IMF (2012). Statement at the Conclusion of an IMF Staff Mission to Malawi Press Release No. 12/115 , April 2, 2012. Technical report, International Monetary Fund.

Justiniano, A. and Preston, B. (2004). Small open economy DSGE models: Specification, estimation and model fit. *Manuscript, Columbia University*, 1(2):3.

Justiniano, A. and Preston, B. (2010). Can structural small open-economy models account for the influence of foreign disturbances? *Journal of International Economics*, 81(1):61–74.

Kose, M. and Riezman, R. (2001). Trade shocks and macroeconomic fluctuations in Africa. *Journal of Development Economics*, 65(1):55–80.

Lensink, R. (1995). Foreign exchange constraints and developing countries. *Economic Modelling*, 12(2):179–191.

Malawi-Government-Publications (2011). 2011/2012 Budget Speech. page Budget-speech2011.

Mangani, R. (2011). The Effects of Monetary Policy in Malawi.

Marquez, J. (1985). Foreign exchange constraints and growth possibilities in the LDCs. *journal of Development Economics*, 19(1):39–57.

Mathisen, J. (2003). Estimation of the equilibrium real exchange rate for Malawi. *IMF Working Paper No. 03/104*.

MCC (2012). MCC Board Approves Zambia Compact, Suspends Compact with Malawi. Technical report, Millenium Challenge Corporation.

McCormack, J. (2015). Why are EM forex reserves in a sustained decline? Electronic Newspaper.

Monacelli, T. (2005). Monetary Policy in a low pass-through environment. *Journal of Money, Credit and Banking*, 37(2):19.

Moran, C. (1989). Imports under a foreign exchange constraint. *The World Bank Economic Review*, 3(2):279–295.

Munthali, T. (2004). The impact of Structural Adjustment Policies (SAPs) on manufacturing growth in Malawi. Technical report, EconWPA.

Mwabutwa, C., Bittencourt, M., and Viegi, N. (2013). Monetary Policy Response to Foreign Aid in an Estimated DSGE Model of Malawi. Technical report, Economic Research Southern Africa.

Ngalawa, H. and Viegi, N. (2013). Interaction of Formal and Informal Financial Markets in Quasi-Emerging Market Economies. *Economic Modelling*, 31:614–624.

Ogaki, M. and Park, J. Y. (1997). A cointegration approach to estimating preference parameters. *Journal of Econometrics*, 82(1):107–134.

Peiris, S. and Saxegaard, M. (2007). An estimated DSGE model for monetary policy analysis in low-income countries. *IMF Working Paper No. 07/282*.

Peiris, S. and Saxegaard, M. (2009). An Estimated Dynamic Stochastic General Equilibrium Model for Monetary Policy Analysis in Mozambique. *IMF Staff Papers*, 57(1):256–280.

Polterovich, V. and Popov, V. (2003). Accumulation of foreign exchange reserves and long term growth.

Porter, R. and Ranney, S. (1982). An eclectic model of recent LDC macroeconomic policy analyses. *World Development*, 10(9):751–765.

Rajan, R. G. and Subramanian, A. (2011). Aid, Dutch disease, and manufacturing growth. *Journal of Development Economics*, 94(1):106–118.

Schmitt-Grohé, S. and Uribe, M. (2003). Closing small open economy models. *Journal of international Economics*, 61(1):163–185.

Senbeta, S. (2011a). A small open economy New Keynesian model for a foreign exchange constrained economy. Technical report, University Library of Munich, Germany.

Senbeta, S. (2011b). How applicable are the new keynesian DSGE models to a typical low-income economy? Technical report, University Library of Munich, Germany.

Senbeta, S. R. (2013). Foreign exchange constraints and macroeconomic dynamics in a small open economy. Technical report, University of Antwerp, Faculty of Applied Economics.

Simwaka, K. (2004). A look at exchange rate and monetary policy in Malawi. Technical report, EconWPA.

Simwaka, K. (2006). The effectiveness of official intervention in foreign exchange market in Malawi. Technical report, University Library of Munich, Germany.

Simwaka, K. and Mkandawire, L. (2008). The efficacy of foreign exchange market intervention in Malawi. Technical report, University Library of Munich, Germany.

Stiglitz, J. and Charlton, A. (2006). Stability with growth: Macroeconomics, liberalization, and development. *otros (2006)*.

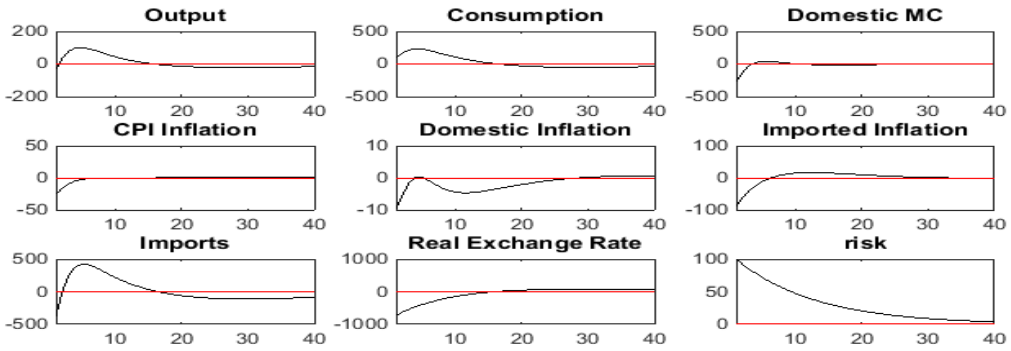
Stiglitz, J. E., Ocampo, J., Spiegel, S., French Davis, R., and Deepak, N. (2006). Stability with Growth Macroeconomics, Liberalization and Development.

WHO, . (2001). Health Topics- Tobacco. Online.

Williams, M. V. (2006). *Foreign Exchange Reserves: How much is Enough?* Caribbean Centre for Monetary Studies.

Appendix

Figure A.1: Impulse Responses to Domestic Risk



Sensitivity Analysis Results

Responses of the variables when the foreign exchange constraint is relaxed.

Figure A.2: Impulse Responses to Foreign Monetary Policy Shock

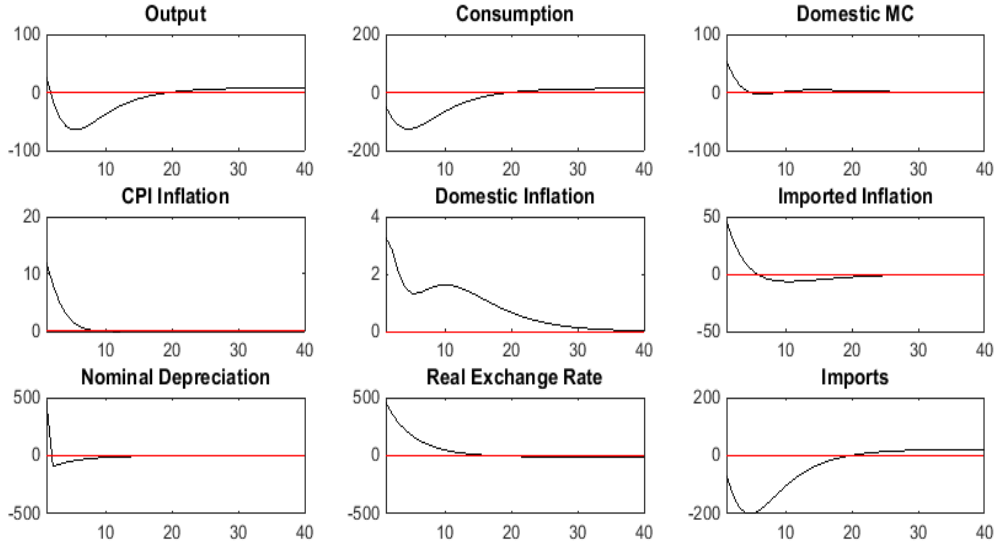


Figure A.3: Impulse Responses to Domestic Monetary Policy

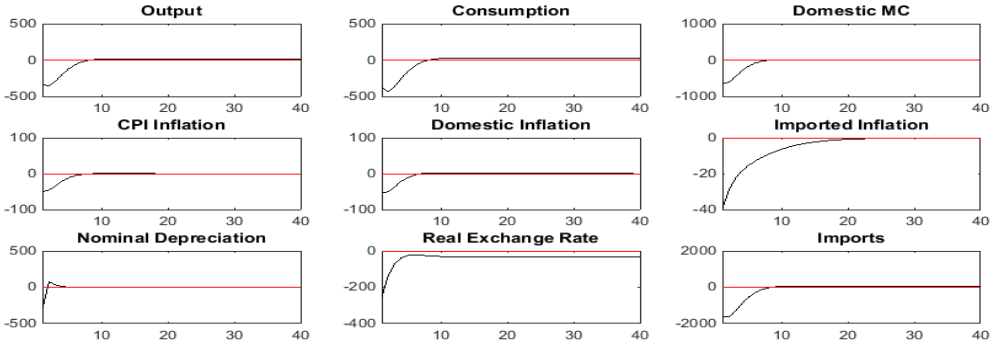


Figure A.4: Impulse Responses to Aid Shock

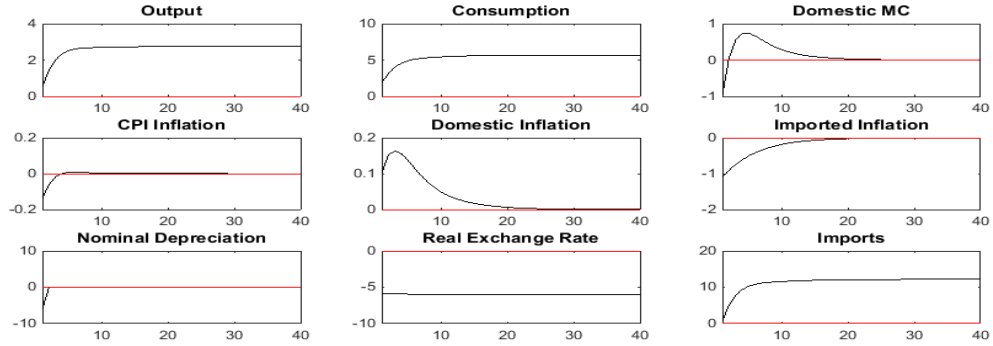
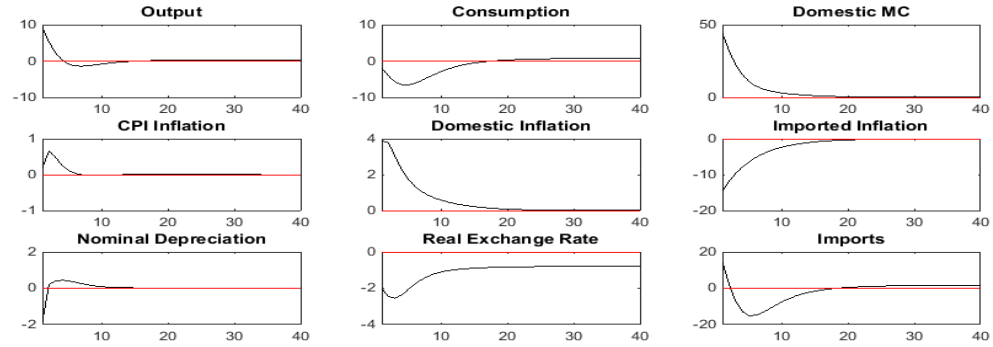


Figure A.5: Impulse Responses to Terms of Trade Shock



7 Parameter Description and Sources

Table B.1: Descriptions and Sources

Parameter	Description and Source
χ_a	Ratio of net aid to imports (Mwabutwa et al., 2013; Ngalawa and Viegi, 2013)
χ_r	Ratio of imports to foreign exchange reserves (World Bank Econ. Indicators)
χ_c	Ratio of imported consumption goods in total imports (NSO)
χ_g	Consumption to GDP ratio (Mwabutwa et al., 2013)
χ_f	Total imports to GDP ratio (IMF country reports, RBM & World Bank Ecoc Indicators)
χ_m	Ratio of imported intermediate inputs in total imports (Senbeta, 2013)
β	Household discount factor (Mwabutwaet al., 2013; Gali and Monacelli, 2005)
ν	Intertemporal elasticity of substitution in consumption (Berg et al. 2007)
φ	Elasticity of labour (Berg et. al., 2010)
χ	Marginal disutility of working (Senbeta, 2013)
ρ^1	Elasticity of substitution between imported and home goods (Calibrated)
θ_f	Elasticity of substitution between varieties of imported goods (Calibrated)
α_1	Share of imported goods in consumption (Senbeta, 2013)
h	Coefficient of habit persistence (Senbeta, 2013)
χ_{risk}	Risk premium parameter (Alpanda, Kotze and Woglom 2010)
σ_1	Share of labour in production of home goods (Mwabutwaet al., 2013)
σ_2	Share of intermediate inputs in production of home goods (Senbeta, 2013)
ς_F	Weight attached to past inflation for importing firms (Senbeta, 2013)
ς_H	Weight attached to past inflation by domestic producers (Senbeta, 2013)
θ_F	Fraction of importing firms that index their prices (Senbeta, 2013)
θ_H	Fraction of domestic producers that index prices (Senbeta, 2013)
ρ_H	Persistence of total productivity shock (Senbeta, 2013)
ρ_r	Persistence of interest rate (Ortiz and Sturz, 2007)
ϕ_π	Inflation stabilization (Alpanda, Kotze and Woglom, 2010)
ϕ_y	Output stabilization (Senbeta, 2013)
ϕ_e	Exchange rate stabilization (Senbeta, 2013)
ρ_a	Persistence of aid shock (IMF Country Report)
ρ_{y^*}	Persistence of foreign income shock (Alpanda, Kotze and Woglom, 2010)
ρ_{π^*}	Persistence of foreign inflation shock (Alpanda, Kotze and Woglom ,2010)
ρ_{r^*}	Persistence of foreign interest rate shock (Alpanda, Kotze and Woglom, 2010)

8 The Estimated Linearised Model

The solution to the structural model presented in the previous sections is the log-linear approximations to steady state of the equations presented in this section and this is the model that is to be calibrated[6]. Inflation is calculated as $\pi_t = lnP_t - lnP_{t-1}$. The estimated equations contain shocks to endogenous variables to capture the dynamic responses of the variables to a percentage change or a unit change in a variable. The full log-linear

[6]Deviations to steady state are taken in such that $x_t = lnX_t - ln\bar{X_t}$, $\bar{X}$ is the steady state value of X_t.

equations are presented in the appendix, however, here we just present the model that is to be calibrated.

Log-linearising the marginal rate of substitution between consumption and labour:

$$w_t - p_t = \varphi l_t + \frac{v}{1-h}c_t - \frac{hv}{(1-h)}c_{t-1} \tag{A.1}$$

the consumption Euler becomes

$c_t - hc_{t-1} = E_t(c_{t+1} - hc_t) - v^{-1}(1-h)(r_t - E_t\pi_{t+1}) + v^{-1}(1-h)$ or

$$c_t = \frac{1}{1+h}E_t c_{t+1} + \frac{h}{1+h}c_{t-1} - \frac{(1-h)}{v(1+h)}(r_t - E_t\pi_{t+1} + \epsilon_{g,t}) \tag{A.2}$$

where $\epsilon_{g,t} = \rho_g \epsilon^g_{t-1} + \varepsilon^g_t, \varepsilon^g_t \sim i.i.d.N(0, \sigma^2_g)$ is a demand shock.

From the uncovered interest parity (UIP) condition $\beta E_t \lambda_{t+1} r_t = \beta E_t \lambda_{t+1} \frac{\varepsilon_{t+1}}{\varepsilon_t} r^*_t \phi_{t+1}$, which is the household's optimal holding of domestic and foreign bonds, it implies that

$$c_t = \frac{1}{1+h}E_t c_{t+1} + \frac{h}{1+h}c_{t-1} - \frac{(1-h)}{\nu(1+h)}(r_t - E_t\pi_{t+1}) \tag{A.3}$$

Using the usual definitions of ϕ_t, then the

$ln\phi_{t+1} = -\eta d_t - \eta\omega_{t+1}$

$ln\phi_{t+1} - ln\bar{\phi} = -\eta d_t - \epsilon_{rps,t})$

Therefore;

$$r_t - E_t\pi_{t+1} = r^*_t - E_t\pi^*_{t+1} + E_t\Delta q_{t+1} - \eta d_t - \varepsilon_{rps,t} \tag{A.4}$$

and q_t is the real exchange rate.

Real Exchange Rate, Terms of Trade and Incomplete Pass-through

Linearising (37) the Law of one price gap and finding its evolution by subtracting one period lag:

$$\psi_t - \psi_{t-1} = e_t - e_{t-1} + \pi_t^* - \pi_{F,t} \tag{A.5}$$

and linearising (38) the real exchange rate

$$q_t = e_t + p_t^* - p_t \tag{A.6}$$

since $p_t = (1-\alpha_1)p_{H,t} + \alpha_1 p_{F,t}$, using the log-linearised definition of the law of one price gap, then the real exchange rate can be written as $q_t = \psi_t + p_{F,t} - (1-\alpha_1)p_{H,t} + \alpha_1 p_{F,t}$ which can be simplified as

$q_t = \psi_t + (p_{F,t} - p_{H,t}) - \alpha_1(p_{F,t} - p_{H,t})$

Linearising the terms of trade condition we have $s_t = p_{F,t} - p_{H,t} \iff \Delta s_t = \pi_{F,t} - \pi_{H,t}$

Using this definition of terms of trade and re-writing the real exchange rate equation, we have:

$$q_t = \psi_t + (1-\alpha_1)s_t \tag{A.7}$$

and the terms of trade definition comprising the terms of trade shock is given as:

$$s_t = s_{t-1} + \pi_{F,t} - \pi_{H,t} + \epsilon_{tot,t} \tag{A.8}$$

Firms

Domestically produced goods inflation (New Keynesian Phillips curve) is given as:

$$\pi_{H,t} - \varsigma_H \pi_{H,t-1} = \beta E_t(\pi_{H,t+1} - \varsigma_H \pi_{H,t}) + \kappa_H mc_{H,t} \tag{A.9}$$

where $\kappa_H = \frac{(1-\theta_k)(1-\theta_k\beta)}{\theta_k}$

Log-linearising the real marginal cost function (22) yields

$mc_{H,t} = \sigma_1 w_t + \sigma_2 p_{F,t} - p_t - a_{H,t}$

using (37) and substituting for w_t gives the marginal cost as:

$mc_{H,t} = \sigma_1 \left(p_t + \varphi l_t + \frac{v}{(1-h)} c_t - \frac{hv}{(1-h)} c_{t-1} \right) + \sigma_2 p_{F,t} - p_t - a_{H,t}$

$mc_{H,t} = \sigma_1 \left(\varphi l_t + \frac{v}{(1-h)} c_t - \frac{hv}{(1-h)} c_{t-1} \right) + \sigma_2 p_{F,t} + \sigma_1 p_t - p_t - a_{H,t}$

$mc_{H,t} = \sigma_1 \left(\varphi l_t + \frac{v}{(1-h)} c_t - \frac{hv}{(1-h)} c_{t-1} \right) + \sigma_2 p_{F,t} - (1-\sigma_1) p_t - a_{H,t}$

$mc_{H,t} = \sigma_1 \left(\varphi l_t + \frac{v}{(1-h)} c_t - \frac{hv}{(1-h)} c_{t-1} \right) + \sigma_2 p_{F,t} - \sigma_2 p_t - a_{H,t}$

$$mc_{H,t} = \sigma_1 \left(\varphi l_t + \frac{v}{(1-h)} c_t - \frac{hv}{(1-h)} c_{t-1} \right) + \sigma_2 (1-\alpha_1) s_t - a_{H,t} \tag{A.10}$$

Importing firms

The New Keynesian Phillips curve for importing firms

$$\pi_{F,t} - \varsigma_H \pi_{F,t-1} = \beta E_t (\pi_{F,t+1} - \varsigma_F \pi_{F,t}) + \kappa_F (mc_{F,t} - p_{F,t}) \tag{A.11}$$

and $\kappa_F = \frac{(1-\iota_F)(1-\iota_F \beta)}{\iota_F}$

Marginal cost for importing firms with the mark-up on price: $MC_{F,t} = \varepsilon_t P_t^* (1 + \varpi_t)$

and log-linearising, it becomes $mc_{F,t} = e_t + p_t^* + \hat{\varpi}_t$.

Subtracting $p_{F,t}$ from both sides, we have

$mc_t - p_{F,t} = (e_t + p_t^* + \hat{\varpi}) - p\psi_{F,t} = \psi_t + \hat{\varpi}$.

But, the log-linearised version, given the output as $(y_{F,t})$ is given as :

$$\hat{\varpi} = \frac{1}{\rho_2} y_{F,t} \tag{A.12}$$

Market Clearing Conditions

Goods market clear when

$Y_{F,t} = C_{H,t} + C_{H,t}^*$

which is log-linearised as

$\bar{Y}_{yt} = \bar{C}_H c_{H,t} + \bar{C}^*_H c^*_{H,t}$

Where

$c_{H,t} = -\rho_1(p_{H,t} - p_t) + c_t$

using $p_t - p_{H,t} = \alpha_1 s_t$, then $C_{H,t} = \rho_1 \left(\frac{P_{H,t}}{\varepsilon_t P^*_t}\right)^{-\rho_1} C^*_t = \alpha_1 \left(\frac{P_{H,t}}{\varepsilon_t P^*_t}\right)^{-\rho_1} C^*_t$

and log-linearising, we have

$c^*_{H,t} = \rho_1\alpha_1 s_t + \rho_1 q_t + c^*_t$

therefore, a balanced trade steady state implies

$\bar{C}^*_H + \bar{A} = \bar{Y}_F \iff \bar{C}^*_H = \bar{C}_F + \bar{M} - \bar{A}$

dividing through by $\bar{C}^*_H$, then $1 = (1-\alpha_1)\chi_g + \alpha_1\chi_g + (\chi_m - \chi_a)\chi_f$,

Letting $\frac{\bar{C}}{\bar{Y}} = \chi_g$, $\frac{\bar{M}}{\bar{Y}_F} = \chi_m$, $\frac{\bar{Y}_F}{\bar{Y}} = \chi_f$, $\frac{\bar{A}}{\bar{Y}_F} = \chi_a$, then

$\bar{C}^*_H/\bar{Y} = \alpha_1\chi_g + (\chi_m - \chi_a)\chi_f$

Therefore, the domestic goods clearing condition becomes:

$y_{H,t} = (1-\alpha_1)\chi_g(\rho_1\alpha_1 s_t + c_t) + (\alpha_1\chi_g + (\chi_m - \chi_a)\chi_f)(\rho_1\alpha_1 s_t + \rho_1 q_t + c^*_t)$

If we define the steady state proportion of imported consumption goods and intermediate goods in total imports as: $\chi_c = \bar{C}_F/\bar{Y}_F$and $(1-\chi_c) = \bar{M}/\bar{Y}_F = \chi_m$,

Then from $Y_{F,t} = C_{F,t} + M_t$, we have the log-linearised function as :

$y_{F,t} = \chi_c c_{F,t} + (1-\chi_c)m_t$,

And using the definition of foreign imported goods consumption and intermediate inputs and substituting for p_t we have:

$c_{F,t} = -\rho_1(1-\alpha_1)s_t + c_t$

The log-linearised imported inputs becomes:

$m_t = -\sigma_1 p_{F,t} + \sigma_1 w_t - a_{H,t} + y_{H,t}$

$$m_t = \sigma_1(w_t - p_{F,t}) - a_{H,t} + y_{H,t}$$

which becomes:

$$m_t = \varphi l_t + \frac{\sigma_1 v}{(1-h)} c_t - \frac{\sigma_1 h v}{(1-h)} c_{t-1} - a_{H,t} + y_{H,t} - \sigma_1(1-\alpha_1)s_t \tag{A.13}$$

This follows from Senbeta (2013) and the debt evolves according to:

$$d_t = \frac{1}{\beta} d_{t-1} + (\chi_a + (1-\chi_a)\rho_1 - 1)q_t + \chi_a a_t + (1-\chi_a)((\rho_1 - 1)\alpha_1 s_t + y_t^*) - y_{Ft} \tag{A.14}$$

and the log-linearised foreign exchange holdings is given as:

$$res_t = \rho_{res} res_t = \rho_{res} res_{t-1} + \rho_{debt}(r_{t-1} + \varepsilon_t + b_{t-1} + \phi_t) + \rho_a(\varepsilon_t + a_t) + \rho_{ex}(p_t + c_{H,t}^*) - \rho_m(\varepsilon_t + p_{F,t} + y_{F,t}) \tag{A.15}$$